



IIT ROORKEE



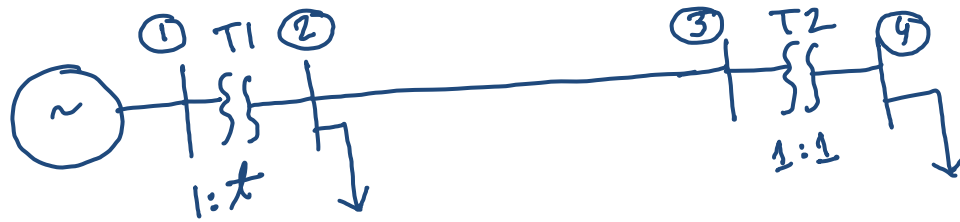
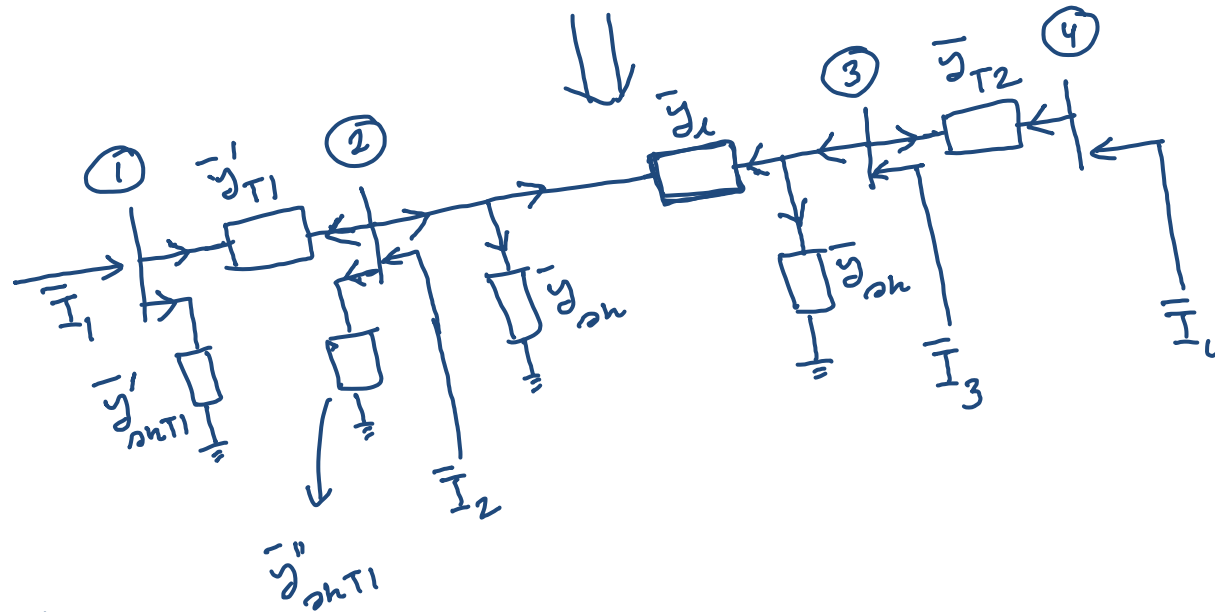
NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

Bus Admittance Matrix

Prof. Biswarup Das
Department of Electrical Engineering





At Bus 1

$$\begin{aligned}\bar{I}_1 &= \bar{V}_1 \bar{y}'_{\text{shT1}} + \bar{y}'_{\text{T1}} (\bar{V}_1 - \bar{V}_2) \\ &= (\bar{y}'_{\text{shT1}} + \bar{y}'_{\text{T1}}) \bar{V}_1 - \bar{y}'_{\text{T1}} \bar{V}_2 \quad \dots (1)\end{aligned}$$

At Bus 2

$$\begin{aligned}\bar{I}_2 &= \bar{V}_2 \bar{y}''_{\text{shT1}} + \bar{y}'_{\text{T1}} (\bar{V}_2 - \bar{V}_1) + \bar{y}_{\text{sh}} \bar{V}_2 + \bar{y}_{\text{L}} (\bar{V}_2 - \bar{V}_3) \\ &= -\bar{y}'_{\text{T1}} \bar{V}_1 + (\bar{y}''_{\text{shT1}} + \bar{y}'_{\text{T1}} + \bar{y}_{\text{sh}} + \bar{y}_{\text{L}}) \bar{V}_2 - \bar{y}_{\text{L}} \bar{V}_3 \quad \dots (2)\end{aligned}$$

At Bus-3

$$\begin{aligned}\bar{I}_3 &= \bar{y}_{\text{sh}} \bar{V}_3 + \bar{y}_{\text{L}} (\bar{V}_3 - \bar{V}_2) + \bar{y}_{\text{T2}} (\bar{V}_3 - \bar{V}_4) \\ &= -\bar{y}_{\text{L}} \bar{V}_2 + (\bar{y}_{\text{sh}} + \bar{y}_{\text{L}} + \bar{y}_{\text{T2}}) \bar{V}_3 - \bar{y}_{\text{T2}} \bar{V}_4 \quad \dots (3)\end{aligned}$$



At Bus 4

$$\bar{I}_4 = \bar{Y}_{T2}(\bar{V}_4 - \bar{V}_3) = -\bar{Y}_{T2}\bar{V}_3 + \bar{Y}_{T2}\bar{V}_4 \quad \dots (4)$$

Injection current vector $\leftarrow$

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \end{bmatrix} = \begin{bmatrix} \bar{Y}'_{2HT1} + \bar{Y}'_{T1} & -\bar{Y}'_{T1} & 0 & 0 \\ -\bar{Y}'_{T1} & \bar{Y}''_{2HT1} + \bar{Y}'_{T1} + \bar{Y}_{2L} & -\bar{Y}_{2L} & 0 \\ 0 & -\bar{Y}_{2L} & \bar{Y}_{2HT} + \bar{Y}_{2L} + \bar{Y}_{T2} & -\bar{Y}_{T2} \\ 0 & 0 & -\bar{Y}_{T2} & \bar{Y}_{T2} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \end{bmatrix} \rightarrow \text{Bus voltage vector}$$

Symmetric, complex Bus admittance matrix



$$\bar{I} = Y \bar{V}$$

Injection
current
vector

Bus
voltage
vector

Bus admittance
matrix

$$\bar{I} = [\bar{I}_1 \quad \bar{I}_2 \quad \bar{I}_3 \quad \bar{I}_4]^T \rightarrow \text{for this example}$$

$$= [\bar{I}_1 \quad \bar{I}_2 \quad \dots \quad \bar{I}_N]^T \rightarrow \text{For a } N\text{-Bus system}$$

(N x 1)

$$\bar{V} = [\bar{V}_1 \quad \bar{V}_2 \quad \bar{V}_3 \quad \bar{V}_4]^T \rightarrow \text{For this example}$$

$$= [\bar{V}_1 \quad \bar{V}_2 \quad \dots \quad \bar{V}_N]^T \rightarrow \text{For a } N\text{-Bus system}$$

(N x 1)

$$\bar{Y} = (4 \times 4) \\ = (N \times N)$$

$$\bar{Y} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & & \\ \bar{Y}_{21} & \bar{Y}_{22} & & \\ & & \ddots & \\ \bar{Y}_{N1} & \bar{Y}_{N2} & & \end{bmatrix}$$

For this example
For a N-Bus system

$$\begin{bmatrix} \dots & \dots & \bar{Y}_{1N} \\ \dots & \dots & \bar{Y}_{2N} \\ & & \ddots \\ \dots & \dots & \bar{Y}_{NN} \end{bmatrix}$$

$\bar{Y}_{ii} \rightarrow i^{\text{th}}$ diagonal element
= sum total of the admittances connected at bus 'i'

$\bar{Y}_{ij} \rightarrow$ off-diagonal element
= 0 if bus 'i' and bus 'j' are not directly connected

= negative of the admittance connected between bus 'i' and bus 'j'.

Observation

In i^{th} row, number of non-zero off-diagonal elements is equal to the number of physical elements connected at bus 'i'.

Suppose $N = 1000$



$$Y = (1000 \times 1000)$$

no. of elements in this matrix = 1.0×10^6 .

Let on an average, every bus is physically directly connected to 4 other buses.

⇒ In each row, number of non-zero off-diagonal element is 4



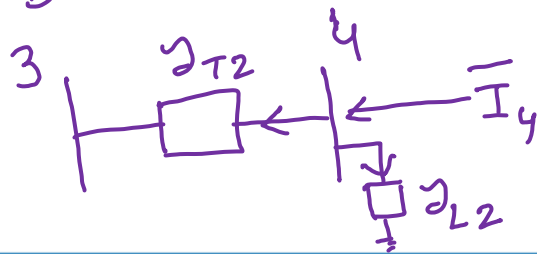
Total number of non-zero element in the matrix
 $= 4 \times 1000 + 1000 = 5000$

$\Rightarrow$ Total number of zero elements $= 1000000 - 5000$
 $= 995000$

$\Rightarrow$ Sparsity

$\% \text{ sparsity} = \frac{995000}{1000000} \times 100 = 99.5\%$

Bus admittance matrix is a highly sparse matrix.



$$\begin{aligned}\bar{I}_4 &= \bar{y}_{T2} (\bar{V}_4 - \bar{V}_3) + \bar{y}_{L2} \bar{V}_4 \\ &= -\bar{y}_{T2} \bar{V}_3 + (\bar{y}_{T2} + \bar{y}_{L2}) \bar{V}_4\end{aligned}$$

$$\bar{I}_i = \bar{Y}_{i1} \bar{V}_1 + \bar{Y}_{i2} \bar{V}_2 + \dots + \bar{Y}_{iN} \bar{V}_N$$

$$= \sum_{k=1}^N \bar{Y}_{ik} \bar{V}_k$$

$$\boxed{\bar{I}_i = \sum \bar{Y}_{ik} \bar{V}_k}$$

