



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

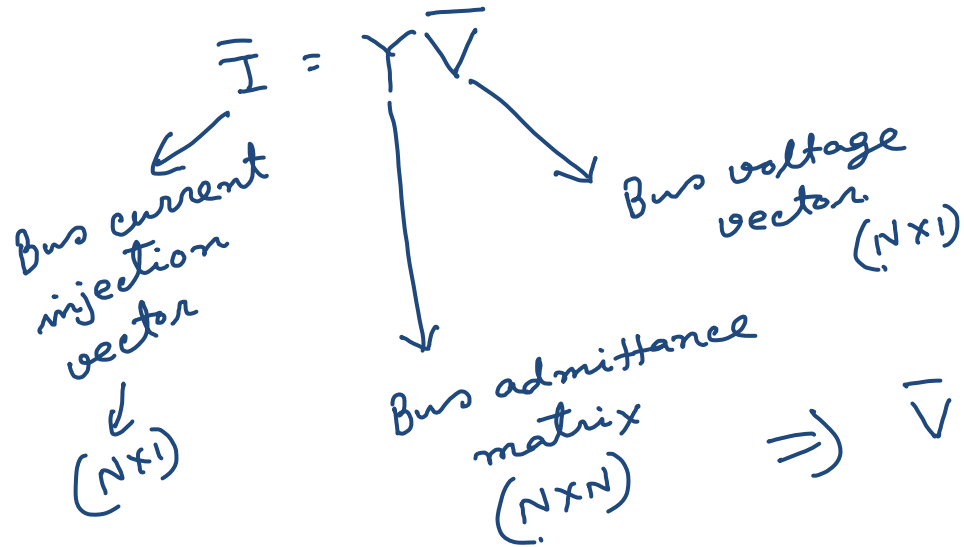
Line outage sensitivity factor

Prof. Biswarup Das

Department of Electrical Engineering



Thevenin Equivalent circuit



$$\bar{I} = [I_1 \quad I_2 \quad \dots \quad I_N]^T$$

$$\bar{V} = [V_1 \quad V_2 \quad \dots \quad V_N]^T$$

$$\Rightarrow \bar{V} = Y^{-1} \bar{I} = Z \bar{I}$$

Bus Impedance matrix



$$\bar{\mathbf{I}} + \Delta \bar{\mathbf{I}} = [\mathbf{I}_1 + \Delta \mathbf{I}_1 \quad \mathbf{I}_2 + \Delta \mathbf{I}_2 \quad \dots \quad \mathbf{I}_N + \Delta \mathbf{I}_N]^T$$

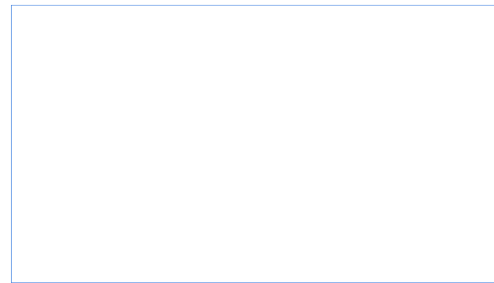
$$\bar{\mathbf{V}} + \Delta \bar{\mathbf{V}} = \mathbf{Z}(\bar{\mathbf{I}} + \Delta \bar{\mathbf{I}}) \Rightarrow \bar{\mathbf{V}} + \Delta \bar{\mathbf{V}} = \mathbf{Z}\bar{\mathbf{I}} + \mathbf{Z}\Delta \bar{\mathbf{I}}$$

$$\Rightarrow \underline{\Delta \bar{\mathbf{V}}} = \mathbf{Z}\Delta \bar{\mathbf{I}} \quad \left[\text{as } \bar{\mathbf{V}} = \mathbf{Z}\bar{\mathbf{I}} \right]$$

$$\Delta \bar{\mathbf{V}} = [\Delta V_1 \quad \Delta V_2 \quad \dots \quad \Delta V_N]^T$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1N} \\ z_{21} & z_{22} & \dots & z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ z_{N1} & z_{N2} & \dots & z_{NN} \end{bmatrix} \begin{bmatrix} \Delta I_1 \\ \Delta I_2 \\ \vdots \\ \Delta I_N \end{bmatrix}$$

Now, suppose $\Delta I_j = 0$
 $\forall j \neq k \quad j=1, \dots, N$



$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1N} \\ z_{21} & z_{22} & \dots & z_{2N} \\ \vdots & \vdots & & \vdots \\ z_{k1} & z_{k2} & \dots & z_{kN} \\ \vdots & \vdots & & \vdots \\ z_{N1} & z_{N2} & \dots & z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \Delta I_k \\ \vdots \\ 0 \end{bmatrix}$$

$$V_k = V_k^{(0)} + \Delta V_k = V_k^{(0)} + z_{kk} \Delta I_k$$

$$\Delta V_i = z_{ik} \Delta I_k$$

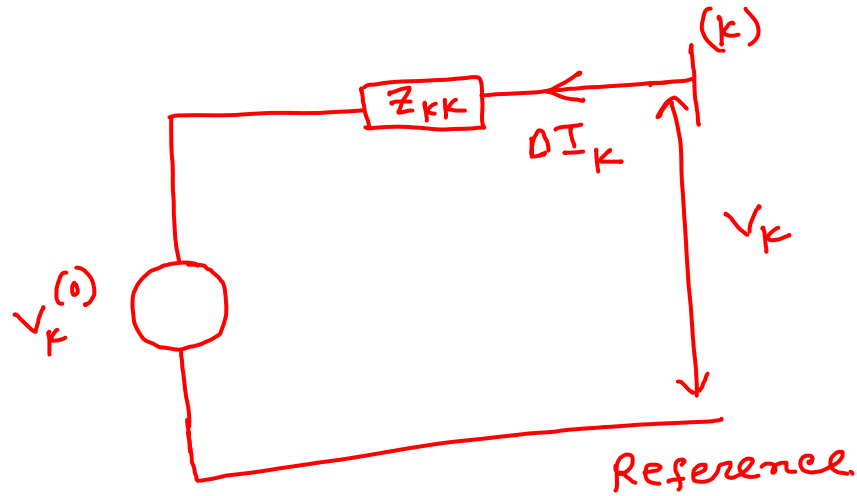
$$\forall i = 1, \dots, N$$

$$\Delta V_k = z_{kk} \Delta I_k$$

If initial voltage at bus 'k' is $V_k^{(0)}$, then the new voltage is



$$V_k = V_k^{(0)} + Z_{kk} \Delta I_k$$



$\Rightarrow$ This is the equivalent circuit of the power network as seen from bus 'k' and reference (ground).

$\Rightarrow$ Thevenin equivalent impedance as seen from bus 'k' and reference is Z_{kk}

Now, we assume that $\Delta I_i = 0 \quad \forall i=1, \dots, N; i \neq m, i \neq n$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & \dots & z_{1N} \\ z_{21} & z_{22} & \dots & z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ z_{N1} & z_{N2} & \dots & z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \Delta I_m \xrightarrow{\text{m}^{\text{th}} \text{ position}} \\ \vdots \\ \Delta I_n \xrightarrow{\text{n}^{\text{th}} \text{ position}} \\ \vdots \\ 0 \end{bmatrix}$$

$$\Delta V_m = z_{mm} \Delta I_m + z_{mn} \Delta I_n$$

$$\Delta V_n = z_{nm} \Delta I_m + z_{nn} \Delta I_n$$



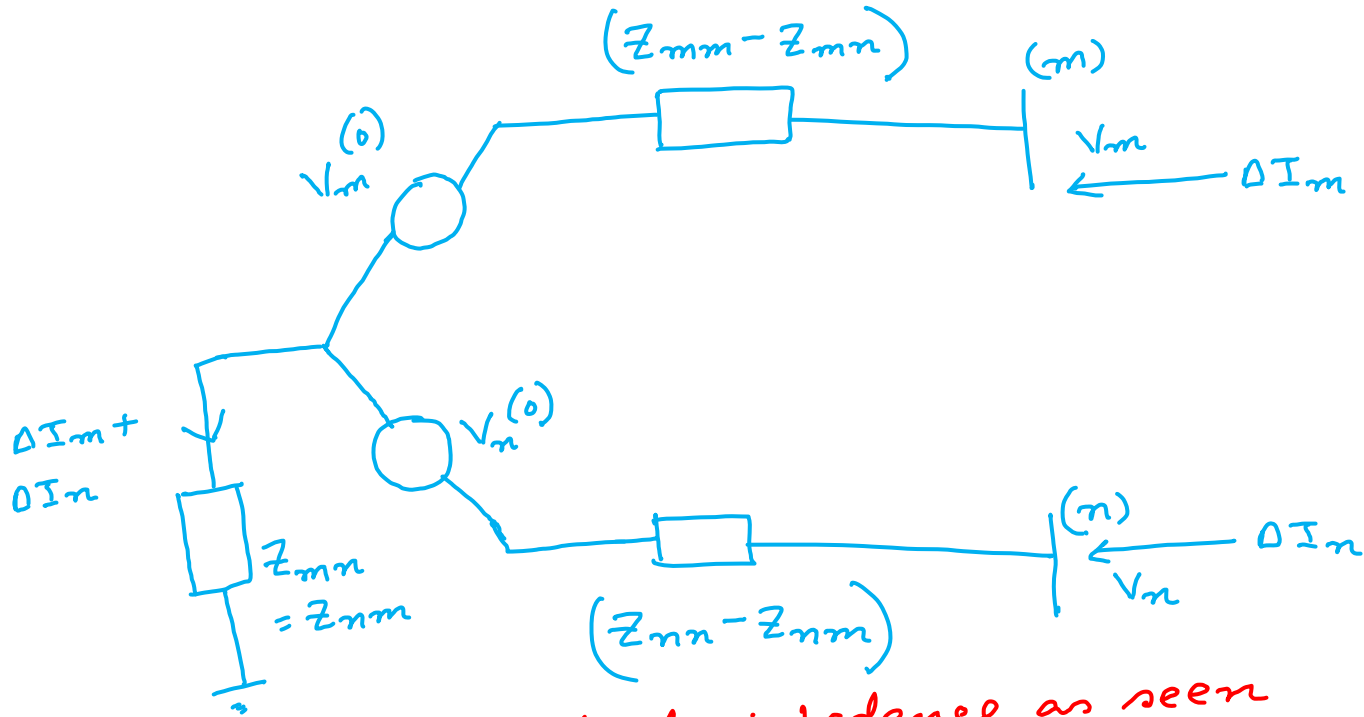
$$V_m = V_m^{(0)} + \Delta V_m = V_m^{(0)} + Z_{mm} \Delta I_m + Z_{mn} \Delta I_n$$

$$V_n = V_n^{(0)} + \Delta V_n = V_n^{(0)} + Z_{nm} \Delta I_m + Z_{nn} \Delta I_n$$

$$V_m = V_m^{(0)} + (Z_{mm} - Z_{mn}) \Delta I_m + Z_{mn} (\Delta I_m + \Delta I_n)$$

$$V_n = V_n^{(0)} + Z_{nm} (\Delta I_m + \Delta I_n) + (Z_{nn} - Z_{nm}) \Delta I_n$$





The Thevenin equivalent ckt. of the power grid as perceived from bus 'm' and 'n'.

Thevenin equivalent impedance as seen between bus 'm' and 'n' is

$$Z_{Th,mn} = Z_{mm} - Z_{mn} + Z_{nn} - Z_{nm}$$

$$\Rightarrow Z_{Th, mn} = Z_{mm} + Z_{nn} - 2Z_{mn}$$

$$\text{or } Z_{mn} = Z_{nm}$$

