IIT ROORKEE

NPTEL ONLINE CERTIFICATION COURSE

Computer aided power system analysis

Basics of Newton Raphson Numerical Method

Prof. Biswarup Das
Department of Electrical Engineering

Basic Newton Raphson Numerical method

n unknowns: $x_1, x_2 \dots x_n \Rightarrow \bar{X} = [x_1, x_2 \dots x_n]^T$

Equations

$$f_1(x_1, x_2 \dots x_n) = 0$$
$$f_2(x_1, x_2 \dots x_n) = 0$$
$$\vdots$$
$$f_n(x_1, x_2 \dots x_n) = 0$$

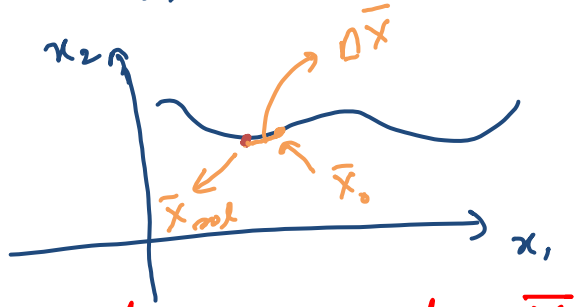
$f_1(\bar{X}) = 0$

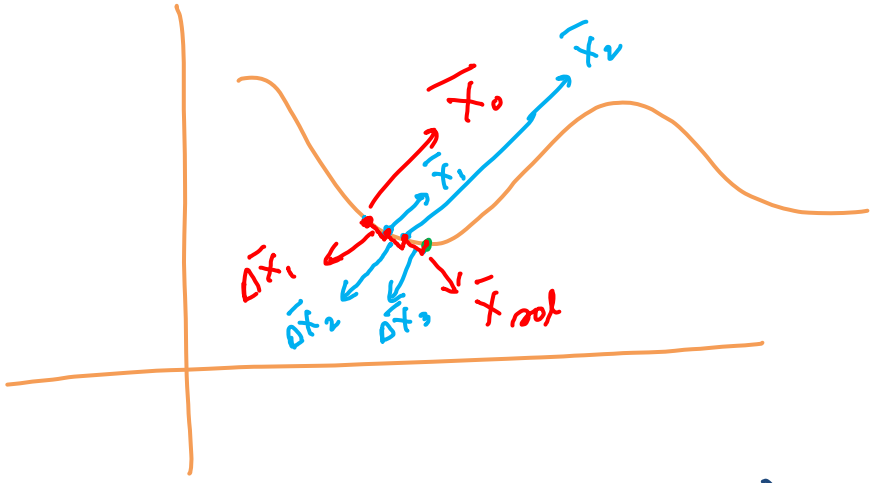
$f_1(\bar{X}_0)$ → We evaluate the function f_1 with the values of vector $\bar{X}_0$

Start with an initial guess $\bar{X}_0$.

If $\bar{X}_0$ is close to $\bar{X}_{sol}$, then we can write that

$$\bar{X}_{sol} = \bar{X}_0 + \Delta\bar{X}$$





$\bar{X}_0$

$\bar{X}_0 + \Delta\bar{X} = \bar{X}_{sol}$

$f_1(\bar{X}_{sol}) = f_1(\bar{X}_0 + \Delta\bar{X}) = 0$

$\Rightarrow f_2(\bar{X}_0 + \Delta\bar{X}) = 0$

$\vdots$

$f_n(\bar{X}_0 + \Delta\bar{X}) = 0$

$f_1(x_1 + \Delta x_1, x_2 + \Delta x_2, \cdots x_n + \Delta x_n) = 0$

$\Rightarrow f_1(\bar{X}_0 + \Delta\bar{X}) = f_1(\bar{X}_0) + \frac{\partial f_1}{\partial x_1}\Delta x_1 + \frac{\partial f_1}{\partial x_2}\Delta x_2 + \cdots + \frac{\partial f_1}{\partial x_n}\cdot\Delta x_n = 0$

$\Rightarrow f_2(\bar{X}_0 + \Delta\bar{X}) = f_2(\bar{X}_0) + \frac{\partial f_2}{\partial x_1}\Delta x_1 + \frac{\partial f_2}{\partial x_2}\Delta x_2 + \cdots + \frac{\partial f_1}{\partial x_n}\Delta x_n = 0$

$\vdots$

$$f_n(\bar{X}_0 + \Delta\bar{X}) = f_n(\bar{X}_0) + \frac{\partial f_n}{\partial x_1}\Delta x_1 + \frac{\partial f_n}{\partial x_2}\Delta x_2 + \cdots + \frac{\partial f_n}{\partial x_n}\Delta x_n = 0$$

$$\Rightarrow \begin{bmatrix} f_1(\bar{X}_0) \\ f_2(\bar{X}_0) \\ \vdots \\ f_n(\bar{X}_0) \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} = 0$$

$\bar{F}(\bar{X}_0)$ $(n \times 1)$ — non-zero vector

$J \rightarrow (n \times n)$

$\Delta\bar{X}$ $(n \times 1)$

$$\Rightarrow \bar{F}(\bar{X}_0) + J\,\Delta\bar{X} = 0$$

$$\bar{F}(\bar{X}_0) = \begin{bmatrix} f_1(\bar{X}_0) & f_2(\bar{X}_0) & \cdots & f_n(\bar{X}_0) \end{bmatrix}^T$$

$$J\,\Delta\bar{X} = -\bar{F}(\bar{X}_0)$$

$$\Rightarrow \Delta\bar{X} = -J^{-1}\bar{F}(\bar{X}_0)$$

This vector is known

J^{-1}: square matrix

Matrix J would be evaluated at $\bar{X} = \bar{X}_0$

$$\bar{X}_1 = \bar{X}_0 + \Delta\bar{X}$$

evaluate $f_1(\bar{X}_1),\ f_2(\bar{X}_1),\ \cdots\cdots\ f_n(\bar{X}_1)$

Calculate $e = \max\left[|f_1(\bar{X}_1)|, |f_2(\bar{X}_1)|, \cdots\cdots |f_n(\bar{X}_1)|\right]$

$$\epsilon = 1.0e^{-12} = 10e^{-12}$$

$$\bar{X}_2 = \bar{X}_1 + \Delta\bar{X}$$

$$f_1(\bar{X}_1 + \Delta\bar{X}) = 0$$
$$\vdots$$
$$f_2(\bar{X}_1 + \Delta\bar{X}) = 0$$
$$\vdots$$
$$f_n(\bar{X}_1 + \Delta X) = 0$$

$$f_1(\bar{X}_2 + \Delta\bar{X}) = 0$$
$$f_2(\bar{X}_2 + \Delta\bar{X}) = 0$$
$$\vdots$$
$$f_n(\bar{X}_2 + \Delta\bar{X}) = 0$$