



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

NRLF in polar co-ordinate (contd..)

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$$J \cdot \Delta \bar{X} = \Delta \bar{M}$$

$$J = \begin{bmatrix} \bar{J}_1 & \bar{J}_2 \\ \bar{J}_3 & \bar{J}_4 \end{bmatrix}$$

$$\bar{J}_2 = \frac{\partial \bar{P}}{\partial \bar{V}}$$

$$P_i = \sum_{k=1}^N V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) \quad \forall i=2, \dots, N$$

$$Q_i = \sum_{k=1}^N V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) \quad \forall i=m+1, \dots, N$$

$$\bar{P} = [P_2 \quad P_3 \quad \dots \quad P_N]^T \rightarrow (N-1) \times 1$$

$$\bar{V} = [V_{m+1} \quad V_{m+2} \quad \dots \quad V_N]^T \rightarrow (N-m) \times 1$$

$$J_2 = \frac{\partial \bar{P}}{\partial \bar{V}} = \begin{bmatrix} \frac{\partial P_2}{\partial V_{m+1}} & \frac{\partial P_2}{\partial V_{m+2}} & \dots & \frac{\partial P_2}{\partial V_N} \\ \vdots & \vdots & & \vdots \\ \frac{\partial P_N}{\partial V_{m+1}} & \frac{\partial P_N}{\partial V_{m+2}} & \dots & \frac{\partial P_N}{\partial V_N} \end{bmatrix} \begin{matrix} \rightarrow \text{Rectangular} \\ \text{matrix} \end{matrix}$$

$\rightarrow (N-1) \times (N-m)$

$\frac{\partial P_i}{\partial V_i} \rightarrow \text{Self-term : ex : } \frac{\partial P_{58}}{\partial V_{58}}, \frac{\partial P_{99}}{\partial V_{99}}$

$N = 100$
 $m = 8$

$\frac{\partial P_i}{\partial V_k} ; k \neq i \rightarrow \text{Non-self-term} \left(\frac{\partial P_2}{\partial V_{10}}, \frac{\partial P_{59}}{\partial V_{98}} \right) - \text{examples}$

$$P_i = \sum_{k=1}^N V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

$$= V_i^2 G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

$$i = 2, \dots, N$$

$$k = m+1, \dots, N$$

$$k \neq i$$

$$\frac{\partial P_i}{\partial V_k}; k \neq i$$

$$= V_i Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik});$$

$$\frac{\partial P_i}{\partial V_i}$$

$$= 2V_i G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik});$$

$$i = 2, \dots, N$$

$$k = m+1, \dots, N$$

$$k = i$$

$$J_3 = \frac{\partial \bar{Q}}{\partial \bar{\theta}} \quad ; \quad \bar{Q} = [Q_{m+1} \quad Q_{m+2} \quad \dots \quad Q_N]^T \rightarrow (N-m) \times 1$$

$$\bar{\theta} = [\theta_2 \quad \theta_3 \quad \dots \quad \theta_N]^T \rightarrow (N-1) \times 1$$

$$J_3 = \frac{\partial \bar{Q}}{\partial \bar{\theta}} = \begin{bmatrix} \frac{\partial Q_{m+1}}{\partial \theta_2} & \frac{\partial Q_{m+1}}{\partial \theta_3} & \dots & \frac{\partial Q_{m+1}}{\partial \theta_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_N}{\partial \theta_2} & \frac{\partial Q_N}{\partial \theta_3} & \dots & \frac{\partial Q_N}{\partial \theta_N} \end{bmatrix} \begin{matrix} \text{Rectangular} \\ \text{matrix} \end{matrix} \rightarrow (N-m) \times (N-1)$$

$$\frac{\partial Q_i}{\partial \theta_i} \rightarrow \text{Self-term};$$

$$\frac{\partial Q_i}{\partial \theta_k} ; k \neq i \rightarrow \text{non-self-term}$$

$$\theta_i = \sum_{k=1}^N V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$

$$= -V_i^2 B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$

$$\begin{aligned} \overline{Y}_{ik} &= Y_{ik} \angle \alpha_{ik} \\ &= G_{ik} + jB_{ik} \\ G_{ik} &= Y_{ik} \cos \alpha_{ik} \\ B_{ik} &= Y_{ik} \sin \alpha_{ik} \end{aligned}$$

$$\frac{\partial \theta_i}{\partial \theta_k}$$

$$= -V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

$$i = m+1, m+2, \dots, N; \quad k = 2, 3, \dots, N; \quad k \neq i$$

$$\frac{\partial \theta_i}{\partial \theta_i}$$

$$= \sum_{\substack{k=1 \\ k \neq i}}^N V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

$$i = m+1, \dots, N$$

$$k = 2, \dots, N$$

$$k = i$$



$$J_y = \frac{\partial \bar{Q}}{\partial \bar{V}} = \begin{bmatrix} \frac{\partial Q_{m+1}}{\partial V_{m+1}} & \frac{\partial Q_{m+1}}{\partial V_{m+2}} & \dots & \frac{\partial Q_{m+1}}{\partial V_N} \\ \vdots & & & \vdots \\ \frac{\partial Q_N}{\partial V_{m+1}} & \frac{\partial Q_N}{\partial V_{m+2}} & \dots & \frac{\partial Q_N}{\partial V_N} \end{bmatrix} \rightarrow \begin{matrix} (N-m) \times \\ (N-m) \\ \downarrow \\ \text{Square} \\ \text{matrix} \end{matrix}$$

$\frac{\partial Q_i}{\partial V_i} \rightarrow$ diagonal term

$\frac{\partial Q_i}{\partial V_k} ; k \neq i \rightarrow$ off-diagonal term

$$Q_i = -V_i^2 B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$

J_4 

$$\frac{\partial Q_i}{\partial V_k}$$

$$\frac{\partial Q_i}{\partial V_i}$$

$$= V_i Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$

$$i = m+1, m+2, \dots, N$$

$$k = m+1, m+2, \dots, N$$

$$k \neq i$$

$$= -2V_i B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^N V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$

$$i = m+1, m+2, \dots, N$$

$$k = m+1, m+2, \dots, N$$

$$k = i$$

