IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Computer aided power system analysis

Bad Data Detection

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BAD DATA DETECTION

$\hat{f}$ follows the chi-square distribution.

$$E[\hat{f}] = E\left[\sum_{i=1}^{m} \frac{(z_i - \hat{z}_i)^2}{\sigma_i^2}\right]$$

We also know, $E[\hat{e}\,\hat{e}^T] = R'$ ($m \times m$)

$\hat{e}_i^2 : \forall i = 1, \ldots, m$ → diagonal elements.

$$E[\hat{e}_i^2] = R'_{ii} \Rightarrow E[(z_i - \hat{z}_i)^2] = R'_{ii}$$

$$\Rightarrow E\left[\frac{(z_i - \hat{z}_i)^2}{R'_{ii}}\right] = \frac{R'_{ii}}{R'_{ii}} = 1$$

$$E[\hat{f}] = E\left[\sum_{i=1}^{m} \frac{(z_i - \hat{z}_i)^2}{\sigma_i^2}\right] = E\left[\sum_{i=1}^{m} \frac{(z_i - \hat{z}_i)^2}{R'_{ii}} \cdot \frac{R'_{ii}}{\sigma_i^2}\right]$$

$$= \sum_{i=1}^{m} E\left[\frac{(z_i - \hat{z}_i)^2}{R'_{ii}} \cdot \frac{R'_{ii}}{\sigma_i^2}\right]$$

$$= \sum_{i=1}^{m} \frac{R'_{ii}}{\sigma_i^2} \cdot E\left[\frac{(z_i - \hat{z}_i)^2}{R'_{ii}}\right] = \sum_{i=1}^{m} \frac{R'_{ii}}{\sigma_i^2}$$

$$\boxed{E[\hat{f}] = \sum_{i=1}^{m} \frac{R'_{ii}}{\sigma_i^2}}$$

Steps for bad data detection

1. Estimate $\hat{\bar{x}}$
2. Calculate $\hat{\bar{z}} = H\hat{\bar{x}}$
3. Calculate $\hat{\bar{e}} = (\bar{z} - \hat{\bar{z}})$
4. Calculate $\hat{f} = \sum_{i=1}^{m} \frac{\hat{e}_i^2}{\sigma_i^2}$
5. Determine the value of K (degree of freedom)

$$K = m - n$$

total no. of measurements

total no. of states

6. Choose an appropriate value of α

$$E(\hat{e}_i^2) = R_{ii}$$

$$E\left[\frac{(z_i - \hat{z}_i)^2}{R_{ii}}\right] = 1$$

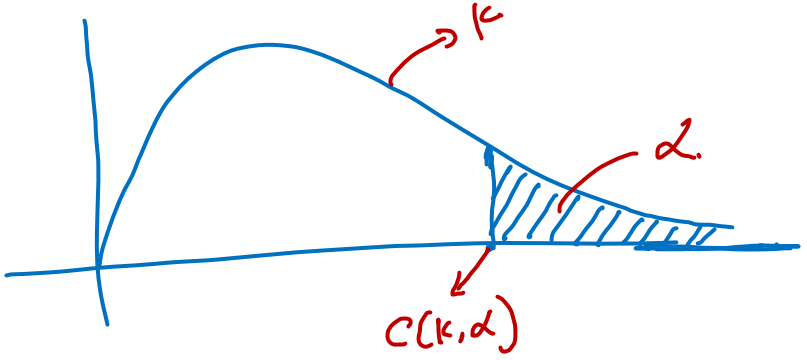
7. Find out the threshold value from the chi-square distribution curve corresponding to the values of k and α. Let this threshold value be denoted as $c(k, \alpha)$

8. If $\hat{f} \leq c(k, \alpha)$, then there is no bad data in the system.

9. If $\hat{f} > c(k, \alpha)$ → presence of bad data is detected.

$\alpha = 0.01$

for $k = 2$, $\alpha = 0.01$
$c(k, \alpha) = 9.21$



10. $$\frac{\hat{e}_i}{R_{ii}} = \frac{(z_i - \hat{z}_i)}{R_{ii}}$$

Chi-square distribution

Values of area α to the right of $\chi^2 = \chi^2_{k,\alpha}$ in Fig. 15.3

	α					α			
k	0.05	0.025	0.01	0.005	k	0.05	0.025	0.01	0.005
1	3.84	5.02	6.64	7.88	11	19.68	21.92	24.73	26.76
2	5.99	7.38	9.21	10.60	12	21.03	23.34	26.22	28.30
3	7.82	9.35	11.35	12.84	13	22.36	24.74	27.69	29.82
4	9.49	11.14	13.28	14.86	14	23.69	26.12	29.14	31.32
5	11.07	12.83	15.09	16.75	15	25.00	27.49	30.58	32.80
6	12.59	14.45	16.81	18.55	16	26.30	28.85	32.00	34.27
7	14.07	16.01	18.48	20.28	17	27.59	30.19	33.41	35.72
8	15.51	17.54	20.09	21.96	18	28.87	31.53	34.81	37.16
9	16.92	19.02	21.67	23.59	19	30.14	32.85	36.19	38.58
10	18.31	20.48	23.21	25.19	20	31.41	34.17	37.57	40.00

Bad data detection

$$\hat{f} = \sum_{j=1}^{4} \frac{\hat{e}_j^2}{\sigma_j^2} = 100\hat{e}_1^2 + 100\hat{e}_2^2 + 50\hat{e}_3^2 + 50\hat{e}_4^2$$

$$= 100(0.00877)^2 + 100(0.00456)^2 + 50(0.02596)^2 + 50(0.00070)^2$$

$$= 0.043507$$

$m = 4 \rightarrow V_A, V_B, I_1, I_2$

$n = 2 \rightarrow V_1, V_2$

$k = 4-2 = 2$, $\alpha = 0.01$ $\Big\}$ $c(k,\alpha) = 9.21$

$\hat{e}_1 = 0.00877$

$\hat{e}_2 = 0.00456$

$\hat{e}_3 = -0.02596$

$\hat{e}_4 = -0.00070$

$\omega_1 = \omega_2 = 100$

$\omega_3 = \omega_4 = 50$

$Z_4 = 5.01$ V

Bad data detection

- Suppose

$$[z_1 \quad z_2 \quad z_3 \quad z_4]^T = [9.01\ A \quad 3.02\ A \quad 6.98\ \text{V} \quad 4.40\ \text{V}]^T$$

$$\hat{f} = \sum_{j=1}^{4} \frac{\hat{e}_j^2}{\sigma_j^2} = 100\hat{e}_1^2 + 100\hat{e}_2^2 + 50\hat{e}_3^2 + 50\hat{e}_4^2$$

$$= 100(0.06228^2 + 0.15439^2) + 50(0.05965^2 + 0.49298^2)$$

$$= 15.1009$$

$$\mathbf{HG}^{-1}\mathbf{H}^T\mathbf{R}^{-1} = \begin{bmatrix} 0.8070 & \times & \times & \times \\ \times & 0.8070 & \times & \times \\ \times & \times & 0.1930 & \times \\ \times & \times & \times & 0.1930 \end{bmatrix}$$

$z_4 = 4.40$ V

$\hat{e}_1 = 0.06228$

$\hat{e}_2 = 0.15439$

$\hat{e}_3 = 0.05965$

$\hat{e}_4 = -0.49298$

$15.1009 > 9.21$

Bad data detection

$$\frac{\hat{e}_1}{\sqrt{R'_{11}}} = \frac{0.06228}{\sqrt{(1-0.807)\sigma_1^2}} = \frac{0.06228}{\sqrt{0.193/100}} = 1.4178$$

$$\frac{\hat{e}_2}{\sqrt{R'_{22}}} = \frac{0.15439}{\sqrt{(1-0.807)\sigma_2^2}} = \frac{0.15439}{\sqrt{0.193/100}} = 3.5144$$

$$\frac{\hat{e}_3}{\sqrt{R'_{33}}} = \frac{0.05965}{\sqrt{(1-0.193)\sigma_3^2}} = \frac{0.05965}{\sqrt{0.807/50}} = 0.4695$$

$$\frac{\hat{e}_4}{\sqrt{R'_{44}}} = \frac{-0.49298}{\sqrt{(1-0.193)\sigma_4^2}} = \frac{-0.49298}{\sqrt{0.807/50}} = -3.8804$$

Revised state estimation

$$\mathbf{H} = \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \end{bmatrix} \qquad \mathbf{H}^T\mathbf{R}^{-1} = \begin{bmatrix} 62.50 & -12.50 & 18.75 \\ -12.50 & 62.50 & 6.25 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} 47.65625 & -13.28125 \\ -13.28125 & 41.40625 \end{bmatrix} \qquad \mathbf{G}^{-1} = \begin{bmatrix} 0.023043 & 0.007391 \\ 0.007391 & 0.026522 \end{bmatrix}$$

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \mathbf{G}^{-1}\mathbf{H}^T\mathbf{R}^{-1}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1.34783 & 0.17391 & 0.47826 \\ 0.13044 & 1.56522 & 0.30435 \end{bmatrix}\begin{bmatrix} 9.01 \\ 3.02 \\ 6.98 \end{bmatrix}$$

$$= \begin{bmatrix} 16.0074\ \text{V} \\ 8.0265\ \text{V} \end{bmatrix}$$

Revised state estimation

$$\begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \hat{z}_3 \end{bmatrix} = \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \end{bmatrix} \begin{bmatrix} 16.0074 \\ 8.0265 \end{bmatrix} = \begin{bmatrix} 9.0013 \text{ A} \\ 3.0157 \text{ A} \\ 7.0061 \text{ V} \end{bmatrix}$$

$$\begin{bmatrix} \hat{e}_1 & \hat{e}_2 & \hat{e}_3 \end{bmatrix}^T = \begin{bmatrix} 0.0087 & 0.0043 & -0.0261 \end{bmatrix}^T$$

$\hat{f} = \Sigma_{j=1}^{3} \hat{e}_j^2 / \sigma_j^2$ = 0.0435 < 6.64 (critical value)

$m = 3$, $n = 2$ } $k = 1$, $\alpha = 0.01$