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CERTIFICATION COURSE

Computer aided power system analysis

Example of NRLF (Rectangular) Method

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Complete NRLF (Rectangular) technique

Steps: 1. Take flat start ($\bar{x}^{(0)}$) and initialise $k = 1$. ↪ iteration count

2. For $i = 2, \dots, M$

i) Calculate $Q_i^{(k)}$

ii) If $Q_i^{\min} \leq Q_i^{(k)} \leq Q_i^{\max}$; then i^{th} bus remains as a PV bus

iii) If $Q_i^{(k)} > Q_i^{\max}$; then $Q_i^{\text{sp}} = Q_i^{\max}$;

iv) If $Q_i^{(k)} < Q_i^{\min}$; then $Q_i^{\text{sp}} = Q_i^{\min}$;

3. change the dimensions of all unknown vectors, specified vectors and Jacobian sub-matrices properly.



Example:

Say $N=100$, $M=10$

Suppose At some iteration it is found that $Q_3^{(k)}$ and $Q_9^{(k)}$ violate the corresponding limits.

$$\bar{P} = [P_2 \ P_3 \ \dots \ P_{100}]^T \rightarrow (99 \times 1) \rightarrow (N-1) \times 1$$

$$\bar{Q} = \left[\underbrace{Q_{11} \ Q_{12} \ \dots \ Q_{100}}_{\text{Original } \bar{Q} \text{ vector}} \quad \underbrace{Q_3 \ Q_9}_{\text{Addition due to violation in } Q \text{ limits.}} \right]^T \rightarrow (92 \times 1)$$

Original $\bar{Q}$
vector
 $\downarrow$
(90x1)

Addition due to violation
in Q limits.
 $\downarrow$ (2x1)

In general, if there are ' l ' ($l \leq m-1$) violations,
 $\bar{Q} \rightarrow (N-m+l) \times 1$

$$(9 \times 1) \leftarrow \bar{V}^2 = [V_2^2 \quad V_3^2 \quad \dots \quad V_9^2 \quad V_{10}^2]^T \rightarrow \text{original } \bar{V}^2 \text{ vector.}$$

↓
original
dimension

Therefore the modified $\bar{V}^2$ vector is

$$\bar{V}^2 = [V_2^2 \quad V_4^2 \quad \dots \quad V_8^2 \quad V_{10}^2]^T \rightarrow (7 \times 1) \rightarrow \text{modified dimension.}$$

In general, for 'l' ($1 \leq m-1$) violations, the dimension of $\bar{V}^2$ vector would be $(m-l-1) \times 1$.

Unknown quantities

$$\bar{e} = [e_2 \quad e_3 \quad \dots \quad e_{100}]^T \rightarrow (99 \times 1) \rightarrow (N-1) \times 1$$

$$\bar{f} = [f_2 \quad f_3 \quad \dots \quad f_{100}]^T \rightarrow (99 \times 1) \rightarrow (N-1) \times 1$$



$$J_3 = \begin{bmatrix} \frac{\partial Q_{11}}{\partial e_2} & \frac{\partial Q_{11}}{\partial e_3} & \dots & \frac{\partial Q_{11}}{\partial e_{100}} \\ \vdots & \vdots & & \vdots \\ \frac{\partial Q_{100}}{\partial e_2} & \frac{\partial Q_{100}}{\partial e_3} & \dots & \frac{\partial Q_{100}}{\partial e_{100}} \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \dots & \frac{\partial Q_3}{\partial e_{100}} \\ \frac{\partial Q_9}{\partial e_2} & \frac{\partial Q_9}{\partial e_3} & \dots & \frac{\partial Q_9}{\partial e_{100}} \end{bmatrix}$$

$(90 \times 1) \rightarrow (N-m) \times 1$
 $(2 \times 1) \rightarrow (L \times 1)$

$$J_5 = \begin{bmatrix} \frac{\partial V_2^2}{\partial e_2} & \frac{\partial V_2^2}{\partial e_3} & \cdots & \frac{\partial V_2^2}{\partial e_{100}} \\ \times \frac{\partial V_3^2}{\partial e_2} & \frac{\partial V_3^2}{\partial e_3} & \cdots & \frac{\partial V_3^2}{\partial e_{100}} \\ \vdots & \vdots & \ddots & \vdots \\ \times \frac{\partial V_9^2}{\partial e_2} & \frac{\partial V_9^2}{\partial e_3} & \cdots & \frac{\partial V_9^2}{\partial e_{100}} \\ \frac{\partial V_{10}^2}{\partial e_2} & \frac{\partial V_{10}^2}{\partial e_3} & \cdots & \frac{\partial V_{10}^2}{\partial e_{100}} \end{bmatrix}$$

original J_5 matrix

earlier $(m-1) \times 1$
modified $(m-l-1) \times 1$

Algorithm (contd.)

4. Calculate $\Delta \bar{m}^{(k)}$ vector

5. We calculate $e_{rr}^{(k)}$

6. If $e_{rr}^{(k)} < \epsilon$, then stop. Else go to step 7.

7. Evaluate $J^{(k)}$ and subsequently $\Delta \bar{x}^{(k)}$

8. update $\bar{x}^{(k)} = \bar{x}^{(0)} + \Delta \bar{x}^{(k)}$.

9. Increment $k = k+1$ and go back to step 2.



NRLF (rectangular) results in 5 bus system without any generator Q limit

$$\mathbf{J}_1 \rightarrow (4 \times 4); \quad \mathbf{J}_2 \rightarrow (4 \times 4); \quad \mathbf{J}_3 \rightarrow (2 \times 4);$$

$$\mathbf{J}_4 \rightarrow (2 \times 4);$$

$$\mathbf{J}_5 \rightarrow (2 \times 4); \quad \mathbf{J}_6 \rightarrow (2 \times 4) \text{ and } \mathbf{J} \rightarrow (8 \times 8)$$

Bus no.	e (p.u)	f (p.u)	V (p.u)	θ (deg)	P_{inj} (p.u)	Q_{inj} (p.u)
1	1	0	1	0	0.56743	0.26505
2	0.99958	0.02893	1	1.65757	0.5	-0.18519
3	0.99987	-0.01592	1	-0.91206	1	0.68875
4	0.89634	-0.13157	0.90594	-8.35088	-1.15	-0.6
5	0.94034	-0.08272	0.94397	-5.02735	-0.85	-0.4
Total iteration = 5						

NRLF (rectangular) results in 5 bus system with generator Q limit

In the third iteration, there is a violation of the reactive power limit of generator 3.

Bus no.	e (p.u)	f (p.u)	V (p.u)	θ (deg)	P_{inj} (p.u)	Q_{inj} (p.u)
1	1	0	1	0	0.56979	0.33935
2	0.99956	0.02961	1	1.69679	0.5	-0.04769
3	0.98244	-0.01097	0.9825	-0.63991	1	0.5
4	0.87973	-0.12927	0.88918	-8.35906	-1.15	-0.6
5	0.93092	-0.08123	0.93445	-4.98675	-0.85	-0.4
Total iteration = 5						