



IIT ROORKEE



**NPTEL ONLINE
CERTIFICATION COURSE**

Computer aided power system analysis

Error Analysis

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Error analysis.

$$\hat{\bar{x}} = \bar{x} + \bar{G}^{-1} H^T W e$$

$$\Rightarrow \hat{\bar{x}} - \bar{x} = \bar{G}^{-1} H^T W \bar{e}$$

$$\Rightarrow E[\hat{\bar{x}} - \bar{x}] = \bar{G}^{-1} H^T W E(\bar{e})$$
$$= 0$$

$$\Rightarrow E[\hat{\bar{x}} - \bar{x}] = 0$$

$$\Rightarrow E(\hat{\bar{x}}) = \bar{x}$$

$$\hat{\bar{e}} = (\mathbf{I} - H \bar{G}^{-1} H^T W) \bar{e}$$

$$E[\hat{\bar{e}}] = (\mathbf{I} - H \bar{G}^{-1} H^T W) E(\bar{e}) = 0$$

$$\Rightarrow E[\hat{\bar{e}}] = 0$$

Because:

- i) Errors are Gaussian with zero mean $\Rightarrow E[\bar{e}] = 0$
- ii) The measurements are independent to each other



$$\hat{e} = z - \hat{z}$$

obtained
measured
values

Estimated values
of the measured
quantities

$$\text{As } E[\hat{e}] = 0 \Rightarrow E[z - \hat{z}] = 0$$

$$\Rightarrow E[\hat{z}] = z$$

Now, let there are 'm' measure-
ments.

Independent measurements.

Two meters (i^{th} and j^{th} say)
are independent to each other

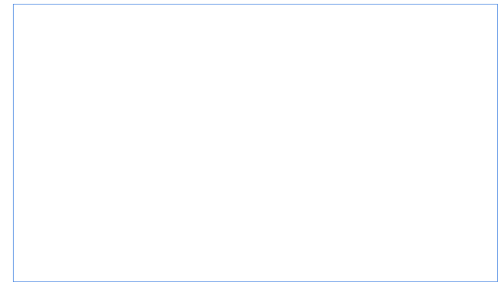
$$\text{if } E[e_i e_j] = 0$$

$e_i \rightarrow$ error associated with
the i^{th} meter.



Then $\bar{e} = [e_1 \ e_2 \ \dots \ e_m]^T$

$$\bar{e}\bar{e}^T = \begin{matrix} \begin{matrix} \downarrow \\ (m+1) \end{matrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} \end{matrix} \begin{matrix} \begin{bmatrix} e_1 & e_2 & \dots & e_m \end{bmatrix} \\ \downarrow \\ (1 \times m) \end{matrix} = \begin{matrix} \begin{matrix} \downarrow \\ (m \times m) \end{matrix} \begin{bmatrix} e_1^2 & e_1 e_2 & \dots & e_1 e_m \\ e_2 e_1 & e_2^2 & \dots & e_2 e_m \\ \vdots & \vdots & \ddots & \vdots \\ e_m e_1 & e_m e_2 & \dots & e_m^2 \end{bmatrix} \end{matrix}$$



$$E[\bar{e}\bar{e}^T] = \begin{bmatrix} E(e_1^2) & E(e_1 e_2) & \dots & E(e_1 e_m) \\ E(e_2 e_1) & E(e_2^2) & \dots & E(e_2 e_m) \\ \vdots & \vdots & \ddots & \vdots \\ E(e_m e_1) & E(e_m e_2) & \dots & E(e_m^2) \end{bmatrix}$$

$$\begin{aligned} E(e_i) &= 0 \\ E[e_i] &= \mu_i \\ \Rightarrow \mu_i &= 0 \\ \Rightarrow E(e_i^2) &= \sigma_i^2 \rightarrow \text{variance} \end{aligned}$$

If all the meters are independent to each other, then

$$E(e_i e_j) = 0 \quad \forall i, j = 1, \dots, m; i \neq j$$

$$\Rightarrow E[\bar{e}\bar{e}^T] = \begin{bmatrix} E(e_1^2) & 0 & \dots & 0 \\ 0 & E(e_2^2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & E(e_m^2) \end{bmatrix}$$



$$E[\bar{e}\bar{e}^T] = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_m^2 \end{bmatrix} = R$$

Also, $w_i = \frac{1}{\sigma_i^2}$

$$W = \begin{bmatrix} 1/\sigma_1^2 & & & 0 \\ & 1/\sigma_2^2 & & \\ & & \ddots & \\ 0 & & & 1/\sigma_m^2 \end{bmatrix} = R^{-1}$$

$$W = R^{-1}$$



we have computed $E[\bar{e}\bar{e}^T] = R$

$G = H^T W H \Rightarrow G^T = [H^T W H]^T = (WH)^T (H^T)^T = H^T W H = G$

↙
Covariance
matrix

$\Rightarrow G^T = G \rightarrow G$ is a symmetric matrix

$\Rightarrow G^{-1}$ is also a symmetric matrix

$\Rightarrow \boxed{G^{-1} = (G^{-1})^T}$

Now, $\hat{\bar{e}} = (I - H\bar{G}^{-1}H^T W)\bar{e}$

$\Rightarrow \hat{\bar{e}}^T = \bar{e}^T (I - H\bar{G}^{-1}H^T W)^T$



$$\Rightarrow \hat{\bar{e}} \hat{\bar{e}}^T = (I - H\bar{G}^{-1}H^TW) \bar{e} \bar{e}^T (I - H\bar{G}^{-1}H^TW)^T$$

$$\text{Now, } (I - H\bar{G}^{-1}H^TW)^T = I - (\underbrace{H\bar{G}^{-1}H^T}_A \underbrace{W}_B)^T = I - (H^TW)^T (H\bar{G}^{-1})^T$$

$$= I - WH\bar{G}^{-1}H^T$$

$$= (I - R^{-1}H\bar{G}^{-1}H^T)$$

$$\Rightarrow \hat{\bar{e}} \hat{\bar{e}}^T = (I - H\bar{G}^{-1}H^TW) \bar{e} \bar{e}^T (I - R^{-1}H\bar{G}^{-1}H^T)$$

$$\Rightarrow E[\hat{\bar{e}} \hat{\bar{e}}^T] = (I - H\bar{G}^{-1}H^TW) \underbrace{E(\bar{e} \bar{e}^T)}_R (I - R^{-1}H\bar{G}^{-1}H^T)$$

$$= (I - H\bar{G}^{-1}H^TW) R (I - R^{-1}H\bar{G}^{-1}H^T)$$



$$\Rightarrow E[\hat{\hat{e}}\hat{\hat{e}}^T] = (I - H\bar{G}^{-1}H^TW)(R - H\bar{G}^{-1}H^T)$$

$$\Rightarrow E[\hat{\hat{e}}\hat{\hat{e}}^T] = (I - H\bar{G}^{-1}H^TW)(I - H\bar{G}^{-1}H^TR^{-1})R$$

$$= \underbrace{(I - H\bar{G}^{-1}H^TW)}_{\text{Idempotent}} (I - H\bar{G}^{-1}H^TW)R \quad [\text{as } W = R^{-1}]$$

Now,

$$(I - H\bar{G}^{-1}H^TW)(I - H\bar{G}^{-1}H^TW)$$

$$= I - H\bar{G}^{-1}H^TW - H\bar{G}^{-1}H^TW + H\bar{G}^{-1}H^TW \underbrace{H\bar{G}^{-1}H^TW}_I$$

$$= I - H\bar{G}^{-1}H^TW - \cancel{H\bar{G}^{-1}H^TW} + \cancel{H\bar{G}^{-1}H^TW}$$

$$= (I - H\bar{G}^{-1}H^TW)$$

$$E[\hat{\hat{e}}\hat{\hat{e}}^T] = (I - H\bar{G}^{-1}H^TW)R$$

$$= R - H\bar{G}^{-1}H^T \quad [\text{as } WR = I]$$

$$E[\hat{\hat{e}}\hat{\hat{e}}^T] = R - H\bar{G}^{-1}H^T$$

