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NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

WLS Examples

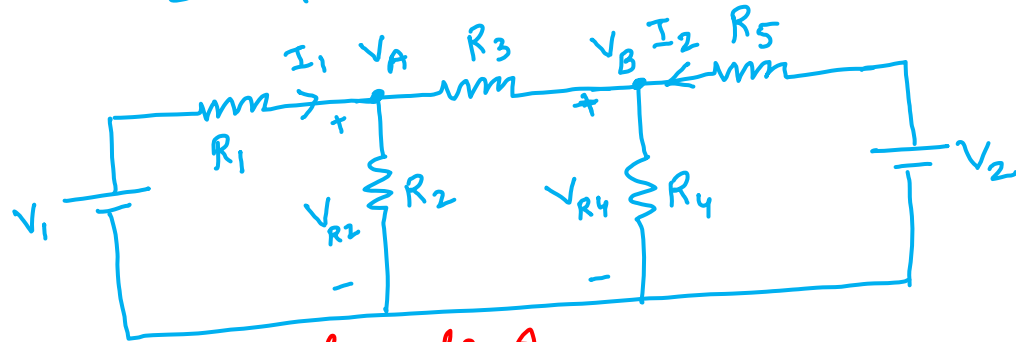
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Measurement model

$$\bar{Z} = H\bar{x} + \bar{E}$$



Applying KCL at node A,

$$\frac{V_A - V_1}{R_1} + \frac{V_A}{R_2} + \frac{V_A - V_B}{R_3} = 0$$

$$\Rightarrow V_A - V_1 + V_A + V_A - V_B = 0$$

$$\Rightarrow 3V_A - V_B = V_1 \dots (1)$$

$$\begin{array}{l|l} Z_1 = I_1 & Z_2 = I_2 \\ Z_3 = V_{R2} & Z_4 = V_{R4} \end{array} \quad \left| \begin{array}{l} V_A = V_{R2} \\ V_B = V_{R4} \end{array} \right.$$

$$\begin{bmatrix} I_1 \\ I_2 \\ V_A \\ V_B \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \\ h_{41} & h_{42} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\text{As } R_1 = R_2 = R_3 = R_4 = R_5 = 1 \text{ ohm}$$



Applying KCL at node B

$$\frac{V_B - V_2}{R_5} + \frac{V_B}{R_4} + \frac{V_B - V_A}{R_3} = 0$$

$$\Rightarrow V_B - V_2 + V_B + V_B - V_A = 0$$

$$\Rightarrow 3V_B - V_A = V_2 \quad \dots (2) \Rightarrow 9V_B - 3V_A = 3V_2 \quad \dots (3)$$

$$\Rightarrow 3V_B - V_A = V_2 \quad \dots (2)$$

$$\Rightarrow 3V_B - V_A = V_2 \quad \dots (2)$$

$$3V_A - V_B = V_1 \quad \dots (1)$$

$$(3) + (1) \Rightarrow 9V_B - 3V_A + 3V_A - V_B = 3V_2 + V_1$$

$$\Rightarrow V_B = \frac{3V_2 + V_1}{8} = 0.125V_1 + 0.375V_2 \quad \dots (4)$$

$$\text{From (1)} \quad 3V_A = V_B + V_1 = \frac{3V_2 + V_1}{8} + V_1 = \frac{9V_1 + 3V_2}{8}$$

$$\Rightarrow V_A = \frac{3V_1 + V_2}{8}$$



$$V_A = \frac{3V_1 + V_2}{8} = 0.375V_1 + 0.125V_2 \dots (5)$$

$$\text{Now, } I_1 = \frac{V_1 - V_A}{R_1} = V_1 - V_A = V_1 - \frac{3V_1 + V_2}{8} = \frac{5V_1 - V_2}{8} = 0.625V_1 - 0.125V_2 \dots (6)$$

$$I_2 = V_2 - V_B = V_2 - \frac{3V_2 + V_1}{8} = \frac{5V_2 - V_1}{8} = -0.125V_1 + 0.625V_2 \dots (7)$$

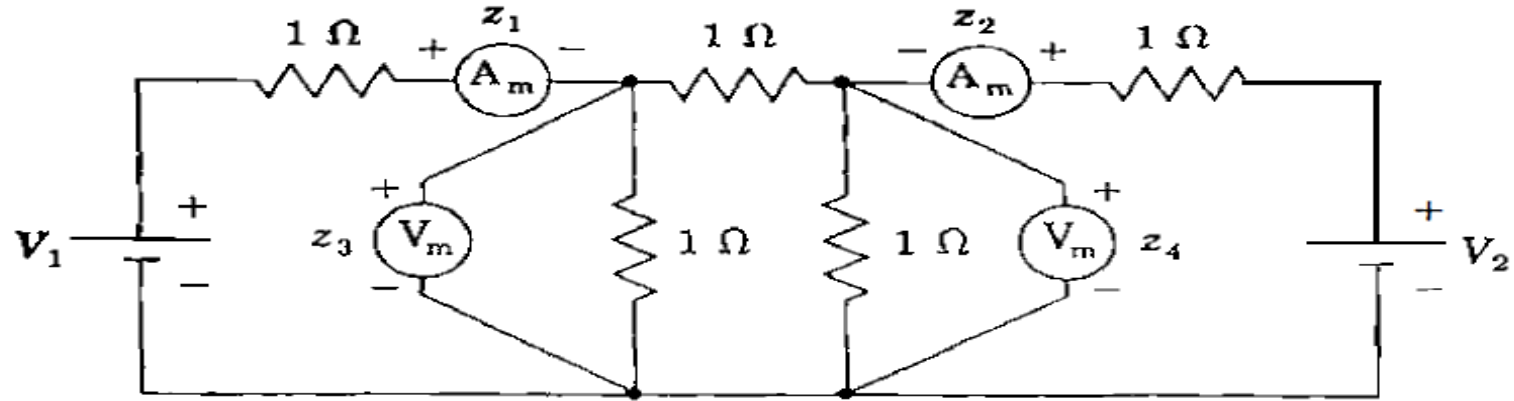
Equations are (in matrix form)

$$\begin{bmatrix} I_1 \\ I_2 \\ V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow H = \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix}$$

$$W = \begin{bmatrix} 100 & 0 \\ 0 & 100 \\ 0 & 50 \\ 50 & 0 \end{bmatrix}$$



WLS estimation



The meter readings are $Z1 = 9.01$ A, $Z2 = 3.02$ A, $z3 = 6.98$ V, and $Z4 = 5.01$ V. Weights are: $w1 = w2 = 100$, $w3 = w4 = 50$.

WLS estimation (contd..)

$$\mathbf{H} = \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix}$$

$$\begin{aligned} \mathbf{H}^T \mathbf{W} &= \begin{bmatrix} 0.625 & -0.125 & 0.375 & 0.125 \\ -0.125 & 0.625 & 0.125 & 0.375 \end{bmatrix} \begin{bmatrix} 100 & \cdot & \cdot & \cdot \\ \cdot & 100 & \cdot & \cdot \\ \cdot & \cdot & 50 & \cdot \\ \cdot & \cdot & \cdot & 50 \end{bmatrix} \\ &= \begin{bmatrix} 62.50 & -12.50 & 18.75 & 6.25 \\ -12.50 & 62.50 & 6.25 & 18.75 \end{bmatrix} \end{aligned}$$

WLS estimation (contd..)

$$\mathbf{G} = \mathbf{H}^T \mathbf{W} \mathbf{H} = \begin{bmatrix} 62.50 & -12.50 & 18.75 & 6.25 \\ -12.50 & 62.50 & 6.25 & 18.75 \end{bmatrix} \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix}$$
$$= \begin{bmatrix} 48.4375 & -10.9375 \\ -10.9375 & 48.4375 \end{bmatrix}$$

WLS estimation (contd..)

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \mathbf{G}^{-1} \mathbf{H}^T \mathbf{W} \mathbf{z}$$

$$= \begin{bmatrix} 48.4375 & -10.9375 \\ -10.9375 & 48.4375 \end{bmatrix}^{-1} \begin{bmatrix} 62.50 & -12.50 & 18.75 & 6.25 \\ -12.50 & 62.50 & 6.25 & 18.75 \end{bmatrix} \mathbf{z}$$

$$= \begin{bmatrix} 0.0218 & 0.0049 \\ 0.0049 & 0.0218 \end{bmatrix} \begin{bmatrix} 62.50 & -12.50 & 18.75 & 6.25 \\ -12.50 & 62.50 & 6.25 & 18.75 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

$$= \begin{bmatrix} 1.2982 & 0.0351 & 0.4386 & 0.2281 \\ 0.0351 & 1.2982 & 0.2281 & 0.4386 \end{bmatrix} \begin{bmatrix} 9.01 \text{ A} \\ 3.02 \text{ A} \\ 6.98 \text{ V} \\ 5.01 \text{ V} \end{bmatrix} = \begin{bmatrix} 16.0072 \text{ V} \\ 8.0261 \text{ V} \end{bmatrix}$$

WLS estimation (contd..)

$$\begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \\ \hat{z}_3 \\ \hat{z}_4 \end{bmatrix} = \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix} \begin{bmatrix} 16.0072 \text{ V} \\ 8.0261 \text{ V} \end{bmatrix} = \begin{bmatrix} 9.00123 \text{ A} \\ 3.01544 \text{ A} \\ 7.00596 \text{ V} \\ 5.01070 \text{ V} \end{bmatrix}$$

$$\begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \hat{e}_4 \end{bmatrix} = \begin{bmatrix} 9.01 \\ 3.02 \\ 6.98 \\ 5.01 \end{bmatrix} - \begin{bmatrix} 9.00123 \\ 3.01544 \\ 7.00596 \\ 5.01070 \end{bmatrix} = \begin{bmatrix} 0.00877 \text{ A} \\ 0.00456 \text{ A} \\ -0.02596 \text{ V} \\ -0.00070 \text{ V} \end{bmatrix}$$

WLS estimation (contd..)

- If $Z_4 = 4.40$ V instead of 5.01 V (other measurements remain same)

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} 1.2982 & 0.0351 & 0.4386 & 0.2281 \\ 0.0351 & 1.2982 & 0.2281 & 0.4386 \end{bmatrix} \begin{bmatrix} 9.01 \text{ A} \\ 3.02 \text{ A} \\ 6.98 \text{ V} \\ 4.40 \text{ V} \end{bmatrix}$$
$$= \begin{bmatrix} 15.86807 \text{ V} \\ 7.75860 \text{ V} \end{bmatrix}$$

WLS estimation (contd..)

- If $Z_4 = 4.40$ V instead of 5.01 V (other measurements remain same)

$$\begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \hat{e}_4 \end{bmatrix} = \begin{bmatrix} 9.01 \\ 3.02 \\ 6.98 \\ 4.40 \end{bmatrix} - \begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.625 \\ 0.375 & 0.125 \\ 0.125 & 0.375 \end{bmatrix} \begin{bmatrix} 15.86807 \text{ V} \\ 7.75860 \text{ V} \end{bmatrix} = \begin{bmatrix} 0.06228 \text{ A} \\ 0.15439 \text{ A} \\ 0.05965 \text{ V} \\ -0.49298 \text{ V} \end{bmatrix}$$