IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Computer aided power system analysis

Error Analysis (cont.)

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$$\Rightarrow E\left[\hat{\underline{e}}\hat{\underline{e}}^T\right] = \left(I - HG^{-1}H^TW\right)\left(R - HG^{-1}H^T\right)$$

$$\Rightarrow E\left[\hat{\underline{e}}\hat{\underline{e}}^T\right] = \left(I - HG^{-1}H^TW\right)\left(I - HG^{-1}H^TR^{-1}\right)R$$

$$= \underbrace{\left(I - HG^{-1}H^TW\right)\left(I - HG^{-1}H^TW\right)R}_{\text{Idempotent}} \quad \left[\text{as } W = R^{-1}\right]$$

Now,

$$\left(I - HG^{-1}H^TW\right)\left(I - HG^{-1}H^TW\right)$$

$$= I - HG^{-1}H^TW - HG^{-1}H^TW + HG^{-1}\underbrace{H^TWH}_{G}G^{-1}H^TW$$

$$= I - HG^{-1}H^TW - H\cancel{G^{-1}H^TW} + H\cancel{G^{-1}H^TW}$$

$$= \left(I - HG^{-1}H^TW\right)$$

$$E\left[\hat{\underline{e}}\hat{\underline{e}}^T\right] = \left(I - HG^{-1}H^TW\right)R$$

$$= R - HG^{-1}H^T \quad \left[\text{as } WR = I\right]$$

$$E\left[\hat{\underline{e}}\hat{\underline{e}}^T\right] = R - HG^{-1}H^T$$

$$E\left[\hat{\bar{e}}\,\hat{\bar{e}}^T\right] = R - HG^{-1}H^T = R'$$

$H \rightarrow (m \times n)$: $G^{-1} \rightarrow (n \times n)$: $H^T \rightarrow n \times m$

$\Rightarrow HG^{-1}H^T \rightarrow (m \times n) \times (n \times n) \times (n \times m) = (m \times m)$

$$\hat{\bar{e}} = \begin{bmatrix} \hat{e}_1 & \hat{e}_2 & \cdots & e_m \end{bmatrix}^T$$

$$\hat{\bar{e}}\,\hat{\bar{e}}^T = \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \vdots \\ \hat{e}_m \end{bmatrix} \begin{bmatrix} \hat{e}_1 & \hat{e}_2 & \cdots & \hat{e}_m \end{bmatrix} = \begin{bmatrix} \hat{e}_1^2 & \hat{e}_1\hat{e}_2 & \cdots & \hat{e}_1\hat{e}_m \\ \hat{e}_2\hat{e}_1 & \hat{e}_2^2 & \cdots & \hat{e}_2\hat{e}_m \\ \vdots & & & \\ \hat{e}_m\hat{e}_1 & \hat{e}_m\hat{e}_2 & \cdots & \hat{e}_m^2 \end{bmatrix} \rightarrow (m \times m)$$

$R \rightarrow$ is a diagonal matrix

$HG^{-1}H^T \rightarrow$ is not a diagonal matrix

$R' = R - HC_r^{-1}H^T$ is not a diagonal matrix

$E[\bar{e}\bar{e}^T] = R \rightarrow$ diagonal $\therefore E[e_i e_j] = 0$

$E[\hat{\bar{e}}\hat{\bar{e}}^T] = R' \Rightarrow$ is not diagonal $\Rightarrow E[\hat{e}_i \hat{e}_j] \neq 0$

Observation : Although the meters are independent, but the estimated errors of all the meters are NOT independent.

$\hat{\bar{x}} = K\bar{z} \Rightarrow$ Any estimated state depends on all measurements

$\hat{\bar{z}} = H\hat{\bar{x}}$

↙ Estimated measurement

As $\hat{\bar{x}}$ depends on all measurements, $\hat{\bar{z}}$ also depends on all measurements.

$\hat{\bar{e}} = \bar{z} - \hat{\bar{z}}$ $\Rightarrow$ As $\hat{\bar{z}}$ depends on all measurements, $\hat{\bar{e}}$ also depends on all measurements.

$\hat{e}_i$

$f = \sum e_i^2 \omega_i$ → original objective function

$\hat{f} = \sum \hat{e}_i^2 \omega_i$ → Estimated value of the objective function

$$\hat{f} = \sum_{i=1}^{m} \hat{e}_i^2 w_i = \sum_{i=1}^{m} \frac{(z_i - \hat{z}_i)^2}{\sigma_i^2} \rightarrow \hat{f} \text{ is random}$$

$\hat{f}$ follows chi-square probability distribution function

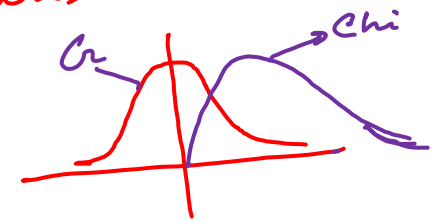
Note : characteristics about chi-square distribution

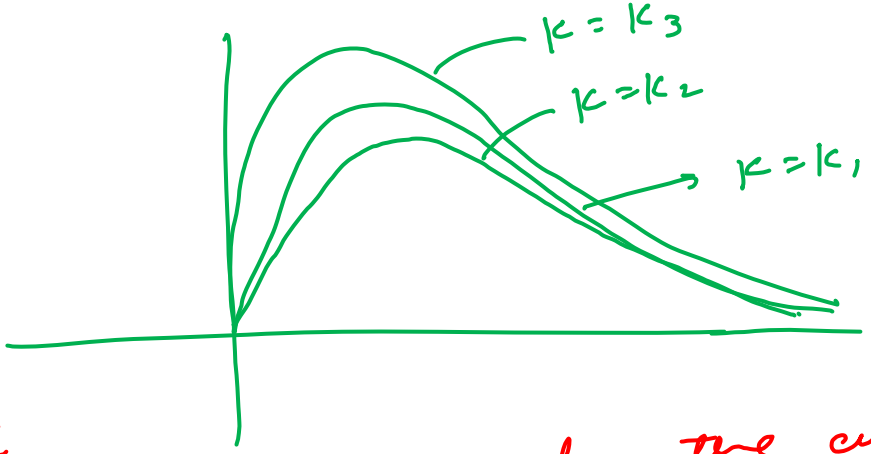
1) It is a one-sided distribution
2) It is quite similar to Gaussian distribution function
3) It is characterised by the degree of freedom

$K = m - n$

$m \rightarrow$ no. of measurement

$n \rightarrow$ no. of state

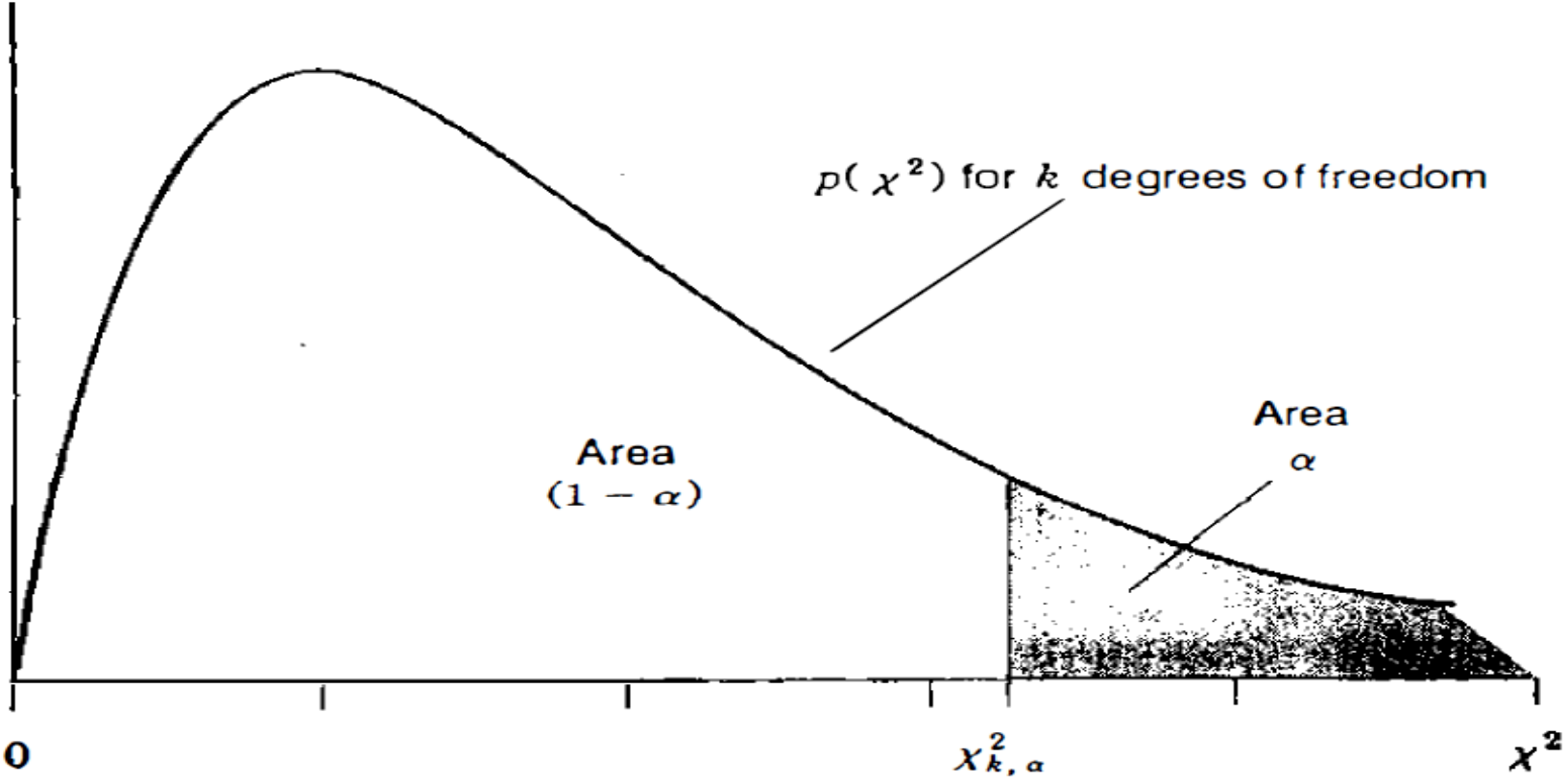




Note

4) Total area under the curve = 1

Chi-square distribution



Chi-square distribution

Values of area α to the right of $\chi^2 = \chi^2_{k,\alpha}$ in Fig. 15.3

	α					α			
k	0.05	0.025	0.01	0.005	k	0.05	0.025	0.01	0.005
1	3.84	5.02	6.64	7.88	11	19.68	21.92	24.73	26.76
2	5.99	7.38	9.21	10.60	12	21.03	23.34	26.22	28.30
3	7.82	9.35	11.35	12.84	13	22.36	24.74	27.69	29.82
4	9.49	11.14	13.28	14.86	14	23.69	26.12	29.14	31.32
5	11.07	12.83	15.09	16.75	15	25.00	27.49	30.58	32.80
6	12.59	14.45	16.81	18.55	16	26.30	28.85	32.00	34.27
7	14.07	16.01	18.48	20.28	17	27.59	30.19	33.41	35.72
8	15.51	17.54	20.09	21.96	18	28.87	31.53	34.81	37.16
9	16.92	19.02	21.67	23.59	19	30.14	32.85	36.19	38.58
10	18.31	20.48	23.21	25.19	20	31.41	34.17	37.57	40.00