28/10/2019

EES311

MODULE-6 - MULTIPLIERS

$$\left.\begin{array}{l} X \rightarrow M \text{ BIT NUMBER} \\ Y \rightarrow N \text{ BIT NUMBER.} \end{array}\right\} \text{UNSIGNED}$$

$$\Rightarrow \quad X = \sum_{i=0}^{M-1} x_i 2^i$$

$$Y = \sum_{j=0}^{N-1} y_j 2^j$$

$$Z = XY$$

$$\sum_{K=0}^{(M+N-1)} z_K 2^K = \sum_{i=0}^{M-1} x_i 2^i \times \sum_{j=0}^{N-1} y_j 2^j = \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (x_i y_j) 2^{(i+j)}$$

$$z_K = \sum_{(i+j=K)} x_i y_j$$

			x_3	x_2	x_1	x_0	Multiplicand	
		×		y_2	y_1	y_0	Multiplier	
			x_3y_0	x_2y_0	x_1y_0	x_0y_0	PP0	→ N-bits long
		x_3y_1	x_2y_1	x_1y_1	x_0y_1		PP1	Partial Products (PP)
	x_3y_2	x_2y_2	x_1y_2	x_0y_2			PP2	(N PP's)
z_6	z_5	z_4	z_3	z_2	z_1	z_0		

- * Partial Product Generation → And Gate
- * PP Shifting → Wiring
- * PP Accumulation → Array of Full Adders.
- * Final Vector Addition → Fast Adder Architecture (CLA)

SIGNED NUMBERS

2'S COMPLEMENT REP

$$\{a_{N-1}\ a_{N-2}\ \cdots\ a_0\} \longrightarrow \sum_{i=1}^{N-1} a_i 2^i \quad \text{— UNSIGNED}$$

$$\longrightarrow \quad -2^{(N-1)} a_{N-1} + \underbrace{\sum_{i=0}^{N-2} a_i 2^i}_{\leq (2^{N-1}-1)}$$

$$\sum_{i=0}^{N-2} a_i 2^i \leq \sum_{i=0}^{N-2} 2^i = (2^{N-1}-1)$$

$$-2^{N-1} + (2^{(N-1)}-1) = -1$$

IF $a_{N-1} = 1$; THEN THE NUMBER $= \begin{pmatrix} \text{1's COMPLEMENT} \\ + \quad 1 \end{pmatrix}$ -ve.

$1101 =$

$-2^3+2^2+0+2^0 = -3$

$$\{a_{N-1}\,a_{N-2} - \cdots a_0\} + \{\overline{a_{N-1}}\,\overline{a_{N-2}} \cdots \overline{a_0}\}$$
$$= (1\,1\,1\,1\,1 \cdots 1) \text{ n times.}$$

1101

$\rightarrow 0010 \rightarrow 1111 - \{a_3 a_2 a_1 a_0\}$

$+ \quad 1 \quad + 1$

$-ve(0011)$

$$\{a_{N-1}\,a_{N-2} \cdots a_0\} = (1\,1 \cdots 1) - \{\overline{a_{N-1}}\,\overline{a_{N-2}} \cdots \overline{a_0}\}$$
$$= -1 - \{\overline{a_{N-1}}\,\overline{a_{N-2}} \cdots \overline{a_0}\}$$
$$= -(1 + \{\overline{a_{N-1}}\,\overline{a_{N-2}} \cdots \overline{a_0}\})$$

$-3 = 1101$ $\qquad$ $+15 \to \underline{0}1111$

$\times\,{-5}$ $\qquad$ $\times\ 1011 \leftarrow -2^3 + (\quad)$

$+15$

N BIT 2's COMP NO. $= [-2^{N-1},\ 2^{N-1} - 1]$

$1000\cdots0$ $\qquad$ 011111

```
          1 1 0 1
        x 1 0 1 1
  -----------------
  1 1 1 1 1 1 0 1        x
  1 1 1 1 1 0 1          x*2^1
  0 0 0 0 0 0            
  0 0 0 1 1              (-x)*2^3
  -----------------
0 0 0 0 0 0 1 1 1 1
```

1011

x

$= (-2^3 + 2^1 + 2^0) = -2^3x + 2^1x + 2^0x$

SIGN EXTENSION OF 2'S COMP NO'S DOES NOT ALTER THE VALUE

$$\{a_{N-1} \ldots a_0\} = \underline{-2^{N-1} a_{N-1}} + \sum_{K=0}^{N-2} a_K 2^K$$

SIGN EXTENDED 'a'

$$\underbrace{\{\underbrace{a_{N-1} \cdots a_{N-1} a_{N-1}}_{M \text{ times.}} a_{N-1} \ldots a_0\}}_{\text{M+N BIT No.}}$$

$$= \underline{-2^{(M+N-1)} a_{N-1} + 2^{(M+N-2)} a_{N-1} + \cdots + 2^{N} a_{N-1} + 2^{N-1} a_{N-1}} + \sum_{K=0}^{N-2} a_K 2^K$$

$$= -2^{N-1} a_{N-1} \left(+2^M - \underbrace{2^{M-1} - 2^{M-2} - \ldots - 1}_{(2^M - 1)} \right\}$$

$$= \underline{-2^{N-1} a_{N-1}} \left(2^M - (2^M - 1)\right)$$