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NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

NRLF in rectangular coordinate (Contd.)

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$$\text{Also, } \bar{S}_i = P_i + jQ_i = \bar{V}_i \bar{I}_i^* = (e_i + jf_i)(a_i - jb_i)$$

$$\Rightarrow P_i + jQ_i = (a_i e_i + f_i b_i) + j(f_i a_i - e_i b_i)$$

$$\Rightarrow P_i = e_i a_i + f_i b_i$$

$$\Rightarrow P_i = \sum_{k=1}^N [e_i (e_k g_{ik} - f_k b_{ik}) + f_i (e_k b_{ik} + f_k g_{ik})]$$

Similarly,

$$Q_i = f_i a_i - e_i b_i$$

$$\Rightarrow Q_i = \sum_{k=1}^N [f_i (e_k g_{ik} - f_k b_{ik}) - e_i (e_k b_{ik} + f_k g_{ik})]$$

$$J \cdot \Delta \bar{X} = \Delta \bar{M}$$

$$J = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \\ J_5 & J_6 \end{bmatrix}; \quad \Delta \bar{X} = \begin{bmatrix} \Delta \bar{e} \\ \Delta \bar{f} \end{bmatrix}; \quad \Delta \bar{M} = \begin{bmatrix} \Delta \bar{P} \\ \Delta \bar{Q} \\ \Delta \bar{V}^2 \end{bmatrix}$$

$$J_3 = \frac{\partial \bar{Q}}{\partial \bar{e}} = \begin{bmatrix} \frac{\partial Q_{m+1}}{\partial e_2} & \frac{\partial Q_{m+1}}{\partial e_3} & \dots & \frac{\partial Q_{m+1}}{\partial e_N} \\ \vdots & & & \vdots \\ \frac{\partial Q_N}{\partial e_2} & \frac{\partial Q_N}{\partial e_3} & \dots & \frac{\partial Q_N}{\partial e_N} \end{bmatrix} \xrightarrow{(N-m) \times (N-1)}$$

Self-term: $\frac{\partial Q_i}{\partial e_k}$; $i = m+1, \dots, N$; $k = 2, \dots, N$; $k = i$

Non-self-term: $\frac{\partial Q_i}{\partial e_k}$; $i = m+1, \dots, N$; $k = 2, \dots, N$; $k \neq i$

We have,

$$Q_i = \sum_{k=1}^N [f_i(e_k g_{ik} - f_k b_{ik}) - e_i(e_k b_{ik} + f_k g_{ik})]$$

$$= -(e_i^2 + f_i^2) b_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N [f_i(e_k g_{ik} - f_k b_{ik}) - e_i(e_k b_{ik} + f_k g_{ik})]$$

Non-self-term

$$\frac{\partial Q_i}{\partial e_k} ; k \neq i = (f_i g_{ik} - e_i b_{ik})$$

→ non-self term for J_3

Self-term: $\frac{\partial Q_i}{\partial e_k}$; $k=i$; $\Rightarrow \frac{\partial Q_i}{\partial e_k} = -2e_i b_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N -(e_k b_{ik} + f_k g_{ik})$

$\Rightarrow \left. \frac{\partial Q_i}{\partial e_k} \right|_{k=i} = -2e_i b_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^N (e_k b_{ik} + f_k g_{ik})$ self-term for J_3

$J_4 = \frac{\partial \bar{Q}}{\partial \bar{f}} = \begin{bmatrix} \frac{\partial Q_{m+1}}{\partial f_2} & \frac{\partial Q_{m+1}}{\partial f_3} & \cdots & \frac{\partial Q_{m+1}}{\partial f_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_N}{\partial f_2} & \frac{\partial Q_N}{\partial f_3} & \cdots & \frac{\partial Q_N}{\partial f_N} \end{bmatrix} \rightarrow (N-m) \times (N-1)$

Self-term: $\frac{\partial Q_i}{\partial f_k}$; $i = m+1, \dots, N$; $k = 2, \dots, N$; $k = i$

Non-self term: $\frac{\partial Q_i}{\partial f_k}$; $i = m+1, \dots, N$; $k = 2, \dots, N$; $k \neq i$

Non-self-term:

$$\left. \frac{\partial Q_i}{\partial f_k} \right|_{k \neq i} = -(f_i b_{ik} + e_i g_{ik})$$

→ Non-self-term for J_y

Self-term: $\left. \frac{\partial Q_i}{\partial f_k} \right|_{k=i}$

$$= -2f_i b_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N (e_k g_{ik} - f_k b_{ik})$$

→ Self-term for J_y

Basic algorithm for NRLF (rectangular)

Steps:

1. Take flat start, initialize $k=1$.
2. For $i = 2, \dots, N$, calculate

$$p_i^{(k)} = \sum_{j=1}^N e_j^{(k-1)} \{ e_j^{(k-1)} g_{ij} - f_j^{(k-1)} b_{ij} \} + f_i^{(k-1)} \{ e_j^{(k-1)} b_{ij} + f_j^{(k-1)} g_{ij} \}$$

$$\bar{X} = \begin{bmatrix} \bar{e} \\ \bar{f} \end{bmatrix}$$

$$\Delta P^{(k)} = \begin{bmatrix} p_2^{op} - p_2^{(k)} & p_3^{op} - p_3^{(k)} & \dots & p_N^{op} - p_N^{(k)} \end{bmatrix}^T$$

3. For $i = M+1, \dots, N$, calculate

$$q_i^{(k)} = \sum_{j=1}^N f_i^{(k-1)} \{ e_j^{(k-1)} g_{ij} - f_j^{(k-1)} b_{ij} \} - e_i^{(k-1)} \{ e_j^{(k-1)} b_{ij} + f_j^{(k-1)} g_{ij} \}$$



$$\Delta \bar{Q}^{(k)} = \begin{bmatrix} Q_{m+1}^{sp} - Q_{m+1}^{(k)} & Q_{m+2}^{sp} - Q_{m+2}^{(k)} & \dots & Q_N^{sp} - Q_N^{(k)} \end{bmatrix}^T$$

4. For $i = 2, \dots, M$, calculate

$$\{V_i^{(k)}\}^2 = \{e_i^{(k-1)}\}^2 + \{f_i^{(k-1)}\}^2$$

$$\{\Delta \bar{V}^{(k)}\}^2 = \left[(V_2^{sp})^2 - \{V_2^{(k)}\}^2 \quad (V_3^{sp})^2 - \{V_3^{(k)}\}^2 \quad \dots \quad (V_m^{sp})^2 - \{V_m^{(k)}\}^2 \right]^T$$

$$5 \quad \Delta \bar{M}^{(k)} = \begin{bmatrix} \Delta \bar{P}^{(k)} \\ \Delta \bar{Q}^{(k)} \\ \{\Delta \bar{V}^{(k)}\}^2 \end{bmatrix} \rightarrow (2N-2) \times 1$$

6. Let $\Delta \bar{m}^{(k)} = [\Delta m_1^{(k)} \quad \Delta m_2^{(k)} \quad \dots \quad \Delta m_{2N-2}^{(k)}]^T$

7. Calculate $e_i^{(k)} = |\Delta m_i^{(k)}| \quad \forall i = 1, \dots, (2N-2)$

8. Calculate $e_{rr}^{(k)} = \max[e_1^{(k)}, e_2^{(k)}, \dots, e_{2N-2}^{(k)}]$

9. If $e_{rr}^{(k)} < \epsilon$, then stop; Else, go to step 10.

10. Evaluate $[J] \bigg|_{\bar{e}^{(k-1)}, \bar{f}^{(k-1)}}$

11. Calculate $\Delta \bar{x}^{(k)} = [J]^{-1} \Delta \bar{m}^{(k)}$

12. update $\bar{x}^{(k)} = \bar{x}^{(k-1)} + \Delta \bar{x}^{(k)}$, and go to step 2.
 $k = k+1$,