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NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

Gaussian Elimination Method

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GAUSSIAN ELIMINATION METHOD

Original
equatio-
ns

a_{11} ^{Pivot} $x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1$ --- (1)

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2$ --- (2)

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3$ --- (3)

$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4$ --- (4)

Step 1

$$x_1 + \frac{a_{12}}{a_{11}}x_2 + \frac{a_{13}}{a_{11}}x_3 + \frac{a_{14}}{a_{11}}x_4 = \frac{b_1}{a_{11}} \quad \text{--- (5)}$$

Step 2 i) By performing (2) - (5) $\times a_{21}$, we get



$$(a_{21} - a_{21})x_1 + \left(a_{22} - \frac{a_{12}}{a_{11}} a_{21}\right)x_2 + \left(a_{23} - \frac{a_{13}}{a_{11}} a_{21}\right)x_3 + \left(a_{24} - \frac{a_{14}}{a_{11}} a_{21}\right)x_4 = \left(b_2 - \frac{b_1}{a_{11}} a_{21}\right)$$

$$\Rightarrow \text{Pivot} \quad a_{22}^{(1)} x_2 + a_{23}^{(1)} x_3 + a_{24}^{(1)} x_4 = b_2^{(1)} \quad \dots \textcircled{6} \checkmark$$

$$\text{ii) } (3) - (5) \times a_{31} \Rightarrow (a_{31} - a_{31})x_1 + \left(a_{32} - \frac{a_{12}}{a_{11}} a_{31}\right)x_2 + \left(a_{33} - \frac{a_{13}}{a_{11}} a_{31}\right)x_3 + \left(a_{34} - \frac{a_{14}}{a_{11}} a_{31}\right)x_4 = \left(b_3 - \frac{b_1}{a_{11}} a_{31}\right)$$

$$\Rightarrow a_{32}^{(1)} x_2 + a_{33}^{(1)} x_3 + a_{34}^{(1)} x_4 = b_3^{(1)} \quad \dots \textcircled{7} \checkmark$$

$$\text{iii) } (4) - (5) \times a_{41} \Rightarrow$$



$$(a_{41} - a_{41})x_1 + \left(a_{42} - \frac{a_{12}}{a_{11}}a_{41}\right)x_2 + \left(a_{43} - \frac{a_{13}}{a_{11}}a_{41}\right)x_3 + \left(a_{44} - \frac{a_{14}}{a_{11}}a_{41}\right)x_4 = \left(b_4 - \frac{b_1}{a_{11}}a_{41}\right)$$

$$\Rightarrow a_{42}^{(1)}x_2 + a_{43}^{(1)}x_3 + a_{44}^{(1)}x_4 = b_4^{(1)} \quad \text{--- (8) } \checkmark$$

Step-3 Divide (6) by $a_{22}^{(1)}$

$$\Rightarrow x_2 + \frac{a_{23}^{(1)}}{a_{22}^{(1)}}x_3 + \frac{a_{24}^{(1)}}{a_{22}^{(1)}}x_4 = \frac{b_2^{(1)}}{a_{22}^{(1)}} \quad \text{--- (9)}$$

Step-4

i) (7) - (9) $\times a_{32}^{(1)}$

$$\Rightarrow \left[a_{32}^{(1)} - a_{32}^{(1)} \right] x_2 + \left[a_{33}^{(1)} - \frac{a_{23}^{(1)}}{a_{22}^{(1)}} a_{32}^{(1)} \right] x_3 + \left[a_{34}^{(1)} - \frac{a_{24}^{(1)}}{a_{22}^{(1)}} a_{32}^{(1)} \right] x_4$$

$$= \left[b_3^{(1)} - \frac{b_2^{(1)}}{a_{22}^{(1)}} a_{32}^{(1)} \right]$$

Pivot

$$\Rightarrow a_{33}^{(2)} x_3 + a_{34}^{(2)} x_4 = b_3^{(2)} \quad \dots (10)$$

ii) (8) - (9) $\times a_{42}^{(1)}$

$$\Rightarrow \left[a_{42}^{(1)} - a_{42}^{(1)} \right] x_2 + \left[a_{43}^{(1)} - \frac{a_{23}^{(1)}}{a_{22}^{(1)}} a_{42}^{(1)} \right] x_3 + \left[a_{44}^{(1)} - \frac{a_{24}^{(1)}}{a_{22}^{(1)}} a_{42}^{(1)} \right] x_4$$

$$= \left[b_4^{(1)} - \frac{b_2^{(1)}}{a_{22}^{(1)}} a_{42}^{(1)} \right]$$

$$\Rightarrow a_{43}^{(2)} x_3 + a_{44}^{(2)} x_4 = b_4^{(2)} \quad \dots\dots (11)$$

Step - 5 Divide (10) by $a_{33}^{(2)}$

$$\Rightarrow x_3 + \frac{a_{34}^{(2)}}{a_{33}^{(2)}} x_4 = \frac{b_3^{(2)}}{a_{33}^{(2)}} \quad \dots\dots (12)$$

Step 6

$$(11) - (12) \times a_{43}^{(2)}$$

$$\Rightarrow \left[a_{43}^{(2)} - a_{43}^{(2)} \right] x_3 + \left[a_{44}^{(2)} - \frac{a_{34}^{(2)}}{a_{33}^{(2)}} \times a_{43}^{(2)} \right] x_4 = b_4^{(2)} - \frac{b_3^{(2)}}{a_{33}^{(2)}} a_{43}^{(2)}$$

$$\Rightarrow a_{44}^{(3)} x_4 = b_4^{(3)} \quad \dots\dots (13)$$

Solution process:

- i) Solve x_4 from eqn (13)
- ii) Substitute x_4 into eqn (12) and solve x_3
- iii) " x_4 and x_3 into eqn (9) and solve x_2
- iv) " x_2, x_3, x_4 into eqn (5) and solve x_1



Gaussian Elimination through Matrix operation

$$\begin{array}{l}
 R_1 \leftarrow \\
 R_2 \leftarrow \\
 R_3 \leftarrow \\
 R_4 \leftarrow
 \end{array}
 \begin{bmatrix}
 a_{11} & a_{12} & a_{13} & a_{14} \\
 a_{21}^{(1)} & a_{22}^{(1)} & a_{23} & a_{24} \\
 a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(2)} & a_{34} \\
 a_{41}^{(1)} & a_{42}^{(1)} & a_{43}^{(2)} & a_{44}^{(3)}
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 x_2 \\
 x_3 \\
 x_4
 \end{bmatrix}
 =
 \begin{bmatrix}
 b_1 \\
 b_2 \\
 b_3 \\
 b_4
 \end{bmatrix}$$

$$iv) R_3 = R_3 / a_{33}^{(2)}$$

$$v) R_4 = R_4 - R_3 \times a_{43}^{(2)}$$

$$\Rightarrow \text{Solve for } x_4 \rightarrow x_3 \rightarrow x_2 \rightarrow x_1$$

Steps:

$$i) R_1 = \frac{R_1}{a_{11}}$$

$$ii) R_2 = (R_2 - R_1 \times a_{21}); R_3 = (R_3 - R_1 \times a_{31}); R_4 = (R_4 - R_1 \times a_{41})$$

$$iii) R_2 = R_2 / a_{22}^{(1)} ; iv) R_3 = [R_3 - R_2 \times a_{32}^{(1)}]; R_4 = [R_4 - R_2 \times a_{42}^{(1)}]$$

