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CERTIFICATION COURSE

Computer aided power system analysis

Gauss-Seidel Load Flow (GSLF)

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$x_1, x_2, \dots, x_n \rightarrow \text{unknowns}$

$$f_1(x_1, x_2, \dots, x_n) = 0$$

$$f_2(x_1, x_2, \dots, x_n) = 0$$

$\vdots$

$$f_n(x_1, x_2, \dots, x_n) = 0$$

$$\left\{ \begin{array}{l} x_1 = g_1(x_1, x_2, \dots, x_n) \\ x_2 = g_2(x_1, x_2, \dots, x_n) \\ \vdots \\ x_n = g_n(x_1, x_2, \dots, x_n) \end{array} \right.$$

1. Take initial condition
2. update $x_1, x_2, \dots, x_n$
3. Check for convergence
4. If converged, stop. Else go to step 2.



N-Bus system

Assume that there is one generator and rest are load buses.

Generator $\rightarrow$ slack bus $\rightarrow V, \theta$ are known

Load buses $\rightarrow P, Q$ are known

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \vdots \\ \bar{I}_N \end{bmatrix} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \dots & \bar{Y}_{1N} \\ \bar{Y}_{21} & \bar{Y}_{22} & \dots & \bar{Y}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{Y}_{N1} & \bar{Y}_{N2} & \dots & \bar{Y}_{NN} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \vdots \\ \bar{V}_N \end{bmatrix}$$

$$\Rightarrow \bar{I}_i = \sum_{k=1}^N \bar{Y}_{ik} \bar{V}_k$$
$$= \bar{Y}_{ii} \bar{V}_i + \sum_{\substack{k=1 \\ k \neq i}}^N \bar{Y}_{ik} \bar{V}_k$$

$$\bar{Y}_{ii} \bar{V}_i = \bar{I}_i - \sum_{\substack{k=1 \\ k \neq i}}^N \bar{Y}_{ik} \bar{V}_k$$

$$\Rightarrow \bar{V}_i = \frac{1}{\bar{Y}_{ii}} \left[\bar{I}_i - \sum_{\substack{k=1 \\ k \neq i}}^N \bar{Y}_{ik} \bar{V}_k \right]$$

$$= \frac{1}{\bar{Y}_{ii}} \left[\frac{P_i - jQ_i}{\bar{V}_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^N \bar{Y}_{ik} \bar{V}_k \right]$$

$$= \frac{1}{\bar{Y}_{ii}} \left[\frac{P_i - jQ_i}{\bar{V}_i^*} - \sum_{k=1}^{i-1} \bar{Y}_{ik} \bar{V}_k - \sum_{k=i+1}^N \bar{Y}_{ik} \bar{V}_k \right]$$

We also know, that

$$\bar{V}_i \bar{I}_i^* = P_i + jQ_i$$

$$\Rightarrow \bar{I}_i^* = \frac{P_i + jQ_i}{\bar{V}_i}$$

$$\Rightarrow \bar{I}_i = \frac{P_i - jQ_i}{\bar{V}_i^*}$$

$$\Rightarrow \bar{V}_i = g_i(\bar{V}_1, \bar{V}_2, \dots, \bar{V}_N)$$

$$\bar{V}_i = \frac{1}{\bar{Y}_{ii}} \left[\frac{P_i - jQ_i}{\bar{V}_i^*} - \sum_{k=1}^{i-1} \bar{Y}_{ik} \bar{V}_k - \sum_{k=i+1}^N \bar{Y}_{ik} \bar{V}_k \right]$$

Let $N=6$, Bus 1 $\rightarrow$ slack bus and buses 2, 3, ..., 6 are load buses.

$$\bar{V}_4 = \frac{1}{\bar{Y}_{44}} \left[\frac{P_4 - jQ_4}{\bar{V}_4^*} - \bar{Y}_{41} \bar{V}_1 - \bar{Y}_{42} \bar{V}_2 - \bar{Y}_{43} \bar{V}_3 - \bar{Y}_{45} \bar{V}_5 - \bar{Y}_{46} \bar{V}_6 \right]$$

$$\stackrel{i=4}{=} \frac{1}{\bar{Y}_{44}} \left[\frac{P_4 - jQ_4}{\bar{V}_4^*} - \sum_{k=1}^3 \bar{Y}_{4k} \bar{V}_k - \sum_{k=5}^6 \bar{Y}_{4k} \bar{V}_k \right]$$

$\xrightarrow{(i-1)}$ $\xrightarrow{(i+1)}$ $\xrightarrow{=N}$

We wish to maintain the voltage magnitude profile in the system at the nominal level $\Rightarrow$ In p.u. we wish to maintain the voltage magnitudes at 1.0 p.u.





$$|\theta_i - \theta_j| > 30^\circ \text{ to } 35^\circ$$

We take initial guess as $\bar{V}_i^{(0)} = 1.0 \angle 0^\circ \quad \forall i = 2, 3, \dots, N$

Flat start

Algorithm steps

1. Assume flat start as the initial guess: $\bar{V}_i = 1.0 \angle 0^\circ \quad \forall i = 2, \dots, N$
2. Set iteration count $j = 1$.
3. update all bus voltages as;

$$\bar{V}_2^{(j)} = \frac{1}{\bar{Y}_{22}} \left[\frac{P_2 - jQ_2}{\bar{V}_2^{*(j-1)}} - \sum_{k=3}^N \bar{Y}_{2k} \bar{V}_k^{(j-1)} - \bar{Y}_{21} \bar{V}_1^{(j)} \right]$$

$$\bar{V}_3^{(j)} = \frac{1}{\bar{Y}_{33}} \left[\frac{P_3 - jQ_3}{\bar{V}_3^{*(j-1)}} - \sum_{k=1}^2 \bar{Y}_{3k} \bar{V}_k^{(j)} - \sum_{k=4}^N \bar{Y}_{3k} \bar{V}_k^{(j-1)} \right]$$

$$\vdots \bar{V}_i^{(j)} = \frac{1}{\bar{Y}_{ii}} \left[\frac{P_i - jQ_i}{\bar{V}_i^{*(j-1)}} - \sum_{k=1}^{i-1} \bar{Y}_{ik} \bar{V}_k^{(j)} - \sum_{k=i+1}^N \bar{Y}_{ik} \bar{V}_k^{(j-1)} \right]$$

$$\vdots \bar{V}_N^{(j)} = \frac{1}{\bar{Y}_{NN}} \left[\frac{P_N - jQ_N}{\bar{V}_N^{*(j-1)}} - \sum_{k=1}^{N-1} \bar{Y}_{Nk} \bar{V}_k^{(j)} \right]$$

4. Calculate error as $e_i^{(j)} = \left| \bar{V}_i^{(j)} - \bar{V}_i^{(j-1)} \right| \quad \forall i = 2, \dots, N$
5. Calculate max error as $e_{\max}^{(j)} = \max \left[e_2^{(j)}, e_3^{(j)}, \dots, e_N^{(j)} \right]$

6. If $e_{\max} < \epsilon$, stop. Else $j = j+1$ and go to step 3.

