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CERTIFICATION COURSE

Computer aided power system analysis

LU Decomposition

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LU Decomposition

$$A = \begin{bmatrix} d_{11} & 0 & 0 & 0 \\ d_{21} & d_{22} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ 0 & \beta_{22} & \beta_{23} & \beta_{24} \\ 0 & 0 & \beta_{33} & \beta_{34} \\ 0 & 0 & 0 & \beta_{44} \end{bmatrix}$$

←
(4x4)

16

←
L

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U

For $(N \times N)$ matrix, we have to find $(n^2 + n)$ elements.



Usually we set

$d_{ii} = 1$ for $i = 1, 2, \dots, N$

$$\begin{bmatrix} d_{11} & 0 & 0 & 0 & 0 \\ d_{21} & d_{22} & 0 & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 \\ d_{41} & d_{42} & d_{43} & d_{44} & 0 \\ d_{51} & d_{52} & d_{53} & d_{54} & d_{55} \end{bmatrix} \begin{bmatrix} \beta_{11} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \beta_{12} & \beta_{13} & \beta_{14} & \beta_{15} \\ \beta_{22} & \beta_{23} & \beta_{24} & \beta_{25} \\ 0 & \beta_{33} & \beta_{34} & \beta_{35} \\ 0 & 0 & \beta_{44} & \beta_{45} \\ 0 & 0 & 0 & \beta_{55} \end{bmatrix}$$



Triangular factorisation

Crout's Algorithm

step 1: Set $\alpha_{ii} = 1$ for $i = 1, 2, 3 \dots N$.

step 2: For each $j = 1, 2, 3 \dots N$, perform the following operation:

a) For $i = 1, 2, 3 \dots N$ solve for β_{ij} as

$$\beta_{ij} = a_{ij} - \sum_{k=1}^{i-1} \alpha_{ik} \beta_{kj}$$

Note: when $i = 1$, the summation term in equation (3.52) is taken to be zero.

E b) For $i = j + 1, j + 2, \dots N$ solve for α_{ij} as;

$$\alpha_{ij} = \frac{1}{\beta_{jj}} \left[a_{ij} - \sum_{k=1}^{j-1} \alpha_{ik} \beta_{kj} \right]$$

Triangular factorisation

Example

$$\text{Let } \mathbf{A} = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ 0 & \beta_{22} & \beta_{23} \\ 0 & 0 & \beta_{33} \end{bmatrix}$$

$$\alpha_{11} = 1; \alpha_{22} = 1; \alpha_{33} = 1;$$

For $j = 1$;

$$i = 1 \rightarrow \beta_{11} = a_{11} = 3;$$

$$i = 2 \rightarrow \alpha_{21} = \frac{1}{\beta_{11}}(a_{21} - 0) = \frac{2}{3};$$

$$i = 3 \rightarrow \alpha_{31} = \frac{1}{\beta_{11}}(a_{31} - 0) = \frac{3}{3} = 1;$$



Triangular factorisation

For $j = 2$;

$$i = 1 \rightarrow \beta_{12} = a_{12} = 2;$$

$$i = 2 \rightarrow \beta_{22} = a_{22} - \alpha_{21}\beta_{12} = \frac{5}{3};$$

$$i = 3 \rightarrow \alpha_{32} = \frac{1}{\beta_{22}}(a_{32} - \alpha_{31}\beta_{12}) = \frac{6}{5};$$

For $j = 3$;

$$i = 1 \rightarrow \beta_{13} = a_{13} = 7;$$

$$i = 2 \rightarrow \beta_{23} = a_{23} - \alpha_{21}\beta_{13} = -\frac{11}{13};$$

$$i = 3 \rightarrow \beta_{33} = a_{33} - \alpha_{31}\beta_{13} - \alpha_{32}\beta_{23} = -\frac{8}{5};$$

$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix}$$