



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

WLS (Contd.)

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WLS Method

$$\hat{\bar{x}} = G^{-1} H^T W \bar{z}$$

$\bar{z} \rightarrow$ measurement vector

$G \rightarrow$ Gain matrix $\rightarrow H^T W H$

$W \rightarrow$ Weight matrix $\rightarrow$ a diagonal matrix

More weightage to more accurate meters and less weightage to less accurate meters.

$W_i = \frac{1}{\sigma_i^2}$ where $\sigma_i^2 \rightarrow$ meter variance

$$\begin{aligned}\hat{\bar{x}} &= G^{-1} H^T W \bar{z} = G^{-1} H^T W (H \bar{x} + \bar{e}) \\ &= G^{-1} \underbrace{H^T W H}_{\rightarrow G} \bar{x} + G^{-1} H^T W \bar{e}\end{aligned}$$



$$\Rightarrow \hat{\bar{x}} = \bar{x} + G^{-1} H^T W \bar{e}$$

Estimated
value

True
value

$$\Rightarrow \hat{\bar{x}} - \bar{x} = G^{-1} H^T W \bar{e}$$

$\bar{e} \rightarrow$ random but unknown vector

$\bar{z} \rightarrow$ Actual measurement vector

$\hat{\bar{z}} = H \hat{\bar{x}} \rightarrow$ estimated measurements

$$\begin{aligned} \hat{\bar{e}} &= \bar{z} - \hat{\bar{z}} = H \bar{x} + \bar{e} - H \hat{\bar{x}} \\ &= \bar{e} - H(\hat{\bar{x}} - \bar{x}) \\ &= \bar{e} - H G^{-1} H^T W \bar{e} \end{aligned}$$

$$\bar{z} = \bar{z}_{true} + \bar{e}$$

$$\bar{z}_{true} = H \bar{x}_{true}$$

$H \hat{\bar{x}} \rightarrow$ estimated value of $\bar{z}_{true}$

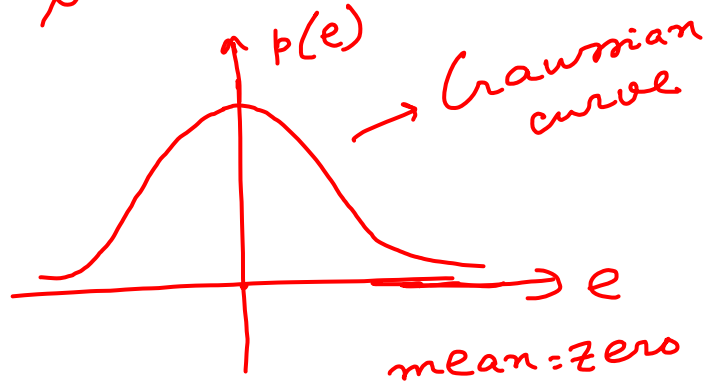
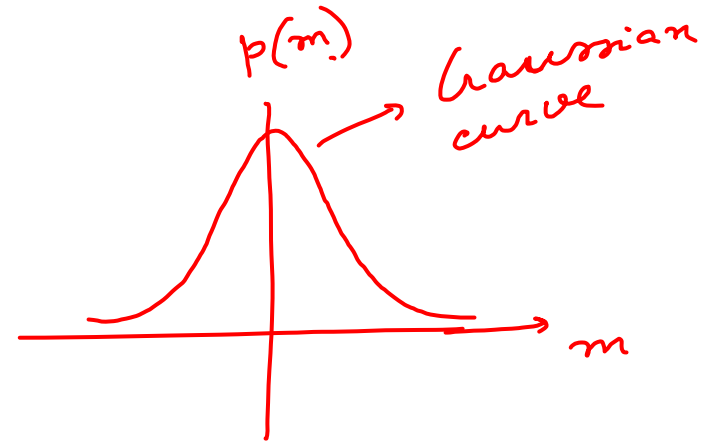
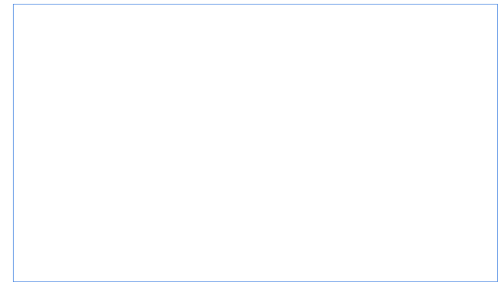


$$\hat{\bar{e}} = (I - H G^{-1} H^T W) \bar{e}$$

Error characteristics

$\bar{m} = x + \bar{e}$
 measured values.

$\downarrow$
 true value (fixed)



Error characteristics

→ zero mean
→ Independent

$$\rightarrow E(e_i) = 0$$

↓ Expected value of e_i

$$\left. \begin{array}{l} E(e_1) = 0 \\ E(e_2) = 0 \\ \vdots \\ E(e_m) = 0 \end{array} \right\} \Rightarrow$$

$$E \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} = 0$$

$$\Rightarrow E(\bar{e}) = 0$$



$$E[\hat{\bar{x}} - \bar{x}] = E(\bar{\omega}' H^T W \bar{e}) = \bar{\omega}' H^T W E(\bar{e}) = 0$$

$$\Rightarrow E(\hat{\bar{x}}) = E(\bar{x}) = \bar{x}$$

$$E(\hat{\bar{x}}) = \bar{x}$$

→

Average values are equal
to the true value

$$E[\hat{\bar{e}}] = (I - H \bar{\omega}' H^T W) E(\bar{e}) = 0$$

$$E(\hat{\bar{e}}) = 0$$

←

