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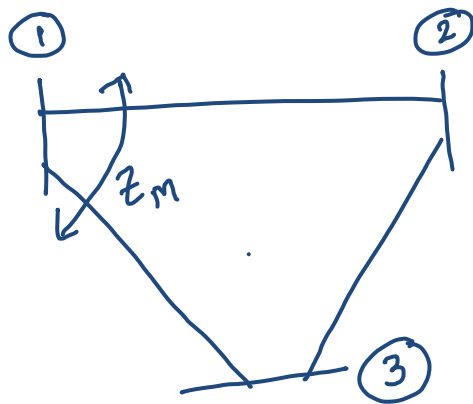
Computer aided power system analysis

Bus Admittance Matrix with mutual impedance

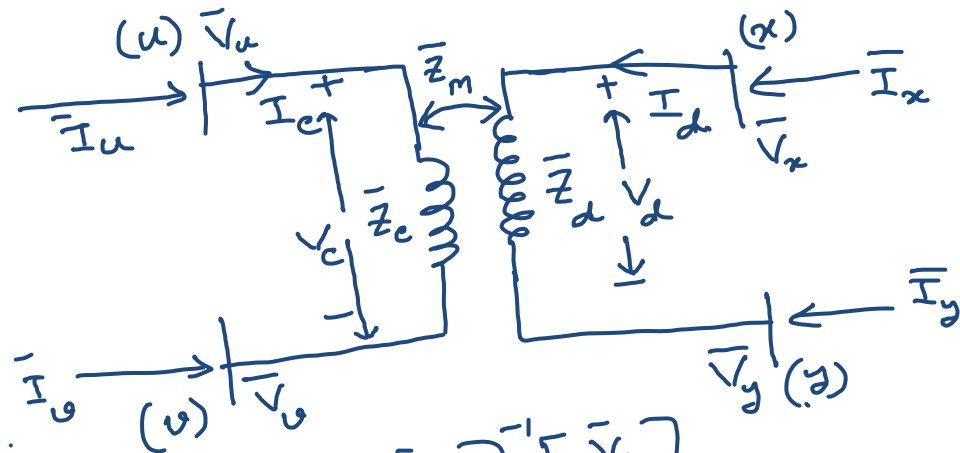
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$$\begin{bmatrix} \bar{V}_c \\ \bar{V}_d \end{bmatrix} = \begin{bmatrix} \bar{z}_c & \bar{z}_m \\ \bar{z}_m & \bar{z}_d \end{bmatrix} \begin{bmatrix} \bar{I}_c \\ \bar{I}_d \end{bmatrix} \Rightarrow$$



$$\begin{bmatrix} \bar{I}_c \\ \bar{I}_d \end{bmatrix} = \begin{bmatrix} \bar{z}_c & \bar{z}_m \\ \bar{z}_m & \bar{z}_d \end{bmatrix}^{-1} \begin{bmatrix} \bar{V}_c \\ \bar{V}_d \end{bmatrix}$$

$$= \frac{1}{\bar{z}_c \bar{z}_d - \bar{z}_m^2} \begin{bmatrix} \bar{z}_d & -\bar{z}_m \\ -\bar{z}_m & \bar{z}_c \end{bmatrix} \begin{bmatrix} \bar{V}_c \\ \bar{V}_d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \bar{I}_c \\ \bar{I}_d \end{bmatrix} = \begin{bmatrix} \bar{Y}_c & \bar{Y}_m \\ \bar{Y}_m & \bar{Y}_d \end{bmatrix} \begin{bmatrix} \bar{V}_c \\ \bar{V}_d \end{bmatrix} \quad \text{--- (1)} \quad \text{where } \Delta = \bar{Z}_c \bar{Z}_d - \bar{Z}_m^2$$

$\bar{Y}_c = \frac{\bar{Z}_d}{\Delta} \quad ; \quad \bar{Y}_m = -\frac{\bar{Z}_m}{\Delta} \quad ; \quad \bar{Y}_d = \frac{\bar{Z}_c}{\Delta}$

$$\begin{bmatrix} \bar{I}_u \\ \bar{I}_v \\ \bar{I}_x \\ \bar{I}_y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \bar{I}_c \\ \bar{I}_d \end{bmatrix} = [C] \begin{bmatrix} \bar{I}_c \\ \bar{I}_d \end{bmatrix} \quad \text{--- (2)} \quad \text{where, } [C] = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} \bar{V}_c \\ \bar{V}_d \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \bar{V}_u \\ \bar{V}_v \\ \bar{V}_x \\ \bar{V}_y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \bar{V}_c \\ \bar{V}_d \end{bmatrix} = [c]^T \begin{bmatrix} \bar{V}_u \\ \bar{V}_v \\ \bar{V}_x \\ \bar{V}_y \end{bmatrix} \dots \textcircled{3}$$

From eqn $\textcircled{1}$

$$\begin{bmatrix} \bar{I}_c \\ \bar{I}_d \end{bmatrix} = \begin{bmatrix} \bar{Y}_c & \bar{Y}_m \\ \bar{Y}_m & \bar{Y}_d \end{bmatrix} \begin{bmatrix} \bar{V}_c \\ \bar{V}_d \end{bmatrix} \Rightarrow [c] \begin{bmatrix} \bar{I}_c \\ \bar{I}_d \end{bmatrix} = [c] \begin{bmatrix} \bar{Y}_c & \bar{Y}_m \\ \bar{Y}_m & \bar{Y}_d \end{bmatrix} \begin{bmatrix} \bar{V}_c \\ \bar{V}_d \end{bmatrix} \dots \textcircled{4}$$

Substituting eqn $\textcircled{3}$ into eqn $\textcircled{4}$ we have,

$$[C] \begin{bmatrix} \bar{I}_e \\ \bar{I}_d \end{bmatrix} = [C] \begin{bmatrix} \bar{Y}_e & \bar{Y}_m \\ \bar{Y}_m & \bar{Y}_d \end{bmatrix} [C]^T \begin{bmatrix} \bar{V}_u \\ \bar{V}_\phi \\ \bar{V}_x \\ \bar{V}_y \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \bar{I}_u \\ \bar{I}_\phi \\ \bar{I}_x \\ \bar{I}_y \end{bmatrix} = [C] \begin{bmatrix} \bar{Y}_e & \bar{Y}_m \\ \bar{Y}_m & \bar{Y}_d \end{bmatrix} [C]^T \begin{bmatrix} \bar{V}_u \\ \bar{V}_\phi \\ \bar{V}_x \\ \bar{V}_y \end{bmatrix} \rightarrow \text{utilizing eqn. (2)}$$

Now, $[C] \begin{bmatrix} \bar{Y}_e & \bar{Y}_m \\ \bar{Y}_m & \bar{Y}_d \end{bmatrix} [C]^T$

$$= \begin{bmatrix} \bar{Y}_e & -\bar{Y}_e & \bar{Y}_m & -\bar{Y}_m \\ -\bar{Y}_e & \bar{Y}_e & -\bar{Y}_m & \bar{Y}_m \\ \bar{Y}_m & -\bar{Y}_m & \bar{Y}_d & -\bar{Y}_d \\ -\bar{Y}_m & \bar{Y}_m & -\bar{Y}_d & \bar{Y}_d \end{bmatrix}$$

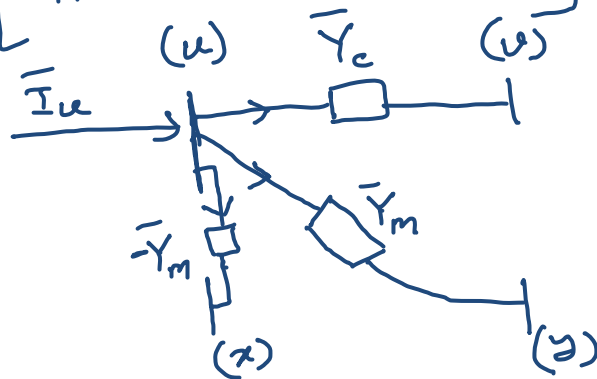
$$= \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \bar{Y}_e & \bar{Y}_m \\ \bar{Y}_m & \bar{Y}_d \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{matrix} \leftarrow (4 \times 2) & = & \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \bar{Y}_e & -\bar{Y}_e & \bar{Y}_m & -\bar{Y}_m \\ \bar{Y}_m & -\bar{Y}_m & \bar{Y}_d & -\bar{Y}_d \end{bmatrix} \rightarrow (2 \times 4) \end{matrix}$$



Therefore,

$$\begin{bmatrix} \bar{I}_u \\ \bar{I}_v \\ \bar{I}_x \\ \bar{I}_y \end{bmatrix} = \begin{bmatrix} \bar{Y}_c & -\bar{Y}_c & \bar{Y}_m & -\bar{Y}_m \\ -\bar{Y}_c & \bar{Y}_c & -\bar{Y}_m & \bar{Y}_m \\ \bar{Y}_m & -\bar{Y}_m & \bar{Y}_d & -\bar{Y}_d \\ -\bar{Y}_m & \bar{Y}_m & -\bar{Y}_d & \bar{Y}_d \end{bmatrix} \begin{bmatrix} \bar{V}_u \\ \bar{V}_v \\ \bar{V}_x \\ \bar{V}_y \end{bmatrix}$$



$$\begin{aligned} \Rightarrow \bar{I}_u &= \bar{Y}_c \bar{V}_u - \bar{Y}_c \bar{V}_v \\ &\quad + \bar{Y}_m \bar{V}_x - \bar{Y}_m \bar{V}_y \\ &= \bar{Y}_c \bar{V}_u - \bar{Y}_c \bar{V}_v + \bar{Y}_m \bar{V}_x \\ &\quad - \bar{Y}_m \bar{V}_y + \bar{Y}_m \bar{V}_u - \bar{Y}_m \bar{V}_u \\ &= \bar{Y}_c (\bar{V}_u - \bar{V}_v) + \bar{Y}_m (\bar{V}_u - \bar{V}_y) \\ &\quad - \bar{Y}_m (\bar{V}_u - \bar{V}_x) \end{aligned}$$