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CERTIFICATION COURSE

Computer aided power system analysis

Newton-Raphson load flow (NRLF) in polar co-ordinate

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Unknowns for load flow solution $\rightarrow \bar{V}_i \quad \forall i = 1, 2, \dots, N$

$$\Rightarrow \bar{V}_i = V_i \angle \theta_i \\ = e_i + jf_i$$

number of
buses in the
system.

$$e_i = V_i \cos \theta_i$$

$$f_i = V_i \sin \theta_i$$

Polar method : V_i, θ_i

Rectangular method : e_i, f_i

Newton-Raphson load flow (NRLF) in Polar co-ordinate.
Load-flow (power flow equations):

$$P_i = \sum_{k=1}^N V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$



$$\theta_i = \sum_{k=1}^N V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$

Three-types of buses

slack-bus —

PV bus —

PQ bus —

known

$V_i, \theta_i = 0$

V_i, P_i

P_i, Q_i

unknown

—

θ_i

V_i, θ_i

$\forall i = 2, \dots, M$

$\forall i = M+1, \dots, N$

N-Bus system

m-generators

1 → slack

2, ... m → PV

m+1, m+2, ... N → PQ.

In total

θ_i to be calculated $\forall i = 2, \dots, N$

V_i to be calculated $\forall i = m+1, \dots, N$

Total number of unknown θ_i
 $= (N-1)$



Total number of unknown $V_i = (N-m)$

$\Rightarrow$ Total number of unknown $= (N-1) + (N-m) = \underline{2N-m-1}$

We denote our unknown vectors as

$$\bar{\mathbf{V}} = [V_{m+1}, V_{m+2}, \dots, V_N]^T \longrightarrow (N-m) \times 1$$

$$\bar{\boldsymbol{\theta}} = [\theta_2, \theta_3, \dots, \theta_N]^T \longrightarrow (N-1) \times 1$$

$$P_i = \sum_{k=1}^N V_i V_k Y_{ik} \cos(\theta_i - \theta_k - d_{ik}) = P_i(\bar{\boldsymbol{\theta}}, \bar{\mathbf{V}})$$

$$Q_i = \sum_{k=1}^N V_i V_k Y_{ik} \sin(\theta_i - \theta_k - d_{ik}) = Q_i(\bar{\boldsymbol{\theta}}, \bar{\mathbf{V}})$$

known quantities

$P_i : \forall i = 2, \dots, m \rightarrow PV \text{ bus}$

$P_i : \forall i = m+1, m+2, \dots, N$

$Q_i : \forall i = m+1, m+2, \dots, N$

} PQ-bus

Total

Number
(m-1)

(N-m)

(N-m)

2N-m-1

$$P_i = P_i(\bar{\theta}, \bar{V}) = P_i(\theta_2, \theta_3, \dots, \theta_N, V_{m+1}, V_{m+2}, \dots, V_N)$$

$$Q_i = Q_i(\bar{\theta}, \bar{V}) = Q_i(\theta_2, \theta_3, \dots, \theta_N, V_{m+1}, V_{m+2}, \dots, V_N)$$

Now, $P_i(\bar{\theta}, \bar{V}) = P_i^{sp} ; \quad Q_i(\bar{\theta}, \bar{V}) = Q_i^{sp}$



Applying Taylor series expansion:

$$P_i^{\text{opt}} = P_i(\bar{\theta}^{(0)}, \bar{V}^{(0)}) + \frac{\partial P_i}{\partial \theta_2} \Delta \theta_2 + \frac{\partial P_i}{\partial \theta_3} \Delta \theta_3 + \dots + \frac{\partial P_i}{\partial \theta_N} \Delta \theta_N + \frac{\partial P_i}{\partial V_{m+1}} \Delta V_{m+1} + \frac{\partial P_i}{\partial V_{m+2}} \Delta V_{m+2} + \dots + \frac{\partial P_i}{\partial V_N} \Delta V_N$$

$$\forall i = 2, \dots, N$$

Taylor series

P_i^{cal}

$$P_i(\bar{\theta}, \bar{V}) = P_i^{\text{opt}}$$

$$\Rightarrow P_i(\bar{\theta}^{(0)} + \Delta \bar{\theta}, \bar{V}^{(0)} + \Delta \bar{V}) = P_i^{\text{opt}}$$

Similarly:

$$Q_i(\bar{\theta}, \bar{V}) = Q_i^{\text{opt}}$$

$$\Rightarrow Q_i(\bar{\theta}^{(0)} + \Delta \bar{\theta}, \bar{V}^{(0)} + \Delta \bar{V}) = Q_i^{\text{opt}}$$

Taking Taylor series expansion;



$$Q_i^{\text{cal}} = Q_i(\bar{\theta}^{(0)}, \bar{V}^{(0)}) + \frac{\partial Q_i}{\partial \theta_2} \Delta \theta_2 + \frac{\partial Q_i}{\partial \theta_3} \Delta \theta_3 + \dots + \frac{\partial Q_i}{\partial \theta_N} \Delta \theta_N$$

$$+ \frac{\partial Q_i}{\partial V_{m+1}} \Delta V_{m+1} + \dots + \frac{\partial Q_i}{\partial V_N} \Delta V_N$$

$\forall i = m+1, \dots, N$

Collecting all the above equations together and putting them in a matrix form, we get

$$\begin{bmatrix}
 \frac{\partial P_2}{\partial \theta_2} & \frac{\partial P_2}{\partial \theta_3} & \dots & \frac{\partial P_2}{\partial \theta_N} & \frac{\partial P_2}{\partial V_{m+1}} & \dots & \frac{\partial P_2}{\partial V_N} \\
 \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 \frac{\partial P_N}{\partial \theta_2} & \frac{\partial P_N}{\partial \theta_3} & \dots & \frac{\partial P_N}{\partial \theta_N} & \frac{\partial P_N}{\partial V_{m+1}} & \dots & \frac{\partial P_N}{\partial V_N}
 \end{bmatrix}
 \begin{bmatrix}
 \Delta \theta_2 \\
 \Delta \theta_3 \\
 \vdots \\
 \Delta \bar{\theta} \\
 \vdots \\
 \Delta \theta_N
 \end{bmatrix}
 +
 \begin{bmatrix}
 \frac{\partial Q_{m+1}}{\partial \theta_2} & \frac{\partial Q_{m+1}}{\partial \theta_3} & \dots & \frac{\partial Q_{m+1}}{\partial \theta_N} & \frac{\partial Q_{m+1}}{\partial V_{m+1}} & \dots & \frac{\partial Q_{m+1}}{\partial V_N} \\
 \vdots & \vdots & & \vdots & \vdots & & \vdots \\
 \frac{\partial Q_N}{\partial \theta_2} & \frac{\partial Q_N}{\partial \theta_3} & \dots & \frac{\partial Q_N}{\partial \theta_N} & \frac{\partial Q_N}{\partial V_{m+1}} & \dots & \frac{\partial Q_N}{\partial V_N}
 \end{bmatrix}
 \begin{bmatrix}
 \Delta V_{m+1} \\
 \vdots \\
 \Delta \bar{V} \\
 \vdots \\
 \Delta V_N
 \end{bmatrix}
 =
 \begin{bmatrix}
 P_2^{op} - P_2^{cal} \\
 P_3^{op} - P_3^{cal} \\
 \vdots \\
 \Delta \bar{P} \\
 \vdots \\
 P_N^{op} - P_N^{cal} \\
 \hline
 Q_{m+1}^{op} - Q_{m+1}^{cal} \\
 \vdots \\
 \Delta \bar{Q} \\
 \vdots \\
 Q_N^{op} - Q_N^{cal}
 \end{bmatrix}$$



$$\begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \bar{\theta} \\ \Delta \bar{v} \end{bmatrix} = \begin{bmatrix} \Delta \bar{p} \\ \Delta \bar{q} \end{bmatrix}$$

