



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

NRLF in rectangular coordinate

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NRLF in Rectangular co-ordinate

$$\bar{V}_i = V_i \angle \theta_i = e_i + jf_i \quad ; \quad e_i = V_i \cos \theta_i \quad ; \quad f_i = V_i \sin \theta_i$$

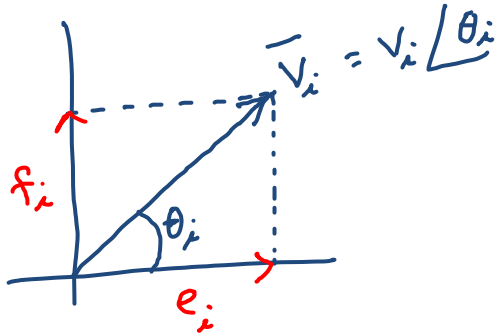
We know $\bar{I}_i = \sum_{k=1}^N \bar{Y}_{ik} \bar{V}_k$

$$\bar{V}_k = e_k + jf_k \quad ; \quad \bar{Y}_{ik} = g_{ik} + jb_{ik}$$

$$\Rightarrow \bar{I}_i = a_i + jb_i = \sum_{k=1}^N (g_{ik} + jb_{ik})(e_k + jf_k)$$

$$\Rightarrow a_i + jb_i = \sum_{k=1}^N (e_k g_{ik} - f_k b_{ik}) + j \sum_{k=1}^N (e_k b_{ik} + f_k g_{ik})$$

$$\Rightarrow a_i = \sum_{k=1}^N (e_k g_{ik} - f_k b_{ik}) \quad \text{and} \quad b_i = \sum_{k=1}^N (e_k b_{ik} + f_k g_{ik})$$



$$\text{Also, } \bar{S}_i = P_i + jQ_i = \bar{V}_i \bar{I}_i^* = (e_i + jf_i)(a_i - jb_i)$$

$$\Rightarrow P_i + jQ_i = (a_i e_i + f_i b_i) + j(f_i a_i - e_i b_i)$$

$$\Rightarrow P_i = e_i a_i + f_i b_i$$

$$\Rightarrow P_i = \sum_{k=1}^N [e_i (e_k g_{ik} - f_k b_{ik}) + f_i (e_k b_{ik} + f_k g_{ik})]$$

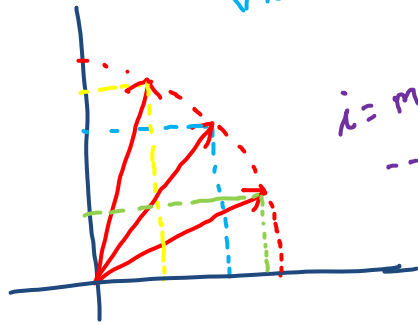
Similarly,

$$Q_i = f_i a_i - e_i b_i$$

$$\Rightarrow Q_i = \sum_{k=1}^N [f_i (e_k g_{ik} - f_k b_{ik}) - e_i (e_k b_{ik} + f_k g_{ik})]$$

Unknown quantities

Slack bus	→	No unknown	→	0
PV bus	→	e_i, f_i are unknown	→	$2(m-1)$
PQ - bus	→	e_i, f_i are unknown	→	$2(N-m)$
<hr/>				
Grand total				$2N-2$
				$= 2(N-1)$

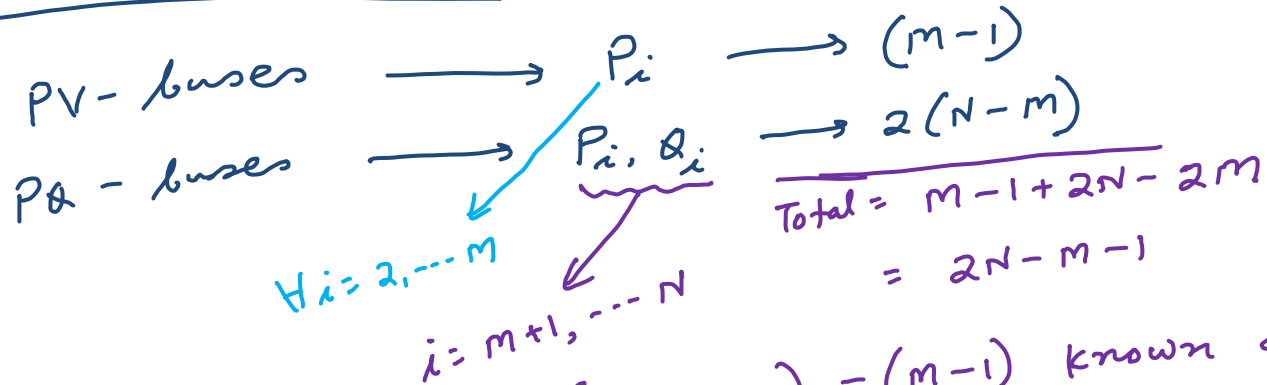


$\forall i=2, \dots, m$

$i = m+1, m+2, \dots, N$

N - Buses
 m → generators
 1 → slack
 $2, \dots, m$ → PV
 $m+1, \dots, N$ → PQ

known quantities



There is a deficit of $(2N-2) - (2N-m-1) = (m-1)$ known quantities
 At each PV-bus, we know $\underline{V_i^2 = E_i^2 + f_i^2} \quad \forall i = 2, \dots, M$
 extra $(m-1)$ equations which are required.

To summarise:
Unknown quantity

$$\left. \begin{matrix} e_i \\ f_i \end{matrix} \right\} \forall i = 2, \dots, N$$

$\rightarrow (N-1)$

$\rightarrow (N-1)$

Total = $2N-2$

Equations

$$\begin{aligned} P_i &: \forall i = 2, \dots, N \\ Q_i &: \forall i = m+1, m+2, \dots, N \\ V_i^2 &: \forall i = 2, \dots, m \end{aligned}$$

$\rightarrow (N-1)$

$\rightarrow (N-m)$

$\rightarrow (m-1)$

Total = $N-1 + N-m + m-1$
 $= 2N-2$



Recollect NRLF (Polar)

$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \bar{\theta} \\ \Delta \bar{v} \end{bmatrix} = \begin{bmatrix} \Delta \bar{p} \\ \Delta \bar{q} \end{bmatrix}$$

NRLF (Rectangular)

We define:

$$\begin{aligned} \bar{e} &= [e_2 \ e_3 \ \dots \ e_N]^T \rightarrow (N-1) \times 1 \\ \bar{f} &= [f_2 \ f_3 \ \dots \ f_N]^T \rightarrow (N-1) \times 1 \\ \bar{p} &= [p_2 \ p_3 \ \dots \ p_N]^T \rightarrow (N-1) \times 1 \\ \bar{q} &= [q_{m+1} \ q_{m+2} \ \dots \ q_N]^T \rightarrow (N-m) \times 1 \\ \bar{v}^2 &= [v_2^2 \ v_3^2 \ \dots \ v_m^2]^T \rightarrow (M-1) \times 1 \end{aligned}$$

} unknown vectors

} known vectors