IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Computer aided power system analysis

Sparsity and Gaussian Elimination

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SPARSITY OF A MATRIX

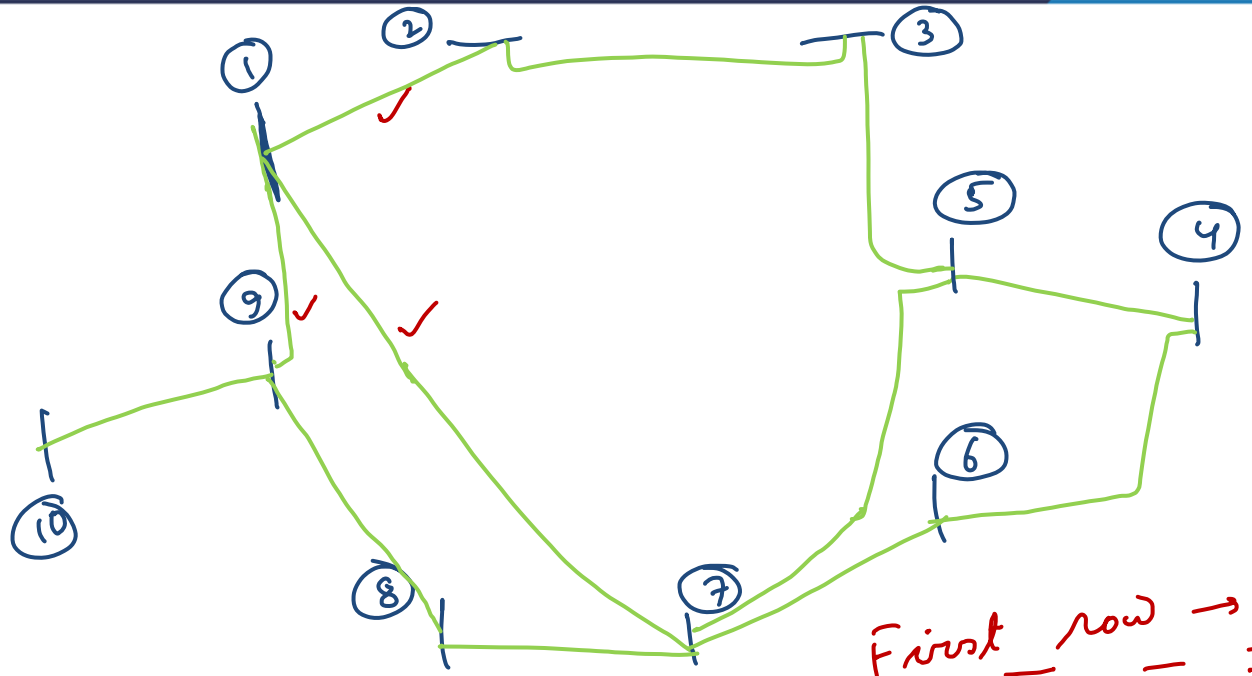
Sparse matrix → A matrix whose most of the elements are zero.

Y_{BUS} matrix

$$Y_{BUS} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \cdots & \bar{Y}_{1N} \\ \bar{Y}_{21} & \bar{Y}_{22} & \cdots & \bar{Y}_{2N} \\ \vdots & \vdots & & \vdots \\ \bar{Y}_{N1} & \bar{Y}_{N2} & \cdots & \bar{Y}_{NN} \end{bmatrix}$$

$\bar{Y}_{ii}$ = Sum total of all admittances connected to bus 'i'.

$\bar{Y}_{ij}$ = Negative of the admittance connected between bus 'i' and bus 'j'.



$Y_{BUS} = (10 \times 10)$

First row → Non-zero elements

$\bar{Y}_{11}, \bar{Y}_{12}, \bar{Y}_{17}, \bar{Y}_{19}$

$\bar{Y}_{BUS}$ matrix is a sparse matrix

NRLF (POLAR)

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\bar{\theta} \\ \Delta\bar{V} \end{bmatrix} = \begin{bmatrix} \Delta\bar{P} \\ \Delta\bar{Q} \end{bmatrix}$$

$\Delta\bar{X}$ $\Delta\bar{M}$

$$J_1 = \frac{\partial P_i}{\partial \theta_i} \; ; \; \left.\frac{\partial P_i}{\partial \theta_j}\right|_{j \neq i}$$

$$P_i = V_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq}}^{N} V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$$

$$\left.\frac{\partial P_i}{\partial \theta_j}\right|_{j \neq i} = V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}) = 0$$

J → Jacobian matrix ⟹ SPARSE

$(2N-M-1) \times (2N-M-1)$

$$J \cdot \Delta\bar{X} = \Delta\bar{M}$$

$$\Delta\bar{X} = [J^{-1}]\,\Delta\bar{M}$$

$1000 = N$

$M = 50$

(1949×1949)

1000 → 5 → in each row

5000 → total non-zero elements.

$\frac{5 \cancel{5000} \times 100}{\cancel{1000} \times \cancel{1000}} = \frac{1}{2}\%$

200

$\bar{J} \cdot \Delta\bar{X} = \Delta\bar{M}$

$$J = \begin{bmatrix} x & x & 0 \cdots 0 & x & \cdots 0 & 0 \\ \cancel{x} & 0 & x \cdots 0 \cdots & & . & \end{bmatrix}$$

GAUSSIAN ELIMINATION METHOD

Solves a set of linear equations

$$A\bar{x} = \bar{b}$$

$$\bar{x} = A^{-1}\bar{b}$$

$$\bar{x} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$$

$$\bar{b} = [b_1 \quad b_2 \quad b_3 \quad b_4]^T$$

$A \rightarrow (4 \times 4)$ matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \rightarrow \text{Real, constant matrix}$$

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 &= b_3 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 &= b_4 \end{aligned}$$

$A \rightarrow (n \times n)$: $\bar{x} \rightarrow (n \times 1) \rightarrow [x_1 \quad x_2 \quad \cdots\cdots \quad x_n]^T$

$\{a_{ij}\}$ $\bar{b} \rightarrow (n \times 1) \rightarrow [b_1 \quad b_2 \quad \cdots\cdots \quad b_n]^T$

element at i^{th} row and j^{th} column

$$a_{11}x_1 + a_{12}x_2 + \cdots\cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots\cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots\cdots + a_{nn}x_n = b_n$$

} n-simultaneous linear equations