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CERTIFICATION COURSE

Computer aided power system analysis

Weighted Least Square (WLS) Method

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Weighted Least square (WLS) method

$$z_1 = h_{11}x_1 + h_{12}x_2 + e_1$$

$$z_2 = h_{21}x_1 + h_{22}x_2 + e_2$$

$$z_3 = h_{31}x_1 + h_{32}x_2 + e_3$$

$$z_4 = h_{41}x_1 + h_{42}x_2 + e_4$$

$$\bar{z} = [z_1 \ z_2 \ z_3 \ z_4]^T \rightarrow (4 \times 1)$$

$\rightarrow$ measurement vector

$$\bar{x} = [x_1 \ x_2]^T \rightarrow (2 \times 1) \rightarrow \text{state vector}$$

$$\bar{e} = [e_1 \ e_2 \ e_3 \ e_4]^T \rightarrow (4 \times 1)$$

$\rightarrow$ error vector

i) $f_1 = \sum_{i=1}^m e_i$ $m \rightarrow$ no. of measurements

$$e_1 = +5, \ e_2 = -3; \quad e_3 = +3, \ e_4 = -5$$

$$f_1 = 0$$



$$ii) f_2 = \sum_{i=1}^m |e_i|$$

$$iii) f_3 = \sum_{i=1}^m w_i e_i^2$$

$w_i \rightarrow$ weight of the i^{th} meter

$$\left. \begin{array}{l} \sqrt{m_1} \rightarrow \pm 1\% \\ m_2 \rightarrow \pm 5\% \\ m_3 \rightarrow \pm 3\% \\ \sqrt{m_4} \rightarrow \pm 10\% \end{array} \right\}$$

m_1, m_2, m_3, m_4
 $\rightarrow$ are the meters

$$f = w_1 e_1^2 + w_2 e_2^2 + w_3 e_3^2 + w_4 e_4^2$$

$$\frac{\partial f}{\partial x_1} = 0 ; \frac{\partial f}{\partial x_2} = 0$$

↓
Weighted square error



$$\left. \begin{aligned} \frac{\partial f}{\partial x_1} &= 2\omega_1 e_1 \frac{\partial e_1}{\partial x_1} + 2\omega_2 e_2 \frac{\partial e_2}{\partial x_1} + 2\omega_3 e_3 \frac{\partial e_3}{\partial x_1} + 2\omega_4 e_4 \frac{\partial e_4}{\partial x_1} = 0 \\ \frac{\partial f}{\partial x_2} &= 2 \left[\omega_1 e_1 \frac{\partial e_1}{\partial x_2} + \omega_2 e_2 \frac{\partial e_2}{\partial x_2} + \omega_3 e_3 \frac{\partial e_3}{\partial x_2} + \omega_4 e_4 \frac{\partial e_4}{\partial x_2} \right] = 0 \end{aligned} \right\}$$

$$\begin{aligned} z_1 &= h_{11}x_1 + h_{12}x_2 + e_1 \\ z_2 &= h_{21}x_1 + h_{22}x_2 + e_2 \\ z_3 &= h_{31}x_1 + h_{32}x_2 + e_3 \\ z_4 &= h_{41}x_1 + h_{42}x_2 + e_4 \end{aligned}$$

$$\vec{z} = H\vec{x} + \vec{e}$$

↓
(4×2)

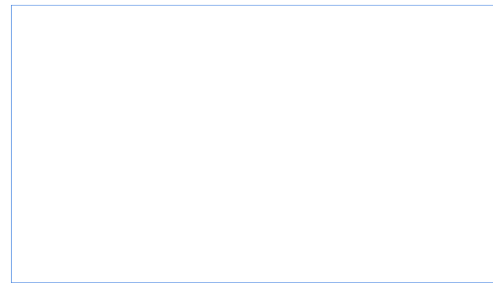
$$\Rightarrow \frac{\partial e_1}{\partial x_1} = -h_{11}; \quad \frac{\partial e_2}{\partial x_1} = -h_{21}$$

$$\frac{\partial e_3}{\partial x_1} = -h_{31}; \quad \frac{\partial e_4}{\partial x_1} = -h_{41}$$

$$\frac{\partial e_1}{\partial x_2} = -h_{12};$$

$$\frac{\partial e_2}{\partial x_2} = -h_{22}; \quad \frac{\partial e_2}{\partial x_3} = -h_{32}$$

$$\frac{\partial e_2}{\partial x_4} = -h_{42}$$



$$\left. \begin{aligned} \omega_1 e_1 h_{11} + \omega_2 e_2 h_{21} + \omega_3 e_3 h_{31} + \omega_4 e_4 h_{41} &= 0 \\ \omega_1 e_1 h_{12} + \omega_2 e_2 h_{22} + \omega_3 e_3 h_{32} + \omega_4 e_4 h_{42} &= 0 \end{aligned} \right\} \rightarrow \text{In a matrix form.}$$

$$\begin{bmatrix} h_{11} & h_{21} & h_{31} & h_{41} \\ h_{12} & h_{22} & h_{32} & h_{42} \end{bmatrix} \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 & \omega_4 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = 0 \quad \dots \textcircled{1}$$

$\downarrow$
 (2×4)

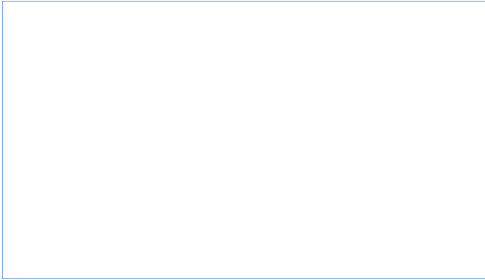
$\downarrow$
 W

$\downarrow$
 Weight matrix

$\downarrow$
 (4×4)

$\downarrow$
 (4×1)

$\rightarrow \bar{e}$



As $H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \\ h_{41} & h_{42} \end{bmatrix}$

; Equation (1) can be written as

$$H^T W \bar{e} = 0 \Rightarrow H^T W (z - H \bar{x}) = 0$$

$$\Rightarrow H^T W \bar{z} = H^T W H \bar{x}$$

$$\Rightarrow \bar{x} = (H^T W H)^{-1} H^T W \bar{z}$$

↓
Estimated state vector

We denote
 $G = H^T W H \rightarrow (2 \times 2)$

$$\Rightarrow \hat{\bar{x}} = G^{-1} H^T W \bar{z}$$



As the weighted square error is made least for estimating the state vector, this method is called **weighted least square (WLS) technique**

$$\hat{\bar{x}} = \bar{C}^{-1} H^T W \bar{z}$$

Now, let

$$\begin{aligned}\bar{x} &\rightarrow (n \times 1) \\ \bar{z} &\rightarrow (m \times 1) \\ \bar{e} &\rightarrow (m \times 1) \\ W &\rightarrow (m \times m) \\ H &\rightarrow (m \times n)\end{aligned}$$

$$C = H^T W H$$

$\swarrow \quad \downarrow \quad \searrow$
 $(n \times m) \quad (m \times m) \quad (m \times n)$

$$\Rightarrow C \rightarrow (n \times n) \rightarrow \text{always a square matrix}$$

$$\hat{\bar{x}} \rightarrow (n \times n) \times (n \times m) \times (m \times m) \times (m \times 1)$$

$$\Rightarrow \hat{\bar{x}} \rightarrow (n \times 1)$$

