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CERTIFICATION COURSE

Computer aided power system analysis

Power flow equations and classification of buses

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N-Bus system

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \vdots \\ \bar{I}_i \\ \vdots \\ \bar{I}_N \end{bmatrix} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \dots & \bar{Y}_{1i} & \dots & \bar{Y}_{1N} \\ \bar{Y}_{21} & \bar{Y}_{22} & \dots & \bar{Y}_{2i} & \dots & \bar{Y}_{2N} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{Y}_{i1} & \bar{Y}_{i2} & \dots & \bar{Y}_{ii} & \dots & \bar{Y}_{iN} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{Y}_{N1} & \bar{Y}_{N2} & \dots & \bar{Y}_{Ni} & \dots & \bar{Y}_{NN} \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \vdots \\ \bar{V}_i \\ \vdots \\ \bar{V}_N \end{bmatrix}$$

$\downarrow$ $(N \times 1)$ $\downarrow$ $(N \times N)$ $\downarrow$ $(N \times 1)$

$\bar{I}_i \rightarrow$ Injected current at bus i , $\forall i = 1, \dots, N$

$\bar{V}_i \rightarrow$ voltage at bus i , $\forall i = 1, 2, \dots, N$

$\bar{Y}_{ij} \rightarrow (i, j)^{\text{th}}$ element of the Y_{BUS} matrix
 $\forall i, j = 1, 2, \dots, N$

$$\begin{aligned} \bar{I}_i &= \bar{Y}_{i1} \bar{V}_1 + \bar{Y}_{i2} \bar{V}_2 + \dots \\ &\quad \dots + \bar{Y}_{ii} \bar{V}_i + \dots + \bar{Y}_{iN} \bar{V}_N \\ &= \sum_{k=1}^N \bar{Y}_{ik} \bar{V}_k \end{aligned}$$

$$\begin{aligned}\text{Let } \bar{V}_i &= V_i \angle \theta_i \\ \bar{V}_k &= V_k \angle \theta_k \\ \bar{Y}_{ik} &= Y_{ik} \angle \alpha_{ik}\end{aligned}$$

$$\begin{aligned}\text{We know,} \\ \bar{S}_i = P_i + jQ_i = \bar{V}_i \bar{I}_i^* = \bar{V}_i \left[\sum_{k=1}^N \bar{Y}_{ik} \bar{V}_k \right]^*\end{aligned}$$

$$= \bar{V}_i \sum_{k=1}^N \bar{V}_k^* \bar{Y}_{ik}^*$$

$$= \sum_{k=1}^N \bar{V}_i \bar{V}_k^* \bar{Y}_{ik}^*$$

$$= \sum_{k=1}^N V_i V_k Y_{ik} \angle \theta_i - \theta_k - \alpha_{ik}$$

$$P_i = \sum_{k=1}^N V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

$$Q_i = \sum_{k=1}^N V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$



$$\left. \begin{aligned} P_i &= \sum_{k=1}^N V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik}) \\ Q_i &= \sum_{k=1}^N V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik}) \end{aligned} \right| \begin{array}{l} \text{Power-flow equations} \\ \forall i = 1, 2, \dots, N \end{array}$$

There are altogether '2N' power-flow equations.
At every bus, we would like to know

→ Total 4 quantities.

$\left. \begin{array}{l} V_i \\ \theta_i \\ P_i \\ Q_i \end{array} \right\}$

Altogether, there are '4N' quantities which we would like to know.

Total number of equations = $2N$

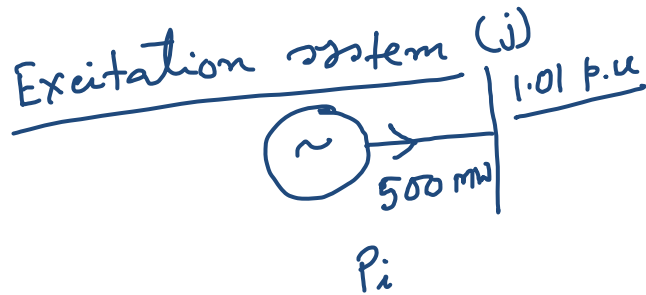
Total number of quantities = $4N$

$\Rightarrow$ We have to specify ' $2N$ ' number of quantities

Classification of buses

Load

Generator



i) PQ-buses
 P_i and Q_i are specified

ii) PV buses
 P_i and V_i are specified.

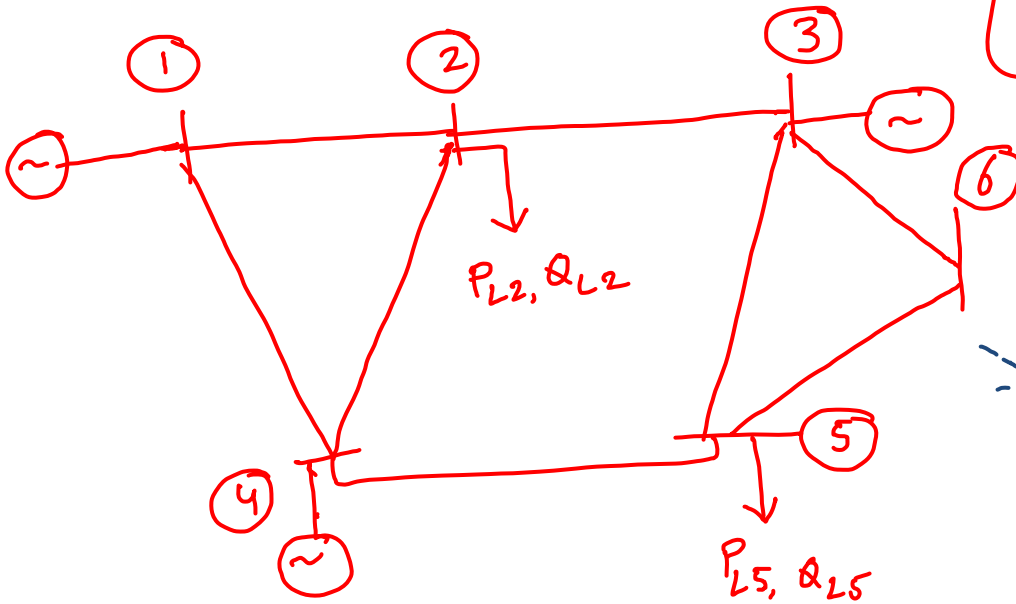
iii) Slack bus



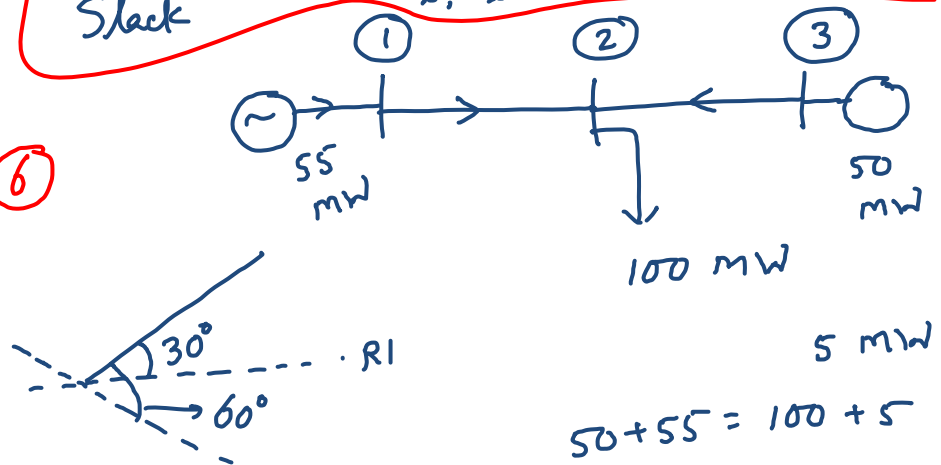
Slack bus

$$\theta_i = 0$$

$$V_i$$



	Specified	Calculated
PQ	P_i, Q_i	V_i, θ_i
PV	V_i, P_i	Q_i, θ_i
Slack	$V_i, \theta_i = 0$	P_i, Q_i



PQ $\rightarrow$ 2, 5, 6
 PV $\rightarrow$ 3, 4
 Slack - 1, $\theta_1 = 0$