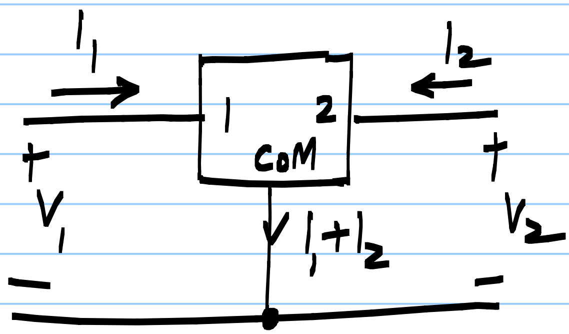
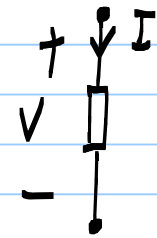
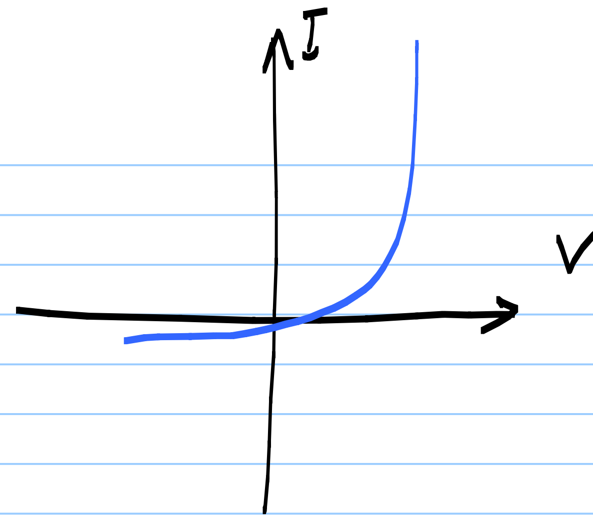
Two port nonlinear elements



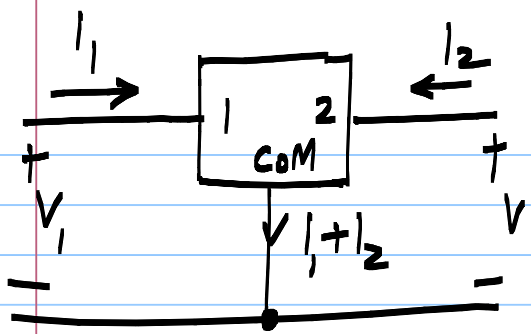


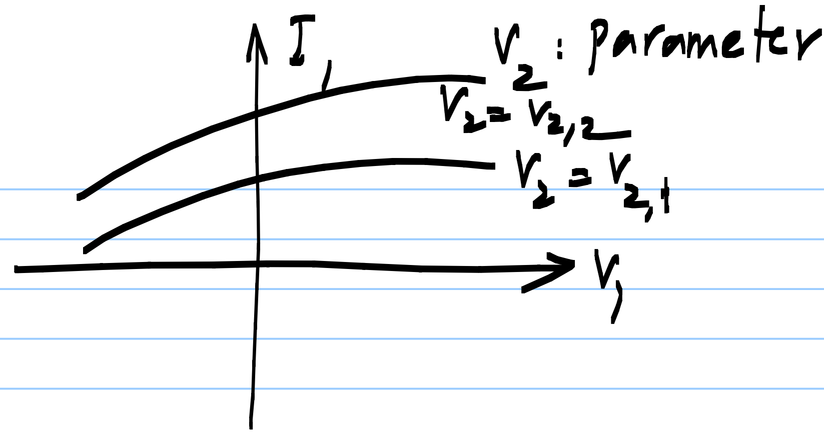


$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2) \\ I_2 &= f_2(V_1, V_2) \end{aligned}\right]$$

Representation of a two port nonlinear element

$I = f(V)$





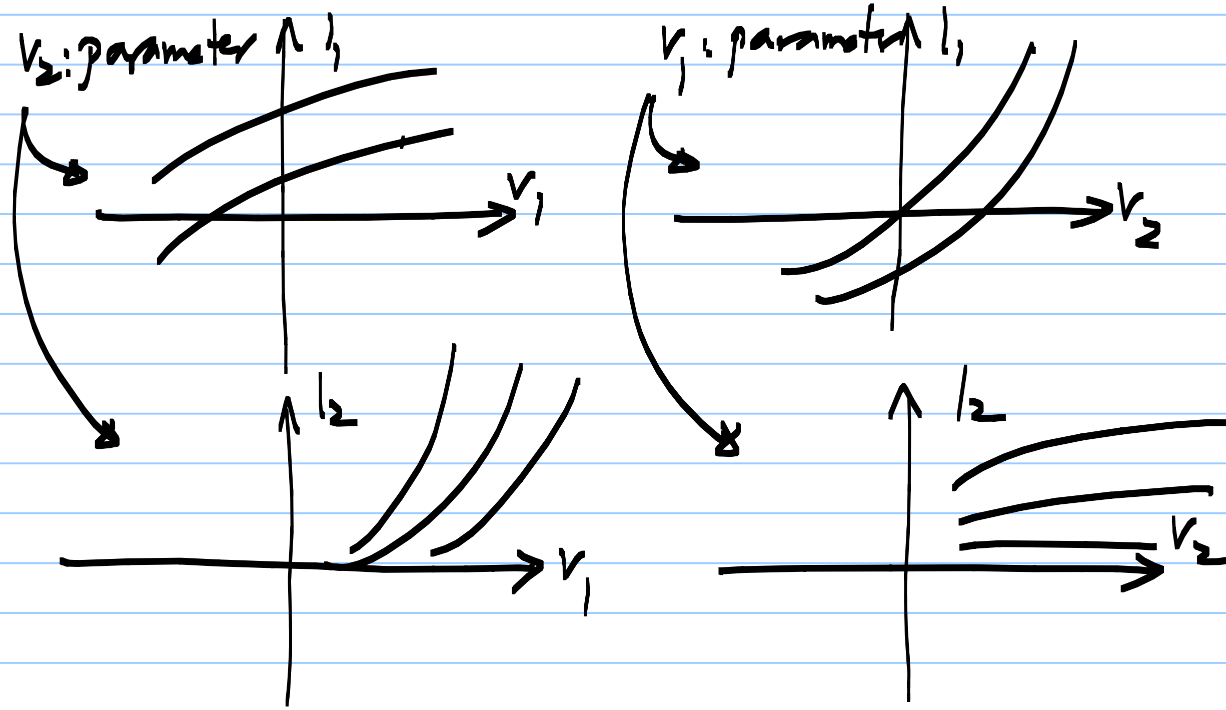
$$I_1 = f_1(V_1, V_2)$$
$$I_2 = f_2(V_1, V_2)$$

plot I_1 vs. V_1 keeping V_2 constant

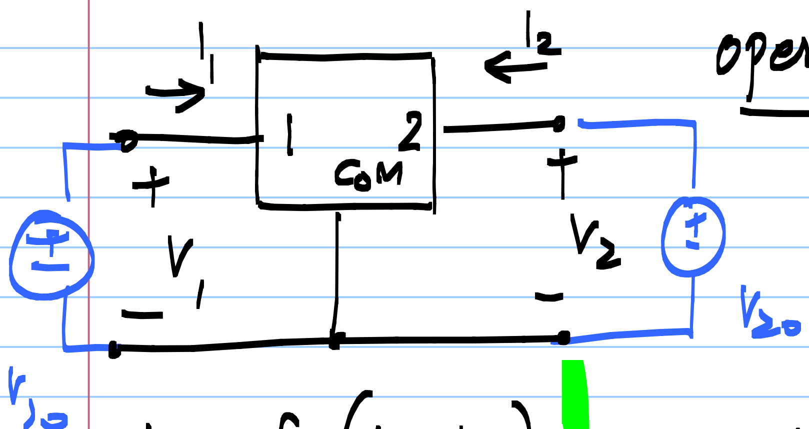
Vary V_2 & repeat the plot

Graphical representation of a two port nonlinearity:

4 plots:



Small signal equivalent of a two port nonlinearity



$$I_1 = f_1(V_1, V_2)$$

$$I_2 = f_2(V_1, V_2)$$

<u>operating point:</u> $V_{10}, V_{20} \Rightarrow I_{10}, I_{20}$

$$I_{10} = f_1(V_{10}, V_{20})$$

$$I_{20} = f_2(V_{10}, V_{20})$$

<u>op. point + increments</u> : $\underbrace{V_{10} + \Delta V_1}_{}^{v_1}, \; V_{20} + \Delta V_2$

$\underbrace{I_{10} + \Delta I_1}_{i_1}, \; I_{20} + \Delta I_2$

$$I_{10} = f_1(V_{10}, V_{20}) \quad \text{op. point}$$

$$I_{20} = f_2(V_{10}, V_{20})$$

$$\cancel{I_{10}} + \underbrace{\Delta I_1} = f_1(V_{10} + \Delta V_1, V_{20} + \Delta V_2)$$

$$I_{20} + \Delta I_2 = f_2(V_{10} + \Delta V_1, V_{20} + \Delta V_2)$$

Expand in a Taylor series around the op. point

$$= \cancel{f_1(V_{10}, V_{20})} \quad \text{op. point}$$

$$+ \left.\frac{\partial f_1}{\partial V_1}\right|_{op} \cdot \underbrace{\Delta V_1} + \left.\frac{\partial f_1}{\partial V_2}\right|_{op} \cdot \underbrace{\Delta V_2}$$

linear (first order) terms

$$+ (\ )\Delta V_1^2 + (\ )\Delta V_2^2 + (\ )\Delta V_1 \Delta V_2$$

higher order terms

neglected ($\Delta V_1, \Delta V_2$ small enough)

Incremental picture of a nonlinear two-port

$$\left.\begin{aligned}
\Delta I_1 &= \left.\frac{\partial f_1}{\partial V_1}\right|_{op} \cdot \Delta V_1 + \left.\frac{\partial f_1}{\partial V_2}\right|_{op} \cdot \Delta V_2 \\
\Delta I_2 &= \left.\frac{\partial f_2}{\partial V_1}\right|_{op} \Delta V_1 + \left.\frac{\partial f_2}{\partial V_2}\right|_{op} \Delta V_2
\end{aligned}\right]$$

Linear two-port

ΔV_1, ΔV_2

ΔI_1, ΔI_2

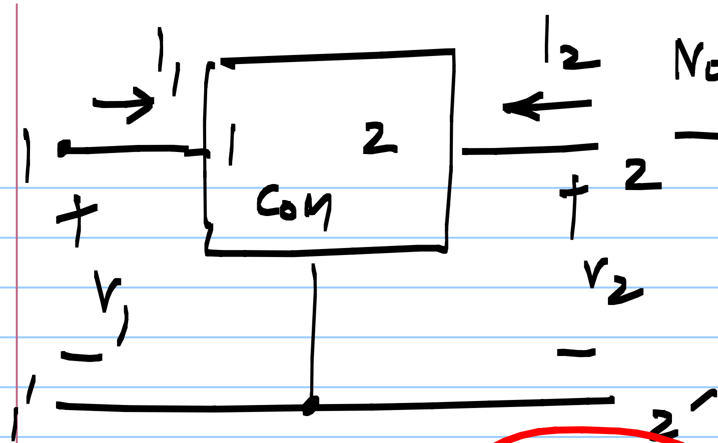
$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} :$$

Small signal (incremental) y-parameters

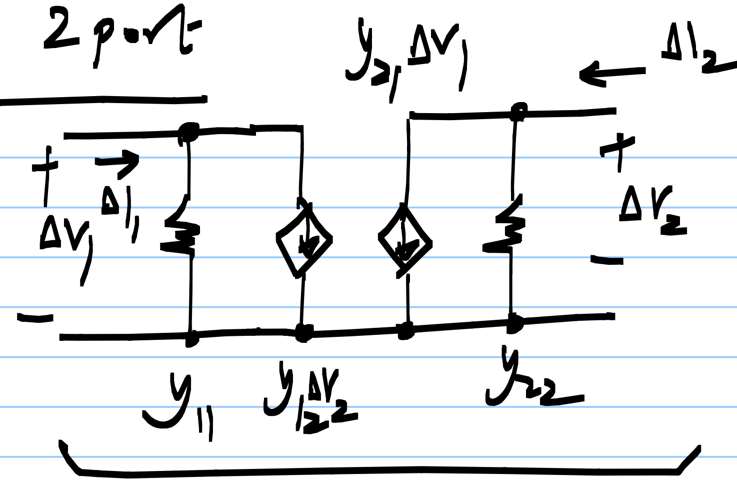
$$\Delta I_1 = y_{11} \cdot \Delta V_1 + y_{12} \Delta V_2$$

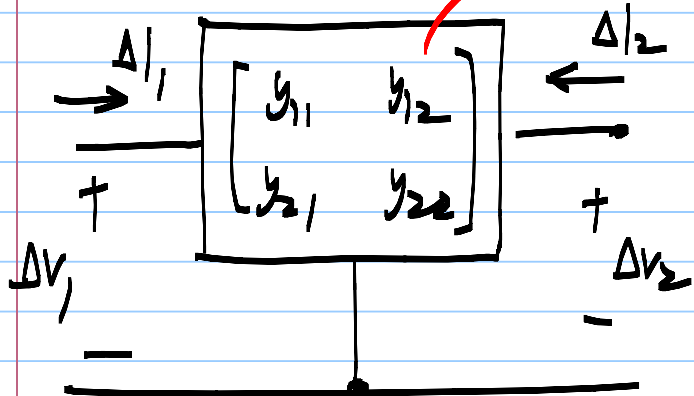
effect — cause

$$\Delta I_2 = y_{21} \Delta V_1 + y_{22} \Delta V_2$$

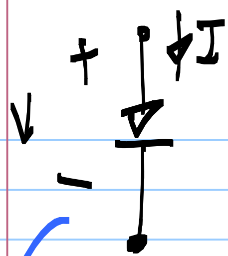


Nonlinear 2port

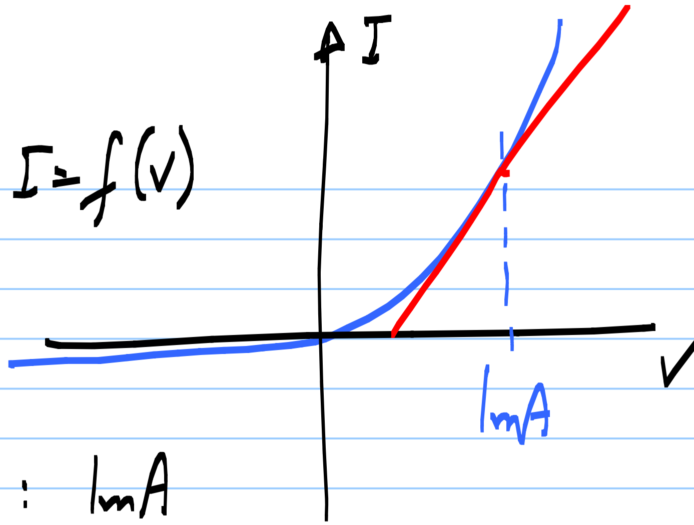




evaluated at the relevant op. point



$I = f(V)$



op. point : 1mA

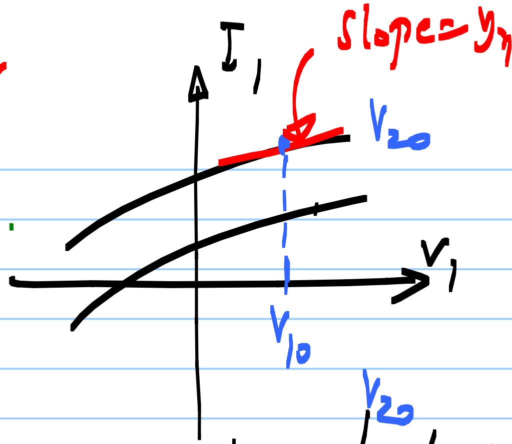
s.s. incr. eq.

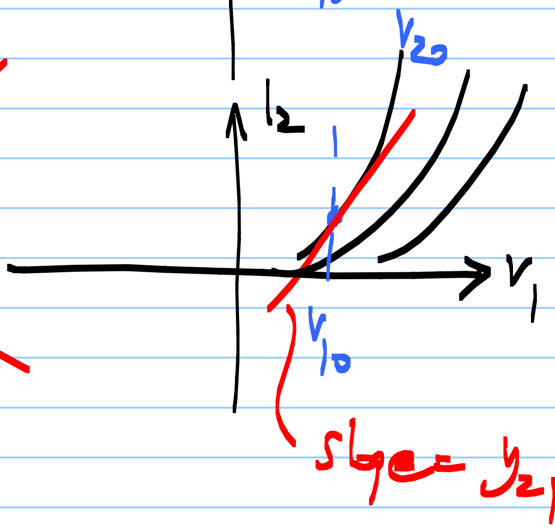
$g = f'\Big|_{op}$

$$y_{11} = \frac{\partial f_1}{\partial V_1}\Big|_{op.}$$

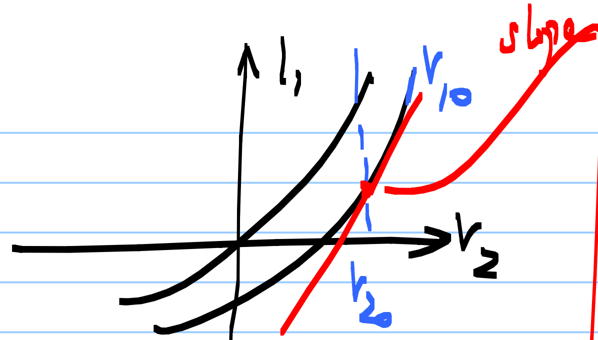
V_{10}, V_{20}

V_2: parameter



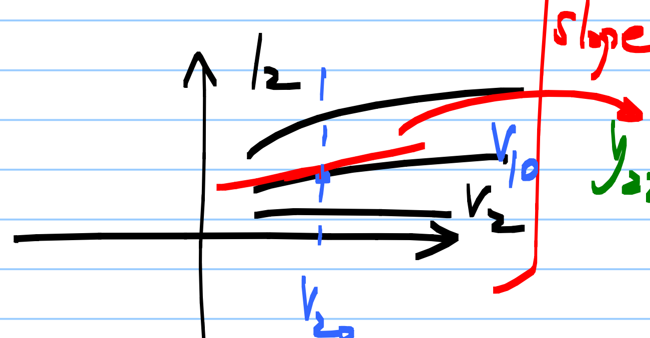


$$y_{21} = \frac{\partial f_2}{\partial V_1}\Big|_{op.}$$



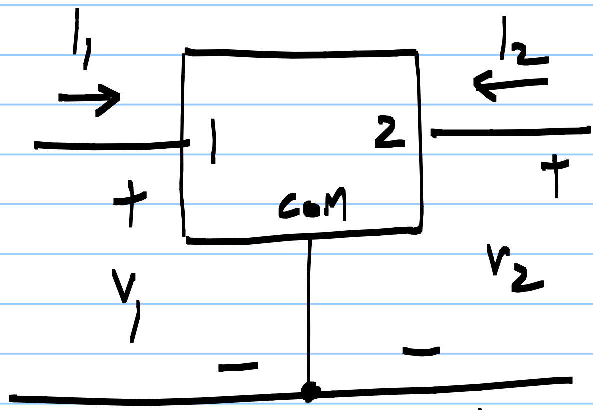
$$y_{12} = \frac{\partial f_1}{\partial V_2}\Big|_{op}$$

V_1: parameter



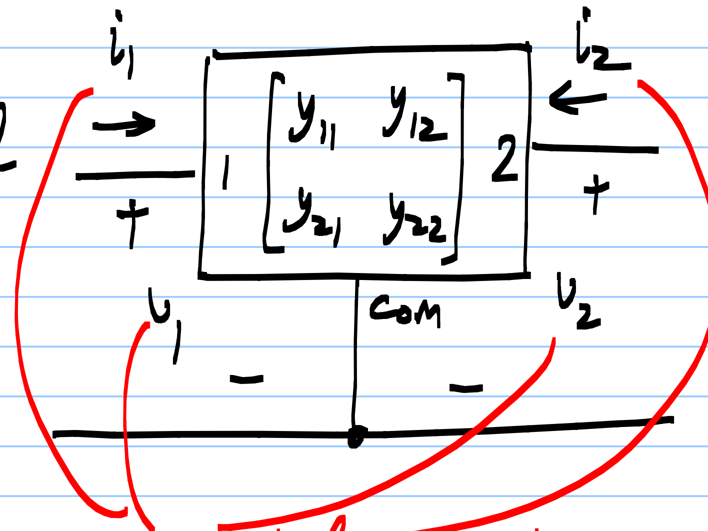
$$y_{22} = \frac{\partial f_2}{\partial V_2}\Big|_{op}$$

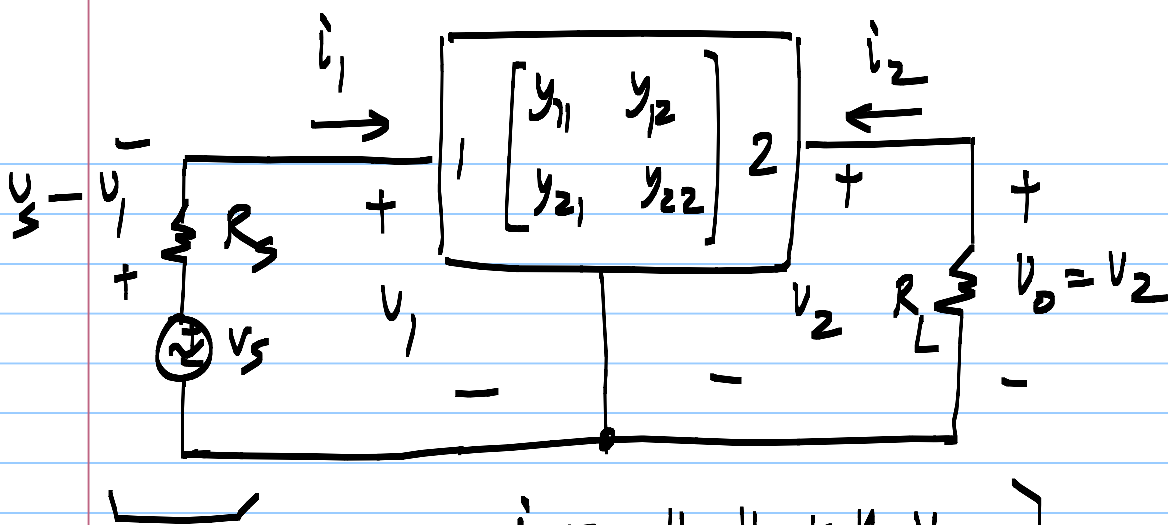
Constraints for maximizing the incremental (power) gain of a two-port network



$$I_1 = f_1(V_1, V_2) \qquad V_{10},\ I_{10}$$
$$I_2 = f_2(V_1, V_2) \qquad V_{20},\ I_{20}$$

Incremental eq. ckt





$$\left.\begin{aligned} i_1 &= y_{11} v_1 + y_{12} v_2 \\ i_2 &= y_{21} v_1 + y_{22} v_2 \\ i_1 &= (v_s - v_1) G_s \\ i_2 &= -G_L \cdot v_2 \end{aligned}\right]$$

$G_s = 1/R_s$

$G_L = 1/R_L$

$$i_1 = y_{11} v_1 + y_{12} v_2$$

$$i_2 = y_{21} v_1 + y_{22} v_2$$

$$i_1 = (v_s - v_1) G_s$$

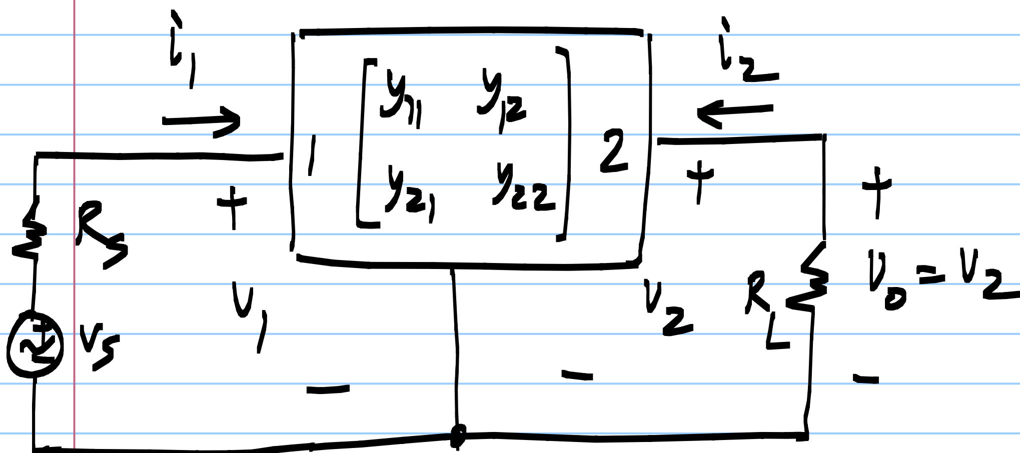
$$i_2 = -G_L \cdot v_2$$

$$v_1 = -\frac{(y_{22} + G_L)}{y_{21}} \cdot v_2$$

$$(v_s - v_1) G_s = y_{11} v_1 + y_{12} v_2$$

$$\frac{v_2}{v_s} = \frac{-y_{21} \cdot G_s}{(y_{11} + G_s)(y_{22} + G_L) - y_{12} y_{21}}$$

$$\underbrace{\text{Voltage gain}}_{\text{Power gain}} \quad \frac{v_2^2 / R_L}{\left[v_s^2 / 4R_s \right]}$$



$$\frac{v_2}{v_s} = \frac{-y_{21} \cdot G_s}{(y_{11}+G_s)(y_{22}+G_L) - y_{12}y_{21}}$$

v_s, R_s : signal source

R_L : load

$y_{11} = 0, \ y_{22} = 0$

y_{21} : as large as possible

$y_{12} = 0$ to avoid the possibility of denominator $\rightarrow 0$

Implications on the large signal (total) characteristics of the two-port

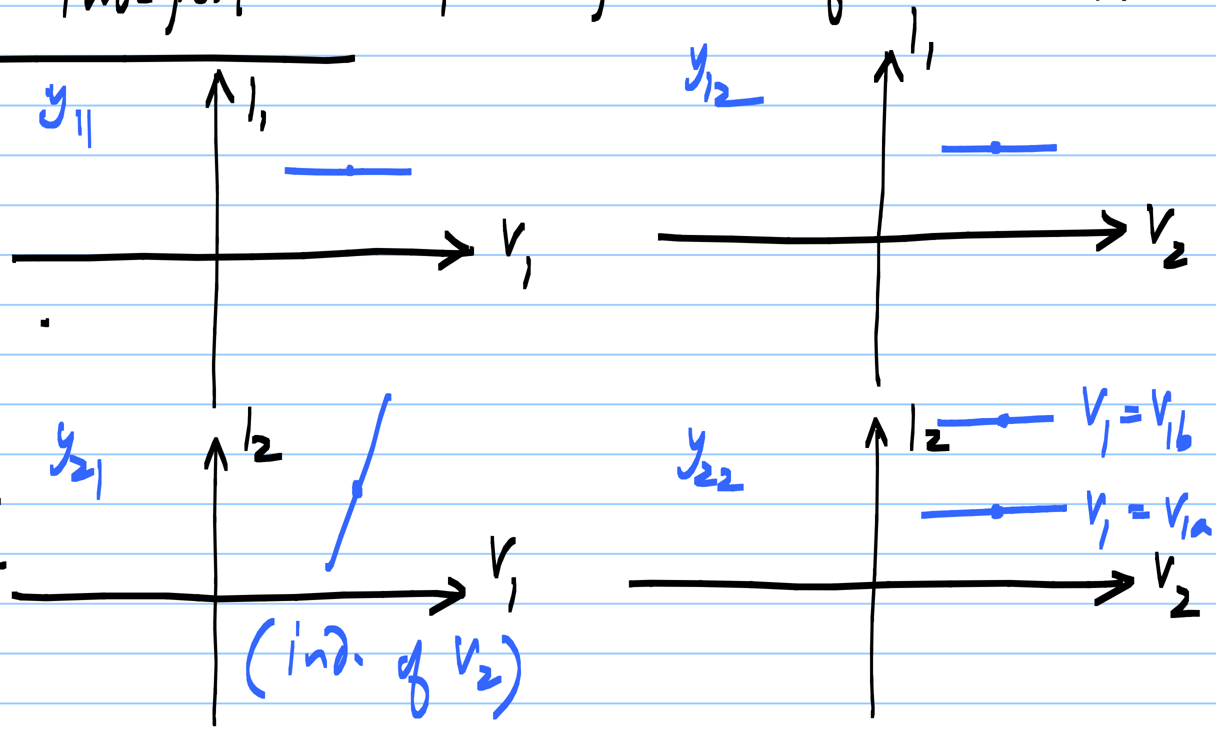
I_1: independent of both V_1, V_2

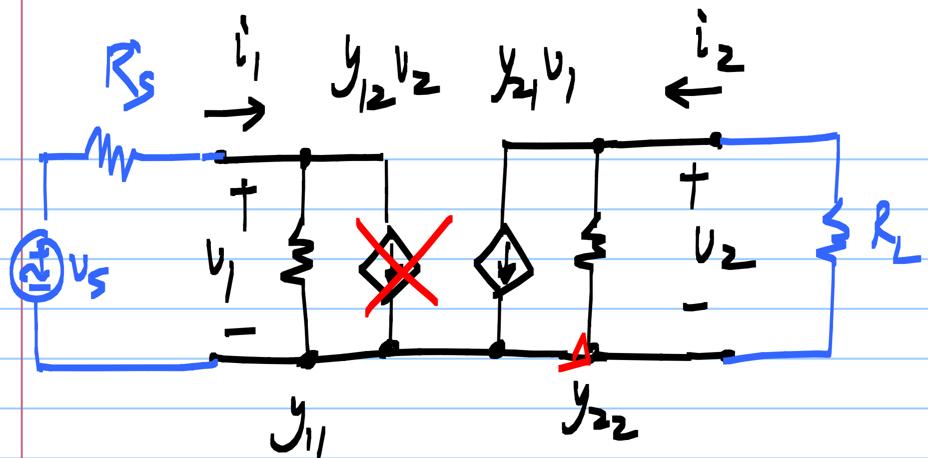
$y_{11} = 0$

$y_{22} = 0$

$y_{12} = 0$

y_{21}: large





$y_{11} = 0$: open circuited

$\Rightarrow$ no voltage division

$y_{22} = 0$: open circuited

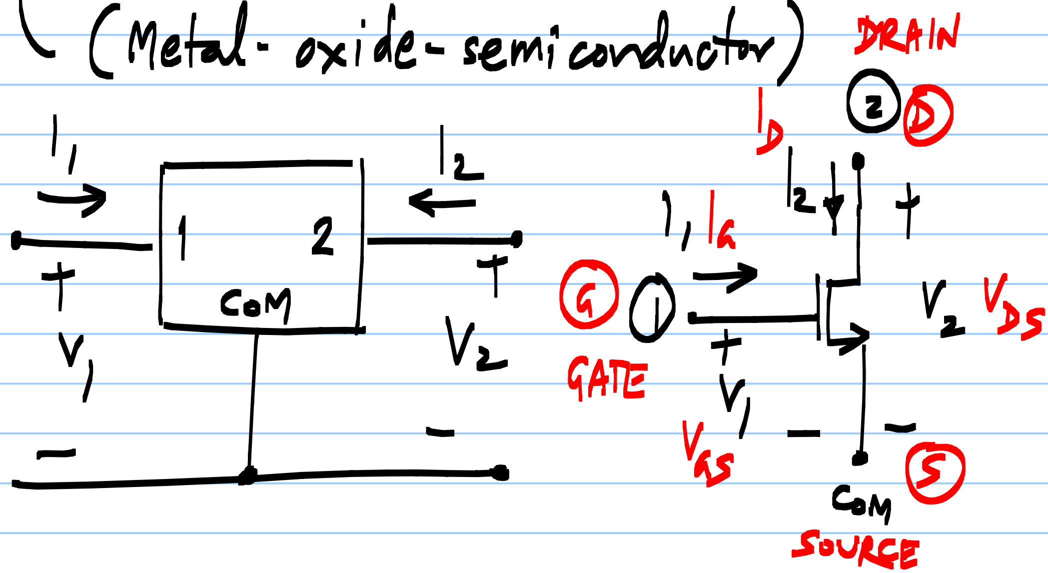
$\Rightarrow$ no current division (between y_{22} & R_L)

$y_{12} = 0$ (unconditional stability

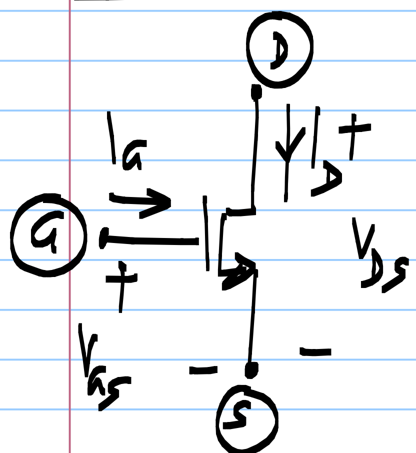
avoid gain $\to \infty$)

MOS transistor

(Metal-oxide-semiconductor)



MOS transistor characteristics:



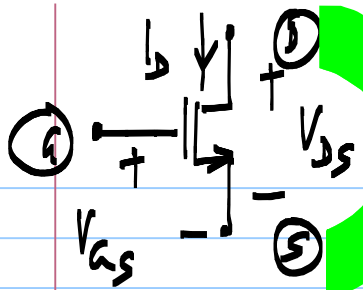
I_G must be constant (independent of both V_{GS} & V_{DS})
(I_1) @ some op. point (V_1) (V_2)

$\underline{I_G = 0}$ $\quad \underline{\text{independent of } V_{GS} \text{ \& } V_{DS}}$

$$I_D = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2$$

at some op. point

y_{21} : large, ✓

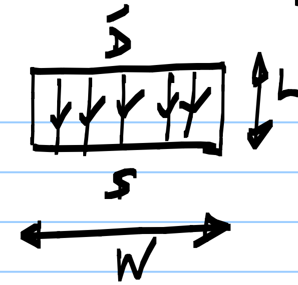


V_T : Threshold voltage

μ_n : Mobility of electrons

C_{ox} : oxide capacitance/unit area

W: width L: length



$\underline{V_{GS}, V_{DS} > 0}$; $\underline{\underline{V_T > 0}}$

cutoff

$I_D = 0$ $\quad V_{GS} \leq V_T$

triode/linear region

$$\mu_n C_{ox} \cdot \frac{W}{L}\left[(V_{GS} - V_T) \cdot V_{DS} - \frac{V_{DS}^2}{2}\right] \qquad V_{GS} > V_T,\ \ V_{DS} \leq V_{GS} - V_T$$

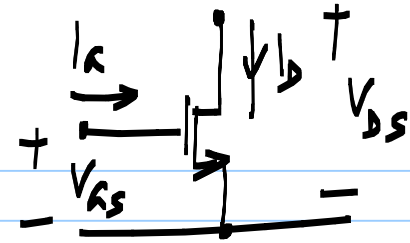
$$\frac{\mu_n C_{ox}}{2} \frac{W}{L}(V_{GS} - V_T)^2 \qquad V_{GS} > V_T,\ \ V_{DS} > V_{GS} - V_T$$

Saturation region

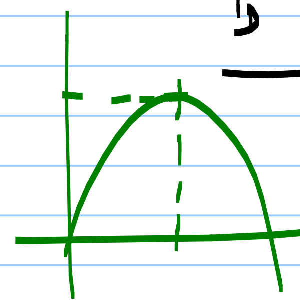
$\mu_n C_{ox} \frac{W}{L} = K_n$

current factor

$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu_n C_{ox} \cdot \frac{W}{L}\left[(V_{GS}-V_T)\cdot V_{DS} - \frac{V_{DS}^2}{2}\right] & V_{GS} > V_T,\ V_{DS} < V_{GS}-V_T \\ \frac{\mu_n C_{ox}}{2}\,\frac{W}{L}\,(V_{GS}-V_T)^2 & V_{GS} > V_T,\ V_{DS} > V_{GS}-V_T \end{cases}$



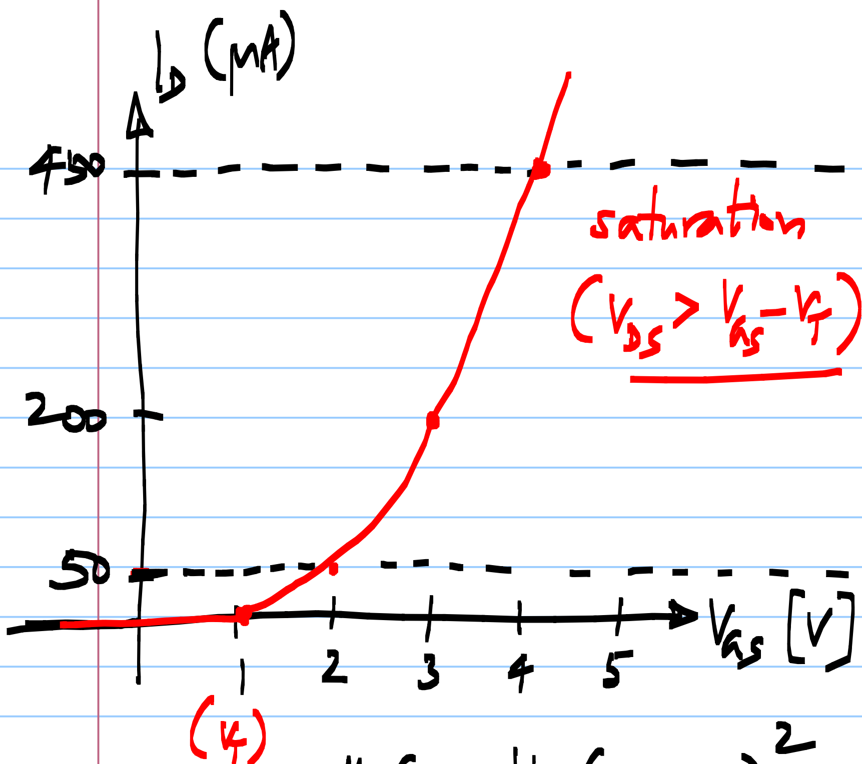
I_D vs. V_{GS} & I_D vs. V_{DS}

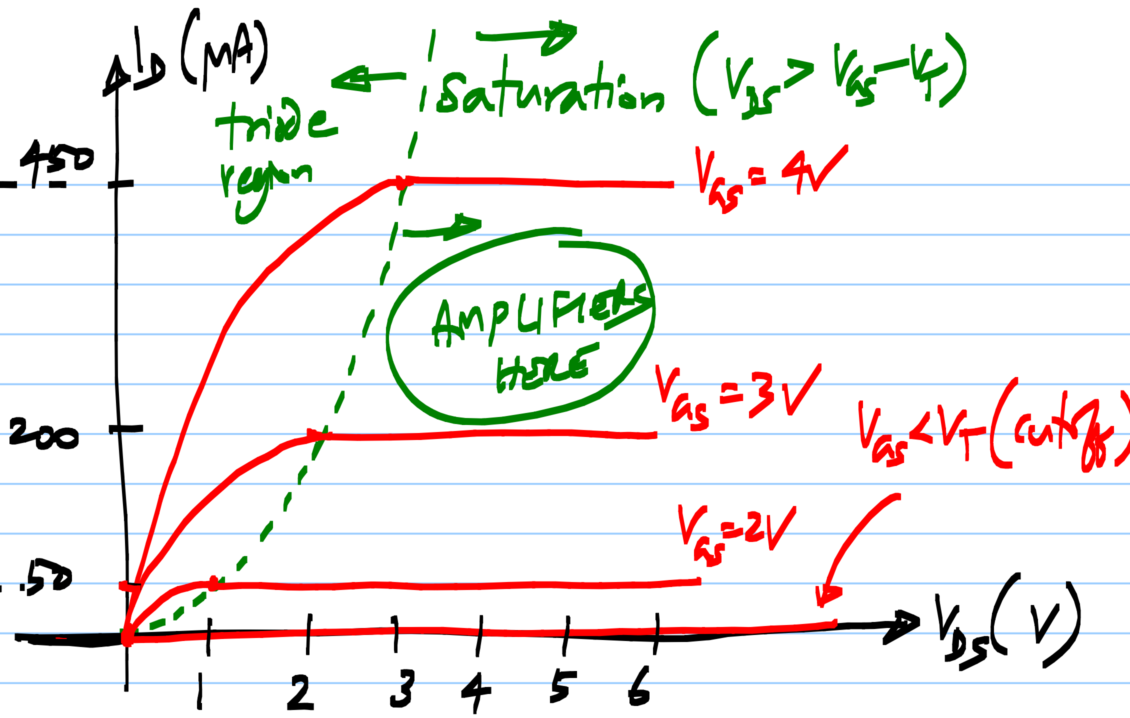


$\mu_n C_{ox} = 100\mu A/V^2$

$W/L = 1$

$K_n = \mu_n C_{ox} \cdot \frac{W}{L} = \underline{100\mu A/V^2}$; $V_T = 1V$;





$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_T)^2$$

$V_{DS} > V_{GS} - V_T$

saturation region

Small signal model of the MOS transistor

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu_n C_{ox} \cdot \frac{W}{L}\left[(V_{GS}-V_T)\cdot V_{DS} - \frac{V_{DS}^2}{2}\right] & V_{GS} > V_T,\ V_{DS} \le V_{GS}-V_T \\ \frac{\mu_n C_{ox}}{2}\,\frac{W}{L}(V_{GS}-V_T)^2 & V_{GS} > V_T,\ V_{DS} > V_{GS}-V_T \end{cases}$$

Small signal incremental model in saturation region

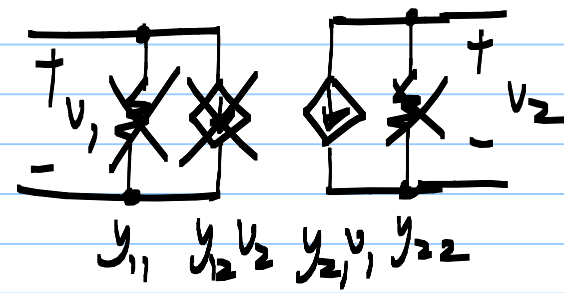
$\mu_n C_{ox} \frac{W}{L} = 100\mu A/V^2$

$V_T = 1V$

$$\frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_T)^2$$

$V_{GS} > V_T$

$V_{DS} > V_{GS} - V_T$



$$y_{21} = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T) = g_m$$

g_m : transconductance

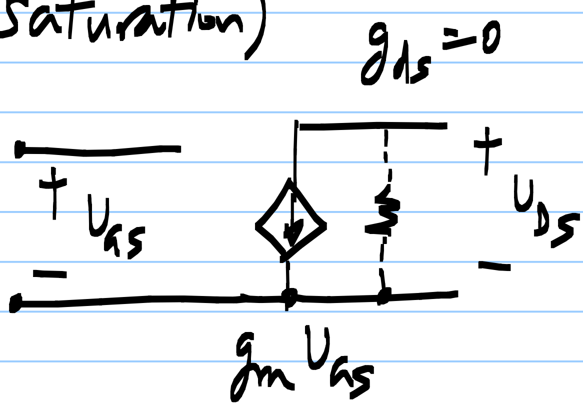
For $V_{GS} = 2V$,

$g_m = 100\mu S$

$$y_{22} = \frac{\partial I_D}{\partial V_{DS}} = 0 = g_{ds} \text{ (drain to source conductance)}$$

5.5 incremental model of the MOS transistor

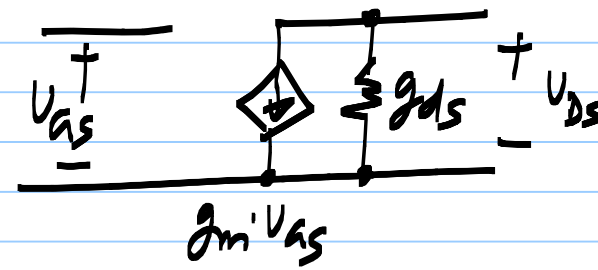
(saturation)



$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$$

small signal incremental model in triode (linear) region

$$I_D = \mu_n C_{ox} \frac{W}{L}\left((V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right)$$



$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} \cdot V_{DS}$$

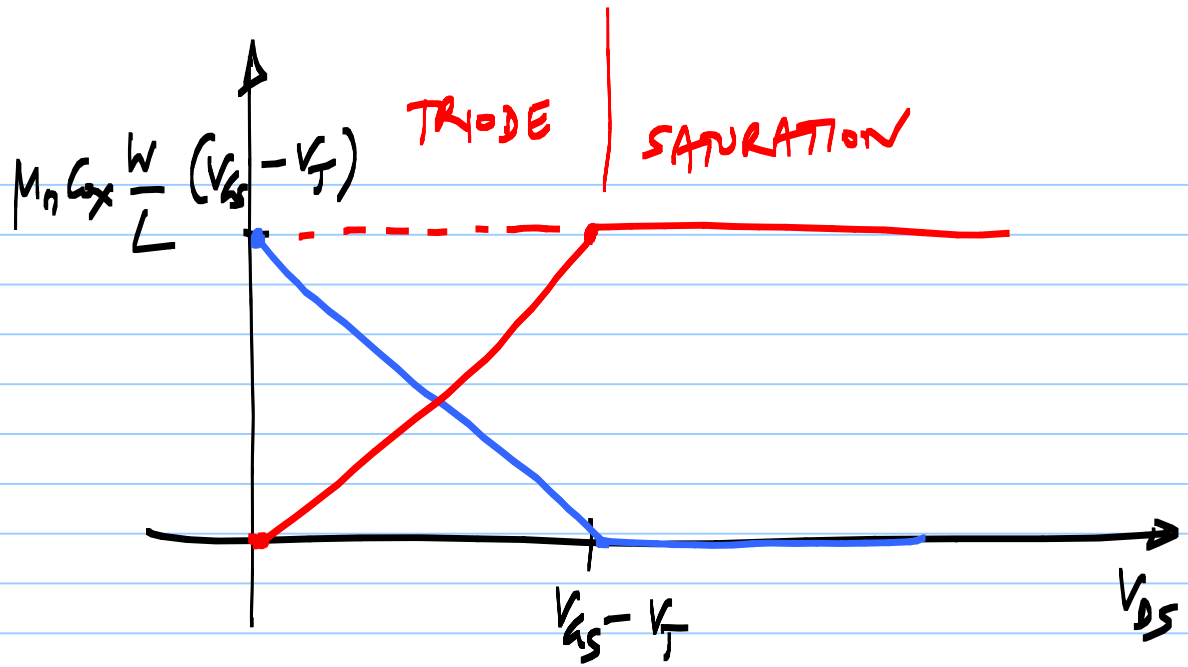
$$g_{ds} = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_T - V_{DS} \right)$$

	Saturation region	Triode (linear) region $V_{DS} < (V_{GS} - V_T)$
g_m :	$\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$	$\mu_n C_{ox} \frac{W}{L} \cdot V_{DS}$ $g_m(\text{triode}) < g_m(\text{sat.})$
g_{DS} :	0	$\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T - V_{DS})$ $\quad 0 < V_{DS} < V_{GS} - V_T$

V_{DS} : 0 to $V_{GS} - V_T$

g_{DS} : $\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$ to zero

g_m : 0 to $\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$


TRIODE
SATURATION
$\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$
$V_{GS} - V_T$
V_{DS}

$$I_D = \mu_n C_{ox} \frac{W}{L} \underbrace{\left((V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right)}$$

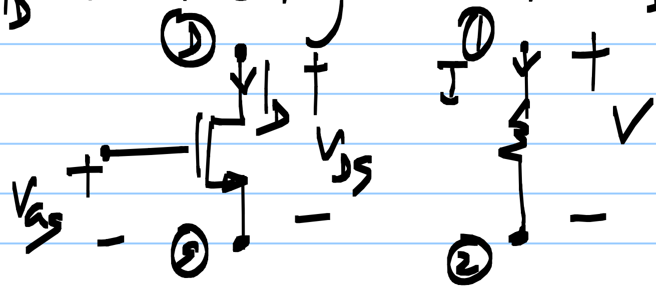
$V_{GS} > V_T$

$V_{DS} < V_{GS} - V_T$

$V_{DS} \ll V_{GS} - V_T$ $\qquad$ $\left(V_{GS} - V_T - \cancel{\frac{V_{DS}}{2}} \right)$ $\qquad$ V_{DS}

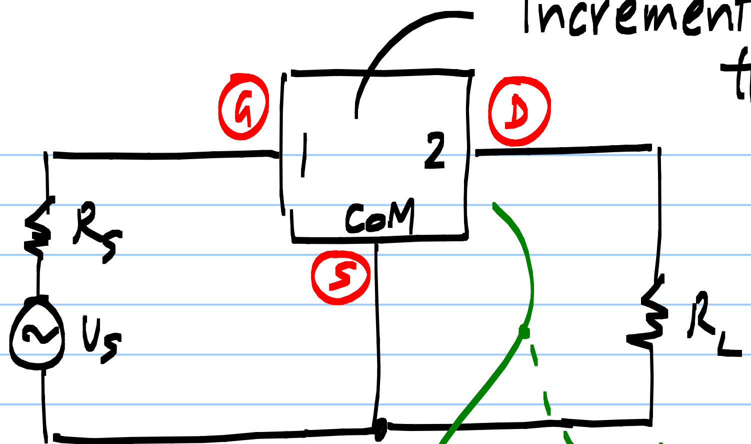
$$I_D \simeq \overbrace{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)}^{G_{mos}} V_{DS}$$

I_D : linearly with V_{DS} $\qquad$ (linear region)



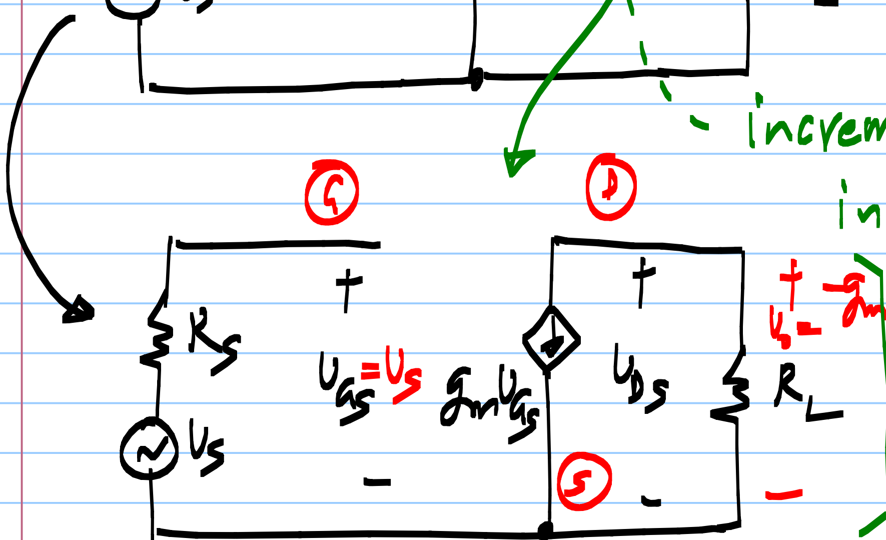
MOS in triode region: Voltage controlled resistor

Incremental eq. of the NL two port



$$v_o = \underbrace{(-g_m R_L)}_{\text{gain}} \cdot v_S$$

incremental eq. of MOS in saturation region



incremental eq. of the amplifier