



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

NRLF in rectangular coordinate (Contd.)

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$$\text{Also, } \bar{S}_i = P_i + jQ_i = \bar{V}_i \bar{I}_i^* = (e_i + jf_i)(a_i - jb_i)$$

$$\Rightarrow P_i + jQ_i = (a_i e_i + f_i b_i) + j(f_i a_i - e_i b_i)$$

$$\Rightarrow P_i = e_i a_i + f_i b_i$$

$$\Rightarrow P_i = \sum_{k=1}^N [e_i (e_k g_{ik} - f_k b_{ik}) + f_i (e_k b_{ik} + f_k g_{ik})]$$

Similarly,

$$Q_i = f_i a_i - e_i b_i$$

$$\Rightarrow Q_i = \sum_{k=1}^N [f_i (e_k g_{ik} - f_k b_{ik}) - e_i (e_k b_{ik} + f_k g_{ik})]$$

Unknown vectors : $\bar{e}, \bar{f}$

known vectors : $\bar{p}, \bar{q}, \bar{v}^2$

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \\ J_5 & J_6 \end{bmatrix} \begin{bmatrix} \Delta \bar{e} \\ \Delta \bar{f} \end{bmatrix} = \begin{bmatrix} \Delta \bar{p} \\ \Delta \bar{q} \\ \Delta \bar{v}^2 \end{bmatrix}$$

$\Delta \bar{x}$ (pointing to $\Delta \bar{f}$) $\Delta \bar{m}$ (pointing to $\Delta \bar{v}^2$)

$$\Delta \bar{e} \rightarrow (N-1) \times 1$$

$$\Delta \bar{f} \rightarrow (N-1) \times 1$$

$$\Delta \bar{p} \rightarrow (N-1) \times 1$$

$$\Delta \bar{q} \rightarrow (N-m) \times 1$$

$$\Delta \bar{v}^2 \rightarrow (m-1) \times 1$$

$$J_1 \rightarrow (N-1) \times (N-1)$$

$$J_2 \rightarrow (N-1) \times (N-1)$$

$$J_3 \rightarrow (N-m) \times (N-1)$$

$$J_4 \rightarrow (N-m) \times (N-1)$$

$$J_5 \rightarrow (m-1) \times (N-1); \quad J_6 \rightarrow (m-1) \times (N-1);$$



$$\Delta \bar{X} = 2(N-1) \times 1 : \Delta \bar{m} \rightarrow 2(N-1) \times 1$$

$$J \rightarrow 2(N-1) \times 2(N-1)$$

$$\Rightarrow J \cdot \Delta \bar{X} = \Delta \bar{m}$$

$$J_1 = \frac{\partial \bar{P}}{\partial \bar{e}} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \cdots & \frac{\partial P_2}{\partial e_N} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \cdots & \frac{\partial P_3}{\partial e_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_N}{\partial e_2} & \frac{\partial P_N}{\partial e_3} & \cdots & \frac{\partial P_N}{\partial e_N} \end{bmatrix}$$

$\rightarrow (N-1) \times (N-1)$
diagonal: $\frac{\partial P_i}{\partial e_i}$

$\forall i = 2, \dots, N$

off-diagonal : $\frac{\partial P_i}{\partial e_k}$; $\forall i = 2, \dots, N$; $\forall k = 2, \dots, N$; $k \neq i$

Now,

$$P_i = \sum_{k=1}^N [e_i(e_k g_{ik} - f_k b_{ik}) + f_i(e_k b_{ik} + f_k g_{ik})]$$
$$= (e_i^2 + f_i^2) g_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N [e_i(e_k g_{ik} - f_k b_{ik}) + f_i(e_k b_{ik} + f_k g_{ik})]$$

$$\frac{\partial P_i}{\partial e_k} ; k \neq i = (e_i g_{ik} + f_i b_{ik}) ; \forall i, k = 2, \dots, N ; k \neq i$$

$$\frac{\partial P_i}{\partial e_i} = 2e_i g_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N (e_k g_{ik} - f_k b_{ik})$$

$\forall i = 2, \dots, N$

$$\bar{J}_2 = \frac{\partial \bar{P}}{\partial \bar{f}} = \begin{bmatrix} \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \cdots & \frac{\partial P_2}{\partial f_N} \\ \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} & \cdots & \frac{\partial P_3}{\partial f_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_N}{\partial f_2} & \frac{\partial P_N}{\partial f_3} & \cdots & \frac{\partial P_N}{\partial f_N} \end{bmatrix}$$

→ (N-1) × (N-1)
Diagonal elements:

$$\frac{\partial P_i}{\partial f_i} \quad \forall i=2, \dots, N$$

Off-diagonal elements:

$$\frac{\partial P_i}{\partial f_k} \quad ; \quad \forall i, k=2, \dots, N; \quad k \neq i$$

$$\frac{\partial P_i}{\partial f_k} = (f_i g_{ik} - e_i b_{ik}) \quad \forall i, k=2, \dots, N; \quad k \neq i$$

$$\frac{\partial p_i}{\partial f_i} = 2f_i g_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N (e_k b_{ik} + f_k g_{ik}) ; \quad \forall i = 2, \dots, N$$

$$J_s \rightarrow \frac{\partial \bar{V}^2}{\partial \bar{e}} = \begin{bmatrix} \frac{\partial V_2^2}{\partial e_2} & \frac{\partial V_2^2}{\partial e_3} & \dots & \frac{\partial V_2^2}{\partial e_N} \\ \frac{\partial V_3^2}{\partial e_2} & \frac{\partial V_3^2}{\partial e_3} & \dots & \frac{\partial V_3^2}{\partial e_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial V_m^2}{\partial e_2} & \frac{\partial V_m^2}{\partial e_3} & \dots & \frac{\partial V_m^2}{\partial e_N} \end{bmatrix}$$

$\rightarrow (m-1) \times (N-1)$
Self-term:

$$\frac{\partial V_i^2}{\partial e_k} = 2e_i$$

$\forall i = 2, \dots, m$
 $\forall k = 2, \dots, N$
 $k = i$

Non-self-term:

$$\frac{\partial V_i^2}{\partial e_k} ; k \neq i = 0$$

$$V_i^2 = e_i^2 + f_i^2$$

$$J_6 = \frac{\partial \bar{V}^2}{\partial \bar{f}} = \begin{bmatrix} \frac{\partial V_2^2}{\partial f_2} & \dots & \frac{\partial V_2^2}{\partial f_N} \\ \vdots & & \vdots \\ \frac{\partial V_M^2}{\partial f_2} & \dots & \frac{\partial V_M^2}{\partial f_N} \end{bmatrix} \rightarrow (M-1) \times (N-1)$$

Self-term

$$\frac{\partial V_i^2}{\partial f_k} ; \begin{matrix} i = 2, \dots, M \\ k = 2, \dots, N \\ k = i \end{matrix}$$

$$\frac{\partial V_i^2}{\partial f_k} = 2f_i$$

Non-self-term

$$\frac{\partial V_i^2}{\partial f_k} ; \begin{matrix} k \neq i = 0 ; \\ i = 2, \dots, M \\ k = 2, \dots, N \end{matrix}$$