



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

Linear Sensitivity Factors (Contd.)

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Line outage Sensitivity Factor (LOSF)

$$LOSF = \frac{\Delta P_{ij}}{\Delta P_{mn}}$$



$\beta_{ij,mn}$

Ratio of change in power flow through line 'i-j' due to change in power flow through line 'm-n'.

If line 'm-n' is tripped, then $\Delta P_{mn} = 0 - P_{mn} = -P_{mn}$

$$\Rightarrow \Delta P_{ij} = -\beta_{ij,mn} P_{mn}$$

$$\begin{aligned} \Rightarrow P_{ij}^{new} &= P_{ij}^{old} + \Delta P_{ij} \\ &= P_{ij}^{old} - \beta_{ij,mn} P_{mn} \end{aligned}$$



1-2	$P_{12,23}$	$P_{23,12}$	$P_{34,12}$	$P_{41,12}$
2-3	$P_{12,34}$	$P_{23,34}$	$P_{34,23}$	$P_{41,23}$
3-4	$P_{12,41}$	$P_{23,41}$	$P_{34,41}$	$P_{41,34}$
4-1				

$$4 \times 3 \\ = 4 \times (4-1)$$

'L' lines in the system,

$L(L-1) \rightarrow$ no. of LOSFs
need to be pre-calculated.

1000 1000×999 ✓

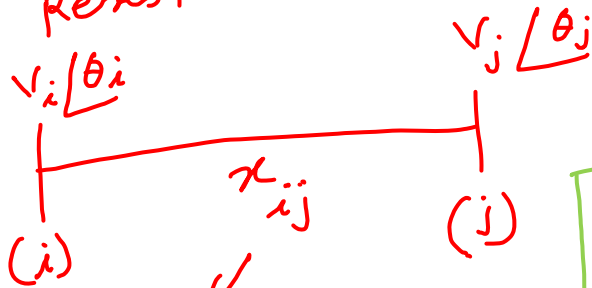
$P_{ij,mn}$



Calculation of GOSF (x_{ij}, k)

Assumptions of FDLF

1. $V_i \approx 1.0 \text{ p.u.}$
2. $\theta_i \approx 0^\circ \Rightarrow (\theta_i - \theta_j) \approx 0^\circ$
3. Resistances are neglected.



Reactance of the line

$$P_{ij} = \frac{V_i V_j}{x_{ij}} \sin(\theta_i - \theta_j)$$

$$\Rightarrow P_{ij} \approx \frac{1}{x_{ij}} (\theta_i - \theta_j)$$

$$\Delta P_{ij} = \frac{\Delta \theta_i - \Delta \theta_j}{x_{ij}}$$

FDLF equations

$$[B'] \cdot \bar{\Delta \theta} = \bar{\Delta P}$$

$$\bar{\Delta \theta} = [\Delta \theta_2 \quad \Delta \theta_3 \quad \dots \quad \Delta \theta_N]^T$$
$$\bar{\Delta P} = [\Delta P_2 \quad \Delta P_3 \quad \dots \quad \Delta P_N]^T$$

$$\Rightarrow \bar{\Delta \theta} = [B']^{-1} \bar{\Delta P} = [X] \cdot \bar{\Delta P}$$

$\downarrow$
 $(N-1) \times (N-1)$ $\downarrow$
 $(N-1) \times (N-1)$



$$\begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \\ \vdots \\ \Delta\theta_N \end{bmatrix} = \begin{bmatrix} x_{22} & x_{23} & \dots & x_{2N} \\ x_{32} & x_{33} & \dots & x_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N2} & x_{N3} & \dots & x_{NN} \end{bmatrix} \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_N \end{bmatrix}$$

$(N-1) \times 1$

$(N-1) \times (N-1)$

$(N-1) \times 1$

$$\Delta\theta_i = \sum_{l=2}^N x_{il} \Delta P_l$$

Note:

x_{ij} occupies the position $(i-1, j-1)$ in the matrix $[X]$.

Let generator at bus ' k ' is out of order only.

$$\Delta P_k = 0 \quad \text{for } k = 2, \dots, N: \neq i$$

$$\Delta P_k = -P_k$$

$$\Delta \theta_i = X_{ik} \Delta P_k$$

Similarly, $\Delta \theta_j = X_{jk} \Delta P_k$

$$\begin{aligned} \text{Therefore, } \Delta P_{ij} &= \frac{\Delta \theta_i - \Delta \theta_j}{x_{ij}} \\ &= \frac{(X_{ik} - X_{jk}) \Delta P_k}{x_{ij}} \end{aligned}$$

$$\text{Hence, } \alpha_{ij, k} = \frac{\Delta P_{ij}}{\Delta P_k} = \frac{X_{ik} - X_{jk}}{x_{ij}}$$



$$\text{GOSF}_{ij}^k = \alpha_{ij,k} = \frac{x_{ik} - x_{jk}}{x_{ij}}$$



Note (Important)

$x_{ik}, x_{jk} \rightarrow$ elements of the $[X]$ matrix, where
 $[X] = [B']^{-1}$

$x_{ij} \rightarrow$ reactance of the line 'i-j'

Bus data for IEEE-14 bus system

Bus no.	Type	$ V $ (p.u)	θ (deg)	P_G (MW)	Q_G (MVAR)	P_L (MW)	Q_L (MVAR)	Q_{MIN} (MVAR)	Q_{MAX} (MVAR)	G_{sh} (p.u)	B_{sh} (p.u)
1	1	1.06	0	0	0	0	0	0	0	0	0
2	2	1.045	0	18.3	5.857	0	0	-500	500	0	0
3	3	1	0	0	0	119	8.762	0	0	0.0002	0.502
4	3	1	0	0	0	47.79	3.9	0	500	0	0
5	3	1	0	0	0	7.599	1.599	0	0	0	0
6	2	1.07	0	11.2	44.2	0	0	-500	500	0	0
7	3	1	0	0	0	0	0	0	0	0	0
8	3	1	0	0	0	0	12.9	0	0	0.0023	0.1325
9	3	1	0	0	0	29.499	16.599	0	0	0	0.0633
10	3	1	0	0	0	9	5.799	0	0	0	0
11	3	1	0	0	0	3.501	1.8	0	500	0	0
12	3	1	0	0	0	6.099	1.599	0	0	0	0
13	3	1	0	0	0	13.5	5.799	0	0	0	0
14	3	1	0	0	0	14.901	5.001	0	0	0	0
Base MVA = 100											

Line data for IEEE-14 bus system

Branch no.	From bus	To bus	R (p.u)	X (p.u)	B/2 (p.u)	Tx. Tap
1	1	2	0.0194	0.0592	0.0528	0
2	1	5	0.054	0.223	0.0492	0
3	2	3	0.047	0.1979	0.0438	0
4	2	4	0.0581	0.1763	0.0374	0
5	2	5	0.0569	0.1738	0.0339	0
6	3	4	0.067	0.171	0.0346	0
7	4	5	0.0134	0.0421	0.0128	0
8	4	7	0	0.209	0	1
9	4	9	0	0.5562	0	1
10	5	6	0	0.2522	0	1
11	6	11	0.095	0.1989	0	0
12	6	12	0.1229	0.2557	0	0
13	6	13	0.0661	0.1302	0	0
14	7	8	0	0.1762	0	1
15	7	9	0	0.011	0	1
16	9	10	0.0318	0.0845	0	0
17	9	14	0.127	0.2703	0	0
18	10	11	0.082	0.192	0	0
19	12	13	0.2209	0.1999	0	0
20	13	14	0.1709	0.3479	0	0

Generator outage analysis in IEEE-14 bus system

Line no.	Pline (ori)	For outage of generator at bus 2				For outage of generator at bus 6			
		GSOF (2)	ΔP_2	Pline (cal)	Pline (ACLF)	GSOF (6)	ΔP_6	Pline (cal)	Pline (ACLF)
1	1.6262	-0.9709	-0.183	1.803875	1.7891	-0.7628	-0.112	1.711634	1.7203
2	0.7464	-0.1935	-0.183	0.781811	0.7776	-0.4344	-0.112	0.795053	0.7753
3	0.8655	0.0089	-0.183	0.863871	0.8606	-0.1568	-0.112	0.883062	0.8888
4	0.5314	0.0556	-0.183	0.521225	0.5207	-0.2928	-0.112	0.564194	0.5678
5	0.3662	0.0821	-0.183	0.351176	0.3511	-0.2977	-0.112	0.399542	0.395
6	-0.358	0.047	-0.183	-0.3666	-0.3622	-0.1203	-0.112	-0.34453	-0.3357
7	-0.7068	0.1064	-0.183	-0.72627	-0.7217	-0.0033	-0.112	-0.70643	-0.7189
8	0.2689	-0.0013	-0.183	0.269138	0.2691	-0.2562	-0.112	0.297594	0.3173
9	0.1063	-0.0004	-0.183	0.106373	0.1064	-0.1009	-0.112	0.117601	0.1255
10	0.2893	-0.0078	-0.183	0.290727	0.2887	-0.6729	-0.112	0.364665	0.3309
11	0.1156	-0.0059	-0.183	0.11668	0.1154	0.2459	-0.112	0.088059	0.0748
12	0.0852	-0.0007	-0.183	0.085328	0.0851	0.0277	-0.112	0.082098	0.0778
13	0.2005	-0.0031	-0.183	0.201067	0.2003	0.1312	-0.112	0.185806	0.1782
14	0.0025	-0.0065	-0.183	0.00369	0.0025	-0.0204	-0.112	0.004785	0.0022
15	0.2664	0.0052	-0.183	0.265448	0.2667	-0.2358	-0.112	0.29281	0.3151
16	0.0123	0.0055	-0.183	0.011294	0.0124	-0.2287	-0.112	0.037914	0.0512
17	0.0655	0.0037	-0.183	0.064823	0.0657	-0.1548	-0.112	0.082838	0.0944
18	-0.0777	0.0057	-0.183	-0.07874	-0.0776	-0.2368	-0.112	-0.05118	-0.039
19	0.0233	-0.0012	-0.183	0.02352	0.0232	0.05	-0.112	0.0177	0.016
20	0.0857	-0.0038	-0.183	0.086395	0.0854	0.1574	-0.112	0.068071	0.0567