IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Computer aided power system analysis

Gaussian Elimination and Optimal Ordering

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$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix} \xrightarrow{\text{1st step}} \begin{bmatrix} 1 & a_{12}' & a_{13}' & a_{14}' \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix}$$

where $a_{1j}' = \frac{a_{1j}}{a_{11}}$; $j = 2, 3, 4$

$$\xrightarrow{\text{2nd step}} \begin{bmatrix} 1 & a_{12}' & a_{13}' & a_{14}' \\ 0 & a_{22}' & a_{23}' & a_{24}' \\ a_{31} & 0 & a_{33} & 0 \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix}$$

$a_{23}' = 0 - a_{13}' a_{21} \neq 0$

$a_{24}' = 0 - a_{14}' a_{21} \neq 0$

$[a_{22}' = a_{22} - a_{12}' a_{22}]$

'Fill-in' process ⟶ Conversion of an original 'zero' element to non-zero element.

Original matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix}$$

$A\bar{x} = \bar{b}$

$[x_1 \quad x_2 \quad x_3 \quad x_4]^T$

$\bar{b} = [b_1 \quad b_2 \quad b_3 \quad b_4]^T$

$A\bar{x} = \bar{b}$ Original equation

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & 0 & a_{33} & 0 \\ a_{41} & 0 & 0 & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$+ a_{33}x_3 = b_3$$

$$a_{31}x_1$$

$$a_{41}x_1 \qquad + a_{44}x_4 = b_4$$

Re-arranged equation

$$\begin{bmatrix} a_{44} & 0 & 0 & a_{41} \\ 0 & 0 & a_{22} & a_{21} \\ 0 & a_{33} & 0 & a_{31} \\ a_{14} & a_{13} & a_{12} & a_{11} \end{bmatrix} \begin{bmatrix} x_4 \\ x_3 \\ x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} b_4 \\ b_2 \\ b_3 \\ b_1 \end{bmatrix}$$

$$a_{44}x_4 + a_{41}x_1 = b_4$$

$$a_{22}x_2 + a_{21}x_1 = b_2$$

$$a_{33}x_3 + a_{31}x_1 = b_3$$

$$a_{14}x_4 + a_{13}x_3 + a_{12}x_2 + a_{11}x_1 = b_1$$

Re-arranged matrix

$$A' = \begin{bmatrix} a_{44} & 0 & 0 & a_{41} \\ 0 & 0 & a_{22} & a_{21} \\ 0 & a_{33} & 0 & a_{31} \\ a_{14} & a_{13} & a_{12} & a_{11} \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 0 & 0 & a'_{41} \\ 0 & 0 & a_{22} & a_{21} \\ 0 & a_{33} & 0 & a_{31} \\ a_{14} & a_{13} & a_{12} & a_{11} \end{bmatrix}$$

$$a'_{41} = \frac{a_{41}}{a_{44}}$$

$$\Longrightarrow \begin{bmatrix} 1 & 0 & 0 & a'_{41} \\ 0 & 0 & a_{22} & a_{21} \\ 0 & a_{33} & 0 & a_{31} \\ 0 & a'_{13} & a'_{12} & a'_{11} \end{bmatrix}$$

$$a'_{13} = a_{13}$$

$$a'_{12} = a_{12}$$

$$a'_{11} = a_{11} - a_{14} a'_{41}$$

Strategies for optimal ordering

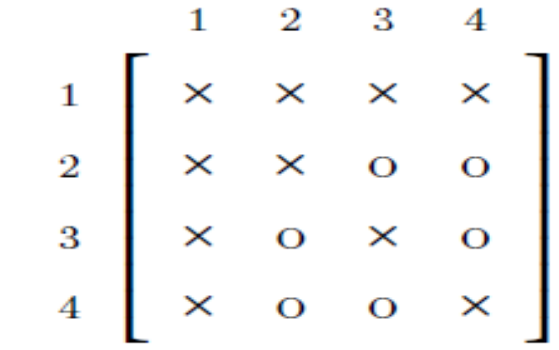
Strategy - 1

Strategy - 2

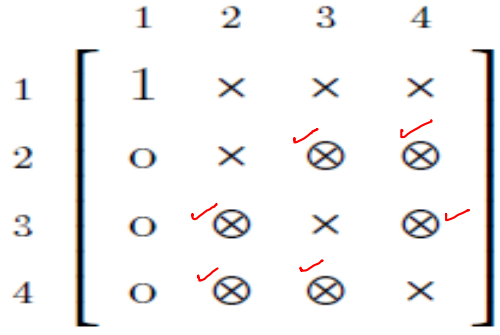
Strategy - 3

Optimal ordering

- Basic concept



a) Initial 'A' matrix



b) 'A' matrix after step 1

Optimal ordering

- Order rearrangement

	4	3	2	1
4	×	o	o	×
2	o	o	×	×
3	o	×	o	×
1	×	×	×	×

a) Rearranged ‘**A**’ matrix

	4	3	2	1
4	1	o	o	×
2	o	o	×	×
3	o	×	o	×
1	o	×	×	×

b) Rearranged ‘**A**’ matrix after step 1