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EE5311

MODULE-6 - MULTIPLIERS

$X \rightarrow$ M BIT NUMBER
$Y \rightarrow$ N BIT NUMBER.
} UNSIGNED

$$\Rightarrow \quad X = \sum_{i=0}^{M-1} x_i 2^i$$

$$Y = \sum_{j=0}^{N-1} y_j 2^j$$

$$Z = XY$$

$$\sum_{K=0}^{(M+N-1)} z_K 2^K = \sum_{i=0}^{M-1} x_i 2^i \times \sum_{j=0}^{N-1} y_j 2^j = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (x_i y_j) 2^{(i+j)}$$

$$z_K = \sum_{(i+j=K)} x_i y_j$$

$$
\begin{array}{cccccccl}
 & & & x_3 & x_2 & x_1 & x_0 & \text{MULTIPLICAND} \\
 & & \times & & y_2 & y_1 & y_0 & \text{MULTIPLIER} \\
\hline
 & & & x_3y_0 & x_2y_0 & x_1y_0 & x_0y_0 & PP0 \rightarrow \text{M-BITS LONG.} \\
 & & x_3y_1 & x_2y_1 & x_1y_1 & x_0y_1 & & PP1 \quad \text{PARTIAL PRODUCTS (PP)} \\
 & x_3y_2 & x_2y_2 & x_1y_2 & x_0y_2 & & & PP2 \quad \text{(N PP's)} \\
\hline
Z_6 & Z_5 & Z_4 & Z_3 & Z_2 & Z_1 & Z_0 & \\
\hline
\end{array}
$$

- ★ PARTIAL PRODUCT GENERATION → AND GATE
- ★ PP SHIFTING → WIRING.
- ★ PP ACCUMULATION → ARRAY OF FULL ADDERS.
- ★ FINAL VECTOR ADDITION → FAST ADDER ARCHITECTURE (CLA)

SIGNED NUMBERS

2's COMPLEMENT REP

$$\{a_{N-1}\ a_{N-2}\ \cdots\ a_0\} \longrightarrow \sum_{i=1}^{N-1} a_i 2^i \quad \text{— UNSIGNED}$$

$$\longrightarrow -2^{(N-1)} a_{N-1} + \underbrace{\sum_{i=0}^{N-2} a_i 2^i}_{\leq (2^{N-1}-1)}$$

$$\sum_{i=0}^{N-2} a_i 2^i \leq \sum_{i=0}^{N-2} 2^i = (2^{N-1}-1)$$

$$-2^{N-1} + (2^{(N-1)}-1) = -1$$

IF $a_{N-1} = 1$; THEN THE NUMBER $= \begin{pmatrix} \text{1's COMPLEMENT} \\ + \quad 1 \end{pmatrix}$ -ve.

$1101 =$

$-2^3+2^2+0+2^0 = -3$

1101

$\rightarrow 0010 \rightarrow 1111 - \{a_3 a_2 a_1 a_0\}$

$+ \quad 1 \quad + \quad 1$

-ve(0011)

$\{a_{N-1}\, a_{N-2} - \cdots a_0\} + \{\bar{a}_{N-1}\, \bar{a}_{N-2} \cdots \bar{a}_0\}$

$= (1\ 1\ \ 1\ 1\ \ 1 \cdots 1)$ N times.

$\{a_{N-1}\, a_{N-2} \cdots a_0\} = (1\,1 \cdots\cdots 1) - \{\bar{a}_{N-1}\, \bar{a}_{N-2} \cdots \bar{a}_0\}$

$= \quad -1 \quad - \{\bar{a}_{N-1}\, \bar{a}_{N-2} \cdots \bar{a}_0\}$

$= -(1 + \{\bar{a}_{N-1}\bar{a}_{N-2} \cdots \bar{a}_0\})$

$-3 = 1101$ $\quad +15 \rightarrow \underline{0}1111$

$\times -5 \quad \times 1011 \leftarrow -2^3 + (\quad)$

$+15$

N BIT 2's COMP NO. $= [-2^{N-1}, 2^{N-1} - 1]$

$1000\cdots0 \qquad 011111$

$$
\begin{array}{r}
1101 \\
\times\ 1011 \\
\hline
11111101 \\
1111101 \\
000000 \\
00011 \\
\hline
000000001111
\end{array}
$$

X

$X \times 2^1$

$(-X) \times 2^3$

1011

X

$= (-2^3 + 2^1 + 2^0) = -2^3 X + 2^1 X + 2^0 X$

SIGN EXTENSION OF 2's COMP NO's DOES NOT ALTER THE VALUE

$$\{a_{N-1} \ldots a_0\} = -2^{N-1}a_{N-1} + \sum_{K=0}^{N-2} a_K 2^K$$

SIGN EXTENDED 'a'

$$\underbrace{\{\underbrace{a_{N-1} \cdots a_{N-1}\, a_{N-1}}_{M \text{ times.}}\, a_{N-1} \ldots a_0\}}_{M+N \text{ BIT No.}}$$

$$= -2^{(M+N-1)} a_{N-1} + 2^{(M+N-2)} a_{N-1} + \cdots + 2^{N} a_{N-1} + 2^{N-1} a_{N-1} + \sum_{K=0}^{N-2} a_K 2^K$$

$$= -2^{N-1}a_{N-1}\,(+2^M - \underbrace{2^{M-1} - 2^{M-2} - \ldots - 1}_{(2^M - 1)})$$

$$= -2^{N-1}a_{N-1}\,(2^M - (2^M - 1))$$

NPTEL