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CERTIFICATION COURSE

Computer aided power system analysis

NRLF (Polar) Algorithm and Example

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Complete algorithm for NRLF (polar) method

1. Assume flat start. **set** $k = 1$ iteration count
2. For $i = 2, \dots, M$, calculate

$$Q_i^{(k)} = \sum_{j=1}^N V_i^{(k-1)} V_j^{(k-1)} Y_{ij} \sin(\theta_i^{(k-1)} - \theta_j^{(k-1)} - \delta_{ij})$$

If $Q_i^{\min} \leq Q_i^{(k)} \leq Q_i^{\max}$; $V_i^{(k)} = V_i^{\text{sp}}$ (remark: i^{th} bus remains as PV bus)

Else
 If $Q_i^{(k)} > Q_i^{\max}$; $Q_i^{\text{sp}} = Q_i^{\max}$
 Else If $Q_i^{(k)} < Q_i^{\min}$; $Q_i^{\text{sp}} = Q_i^{\min}$

Remark:
 In these cases,
 PV buses are
 switched to PQ
 buses

Remark.

If there are ' l ' limit violations (of the generator reactive power limits), where $1 \leq l \leq (M-1)$, then there will be extra ' l ' unknowns. These extra unknowns are the bus voltage magnitudes at all the buses where the limit violations take place. To solve for these extra ' l ' unknowns, extra ' l ' equations are also available in terms of Q_i at these buses.

3. If there are ' l ' violations, then

$$\overline{V}^{(k)} = \begin{bmatrix} V_{m+1}^{(k)} & V_{m+2}^{(k)} & \dots & V_N^{(k)} & V_a^{(k)} & V_b^{(k)} & \dots & V_p^{(k)} \end{bmatrix}^T$$

where $a, b, \dots, p$ are the buses at which violations take place.

Further $\bar{Q} = [Q_{m+1}^{(k)} \quad Q_{m+2}^{(k)} \quad \dots \quad Q_N^{(k)} \quad Q_a^{(k)} \quad Q_b^{(k)} \quad \dots \quad Q_p^{(k)}]^T$

$\Rightarrow$ Dimension of $\bar{V}$ vector increases from $(N-m) \times 1$ to $(N-m+1) \times 1$

Similarly dimension of $\bar{Q}$ vector also increases from $(N-m) \times 1$ to $(N-m+1) \times 1$

4. Dimensions of the Jacobian sub-matrices:

$$\left. \begin{array}{l} J_1 \rightarrow \frac{\partial \bar{P}}{\partial \bar{\theta}} \rightarrow (N-1) \times (N-1) \\ J_2 \rightarrow \frac{\partial \bar{P}}{\partial \bar{V}} \rightarrow (N-1) \times (N-m+1) \\ J_3 \rightarrow \frac{\partial \bar{Q}}{\partial \bar{\theta}} \rightarrow (N-m+1) \times (N-1) \end{array} \right\} \begin{array}{l} J_4 \rightarrow \frac{\partial \bar{Q}}{\partial \bar{V}} \\ \rightarrow (N-m+1) \times (N-m+1) \end{array}$$

Overall dimension of the Jacobian matrix increases from $(2N-M-1) \times (2N-M-1)$ to $(2N-M+L-1) \times (2N-M+L-1)$

$$\Delta \bar{X}^{(k)} = \begin{bmatrix} \Delta \theta_2^{(k)} & \dots & \Delta \theta_N^{(k)} & \Delta V_{m+1}^{(k)} & \dots & \Delta V_N^{(k)} & \Delta V_a^{(k)} & \dots & \Delta V_p^{(k)} \end{bmatrix}^T$$

$$\Delta \bar{M}^{(k)} = \begin{bmatrix} \Delta P_2^{(k)} & \dots & \Delta P_N^{(k)} & \Delta Q_{m+1}^{(k)} & \Delta Q_{m+2}^{(k)} & \dots & \Delta Q_N^{(k)} & \Delta Q_a^{(k)} & \dots & \Delta Q_p^{(k)} \end{bmatrix}^T$$

$(2N-M+L-1) \times 1$

5. Calculate vector $\Delta \bar{M}^{(k)} = \begin{bmatrix} \Delta M_1^{(k)} & \Delta M_2^{(k)} & \dots & \Delta M_{2N-M+L-1}^{(k)} \end{bmatrix}^T$

← element of the mismatch vector
↓ no. of Gen.

6. Calculate $e_i^{(k)}$ as $e_i^{(k)} = |\Delta \bar{M}_i^{(k)}| \quad \forall i = 1, 2, \dots, (2N - M + L - 1)$
7. Calculate $e_{rr}^{(k)} = \max(e_i^{(k)})$
8. If $e_{rr}^{(k)} < \epsilon$, then stop. Else go to step 9.
9. Evaluate $J^{(k)} \Big|_{\bar{V}^{(k-1)}, \bar{\theta}^{(k-1)}}$
10. Calculate $\Delta \bar{X}^{(k)} = [J^{(k)}]^{-1} \Delta \bar{M}^{(k)}$
11. Update $\bar{X}^{(k)} = \bar{X}^{(k-1)} + \Delta \bar{X}^{(k)}$; $k = k+1$ and go to step 2.

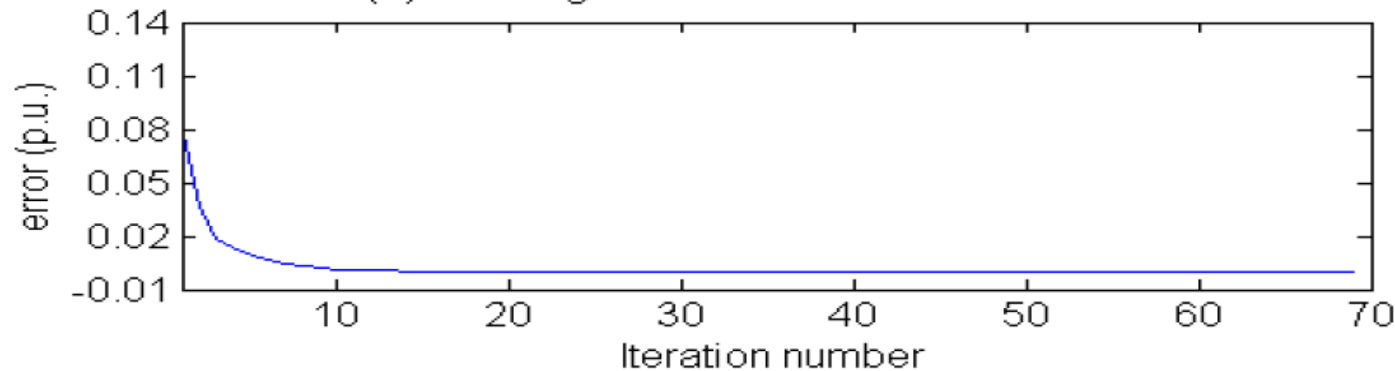
NRLF results in 5 bus system without any generator Q limit

$\mathbf{J}_1 \rightarrow (4 \times 4)$, $\mathbf{J}_2 \rightarrow (4 \times 2)$, $\mathbf{J}_3 \rightarrow (2 \times 4)$ and $\mathbf{J}_4 \rightarrow (2 \times 2)$.

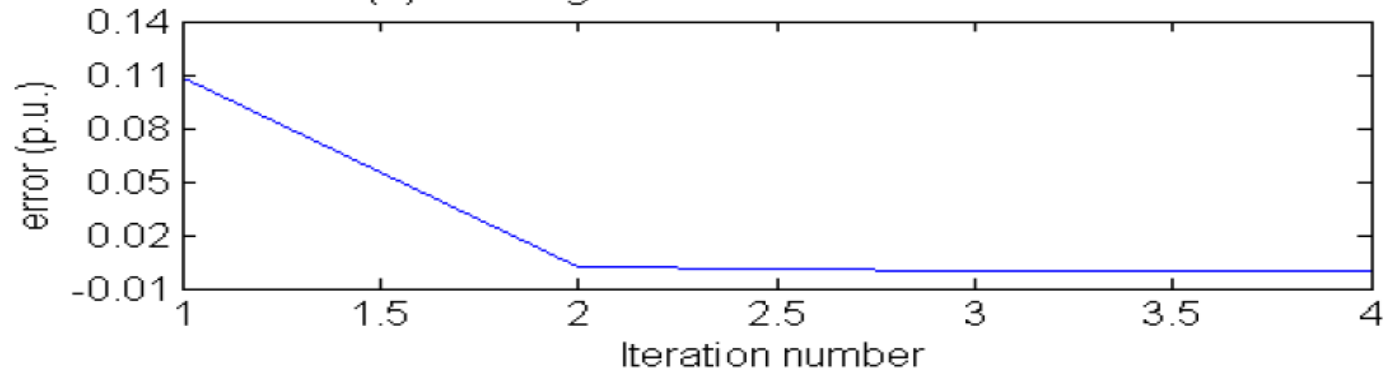
Bus no.	Without generator Q limit			
	$ V $ (p.u)	θ (deg)	P_{inj} (p.u)	Q_{inj} (p.u)
1	1.0	0	0.56743	0.26505
2	1.0	1.65757	0.5	-0.18519
3	1.0	-0.91206	1.0	0.68875
4	0.90594	-8.35088	-1.15	-0.6
5	0.94397	-5.02735	-0.85	-0.4
	Total iteration = 4			

Comparison of convergence characteristics

(a) Convergence characteristics with GSLF



(b) Convergence characteristics with NRLF



Final Results of the 5 bus system with NRLF (polar) with generator Q limit

Bus no.	Without generator Q limit				With generator Q limit			
	$ V $ (p.u)	θ (deg)	P_{inj} (p.u)	Q_{inj} (p.u)	$ V $ (p.u)	θ (deg)	P_{inj} (p.u)	Q_{inj} (p.u)
1	1.0	0	0.56743	0.26505	1.0	0	0.56979	0.33935
2	1.0	1.65757	0.5	-0.18519	1.0	1.69679	0.5	-0.04769
3	1.0	-0.91206	1.0	0.68875	0.9825	-0.63991	1.0	0.5
4	0.90594	-8.35088	-1.15	-0.6	0.88918	-8.35906	-1.15	-0.6
5	0.94397	-5.02735	-0.85	-0.4	0.93445	-4.98675	-0.85	-0.4
	Total iteration = 4				Total iteration = 5			