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CERTIFICATION COURSE

Computer aided power system analysis

Triangular Factorization

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$$A\bar{x} = \bar{b}$$

$$A \rightarrow (4 \times 4) \Rightarrow A =$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

Triangular Factorisation

$$A = LU$$

$(n \times n)$

$(n \times n)$

Lower triangular matrix

$(n \times n)$

upper triangular matrix



$$L = \begin{bmatrix} \alpha_{11} & 0 & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 0 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \end{bmatrix}$$

$$U = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ 0 & \beta_{22} & \beta_{23} & \beta_{24} \\ 0 & 0 & \beta_{33} & \beta_{34} \\ 0 & 0 & 0 & \beta_{44} \end{bmatrix}$$

$$A\bar{x} = \bar{b}$$

$$\Rightarrow (LU)\bar{x} = \bar{b} \quad \left[\text{as } A = LU \right]$$

$$\Rightarrow L(U\bar{x}) = \bar{b}$$

$$\Rightarrow L\bar{y} = \bar{b}; \text{ where } \bar{y} = U\bar{x}$$

$$L\bar{y} = \bar{b}$$

$$U\bar{x} = \bar{y}$$

Now let $\bar{y} = [y_1 \ y_2 \ y_3 \ y_4]^T$
 intermediate vector

$$\begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$\downarrow$ known $\downarrow$ known

$$y_1 = \frac{b_1}{d_{11}}$$

$$y_2 = \frac{1}{d_{22}} (b_2 - d_{21}y_1)$$

$$y_3 = \frac{1}{d_{33}} (b_3 - d_{31}y_1 - d_{32}y_2)$$



$$y_4 = \frac{1}{\alpha_{44}} [\alpha_{41} y_1 - \alpha_{42} y_2 - \alpha_{43} y_3]$$

$$U \bar{x} = \bar{y}$$

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ 0 & \beta_{22} & \beta_{23} & \beta_{24} \\ 0 & 0 & \beta_{33} & \beta_{34} \\ 0 & 0 & 0 & \beta_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

known

$$x_4 = \frac{y_4}{\beta_{44}}$$

$$x_3 = \frac{1}{\beta_{33}} [y_3 - \beta_{34} x_4]$$

$$x_2 = \frac{1}{\beta_{22}} [y_2 - \beta_{23} x_3 - \beta_{24} x_4]$$

$$x_1 = \frac{1}{\beta_{11}} [y_1 - \beta_{12} x_2 - \beta_{13} x_3 - \beta_{14} x_4]$$



$$A \rightarrow (n \times n)$$

Total number of co-efficients of L and U matrices together = $n^2 + n$.



Triangular Factorisation

$A \rightarrow (N \times N)$ matrix

Intermediate
solution

$$\begin{aligned} y_1 &= \frac{b_1}{\alpha_{11}} \\ y_i &= \frac{1}{\alpha_{ii}} \left[b_i - \sum_{j=1}^{i-1} \alpha_{ij} y_j \right]; \quad i = 2, 3, \dots, N \end{aligned}$$

Final
solution

$$\begin{aligned} x_N &= \frac{y_N}{\beta_{NN}} \\ x_i &= \frac{1}{\beta_{ii}} \left[y_i - \sum_{j=i+1}^N \beta_{ij} x_j \right]; \quad i = (N-1), (N-2), \dots, 1 \end{aligned}$$



Triangular Factorisation

Crout's Algorithm

step 1: Set $\alpha_{ii} = 1$ for $i = 1, 2, 3 \dots N$.

step 2: For each $j = 1, 2, 3 \dots N$, perform the following operation:

a) For $i = 1, 2, 3 \dots N$ solve for β_{ij} as

$$\beta_{ij} = a_{ij} - \sum_{k=1}^{i-1} \alpha_{ik} \beta_{kj}$$

Note: when $i = 1$, the summation term in equation (3.52) is taken to be zero.

E b) For $i = j + 1, j + 2, \dots N$ solve for α_{ij} as;

$$\alpha_{ij} = \frac{1}{\beta_{jj}} \left[a_{ij} - \sum_{k=1}^{j-1} \alpha_{ik} \beta_{kj} \right]$$