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CERTIFICATION COURSE

Computer aided power system analysis

Example of Gaussian Elimination Method

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$$A\bar{x} = \bar{b}$$

$$\begin{matrix} R_1 \leftarrow \\ R_2 \leftarrow \\ R_3 \leftarrow \\ R_4 \leftarrow \end{matrix} \begin{bmatrix} 1 & a'_{12} & a'_{13} & a'_{14} \\ 0 & 1 & a''_{23} & a''_{24} \\ 0 & 0 & 1 & a'''_{34} \\ 0 & 0 & 0 & a''''_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

Step - 3 $R_2 = \frac{R_2}{a'_{22}}$ ✓

Step - 4

i) $R_3 = (R_3 - R_2 \times a'_{32})$
 ii) $R_4 = (R_4 - R_2 \times a'_{42})$

Step 1

$$R_1 = \frac{R_1}{a_{11}} \checkmark$$

Step - 2

i) $R_2 = (R_2 - R_1 \times a_{21})$

ii) $R_3 = (R_3 - R_1 \times a_{31})$

iii) $R_4 = (R_4 - R_1 \times a_{41})$



Step-5 : $R_3 = \frac{R_3}{a_{33}''}$ ✓

Step-6 $R_4 = (R_4 - R_3 \times a_{43}'')$

Final solutions

$$x_4 = \frac{b_4'''}{a_{44}'''}$$

$$x_3 = \frac{b_3'''}{a_{33}'''} - a_{34}''' x_4$$

$$x_2 = \frac{b_2''}{a_{22}''} - a_{23}'' x_3 - a_{24}'' x_4$$

$$x_1 = \frac{b_1'}{a_{11}'} - a_{12}' x_2 - a_{13}' x_3 - a_{14}' x_4$$

Assumptions:
All pivot elements;
i.e. $a_{11}', a_{22}', a_{33}'' \neq 0$



Example of Gaussian Elimination

- Given set of equations

$$\begin{bmatrix} 11 & 17 & 18 & 16 \\ 23 & 27 & 25 & 28 \\ 22 & 32 & 34 & 36 \\ 12 & 15 & 41 & 36 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 30 \\ 40 \end{bmatrix}$$

- Step 1:

$$R1/A(1,1) \quad (\text{where, } A(1,1) = 11)$$

$$\begin{bmatrix} 1 & 1.5455 & 1.6364 & 1.4545 \\ 23 & 27 & 25 & 28 \\ 22 & 32 & 34 & 36 \\ 12 & 15 & 41 & 36 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.9091 \\ 20 \\ 30 \\ 40 \end{bmatrix}$$

Example of Gaussian Elimination

- Step 2:

$(R2 - R1 * A(2, 1)), (R3 - R1 * A(3, 1))$ and $(R4 - R1 * A(4, 1))$
(where $A(2, 1) = 23$, $A(3, 1) = 22$ and $A(4, 1) = 12$)

$$\begin{bmatrix} 1 & 1.5455 & 1.6364 & 1.4545 \\ 0 & -8.5455 & -12.6364 & -5.4545 \\ 0 & -2.0 & -2.0 & 4.0 \\ 0 & -3.5455 & 21.3636 & 18.5455 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.9091 \\ -0.9091 \\ 10 \\ 20.0909 \end{bmatrix}$$

Example of Gaussian Elimination

- Step 3: $R2/A(2, 2)$ (where, $A(2, 2) = -8.5455$)

$$\begin{bmatrix} 1 & 1.5455 & 1.6364 & 1.4545 \\ 0 & 1 & 1.4787 & 0.6383 \\ 0 & -2.0 & -2.0 & 4.0 \\ 0 & -3.5455 & 21.3636 & 18.5455 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.9091 \\ 0.1064 \\ 10 \\ 20.0909 \end{bmatrix}$$

- Step 4:

$(R3 - R2 * A(3, 2))$ and $(R4 - R2 * A(4, 2))$
(where $A(3, 2) = -2.0$ and $A(4, 2) = -3.5455$)

$$\begin{bmatrix} 1 & 1.5455 & 1.6364 & 1.4545 \\ 0 & 1 & 1.4787 & 0.6383 \\ 0 & 0 & 0.9574 & 5.2766 \\ 0 & 0 & 26.6065 & 20.8085 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.9091 \\ 0.1064 \\ 10.2128 \\ 29.4681 \end{bmatrix}$$

Example of Gaussian Elimination

- Step 5:

$R3/A(3,3)$ (where $A(3,3) = 0.9574$)

$$\begin{bmatrix} 1 & 1.5455 & 1.6364 & 1.4545 \\ 0 & 1 & 1.4787 & 0.6383 \\ 0 & 0 & 1 & 5.5114 \\ 0 & 0 & 26.6065 & 20.8085 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.9091 \\ 0.1064 \\ 10.6672 \\ 29.4681 \end{bmatrix}$$

- Step 6:

$(R4 - R3 * A(4,3))$ (where $A(4,3) = 26.6065$)

$$\begin{bmatrix} 1 & 1.5455 & 1.6364 & 1.4545 \\ 0 & 1 & 1.4787 & 0.6383 \\ 0 & 0 & 1 & 5.5114 \\ 0 & 0 & 0 & -125.83 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0.9091 \\ 0.1064 \\ 10.6672 \\ -254.3484 \end{bmatrix}$$

Example of Gaussian Elimination

• Final Solution:

$$x_4 = 254.3484/125.83 = 2.0214.$$

$$x_3 = 10.6672 - 5.5114 \times 2.0214 = -0.4735.$$

$$x_2 = 0.1064 + 1.4787 \times 0.4735 - 0.6383 \times 2.0214 = -0.4837.$$

$$x_1 = 0.9091 + 1.5455 \times 0.4837 + 1.6364 \times 0.4735 - 1.4545 \times 2.0214 = -0.5086.$$

