



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

NRLF in polar co-ordinate (contd..)

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$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \bar{\theta} \\ \Delta \bar{V} \end{bmatrix} = \begin{bmatrix} \Delta \bar{P} \\ \Delta \bar{Q} \end{bmatrix}$$

$$J_2 = \begin{bmatrix} \frac{\partial P_2}{\partial V_{m+1}} & \cdots & \frac{\partial P_2}{\partial V_N} \\ \vdots & & \\ \frac{\partial P_N}{\partial V_{m+1}} & \cdots & \frac{\partial P_N}{\partial V_N} \end{bmatrix}$$

$$J_1 = \begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \cdots & \frac{\partial P_2}{\partial \theta_N} \\ \vdots & & \vdots \\ \frac{\partial P_N}{\partial \theta_2} & \cdots & \frac{\partial P_N}{\partial \theta_N} \end{bmatrix} = \frac{\partial \bar{P}}{\partial \bar{\theta}} \rightarrow (N-1) \times (N-1)$$

$$J_2 = \frac{\partial \bar{P}}{\partial \bar{V}} \rightarrow (N-1) \times (N-m)$$

To recall :

$$\bar{\theta} = [\theta_2 \quad \theta_3 \quad \dots \quad \theta_N]^T \rightarrow (N-1) \times 1; \quad \bar{V} = [\bar{v}_{m+1} \quad \bar{v}_{m+2} \quad \dots \quad \bar{v}_N]^T \rightarrow (N-m) \times 1$$

$$\bar{P} = [p_2 \quad p_3 \quad \dots \quad p_N]^T \rightarrow (N-1) \times 1; \quad \bar{Q} = [q_{m+1} \quad q_{m+2} \quad \dots \quad q_N]^T \rightarrow (N-m) \times 1$$

$$J_3 = \begin{bmatrix} \frac{\partial Q_{m+1}}{\partial \theta_2} & \dots & \frac{\partial Q_{m+1}}{\partial \theta_N} \\ \vdots & & \\ \frac{\partial Q_N}{\partial \theta_2} & \dots & \frac{\partial Q_N}{\partial \theta_N} \end{bmatrix} = \frac{\partial \bar{Q}}{\partial \bar{\theta}} \rightarrow (N-m) \times (N-1)$$



$$\bar{J}_y = \begin{bmatrix} \frac{\partial Q_{m+1}}{\partial V_{m+1}} & \cdots & \frac{\partial Q_{m+1}}{\partial V_N} \\ \vdots & & \vdots \\ \frac{\partial Q_N}{\partial V_{m+1}} & \cdots & \frac{\partial Q_N}{\partial V_N} \end{bmatrix} = \frac{\partial \bar{Q}}{\partial \bar{V}} \rightarrow (N-m) \times (N-m)$$

$$\Delta \bar{\theta} = [\Delta \theta_2 \quad \Delta \theta_3 \quad \cdots \quad \Delta \theta_N]^T \rightarrow (N-1) \times 1$$

$$\Delta \bar{V} = [\Delta V_{m+1} \quad \Delta V_{m+2} \quad \cdots \quad \Delta V_N]^T \rightarrow (N-m) \times 1$$

$$\Delta \bar{P} = [\Delta P_2 \quad \Delta P_3 \quad \cdots \quad \Delta P_N]^T \rightarrow (N-1) \times 1$$

where $\Delta P_i = P_i^{\text{exp}} - P_i^{\text{cal}}$
 $\forall i = 2, \dots, N$

$$\Delta \bar{Q} = [\Delta Q_{m+1} \quad \Delta Q_{m+2} \quad \dots \quad \Delta Q_N]^T \rightarrow (N-m) \times 1 \quad \text{where, } \Delta Q_i = Q_i^p - Q_i^{\text{cal}}$$

$\forall i = m+1, \dots, N$

Therefore to summarise

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{bmatrix} \begin{bmatrix} \Delta \bar{\theta} \\ \Delta \bar{V} \\ \Delta \bar{X} \\ \Delta \bar{Q} \end{bmatrix} = \begin{bmatrix} \Delta \bar{P} \\ \Delta \bar{m} \\ \Delta \bar{Q} \end{bmatrix}$$

$(N-1) \times (N-1)$ (for J_1)
 $(N-1) \times (N-m)$ (for J_2)
 $(N-m) \times (N-1)$ (for J_3)
 $(N-m) \times (N-m)$ (for J_4)
 $(N-1) \times 1$ (for $\Delta \bar{\theta}$)
 $(N-m) \times 1$ (for $\Delta \bar{V}$)
 $(N-m) \times 1$ (for $\Delta \bar{X}$)
 $(N-1) \times 1$ (for $\Delta \bar{P}$)
 $(N-m) \times 1$ (for $\Delta \bar{Q}$)
 $(2N-m-1) \times 1$ (for $\Delta \bar{m}$)

$$J = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \rightarrow (2N-M-1) \times (2N-M-1)$$

$\swarrow$ Jacobian matrix
 $\searrow$ square matrix

We further concisely write as

$$J \cdot \Delta X = \Delta M$$

$\swarrow$ Jacobian matrix
 $\searrow$ correction vector
 $\rightarrow$ mismatch vector

J matrix would be evaluated at the present known value of $\bar{\theta}$ and $\bar{V}$

To find analytical expressions of all elements of

J_1, J_2, J_3, J_4

$J_1 =$ diagonal elements $\rightarrow \frac{\partial P_i}{\partial \theta_i}$
off-diagonal elements $\rightarrow \frac{\partial P_i}{\partial \theta_k} ; i \neq k$

$$P_i = \sum_{k=1}^N V_i V_k \gamma_{ik} \cos(\theta_i - \theta_k - d_{ik})$$

$$= V_i^2 \gamma_{ii} \cos(d_{ii}) + \sum_{\substack{k=1 \\ k \neq i}}^N V_i V_k \gamma_{ik} \cos(\theta_i - \theta_k - d_{ik})$$

$$= V_i^2 \gamma_{ii} + \sum_{k=1, k \neq i}^N V_i V_k \gamma_{ik} \cos(\theta_i - \theta_k - d_{ik})$$



$$\begin{aligned}\bar{Y}_{ik} &= Y_{ik} \angle d_{ik} = Y_{ik} \cos d_{ik} + j Y_{ik} \sin d_{ik} \\ &= G_{ik} + j B_{ik}\end{aligned}$$

$$\left. \frac{\partial P_3}{\partial \theta_4} \right|$$

off-diagonal terms

$$\frac{\partial P_i}{\partial \theta_k} : \quad \boxed{\frac{\partial P_i}{\partial \theta_k} = V_i V_k Y_{ik} \sin(\theta_i - \theta_k - d_{ik})}$$

As an example: Say $N = 5$; $\frac{\partial P_3}{\partial \theta_4}$

$$\begin{aligned}P_3 &= V_3^2 G_{33} + \sum_{\substack{k=1 \\ k \neq 3}}^5 V_3 V_k Y_{3k} \cos(\theta_3 - \theta_k - d_{3k}) \\ &= V_3^2 G_{33} + V_3 V_1 Y_{31} \cos(\theta_3 - \theta_1 - d_{31}) + V_3 V_2 Y_{32} \cos(\theta_3 - \theta_2 - d_{32}) \\ &\quad + V_3 V_4 Y_{34} \cos(\theta_3 - \theta_4 - d_{34}) + V_3 V_5 Y_{35} \cos(\theta_3 - \theta_5 - d_{35})\end{aligned}$$



Diagonal term

$$\frac{\partial P_i}{\partial \theta_i} : \quad \frac{\partial P_i}{\partial \theta_i} = - \sum_{\substack{k=1 \\ k \neq i}}^N V_i V_k \gamma_{ik} \sin(\theta_i - \theta_k - d_{ik})$$

For J_2 and J_3 (rectangular matrices)

Self-term $\rightarrow \frac{\partial P_i}{\partial V_i}$

Mutual-term $\rightarrow \frac{\partial P_i}{\partial V_k} ; k \neq i$

$J_4 \rightarrow$ Diagonal and off-diagonal terms

J_3

$$\frac{\partial \theta_i}{\partial \theta_i}$$

$$J_3 \rightarrow \frac{\partial \theta_i}{\partial \theta_k} ; k \neq i$$