

30/10/2019

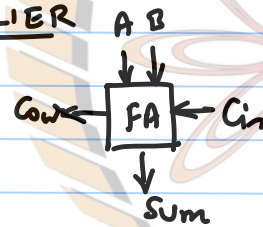
EE5311

MODULE-6 - MULTIPLIERS

ARRAY MULTIPLIER

$$\begin{array}{r} x_3 \ x_2 \ x_1 \ x_0 \\ \times y_3 \ y_2 \ y_1 \ y_0 \\ \hline \end{array}$$

$$\begin{array}{r} x_3 y_0 \ x_2 y_0 \ x_1 y_0 \ x_0 y_0 \\ x_3 y_1 \ x_2 y_1 \ x_1 y_1 \ x_0 y_1 \\ x_3 y_2 \ x_2 y_2 \ x_1 y_2 \ x_0 y_2 \\ x_3 y_3 \ x_2 y_3 \ x_1 y_3 \ x_0 y_3 \end{array}$$



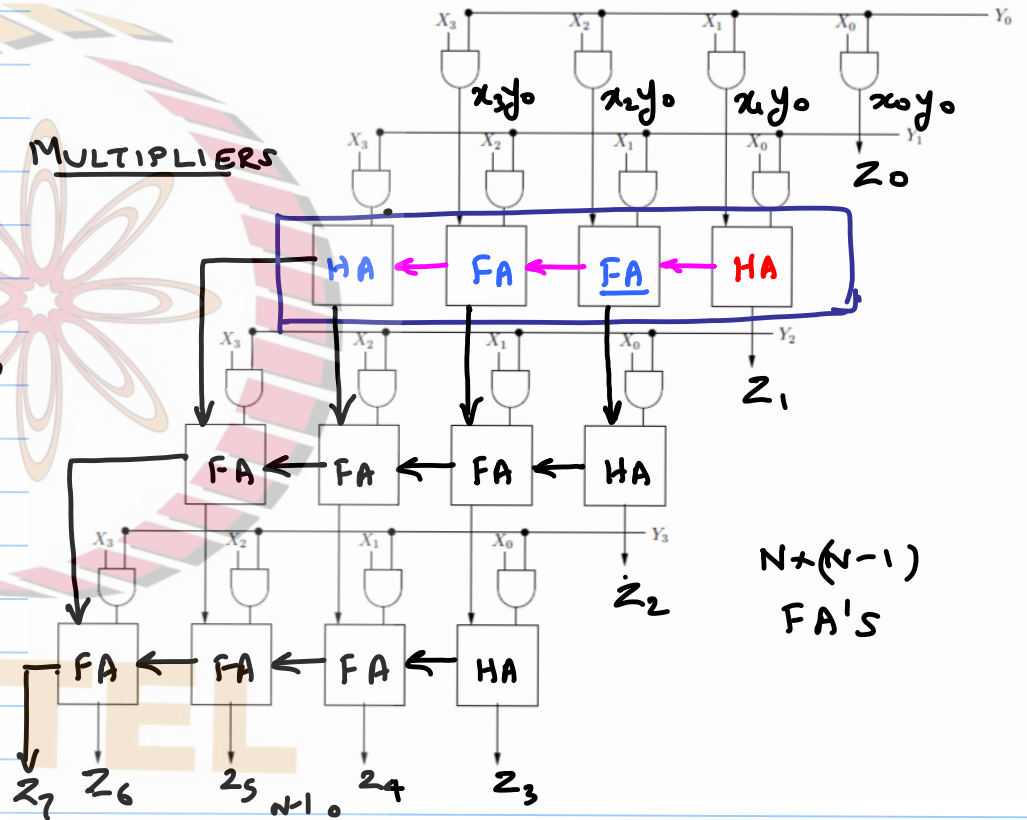
$x y_0$

$2x y_1$

$4x y_2$

$8x y_3$

$$\begin{array}{r} x_3 y_3 \ x_2 y_3 \ x_1 y_3 \ x_0 y_3 \\ \hline z_7 \ z_6 \ z_5 \ z_4 \ z_3 \ z_2 \ z_1 \ z_0 \end{array}$$



$N \times (N-1)$
FA'S

$$X \times Y = X \times \sum_{j=0}^{N-1} 2^j y_j = y_0 \cdot X + 2x \cdot y_1 + 4x \cdot y_2 + 8x \cdot y_3$$

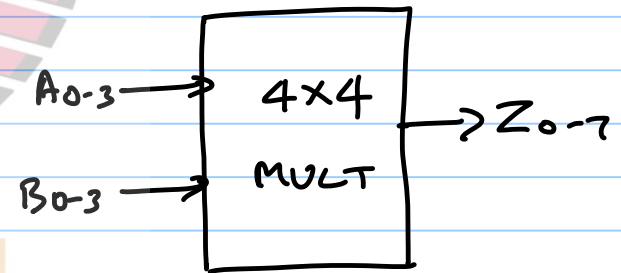
$$t_{\text{ARR-MULT}} = t_{\text{AND}} + (M+N-3)t_{\text{CARRY}} + (M-1)t_{\text{SUM}}$$

$$t_{\text{SUM}} = 1 \rightarrow 2$$

$$t_{\text{CARRY}} = 1$$

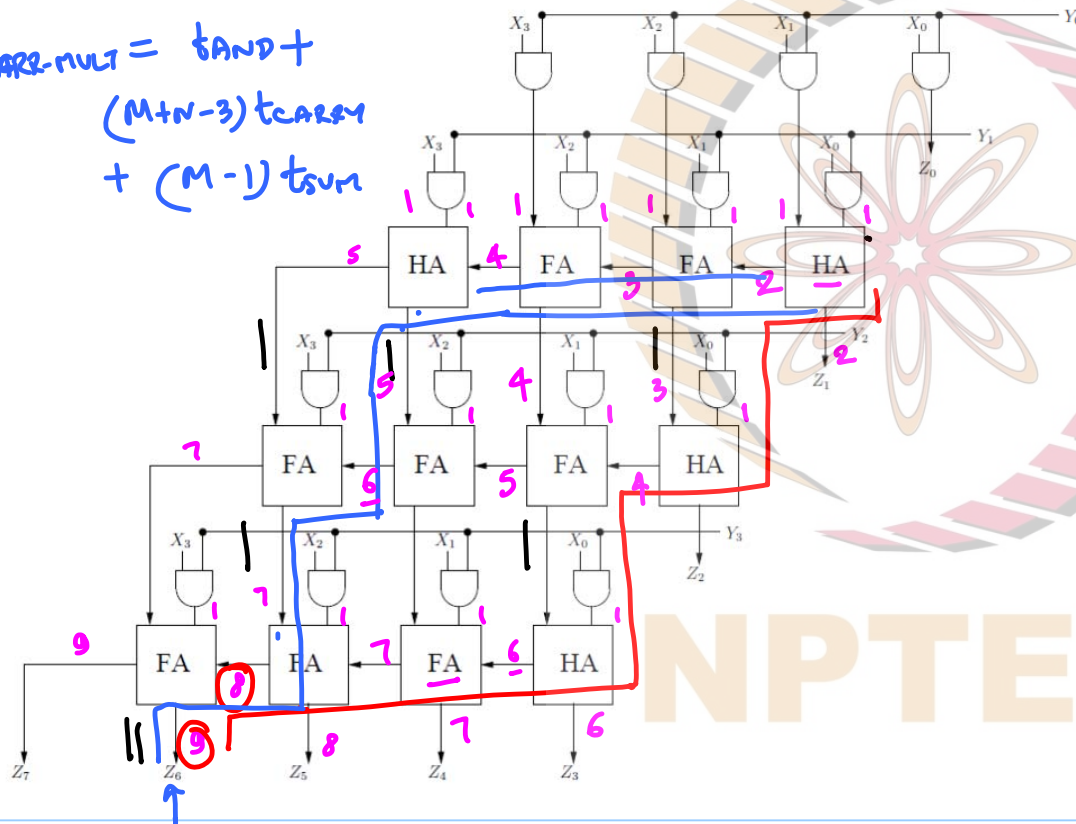
$$t_{\text{AND}} = 1$$

X_{0-3}, Y_{0-3} ARRIVAL TIME = 0
 N BITS M BITS

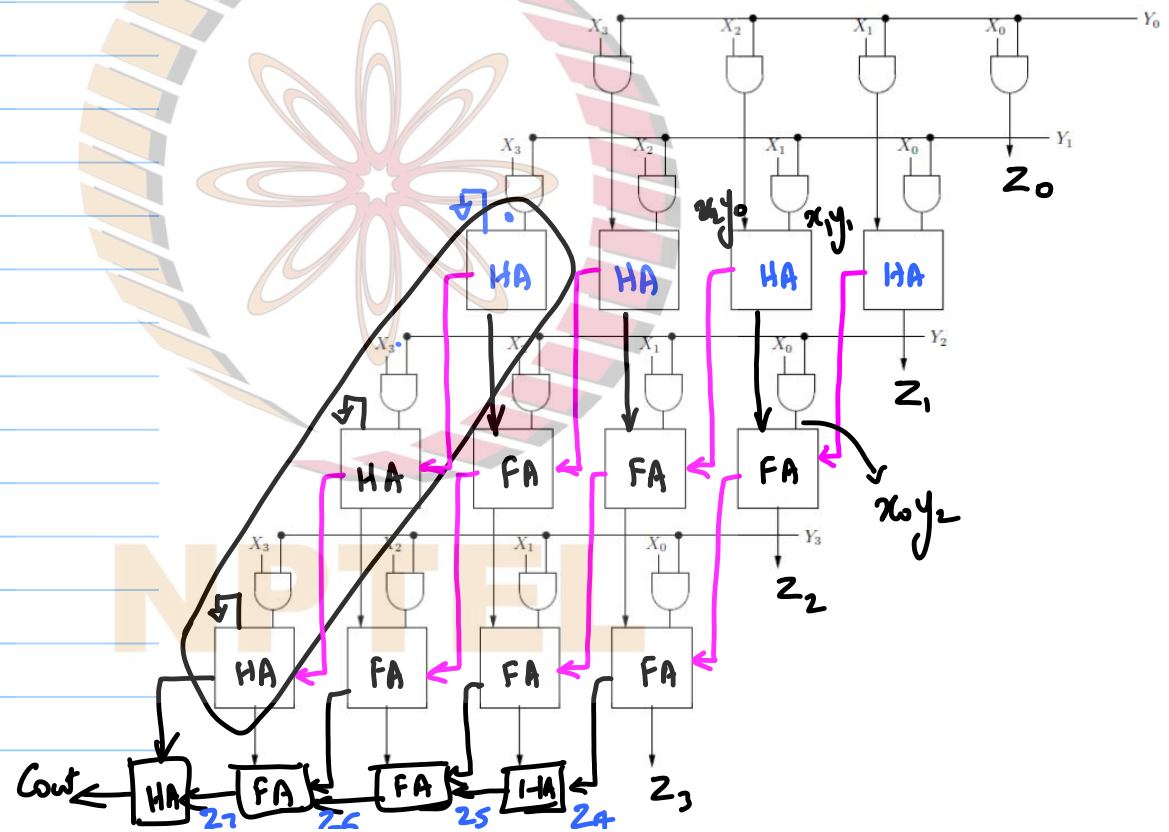


$$t_{\text{ARR-MULT}} = t_{\text{AND}} + (N+M-2)t_{\text{CARRY}} + (M-2)t_{\text{SUM}}$$

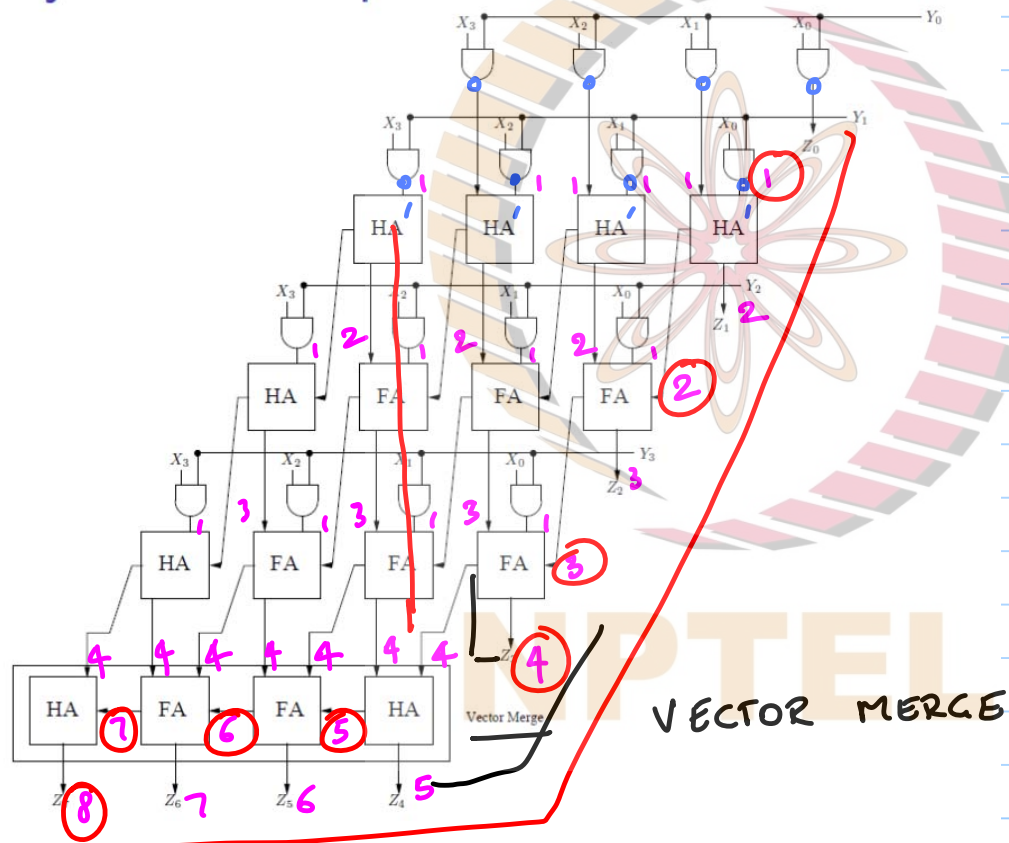
$$\rightarrow t_{\text{AND}} + (M-2+N-1)t_{\text{CARRY}} + (M-1)t_{\text{SUM}}$$



CARRY SAVE MULTIPLIER



Carry Save Multiplier Critical Path



$$t_{AND} = 1$$

$$t_{SUM} = 1$$

$$t_{CARRY} = 1$$

$$t_{CARRY-SAVE} = t_{AND}$$

$$+ (N-1) t_{SUM}$$

$$+ \frac{t_{MERGE}}{N}$$

$$\downarrow$$

$$(N-1) t_{CARRY} + t_{SUM}$$

$$\begin{array}{r}
 x_3 x_2 x_1 x_0 \\
 \times y_3 y_2 y_1 y_0 \\
 \hline
 x_3 y_0 \quad x_3 y_1 \quad x_3 y_2 \quad x_3 y_3 \\
 x_2 y_0 \quad x_2 y_1 \quad x_2 y_2 \quad x_2 y_3 \\
 x_1 y_0 \quad x_1 y_1 \quad x_1 y_2 \quad x_1 y_3 \\
 x_0 y_0 \quad x_0 y_1 \quad x_0 y_2 \quad x_0 y_3
 \end{array}$$

$$Z = X Y$$

$$= \left(-2^{N-1} x_{N-1} + \sum_{i=0}^{N-2} x_i 2^i \right) \times \left(-2^{N-1} y_{N-1} + \sum_{j=0}^{N-2} y_j 2^j \right)$$

$$= \underbrace{2^{(2N-2)}}_{\text{}} \left(x_{N-1} y_{N-1} \right) - 2^{N-1} x_{N-1} \sum_{j=0}^{N-2} y_j 2^j - 2^{N-1} y_{N-1} \sum_{i=0}^{N-2} x_i 2^i + \underbrace{\sum x_i 2^i \sum y_j 2^j}_{\text{}}$$

NPTEL

$$\begin{array}{cccc} x_3 & x_2 & x_1 & x_0 \\ \hline y_3 & y_2 & y_1 & y_0 \end{array}$$

$$\textcircled{i} \quad x_2 y_0 \quad x_1 y_0 \quad x_0 y_0$$

$$x_2 y_1 \quad x_1 y_1 \quad x_0 y_1$$

$$x_2 y_2 \quad x_1 y_2 \quad x_0 y_2$$

$$\begin{array}{cccc} \overline{x_2 y_3} & \overline{x_1 y_3} & \overline{x_0 y_3} & 1 \\ \overline{x_2 y_2} & \overline{x_1 y_2} & \overline{x_0 y_2} & 1 \\ \overline{x_2 y_1} & \overline{x_1 y_1} & \overline{x_0 y_1} & 1 \\ \overline{x_2 y_0} & \overline{x_1 y_0} & \overline{x_0 y_0} & 1 \end{array}$$

$$\begin{array}{ccccccccc} 0 & x_3 y_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline \textcircled{i} & \textcircled{0} & & & & & & & \end{array}$$

\Rightarrow

$$\overline{x_3 y_0} \quad x_2 y_0 \quad x_1 y_0 \quad x_0 y_0$$

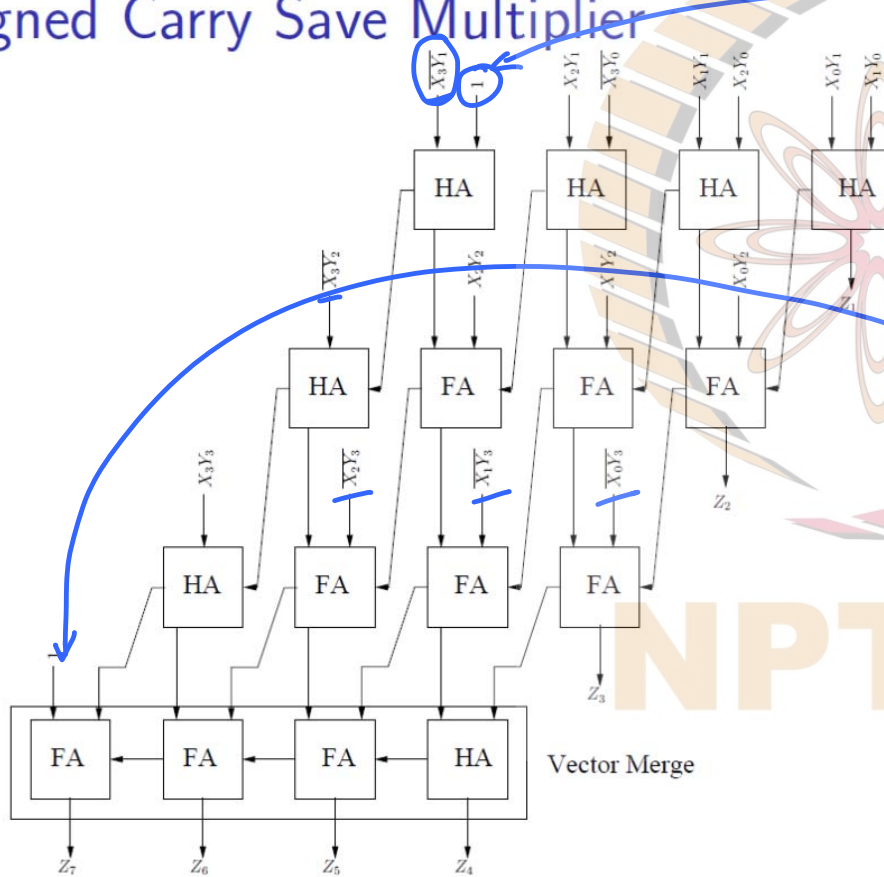
$$\overline{x_3 y_1} \quad x_2 y_1 \quad x_1 y_1 \quad x_0 y_1$$

$$\overline{x_3 y_2} \quad x_2 y_2 \quad x_1 y_2 \quad x_0 y_2$$

$$\begin{array}{cccc} 1 & x_3 y_2 & \overline{x_2 y_2} & \overline{x_1 y_2} & \overline{x_0 y_2} \\ \hline \end{array}$$

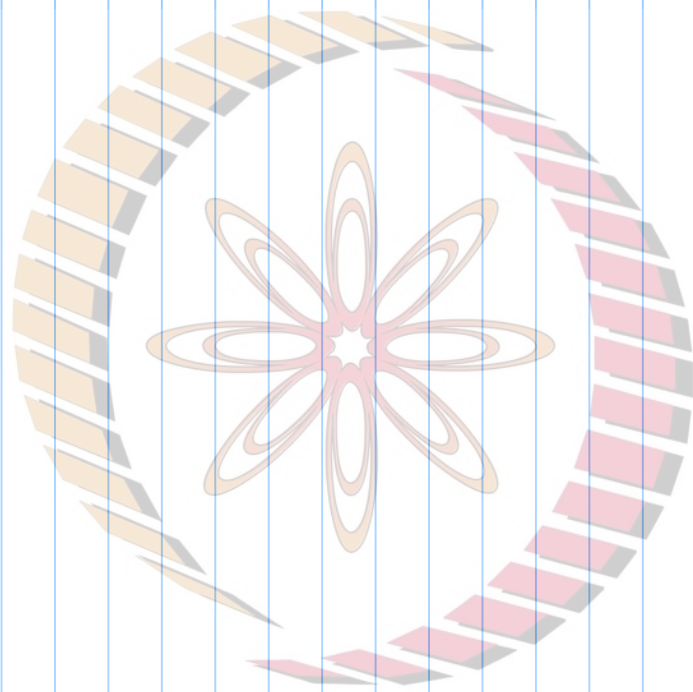
NPTTEL

Signed Carry Save Multiplier



$$\begin{aligned} & \textcircled{1} \quad \overline{x_3y_0} \quad x_2y_0 \quad x_1y_0 \quad x_0y_0 \\ & \Rightarrow \quad \overline{x_3y_1} \quad x_2y_1 \quad x_1y_1 \quad x_0y_1 \\ & \quad \overline{x_3y_2} \quad x_2y_2 \quad x_1y_2 \quad x_0y_2 \\ & \textcircled{1} \quad \overline{x_3y_3} \quad \overline{x_2y_3} \quad \overline{x_1y_3} \quad \overline{x_0y_3} \end{aligned}$$

Vector Merge



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