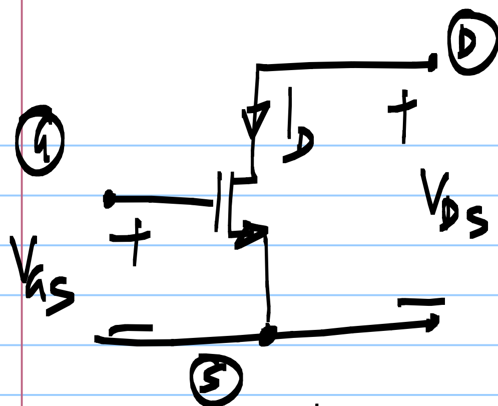
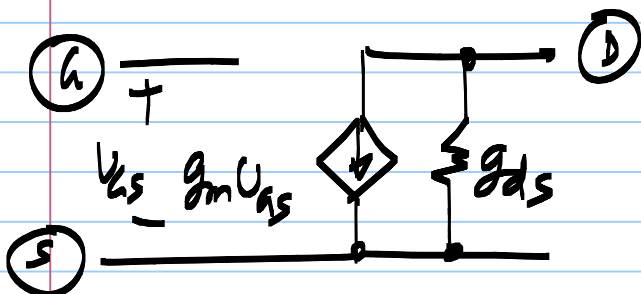
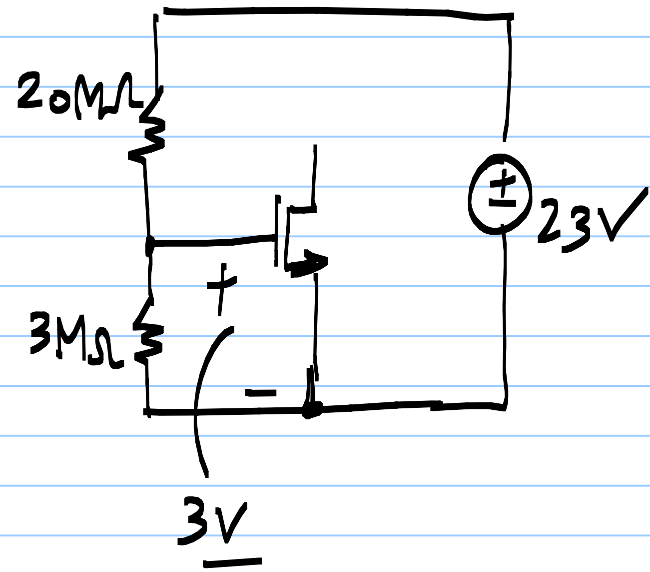
$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T) \; ; \; g_{ds} = \lambda \cdot I_D$$



op. point : V_{GS}, V_{DS}, I_D



Fix V_{GS} to the desired value



$$g_m = \underbrace{\mu_n C_{ox} \frac{W}{L}}\,(\underset{\text{fixed @ 3V}}{V_{GS}} - V_T) \; ; \qquad V_T = 1V, \quad \mu_n C_{ox} \frac{W}{L} = 100\mu A/V^2$$

$$200\mu S$$

$$g_m \propto \underbrace{\mu_n C_{ox}}\frac{W}{L}$$

$V_T = 0.8V \qquad V_{GS} - V_T = 2.2V$

$g_m = 220\mu S$

$V_T = 1.2V \qquad V_{GS} - V_T = 1.8V$

$g_m = 180\mu S$

g_m varies with V_T & $\mu_n C_{ox} \frac{W}{L}$

V_T & $\mu_n C_{ox}$ vary with temperature

$$V_{GS} - V_T = \frac{2 I_D}{(V_{GS} - V_T)} \cdot \frac{1}{\mu_n C_{ox} \frac{W}{L}}$$

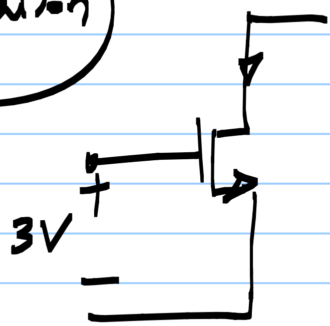
$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_T)^2 \longrightarrow V_{GS} - V_T = \sqrt{\frac{2 \cdot I_D}{\mu_n C_{ox} W/L}}$$

$$g_m = \mu_n C_{ox} \cdot \frac{W}{L} (V_{GS} - V_T)$$

$$= \sqrt{2 \cdot \mu_n C_{ox} \frac{W}{L} \cdot I_D} = \frac{2 I_D}{V_{GS} - V_T}$$

$$g_m = \underbrace{\overbrace{\mu_n C_{ox} \frac{W}{L}}^{K_n} \left(V_{GS} - V_T\right)}_{\text{Both } K_n \text{ \& } V_T \text{ appear}} = \underbrace{\sqrt{2 \cdot \mu_n C_{ox} \frac{W}{L} \cdot I_D}}_{\text{Only } K_n \text{ appears}}$$

Saturation



V_{GS} fixed @ 3V

$$V_T' = V_T + \Delta V_T$$

$$g_m' = \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_T - \Delta V_T \right)$$

$$\frac{g_m'}{g_m} = 1 - \frac{\Delta V_T}{V_{GS} - V_T}$$

Fixed V_{GS}

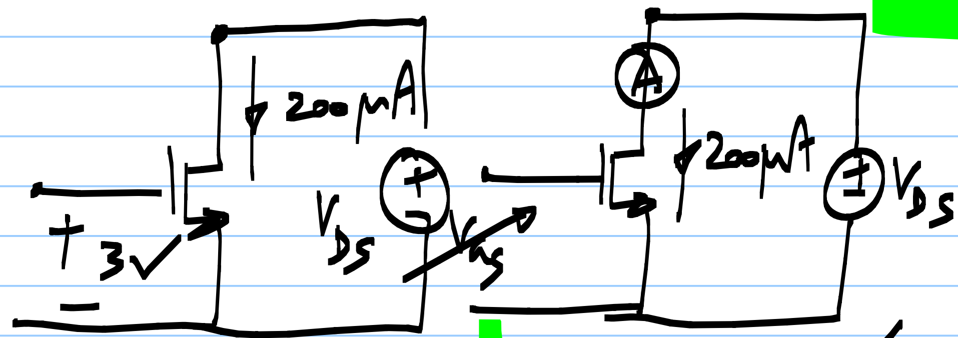
$$K_n \rightarrow K_n'$$

$$g_m' = K_n' \left(V_{GS} - V_T \right)$$

$$\frac{g_m'}{g_m} = \frac{K_n'}{K_n}$$

$\boxed{\text{Fixed } I_D} = 200\mu A$

$$g_m = \sqrt{2 \cdot \mu_n C_{ox} \frac{W}{L} \cdot I_D} = \sqrt{2\, k_n' I_D}$$



$V_T' = V_T + \Delta V_T$

$g_m' = g_m$

$K_n \longrightarrow K_n'$

$$\frac{g_m'}{g_m} = \sqrt{\frac{K_n'}{K_n}}$$

nominal V_T, k_n (1V) (100µA/V²)	Fixed V_{GS} (e.g. 3V)	Fixed I_D (100µA)
$V_T' = V_T + \Delta V_T$ $k_n' = k_n$	$\dfrac{g_m'}{g_m} = 1 - \dfrac{\Delta V_T}{V_{GS} - V_T}$	$\dfrac{g_m'}{g_m} = 1$
$V_T' = V_T$ $k_n' \neq k_n$ $k_n' = 1.1 \cdot k_n$	$\underbrace{\dfrac{g_m'}{g_m} = \dfrac{k_n'}{k_n}}_{}$ $= 1.1$	$\dfrac{g_m'}{g_m} = \sqrt{\dfrac{k_n'}{k_n}} \simeq 1.05$

$$\sqrt{1+x} \simeq 1 + \frac{x}{2}$$

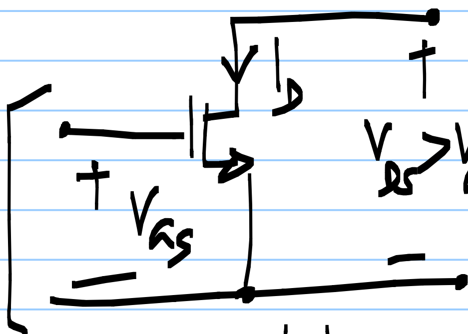
$$\sqrt{1.1} = \sqrt{1+0.1} \simeq 1.05$$

$\dfrac{g_m'}{g_m}$: much closer to 1 in this case

Biasing the transistor at a given I_D: lower sensitivity of g_m to transistor parameters

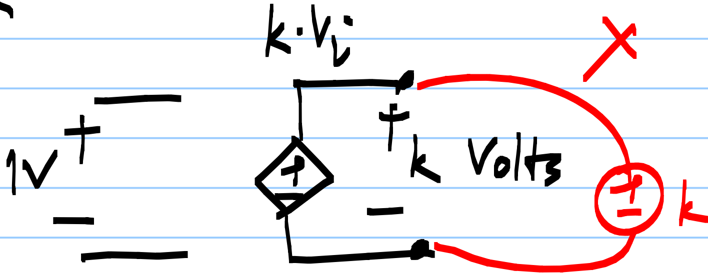
How?

Transistor: Unilateral



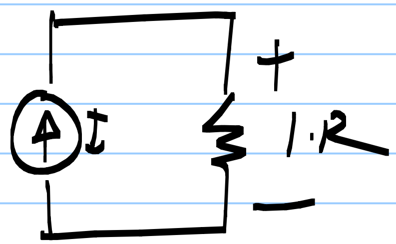
$V_{DS} > V_{GS} - V_T$

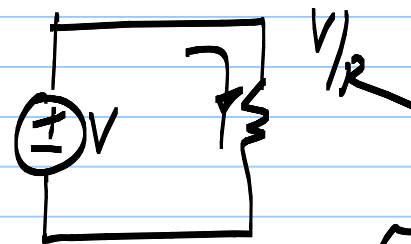
$I_D = f(V_{GS})$



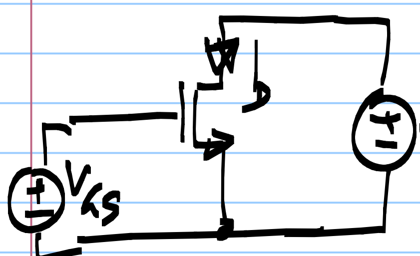
$V = I \cdot R$ $\quad$ $I = V/R$

Apply V_{GS}: $I_D = \frac{K_n}{2}(V_{GS}-V_T)^2$ flows into the drain

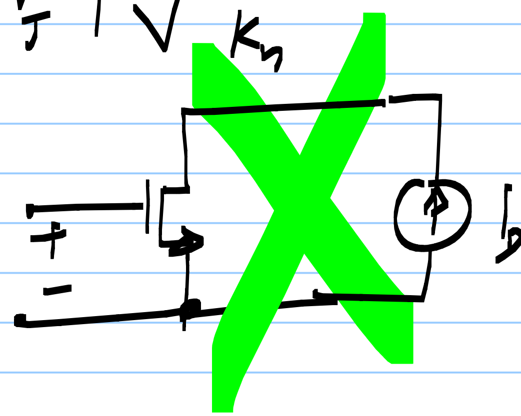




$$I_D = \frac{K_n}{2}(V_{GS}-V_T)^2 \qquad V_{GS} = V_T + \sqrt{\frac{2I_D}{K_n}}$$

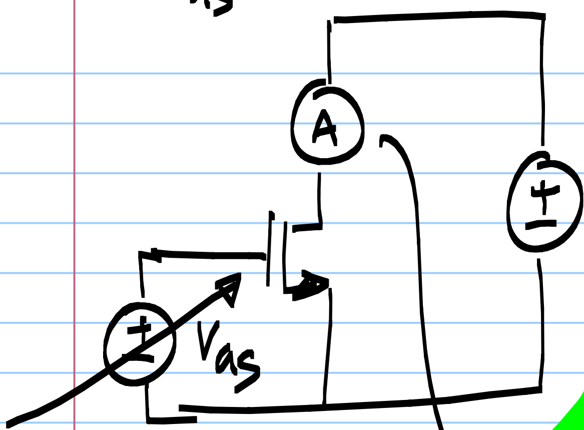


$V_{DS} > V_{GS} - V_T$



V_{GS} controls I_D, not vice versa

target $I_D = 200\mu A$

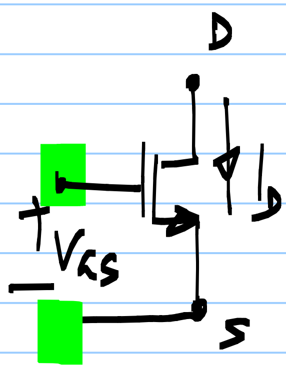


$V_{DS} > V_{GS} - V_T$

negative feedback

observe the actual drain current,
compare it to the desired I_D $(200\mu A)$
increase or decrease V_{GS} as required

Biasing a transistor at a given current



I_D: accessed at the drain OR source

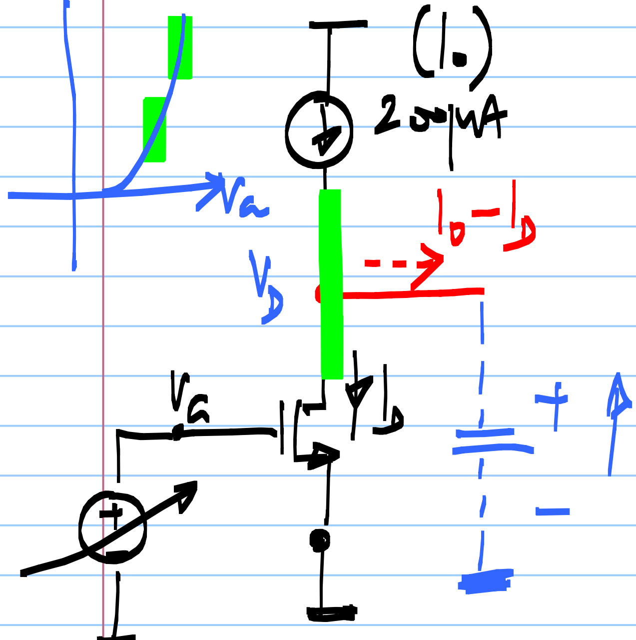
Varying voltage: applied to gate OR source

4 possible bias circuits

keep one of $\{G, S\}$
at a constant voltage
& vary the other terminal

measure (compare) @ drain, control the gate voltage
measure (") @ source, control the source voltage
" " @ drain, control the source voltage
" " @ source, control the gate voltage

measure (compare) @ drain, control the gate voltage



Compare I_D with I_0 $\underline{I_0 - I_D}$ or $I_D - I_0$

adjust V_G

If $I_D < I_0$ ($I_0 - I_D > 0$), $V_D \uparrow$

If $I_D > I_0$ ($I_0 - I_D < 0$), $V_D \downarrow$

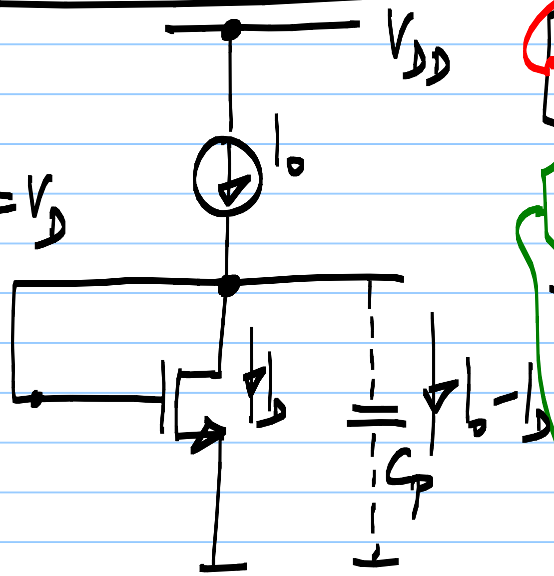
If $I_D < I_0$: we must increase V_G

If $I_D > I_0$: we must reduce V_G

Connect gate to the drain

$V_G \uparrow \Rightarrow I_D \uparrow$

$V_G = V_D$



$I_D < I_0$: V_D increases continuously

$I_D > I_0$: V_D decreases continuously

$I_D = I_0$: V_D stays constant

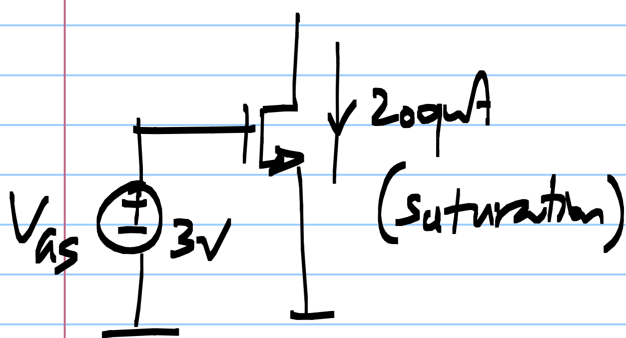
stops here

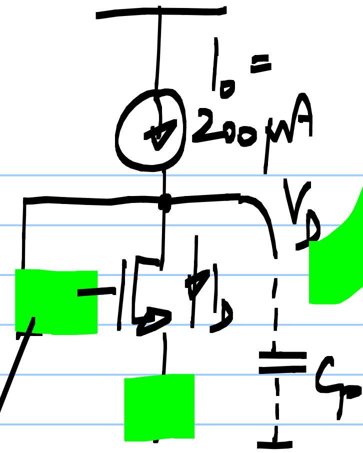
$V_G \downarrow \Rightarrow I_D \downarrow$

Drain feedback

$V_{DS} = V_{GS} \Rightarrow V_{DS} > V_{GS} - V_T$

$(V_T > 0)$





$$g_m = K_n (V_{GS} - V_T)$$

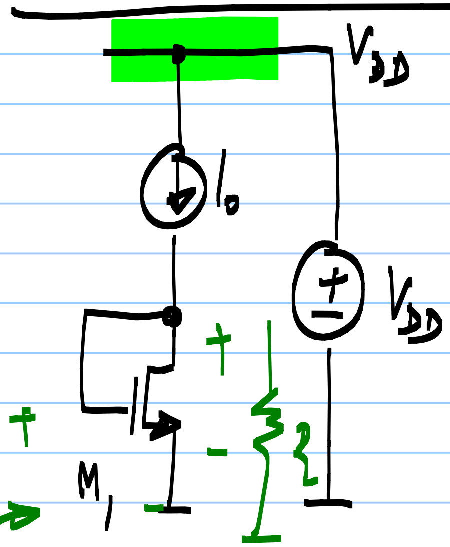
$$= \sqrt{2 \cdot K_n \cdot I_D}$$

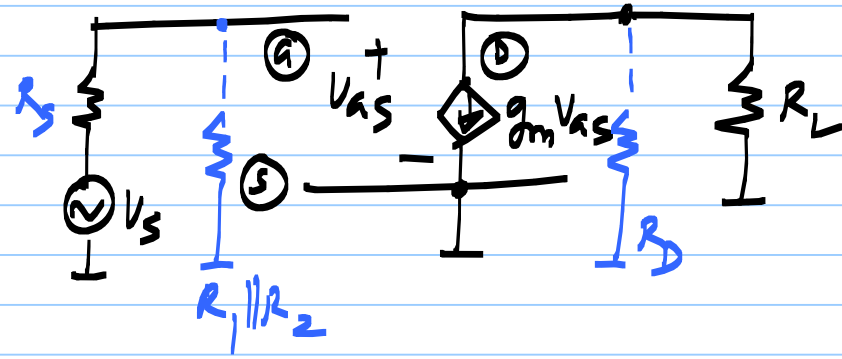
$$I_D = 200\mu A$$

$$I_o = I_D = \frac{K_n}{2}(V_{GS} - V_T)^2$$

$$V_{GS} = V_T + \sqrt{\frac{2 \cdot I_D}{K_n}}$$

Common source amplifier using drain feedback bias



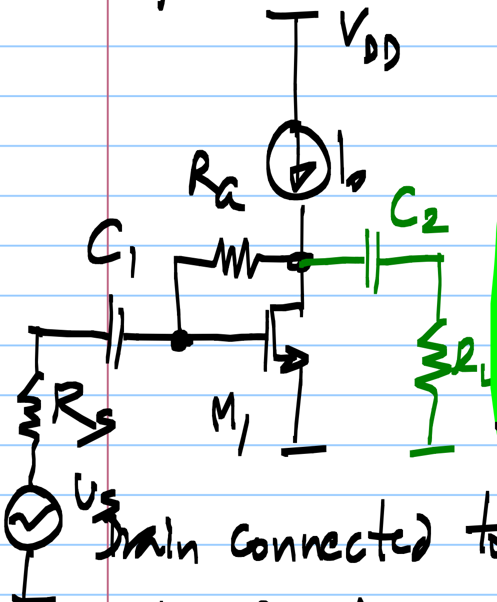


V_S appears across gate-source

load appears across drain source

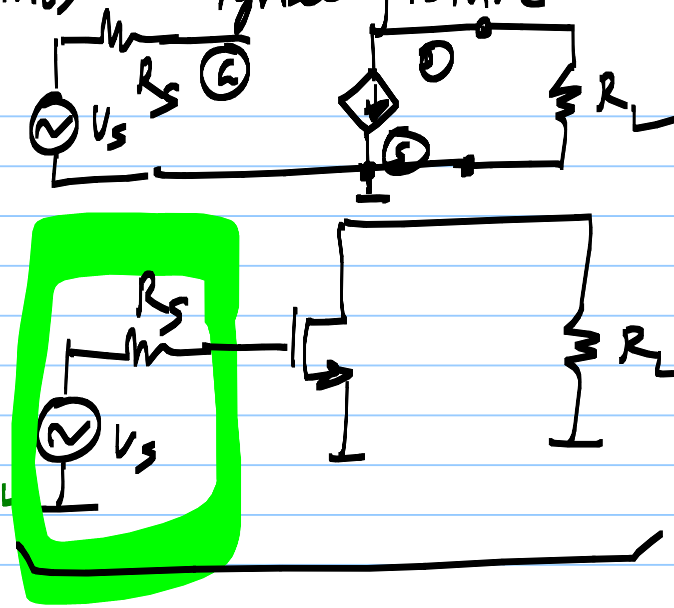
signal is at a frequency $\geq \underline{\omega_0}$

Biasing (op. point) picture



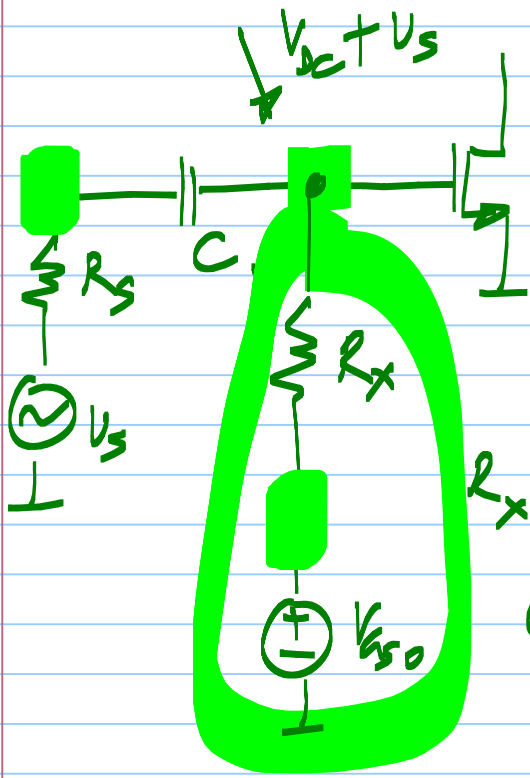
Drain connected to gate for biasing

Signal picture



Gate — dc → to drain; ac → (v_s, R_s) signal source

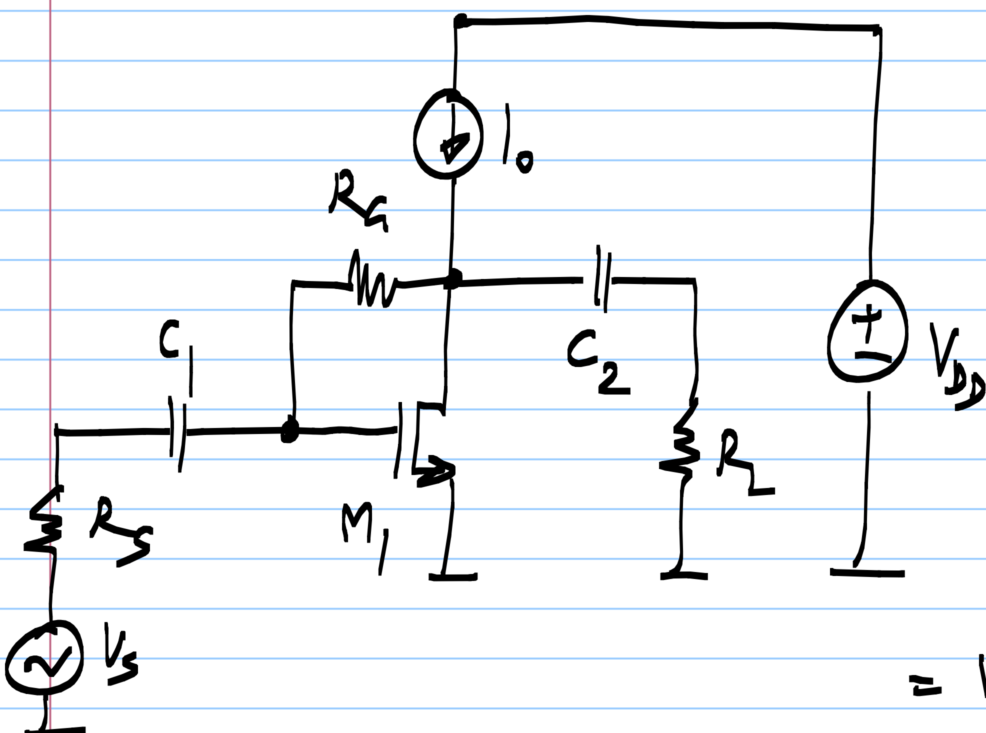
CS amplifier



$R_x \gg R_s$

$C_1 \gg \dfrac{1}{\omega_0 (R_s + R_x)}$

CS amplifier using drain feedback:

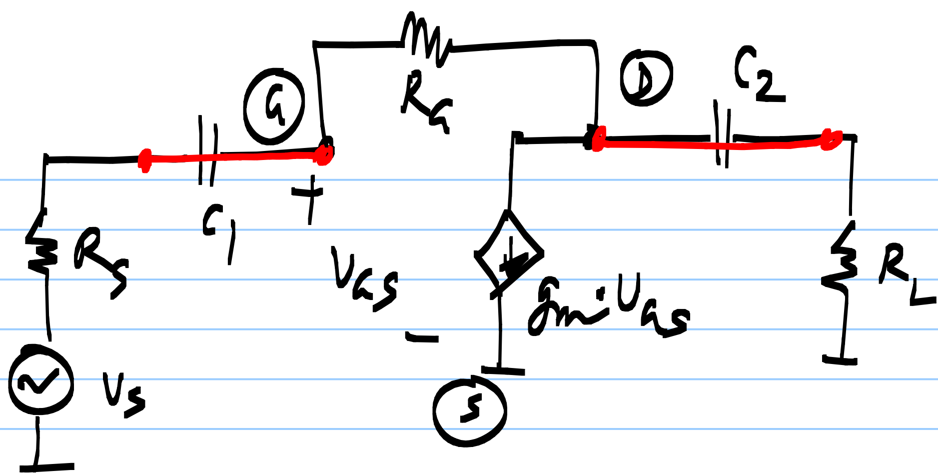


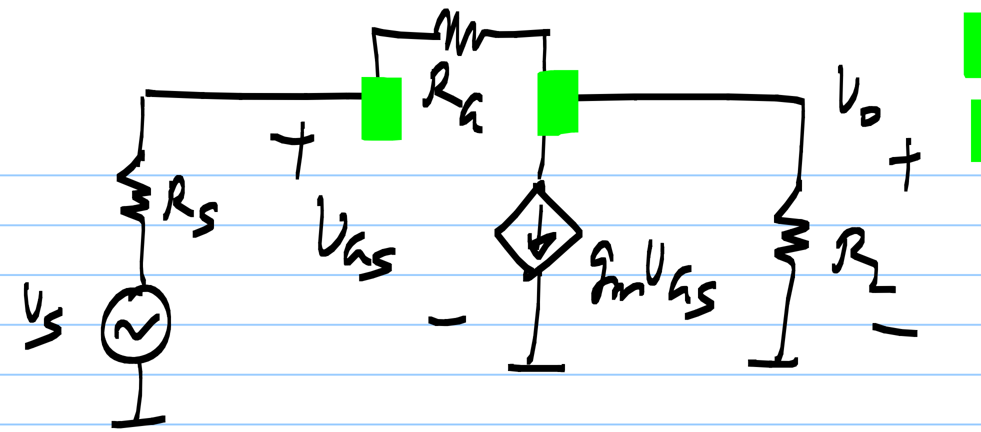
Determine constraints on C_1 & C_2

Analyze the effect of R_G

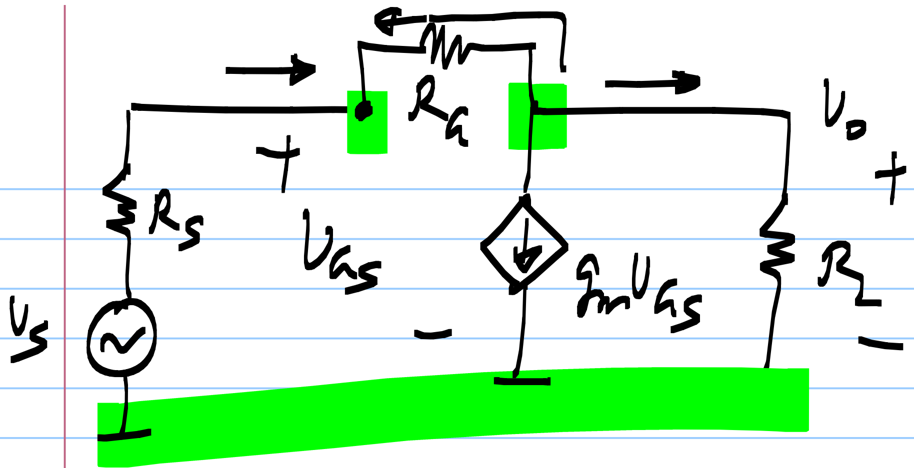
op. point

$$V_{GS} = V_{DS}$$
$$= V_T + \sqrt{\frac{2 \cdot I_0}{\mu_n C_{ox} W/L}}$$





C_1, C_2: short circuits for the desired signal frequency w_0



$$\frac{v_s - v_{GS}}{R_s} + \frac{v_o - v_{GS}}{R_G} = 0$$

$$g_m v_{GS} + \frac{v_o - v_{GS}}{R_G} + \frac{v_o}{R_L} = 0$$

Solve for v_{GS}, v_o

$$\frac{v_o}{v_s} = -\frac{(g_m R_G R_L - R_L)}{g_m R_L R_s + R_s + R_L + R_G}$$

CS. Amp: $\frac{v_o}{v_s} = -g_m R_L$

$$\frac{v_o}{v_s} = -\frac{(g_m R_a R_L - \cancel{R_L})}{g_m \cancel{R_L} R_s + \cancel{R_s} + \cancel{R_L} + R_a} \simeq -\frac{g_m R_a R_L}{R_a} = -g_m R_L$$

$R_s = 0$

$$-\frac{g_m R_a R_L - \cancel{R_L}}{\cancel{R_L} + R_a}$$

$R_a \gg g_m R_L R_s,\ R_s,\ R_L$

$R_a \gg g_m R_L R_s$ OR R_L

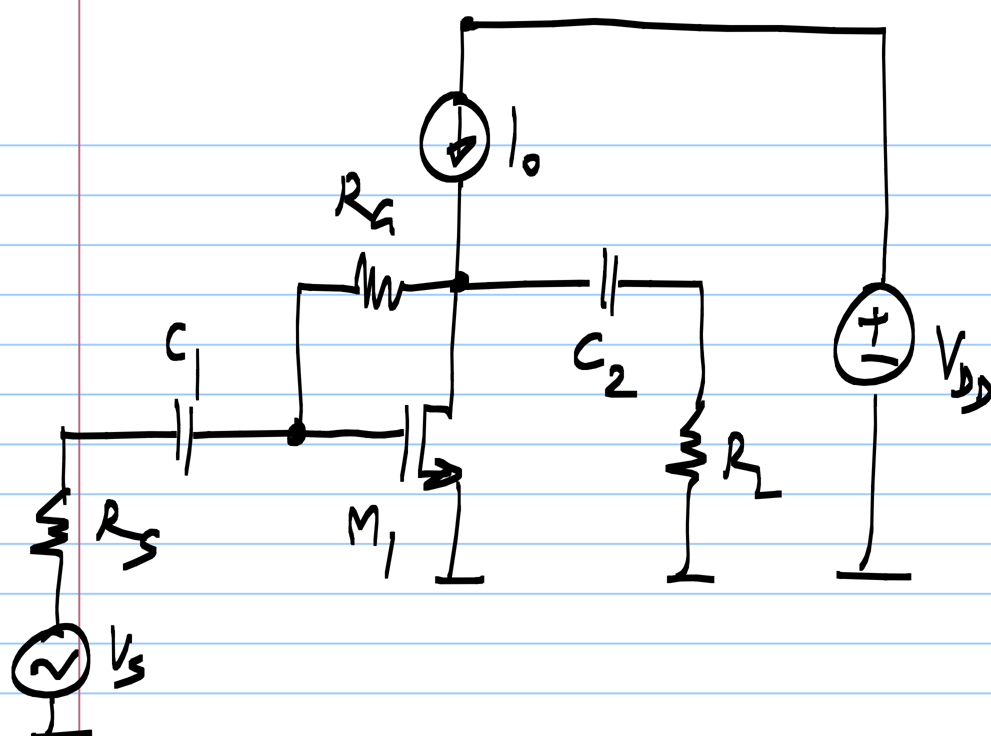
$g_m R_a R_L \gg R_L$

$g_m R_a \gg 1$

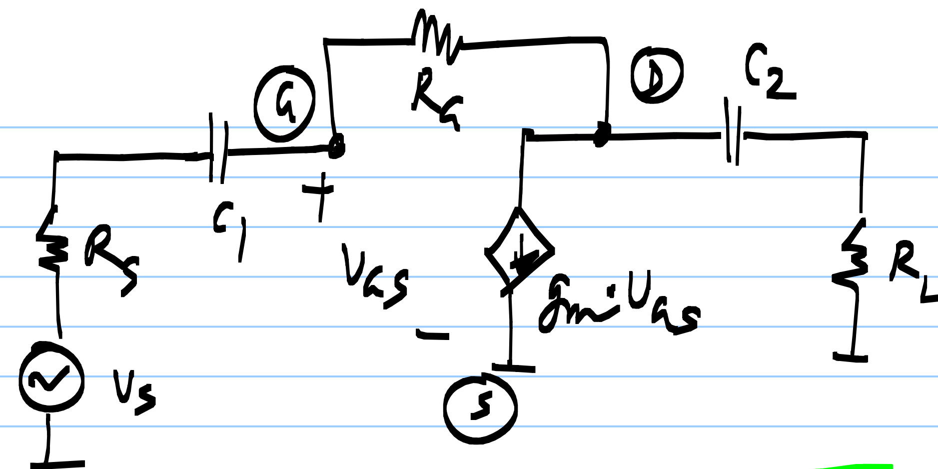
$g_m R_L > 1$

$R_a \gg g_m R_L R_s$

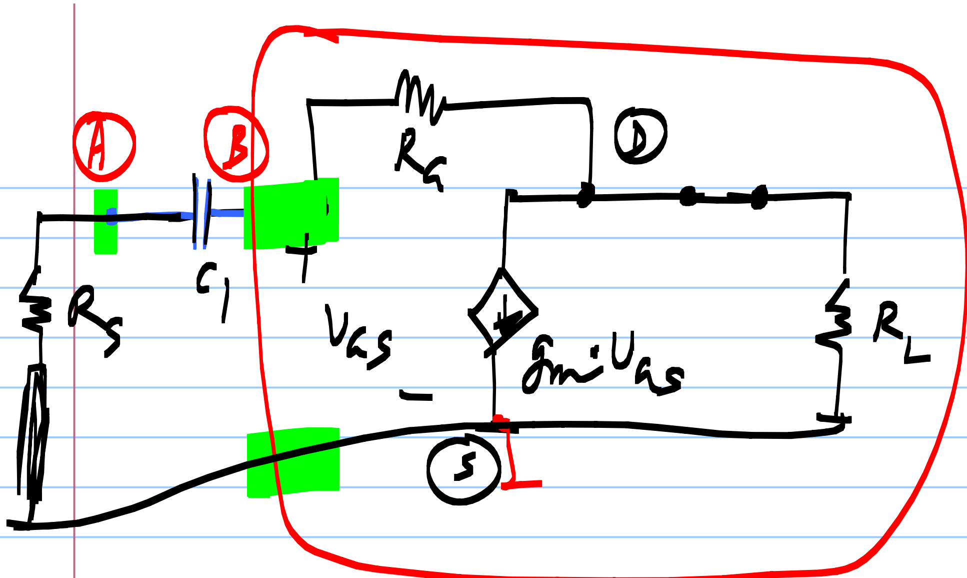
If R_s is very small, $R_a \gg R_L$

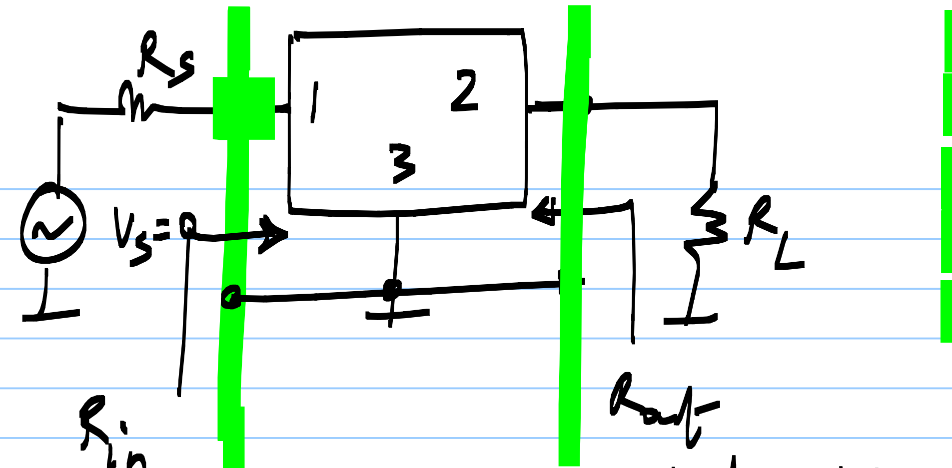


I_0
R_G
C_1
C_2
V_{DD}
M_1
R_L
R_S
V_S



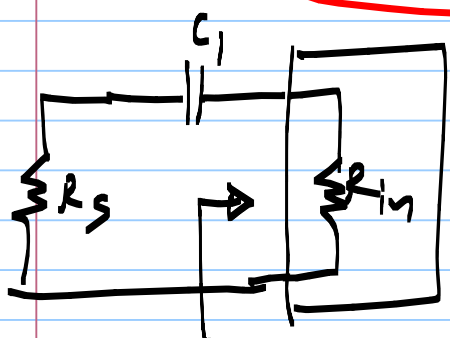
R_G
G
D
C_2
C_1
R_S
V_{GS}
+
−
$g_m \cdot V_{GS}$
R_L
V_S
S





input resistance

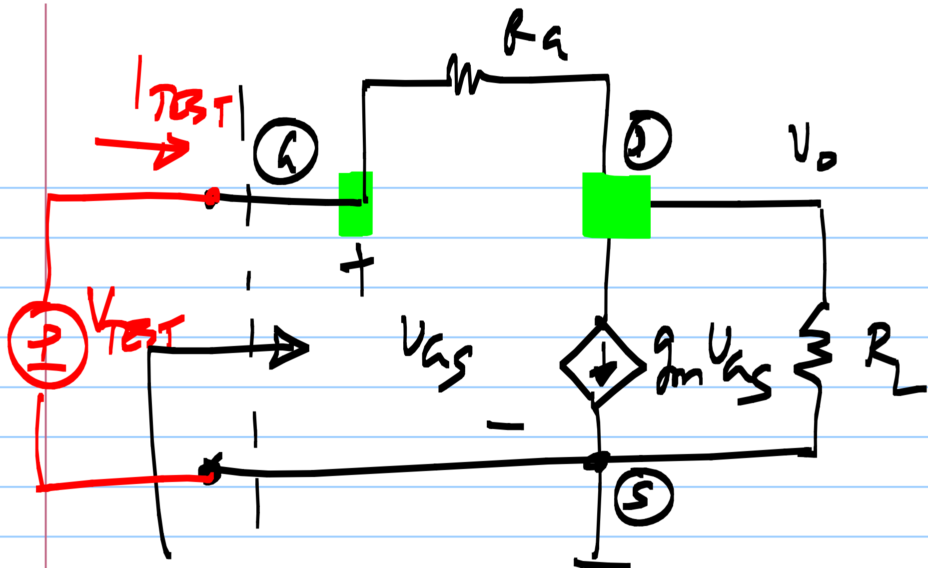
output resistance



$R_s + R_{in}$

$$\frac{1}{\omega_0 C_1} \ll R_s + R_{in}$$

$$\underbrace{C_1 \gg \frac{1}{\omega_0 (R_s + R_{in})}}$$



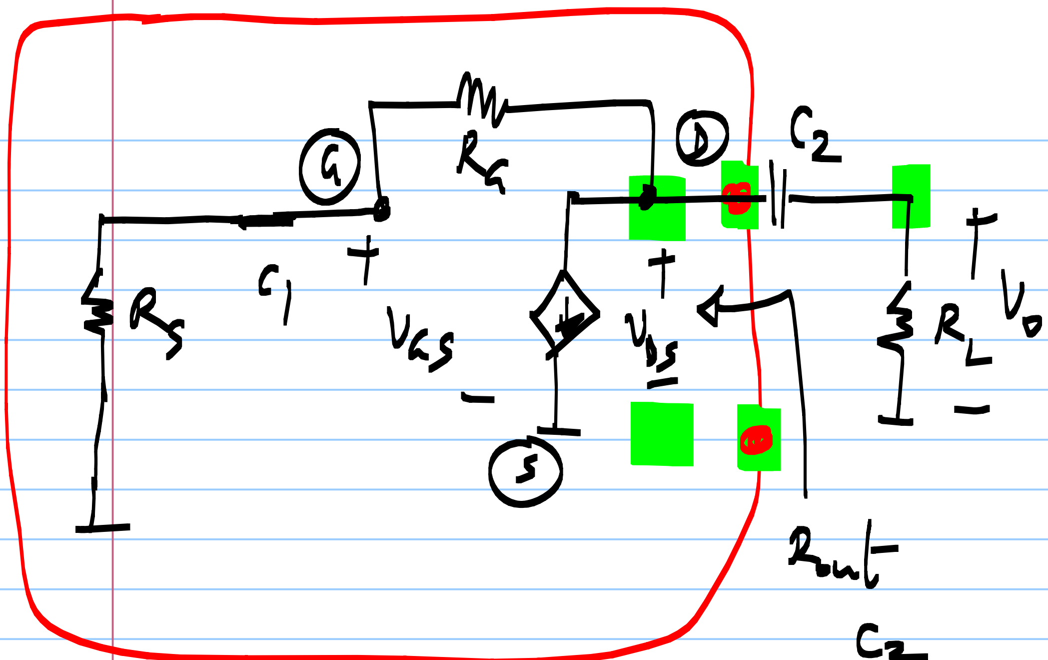
$$R_{in} = \frac{V_{TEST}}{I_{TEST}}$$

$$\frac{V_{TEST} - V_o}{R_G} = g_m \cdot V_{TEST} + \frac{V_o}{R_L}$$

$$I_{TEST} = \frac{V_{TEST} - V_o}{R_G}$$

$$R_{in} = \frac{V_{TEST}}{I_{TEST}} = \frac{R_G + R_L}{g_m R_L + 1}$$

$$C_1 \gg \frac{1}{\frac{1}{w_0\left(R_S + \frac{R_G + R_L}{g_m R_L + 1}\right)}}$$

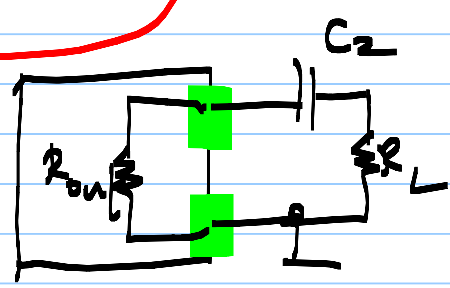


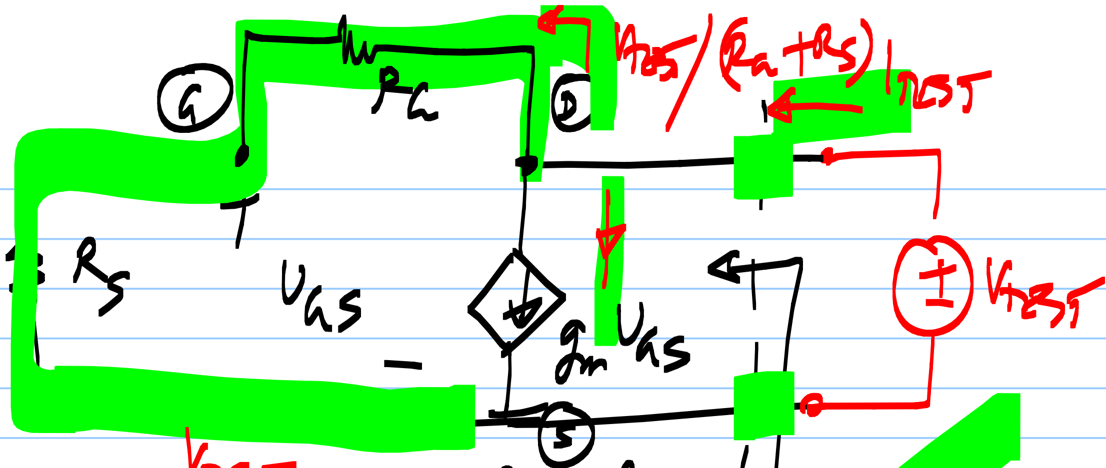
$$\frac{1}{\omega_0 C_2} \ll R_L + R_{out}$$

$$C_2 \gg \frac{1}{\omega_0 (R_L + R_{out})}$$

$$C_2 \gg \frac{1}{\omega_0 R_L}$$

(stronger condition)

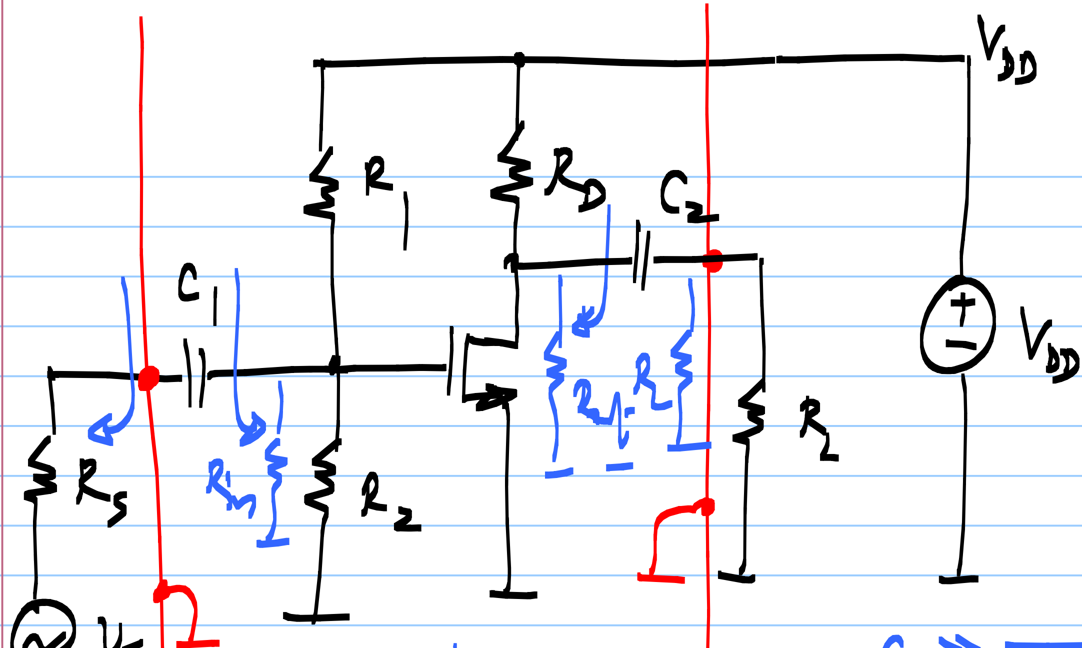


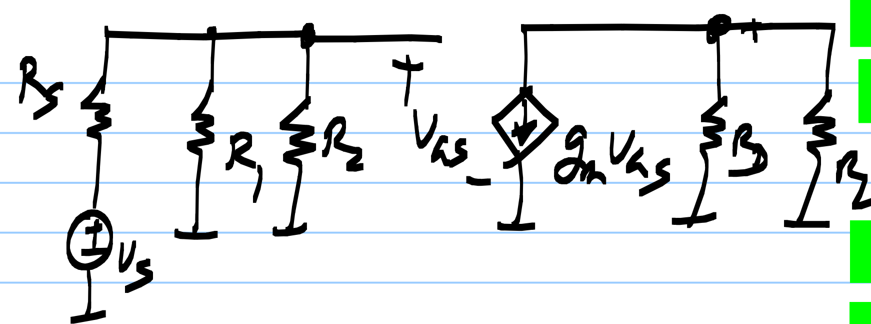


$$I_{TEST} = \frac{V_{TEST}}{R_G+R_S} + g_m \cdot V_{TEST} \cdot \frac{R_S}{R_G+R_S}$$

$$R_{out} = \frac{V_{TEST}}{I_{TEST}} = \frac{R_G+R_S}{g_m R_S + 1}$$

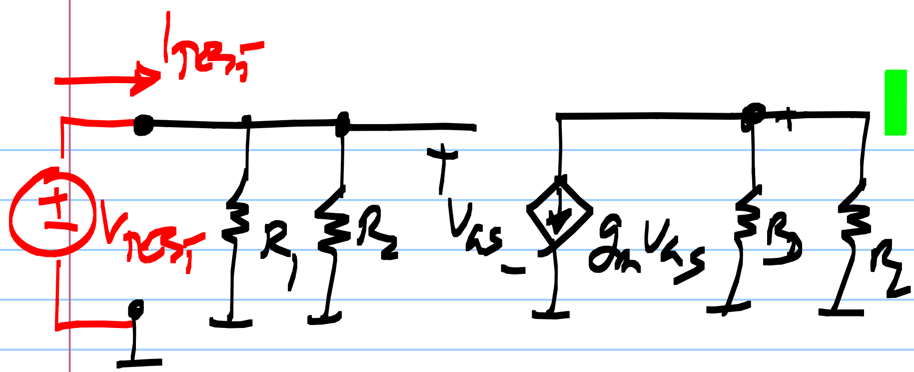
$$V_{GS} = \underbrace{\frac{R_S}{R_G+R_S}} \cdot V_{TEST}$$

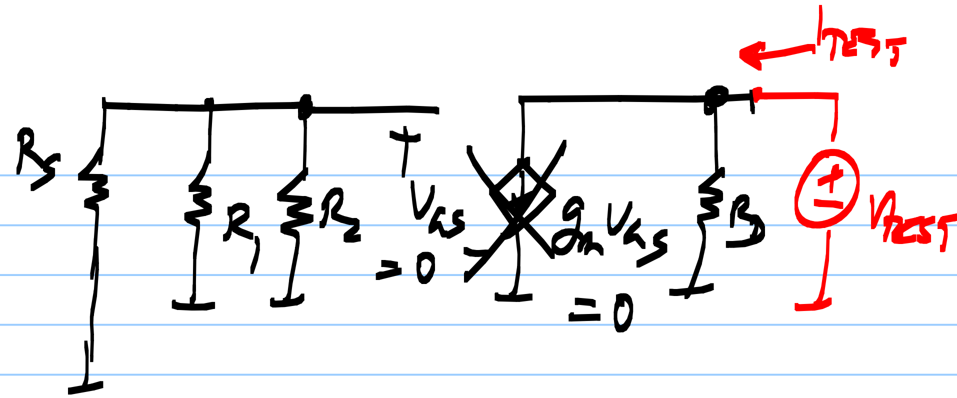




$$C_1 \gg \underbrace{\frac{1}{\omega_o (R_s + R'_{in})}}$$

$$C_2 \gg \underbrace{\frac{1}{\omega_o (R_{out} + R_L)}}$$





$$R_{in} = \frac{V_{TEST}}{I_{TEST}} = \underbrace{\underline{R_1 \| R_2}}$$

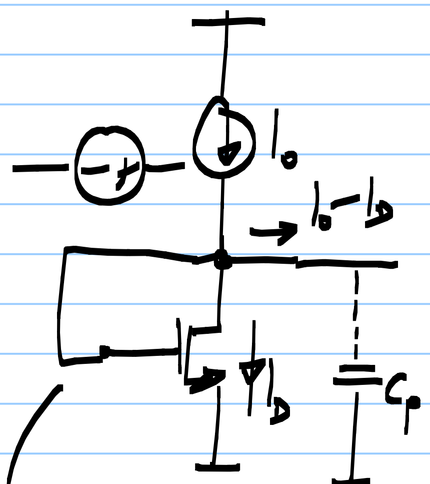
$$\underline{R_{out} = R_D}$$

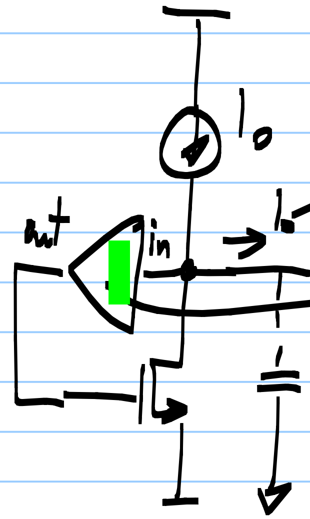
Sense @ drain

fb to gate

If $I_D < I_0$, V_D increases

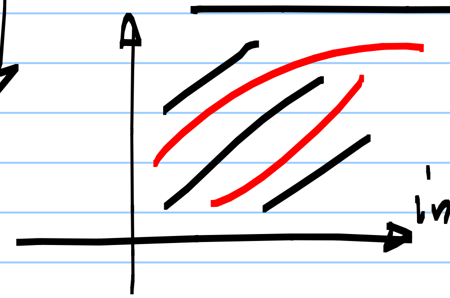
V_G must increase





V_G must increase when V_D increases

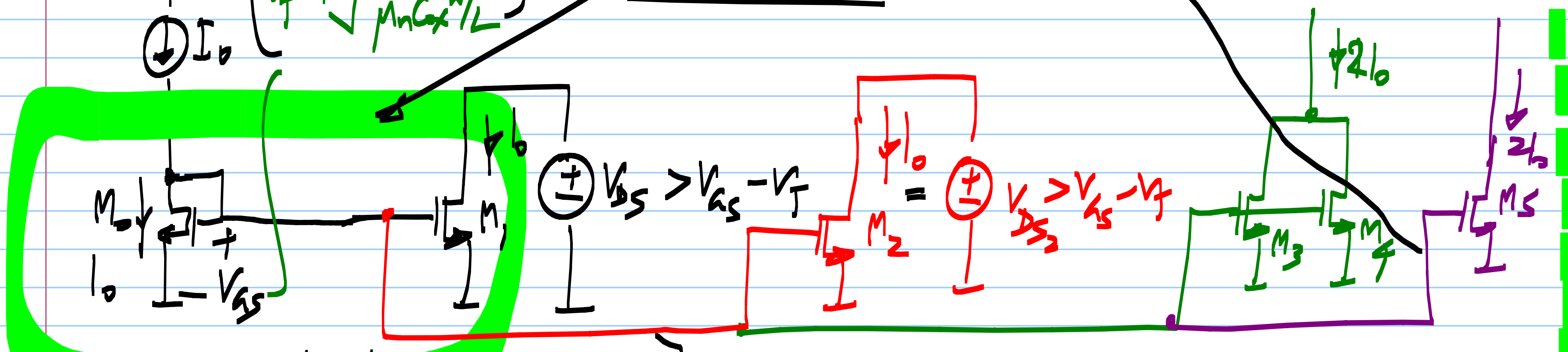
+ve incremental gain



$I_D = I_0$: $V_T + \sqrt{\frac{2 \cdot I_0}{\mu_n C_{ox} W/L}}$

$$\left(V_T + \sqrt{\frac{2I_0}{\mu_n C_{ox} W/L}}\right)$$

Current mirror — replicate & scale



On a chip, closely spaced transistors, their characteristics will be nearly identical

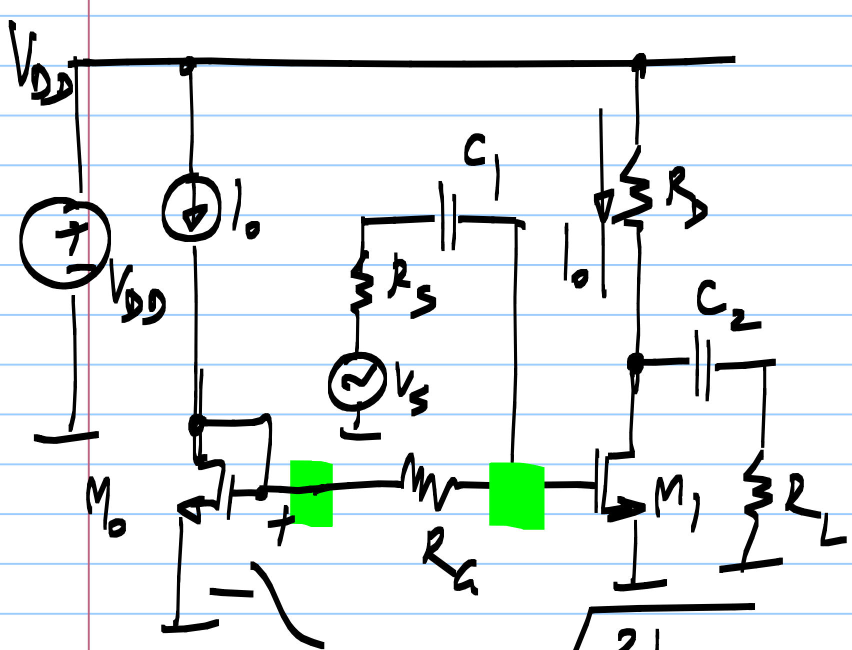
$M_0 = M_1 = M_2$

$M_0 = M_3 = M_4$

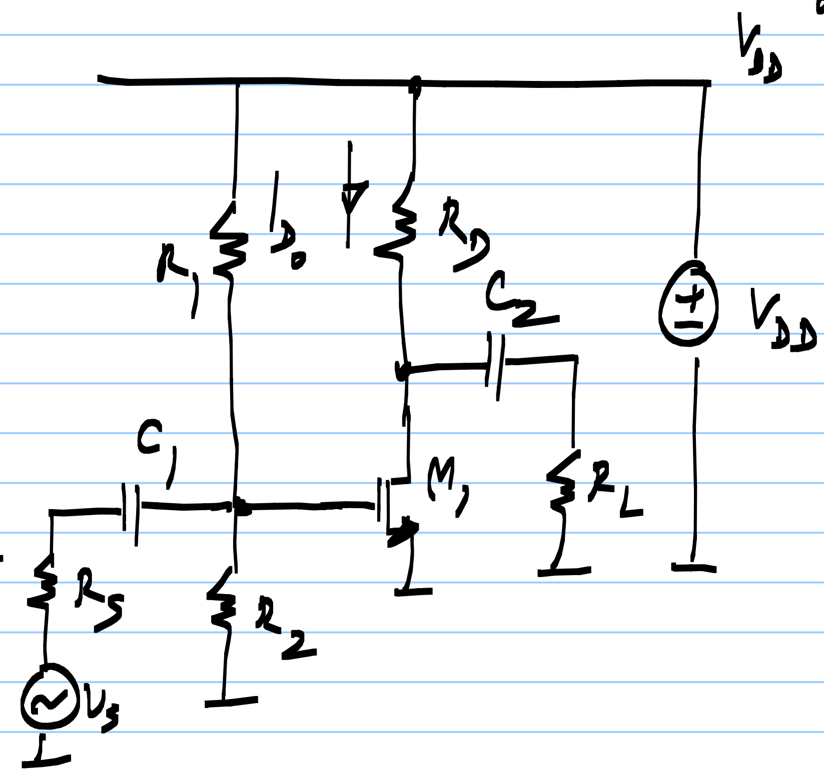
$M_0 : W/L$; $M_5 : 2W/L$

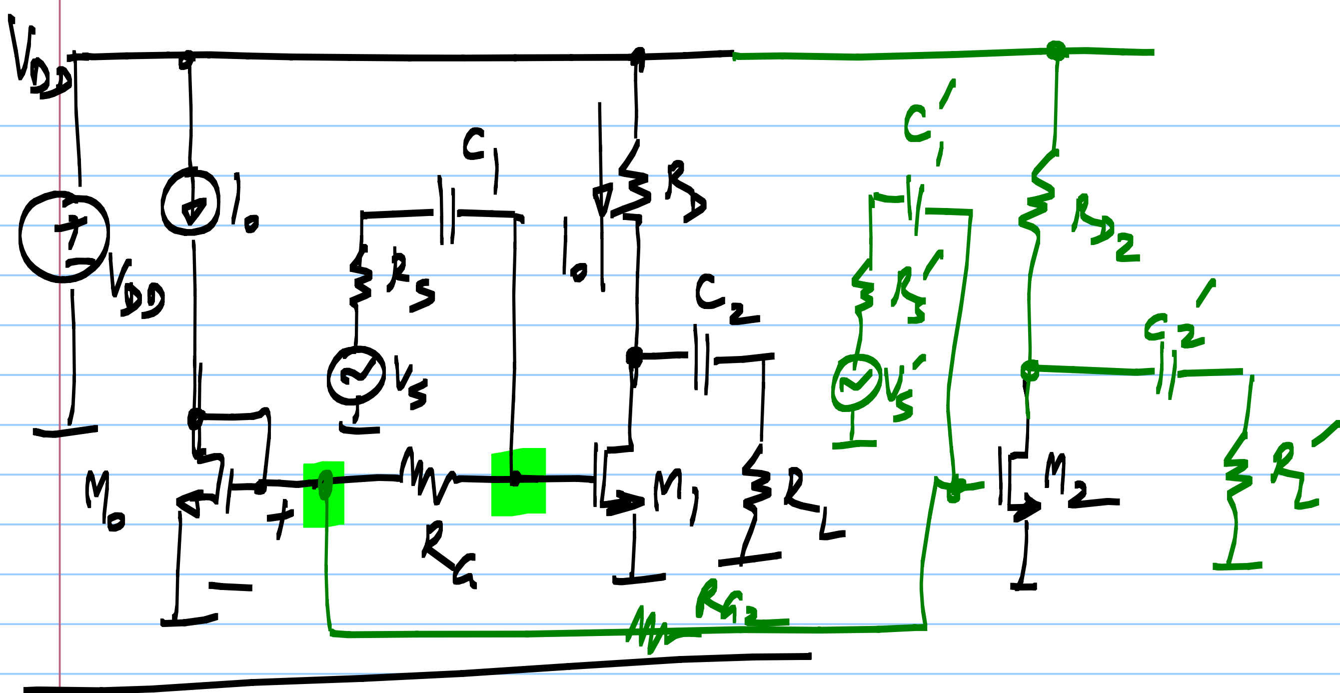
CS amplifier using current mirror bias

$V_{GS,\text{required}}\Big|_{o/p} = \dfrac{R_2}{R_1+R_2} \cdot V_{DD}$

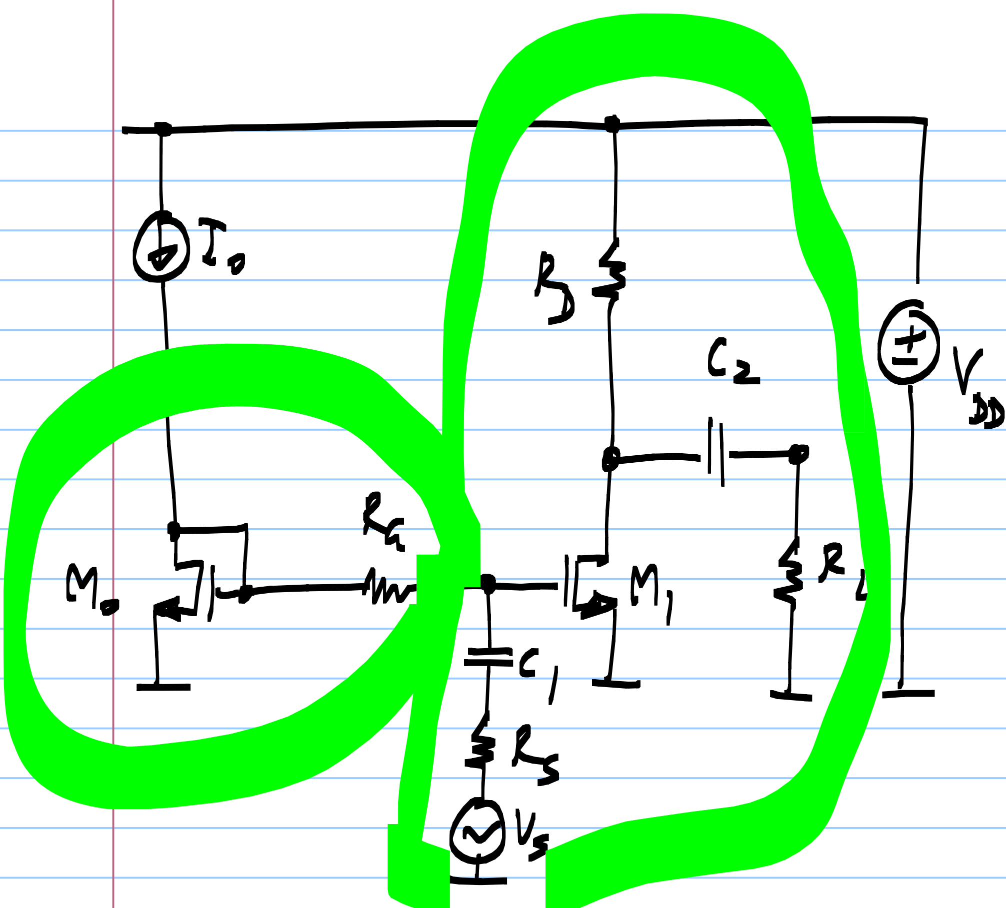


$M_1 = M_0 \qquad V_{GS} = V_T + \sqrt{\dfrac{2 I_0}{\mu_n C_{ox} W/L}}$

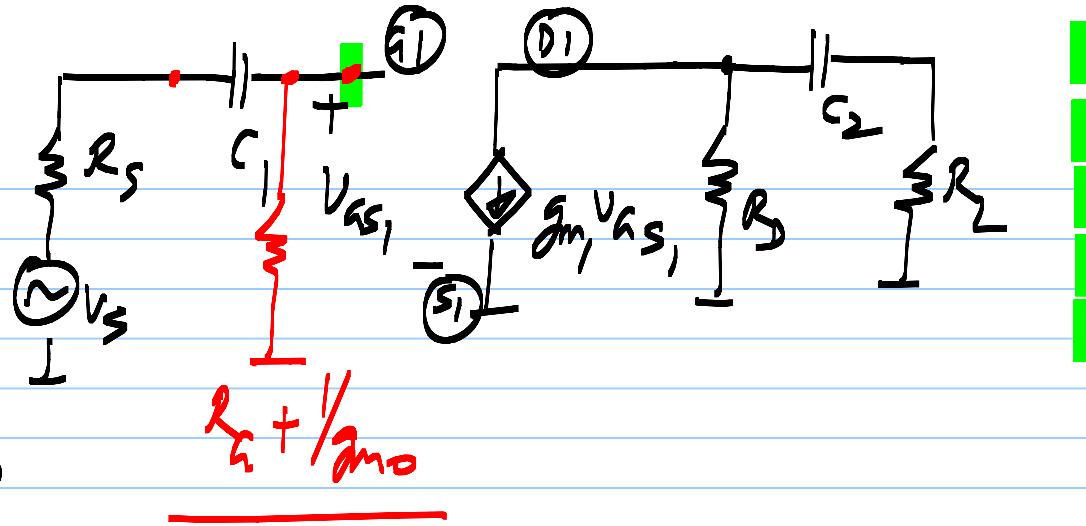




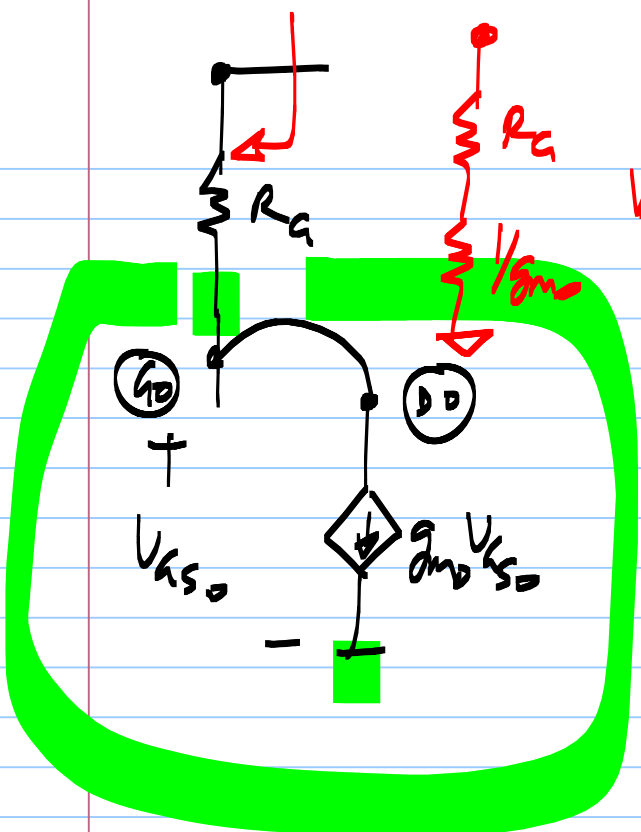
CS amplifier using current mirror bias.



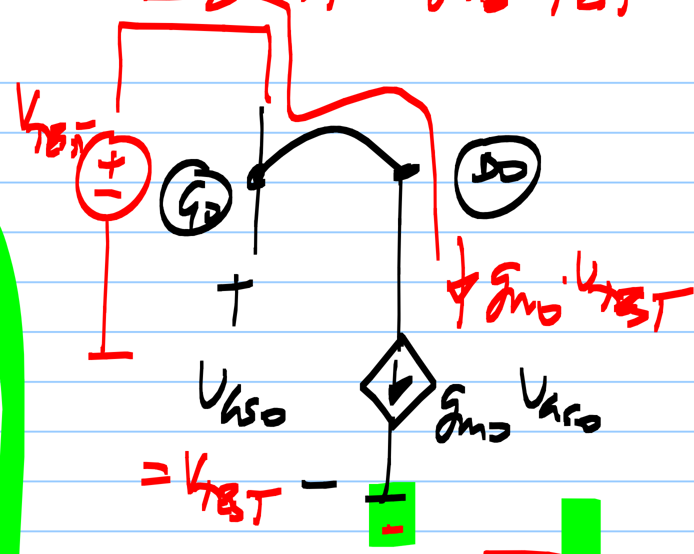
I_o
V_DD
R_D
C_2
R_G
M_o
M_1
R_L
C_1
R_S
V_s



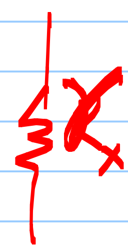
G1
D1
R_S
V_s
C_1
+
V_GS1
−
S1
g_m1 V_GS1
R_D
C_2
R_L
R_G + 1/g_m0



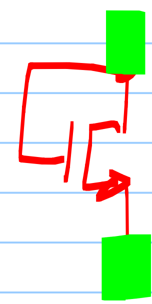
$I_{TEST} = g_{m0} \cdot V_{TEST}$

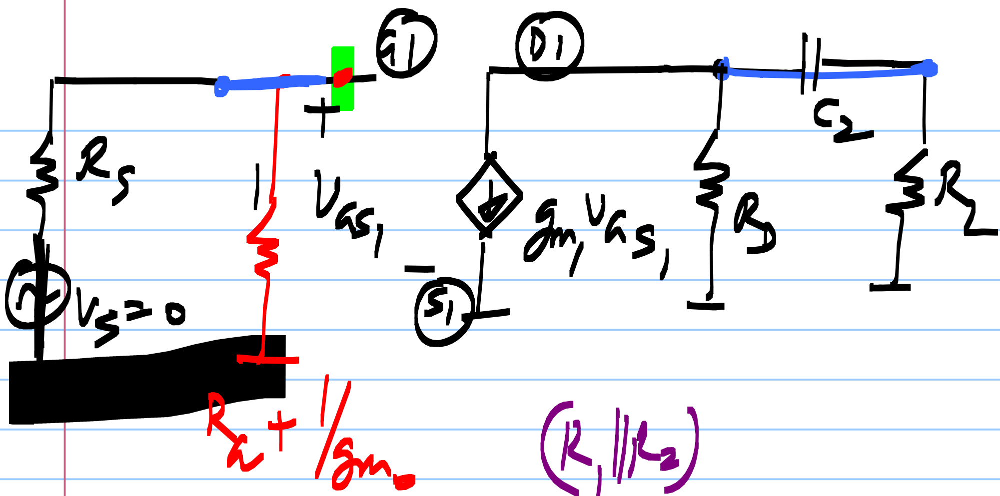


$$\frac{V_{TEST}}{I_{TEST}} = \frac{1}{g_{m0}}$$



$= \frac{1}{g_{m0}}$





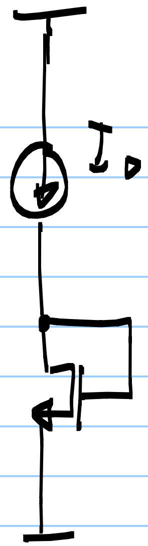
$$\frac{v_{GS_1}}{v_s} = \frac{R_G + 1/g_{m0}}{R_G + 1/g_{m0} + R_s} \simeq 1$$

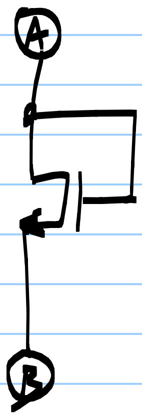
$\underbrace{R_G + 1/g_{m0} \gg R_s}$; $R_G \gg R_s$

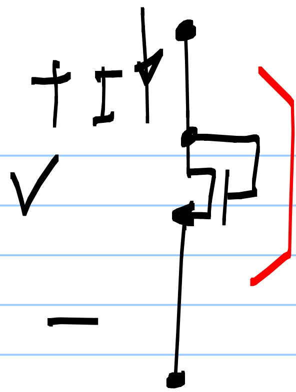
$R_1, R_2 \gg R_s$

C_1: $\frac{1}{\omega C_1} \ll \underbrace{R_s + R_G + 1/g_{m0}}$

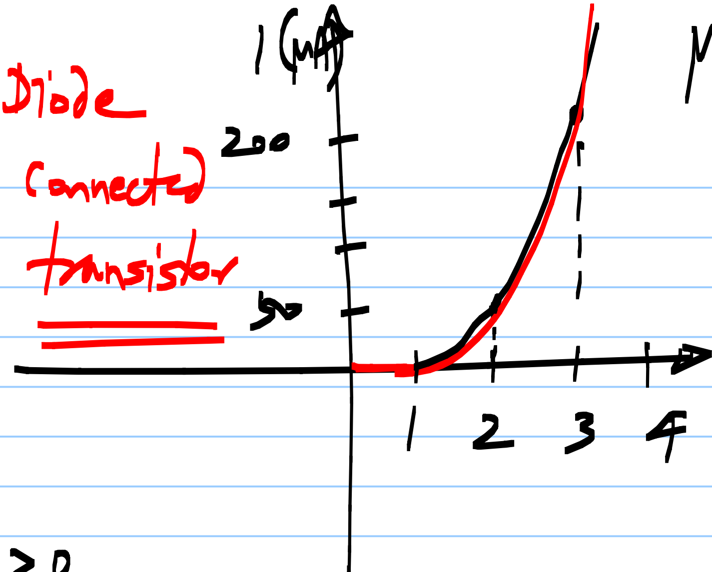
C_2: $\frac{1}{\omega C_2} \ll R_D + R_L$; $\frac{1}{\omega C_2} \ll R_L$







Diode connected transistor



$\mu_n C_{ox} W/L = 100 \mu A/V^2$

$V_T = 1V$

$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_T)^2$

$(I) \qquad (V)$

$V_{DS} > 0, \ V_{GS} > 0$

$V_T > 0$

saturation

cutoff

$V_{DS} < V_{GS} - V_T$ ✗

$V_{DS} = V_{GS}$

Diode connected transistor: incremental picture