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NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

FDLF (Contd.)

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$$J_3 = \frac{\partial \bar{\theta}}{\partial \theta} \quad ; \quad \theta_i = \sum_{j=1}^N v_i v_j \gamma_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$= -B_{ii} v_i^2 + \sum_{\substack{j=1 \\ j \neq i}}^N v_i v_j \gamma_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$\left. \frac{\partial \theta_i}{\partial \theta_j} \right|_{j \neq i} = \frac{-v_i v_j \gamma_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})}{-v_i v_j \left[\underbrace{\gamma_{ij} \cos(\theta_i - \theta_j)}_{\substack{\neq 0 \\ \text{red}}} + B_{ij} \underbrace{\sin(\theta_i - \theta_j)}_{\substack{\neq 0 \\ \text{red}}} \right]} \approx 0$$

$$\boxed{\left. \frac{\partial \theta_i}{\partial \theta_j} \right|_{j \neq i} \approx 0}$$

$$\left. \frac{\partial \theta_i}{\partial \theta_j} \right|_{j=i} = \sum_{\substack{j=1 \\ j \neq i}}^N \frac{v_i v_j \gamma_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})}{\underbrace{\quad}_{\substack{\neq 0 \\ \text{blue}}}}$$

$$\left. \frac{\partial \theta_i}{\partial \theta_j} \right|_{j=i} \approx 0$$

$$J_3 \approx 0$$

$$J_2 \approx 0$$

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \bar{\theta} \\ \Delta \bar{v} \end{bmatrix} = \begin{bmatrix} \Delta \bar{p} \\ \Delta \bar{q} \end{bmatrix} \Rightarrow$$

$\left. \begin{array}{l} \text{"}\theta\text{-}p\text{" coupling} \\ \text{"}\nu\text{-}\theta\text{" coupling} \end{array} \right\}$

$$\left. \begin{array}{l} J_1 \Delta \bar{\theta} = \Delta \bar{p} \\ J_4 \Delta \bar{v} = \Delta \bar{q} \end{array} \right\} \text{Decoupled equations.}$$

$$J_1 = \frac{\partial \bar{P}}{\partial \theta} ; \left. \frac{\partial P_i}{\partial \theta_j} \right|_{j \neq i} \quad \text{and} \quad \left. \frac{\partial P_i}{\partial \theta_j} \right|_{j=i} \quad i, j = 2, \dots, N$$

$$P_i = V_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^N V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$$

$$\begin{aligned} \left. \frac{\partial P_i}{\partial \theta_j} \right|_{j \neq i} &= V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}) \\ &= V_i V_j Y_{ij} \left[\sin(\theta_i - \theta_j) \cos \alpha_{ij} - \cos(\theta_i - \theta_j) \sin \alpha_{ij} \right] \\ &= V_i V_j G_{ij} \underbrace{\sin(\theta_i - \theta_j)}_{\substack{\{ \} \\ 0}} - V_i V_j B_{ij} \cos(\theta_i - \theta_j) \\ &\approx -V_i V_j B_{ij} \end{aligned}$$



$$\frac{\partial P_i}{\partial \theta_j} = -v_i v_j B_{ij} \quad ; j \neq i$$

$$\Rightarrow \frac{1}{v_i} \frac{\partial P_i}{\partial \theta_j} = -v_j B_{ij} \quad ; j \neq i$$

$$\Rightarrow \boxed{\frac{1}{v_i} \frac{\partial P_i}{\partial \theta_j} = -B_{ij} \quad ; j \neq i}$$

$$\frac{\partial P_i}{\partial \theta_i} = - \sum_{\substack{j=1 \\ j \neq i}}^N v_i v_j \gamma_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$= v_i v_i \gamma_{ii} \sin(\theta_i - \theta_i - \alpha_{ii}) - \sum_{\substack{j=1 \\ j \neq i}}^N v_i v_j \gamma_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$



$$\Rightarrow \frac{\partial P_i}{\partial \theta_i} = -B_{ii} V_i^2 - \sum_{j=1}^N V_i V_j Y_{ij} \sin(\theta_i - \theta_j - d_{ij})$$

$$= -B_{ii} V_i^2 - Q_i \quad \approx -B_{ii} V_i^2$$

$$\Rightarrow \frac{1}{V_i} \frac{\partial P_i}{\partial \theta_i} \approx -V_i B_{ii}$$

$\Rightarrow$

$$\frac{1}{V_i} \frac{\partial P_i}{\partial \theta_i} \approx -B_{ii}$$

$$\frac{1}{V_i} \frac{\partial P_i}{\partial \theta_j} \approx -B_{ij} \quad ; j \neq i$$

$$J_4 \Rightarrow \frac{\partial \bar{Q}}{\partial \bar{V}} ; \quad \frac{\partial \theta_i}{\partial V_i} ; \quad \frac{\partial \theta_i}{\partial V_j} \Big|_{j \neq i} \quad i, j = m+1, \dots, N.$$

$$\theta_i = -B_{ii} V_i^2 + \sum_{\substack{j=1 \\ j \neq i}}^N V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$\begin{aligned} \frac{\partial \theta_i}{\partial V_j} \Big|_{j \neq i} &= V_i Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}) \\ &= V_i \left[G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j) \right] \\ &\approx -V_i B_{ij} \end{aligned}$$

$$\Rightarrow \boxed{\frac{\partial \theta_i}{\partial V_j} \approx -B_{ij} ; j \neq i}$$

$$\frac{\partial \theta_i}{\partial V_i} = -2V_i B_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^N V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$$\begin{aligned} \Rightarrow V_i \frac{\partial \theta_i}{\partial V_i} &= -2V_i^2 B_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^N V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}) \\ &= -V_i^2 B_{ii} - V_i^2 B_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^N V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}) \\ &= -V_i^2 B_{ii} + \sum_{j=1}^N V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij}) \\ &= -V_i^2 B_{ii} + Q_i \end{aligned}$$

$$\Rightarrow v_i \frac{\partial Q_i}{\partial v_i} \approx -v_i^2 B_{ii}$$

$$\Rightarrow \frac{\partial Q_i}{\partial v_i} \approx -v_i B_{ii}$$

 $\Rightarrow$

$$\frac{\partial Q_i}{\partial v_i} \approx -B_{ii}$$

$$\frac{\partial Q_i}{\partial v_j} \approx -B_{ij}$$

 J_4

$$J_1 \left\{ \begin{array}{l} \frac{1}{v_i} \frac{\partial P_i}{\partial \theta_j} = -B_{ij} \quad j \neq i \\ \frac{1}{v_i} \frac{\partial P_i}{\partial \theta_i} \approx -B_{ii} \end{array} \right.$$

$$J_2 = 0$$

$$J_3 = 0$$