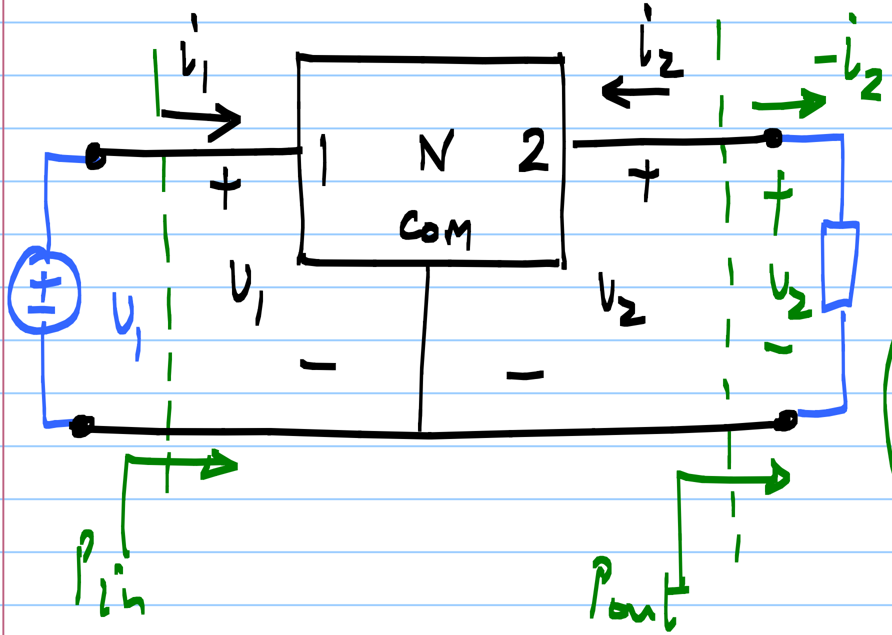
Note Title 6/9/2015

Amplifier characteristics

Two port (input & output) network N is passive



$v_1 i_1 + v_2 i_2 \geq 0$

Amplifier: $P_{out} > P_{in}$

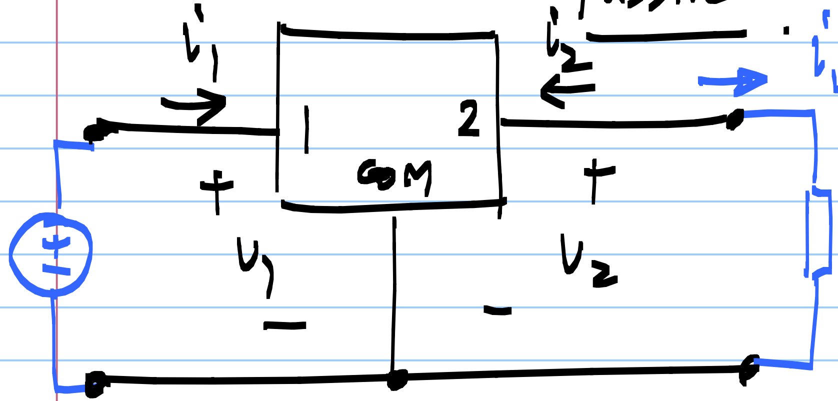
$P_{in} = v_1 \cdot i_1$; $P_{out} = -v_2 i_2$

$-v_2 i_2 \leq v_1 i_1$; $P_{out} \leq P_{in}$

N: Linear two port network

Passive

$i_L = \hat{I}_L \cdot \cos(\omega t)$

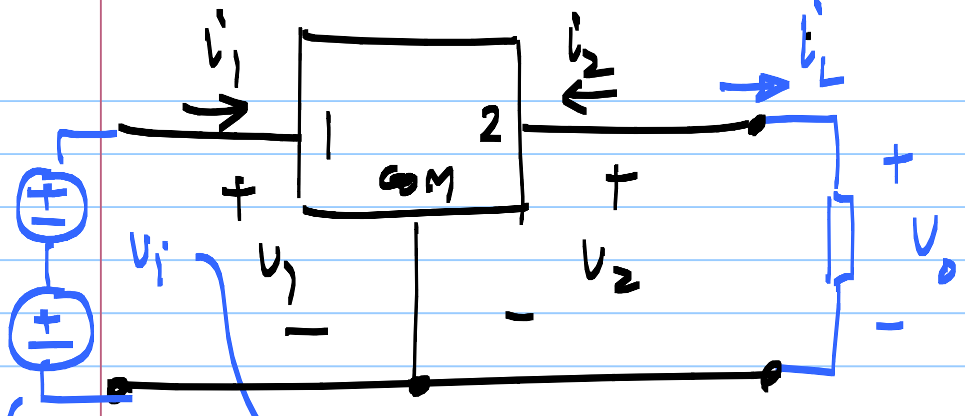


$v_o = \hat{V}_o \cdot \cos(\omega t)$

$v_i = \hat{V}_i \cdot \cos \omega t$

$-v_2 i_2 \le v_1 i_1$

Linear network

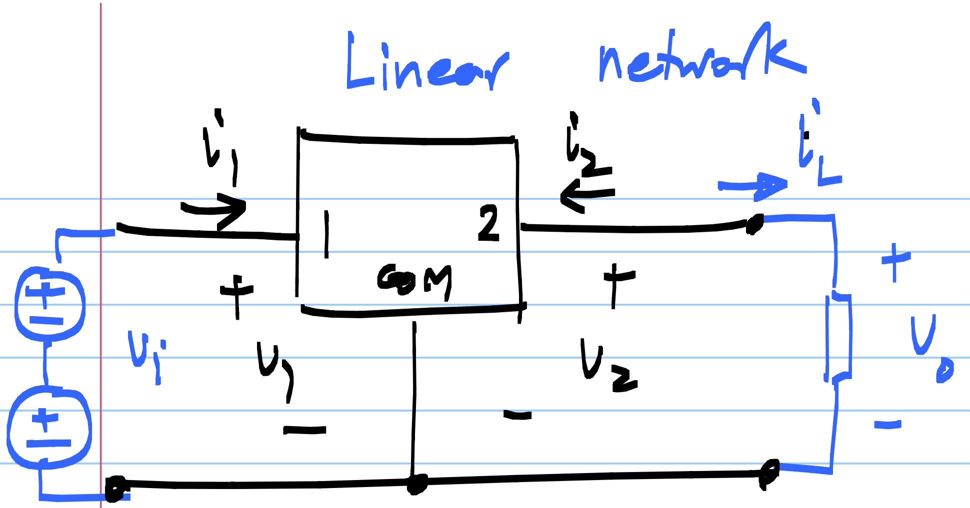


Component due to v_i'

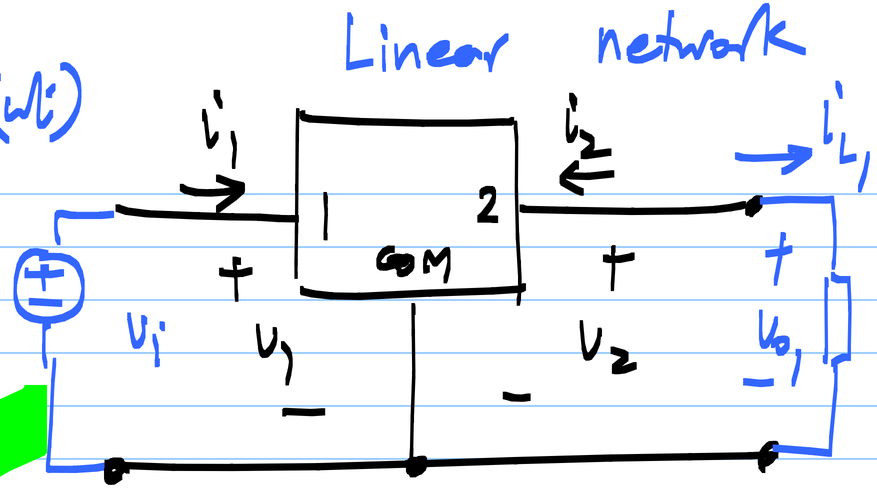
Component due to V_{10}

input signal source $\hat{V}_i \cos \omega t$

Additional power source V_{10}

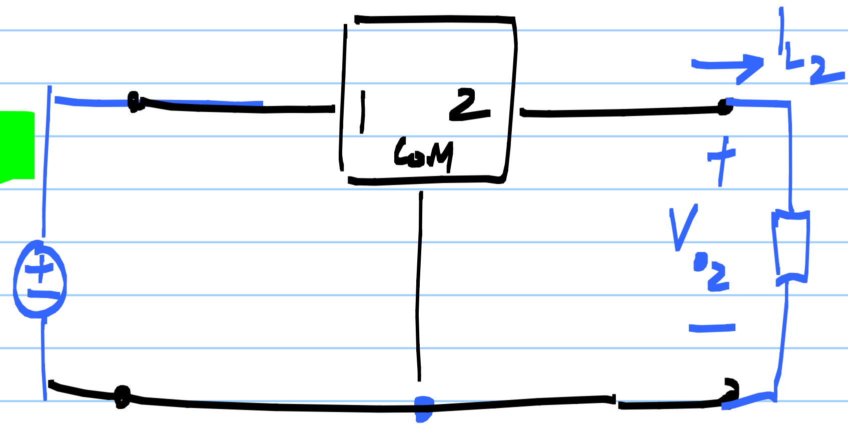


$\hat{V}_i \cos(\omega t)$



$-v_2 i_2' \le v_1 i_1'$

$P_{o,signal} \le P_{i,signal}$



* Passive devices $\Rightarrow$ $P_o \leq P_{in}$ $\Rightarrow$ No power amplification if signal source is the only power source

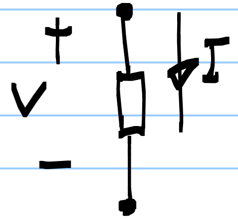
* Additional power source $\Rightarrow$ Can convert some power from this into signal component

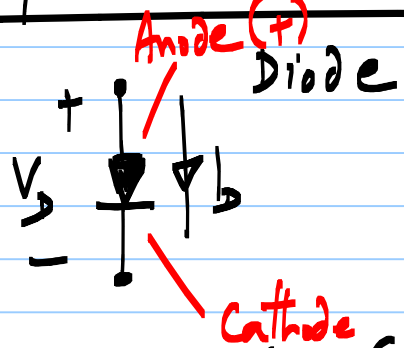
Linear, passive device $\longrightarrow$ $\underline{P_{o,signal} \leq P_{in,signal}}$

output due to the signal source independent of the additional source

We need nonlinear devices!

Analysis of nonlinear circuits





$I = \underline{f}(V)$

$I = \underline{\underline{G}} \cdot V$

$$I_D = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$

I_s : saturation current

$I_s = 10^{-15}A$;

$V_t = \dfrac{kT}{q}$

k: Boltzmann's constant

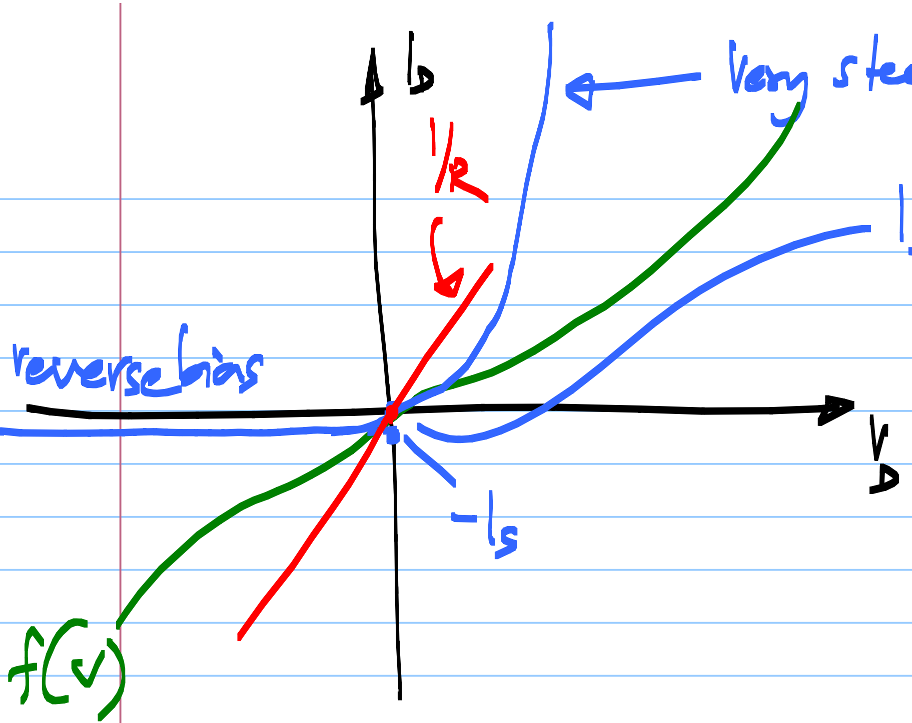
T: Absolute temperature

q: Electron charge

$\dfrac{kT}{q}$: Thermal voltage

room temp. $V_t = \dfrac{kT}{q} \simeq 25.9\,mV$

$\simeq 25\,mV$



Very steep increase

$I_D = 0$ when $V_D = 0$

$$I_D = I_s\left(\exp\left(\frac{V_D}{V_T}\right) - 1\right)$$

$V_D > 0$: Significant currents can flow : forward bias

$V_D < 0$: Very small currents reverse bias

V_D : large & negative

$-V_D \gg V_T$

$I_D \simeq -I_s$

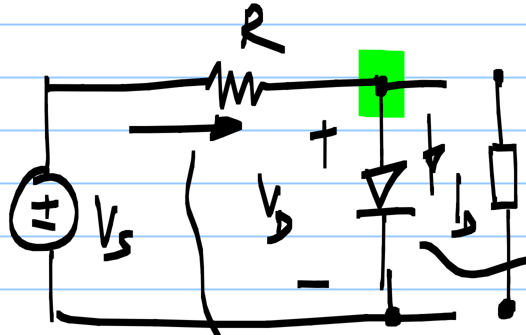
passive element ⇒ characteristic only in 1st & 3rd quadrants

⇒ has to pass through the origin

Nonlinear circuit analysis

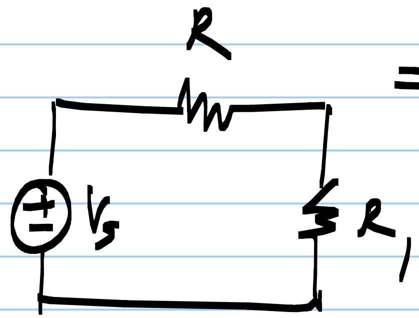
V_s changes

6.7V



$$\frac{V_s - V_D}{R}$$

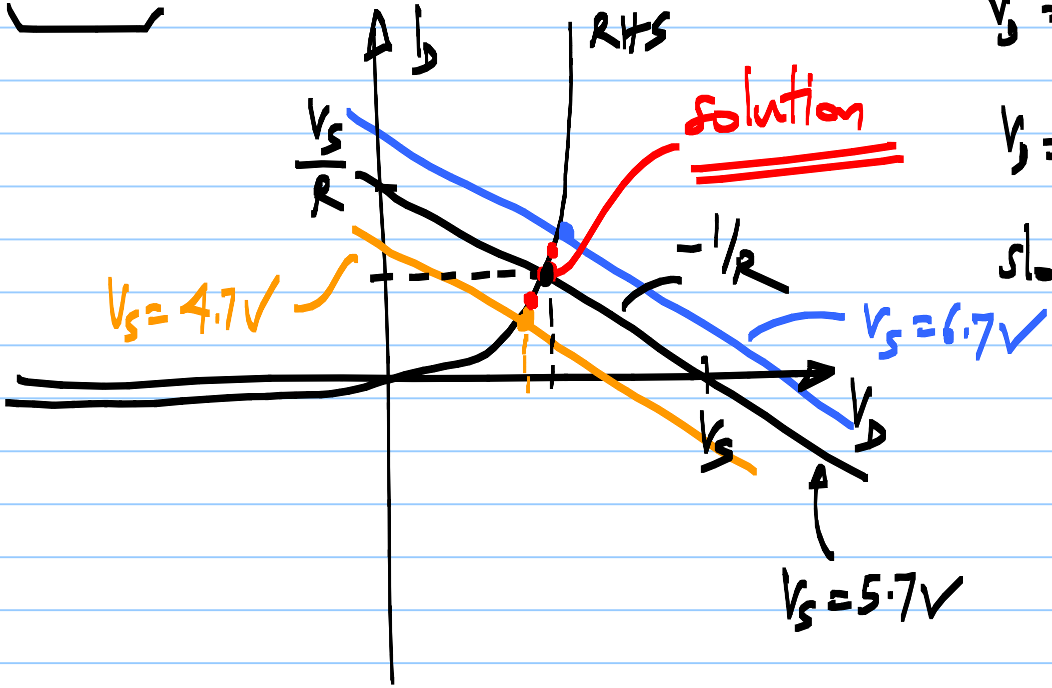
$$I_D = f(V_D) = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$



Can be solved numerically

$$\frac{V_s - \quad}{R} = f(\quad) = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$

$$\underbrace{\frac{V_s - V_D}{R}} = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$



$\frac{V_s - V_D}{R}$ vs. V_D

$V_D = 0$ ~ $\frac{V_s}{R}$

$V_D = V_s$ ~ 0

slope $\left(-\frac{1}{R}\right)$

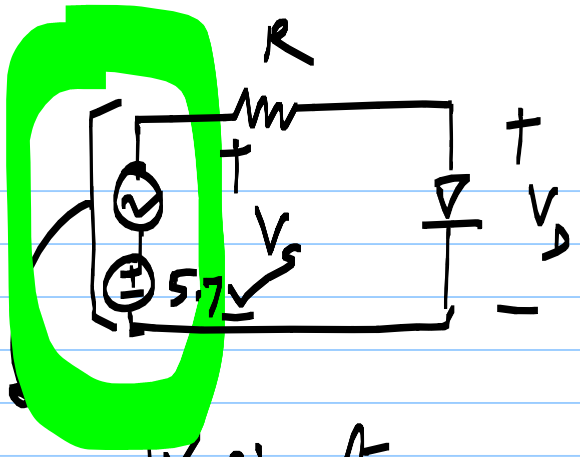
$V_s = 5.7V$, $R = 5k\Omega$

$I_s = 10^{-15} A$

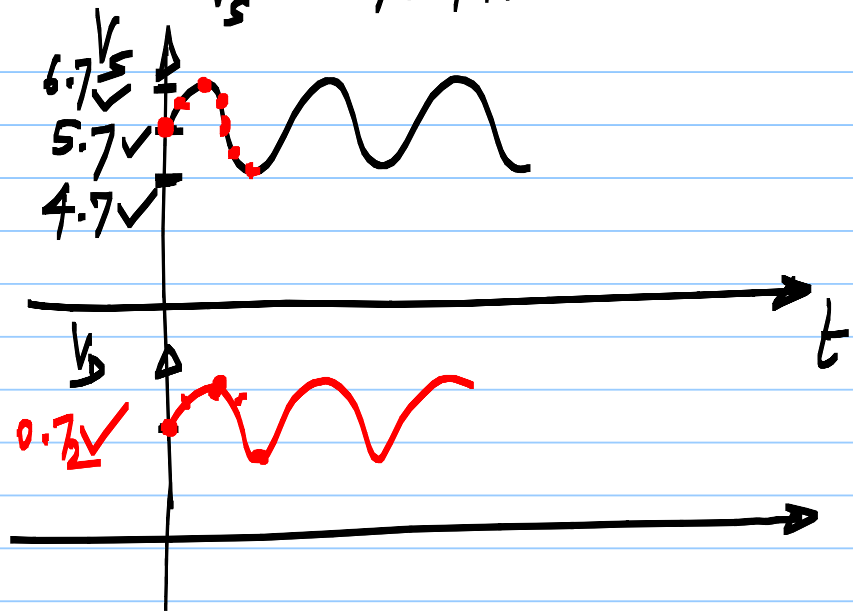
$V_D \simeq 0.72V$

$I_D \simeq 1mA$

solution

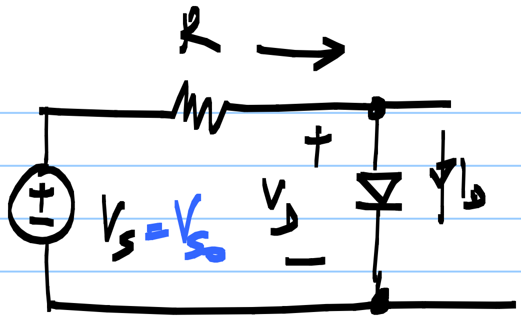


$V_s = 5.7V + 1V \sin \omega t$

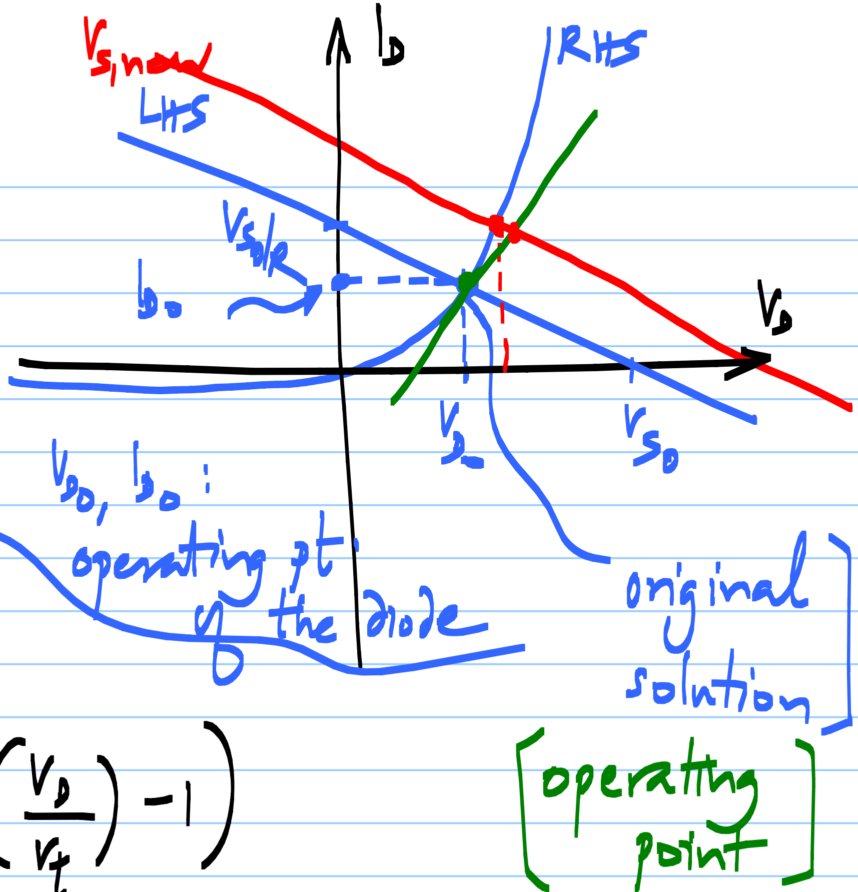


V_s : $1V \cdot \sin \omega t$

"signal"



- Numerically
- Graphically



$$\frac{V_S - V_D}{R} = I_S\left(\exp\left(\frac{V_D}{V_T}\right) - 1\right)$$

If the change is "small", approximating n.l. curve by its tangent yields a good enough solution

$$\frac{V_S - V_D}{R} = f(V_D) = I_S\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$

$I_S = 10^{-15} A$, $V_S = 5.7V$, $R = 5k\Omega$

$V_{D_0} = 0.716V$; $I_{D_0} = 1mA$

1st solution: operating point : $V_S = V_{S_0}$

Solve using nonlinear analysis

@ op. point $V_D = V_{D_0}$; $I_D = I_{D_0}$

$V_S = V_{S_0} + v_s$ new V_S represented as old value

$6.7V = 5.7V + 1.0V$ + increment v_s

$V_D = V_{D_0} + v_D$

All quantities represented as (original) operating point + increments

$$\frac{V_S - V_D}{R} = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$

$$\frac{V_{S_0} - V_{D_0}}{R} = I_s\left(\exp\left(\frac{V_{D_0}}{V_t}\right) - 1\right)$$

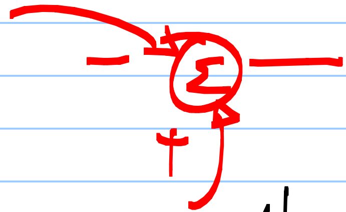
$$\frac{V_{S_0} + v_s - V_{D_0} - v_D}{R} = I_s\left(\exp\left(\frac{V_{D_0} + v_D}{V_t}\right) - 1\right)$$

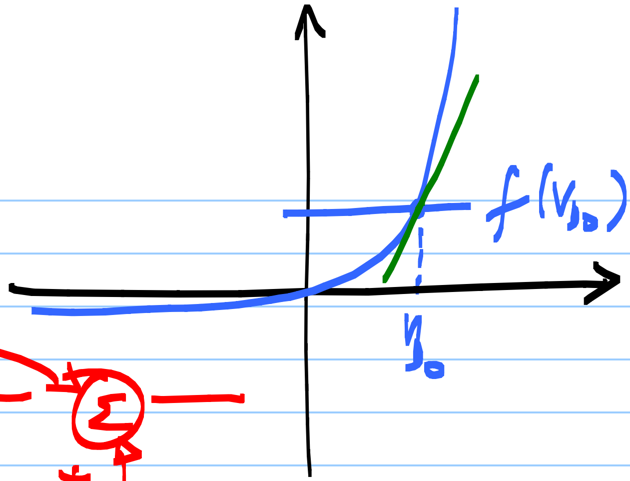
$$\frac{V_S - V_D}{R} = f(V_D)$$

gen. equation

$$\frac{V_{S0} - V_{D0}}{R} = f(V_{D0})$$

op. point





$$\frac{V_{S0} + v_s - V_{D0} - v_D}{R} = \underbrace{f(V_{D0} + v_D)}_{\text{Taylor series about } V_{D0}} = f(V_{D0}) + f'\Big|_{V_{D0}} \cdot v_D + \cancel{h.o.t}$$

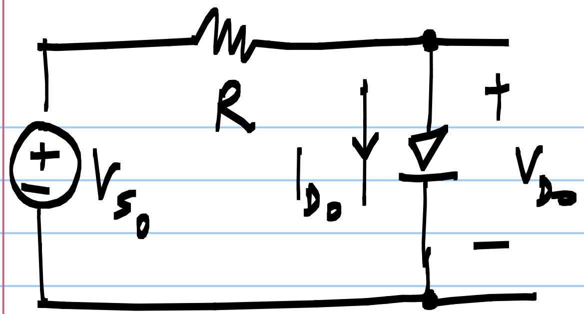
$$f(V_D) = f(V_{D_0} + v_D) = f(V_{D_0}) + f'\Big|_{V_{D_0}} \cdot v_D + \frac{f''}{2!}\Big|_{V_{D_0}} v_D^2 + \frac{f'''}{3!}\Big|_{V_{D_0}} \cdot v_D^3 + \ldots$$

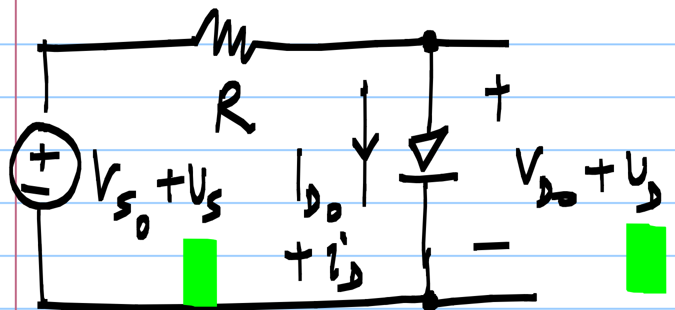
Can always find v_D small enough to neglect h.o.t

$$\frac{v_S - v_D}{R} = \left[f' \Big|_{V_D} v_D \right]$$

Linear in the increment v_D

Incremental quantities related by a linear equation



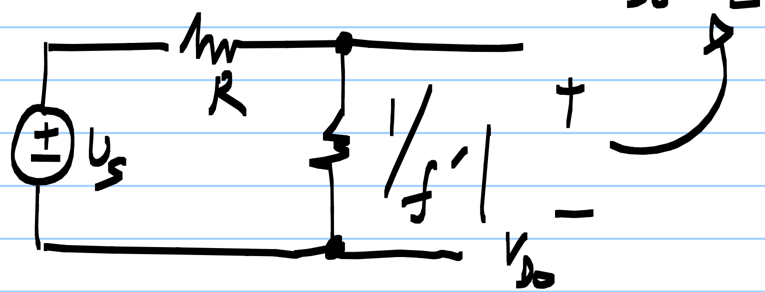


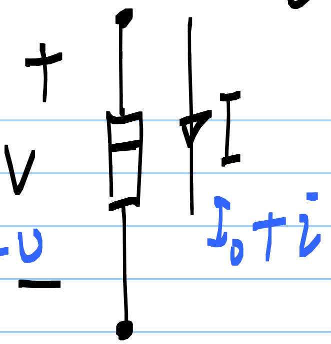
$$\frac{v_S - v_D}{R} = f'\Big|_{V_{D_0}} \cdot v_D$$

$$f(V_D) = I_D$$

$$f' = \frac{\partial I_D}{\partial V_D} \doteq \text{conductance}$$

$$v_D = v_S \cdot \frac{1/f'\big|_{V_{D_0}}}{R + 1/f'\big|_{V_{D_0}}} =$$



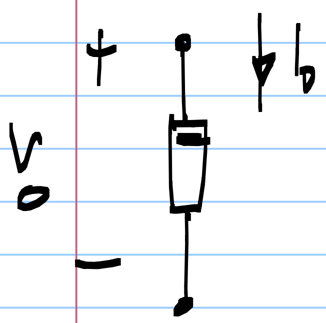


$$I = f(V) \qquad I = V \cdot G$$

$$V = g(I) \qquad V = I \cdot R$$

$$V = V_0 + u$$

$$I = I_0 + i$$

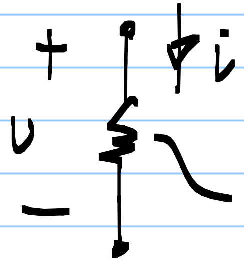


$$I_0 = f(V_0)$$

op. point

$$I_0 + i = \underbrace{f(V_0 + u)} = f(V_0) + f'\big|_{V_0} \cdot u + \cancel{h.o.t}$$

$$\underbrace{i = f'\big|_{V_0} \cdot u}$$

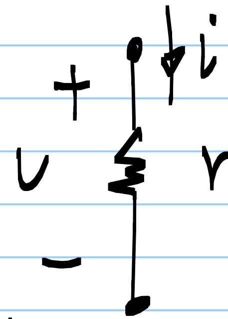


$$r = \frac{1}{f'\big|_{V_0}} ; \; g = f'\big|_{V_0}$$

Every two-terminal non-linear element is a resistor for incremental quantities provided the increment is small.

$$g = f'\Big|_{v_0}$$

$$\underbrace{\qquad\qquad}_{\text{incremental conductance}}$$



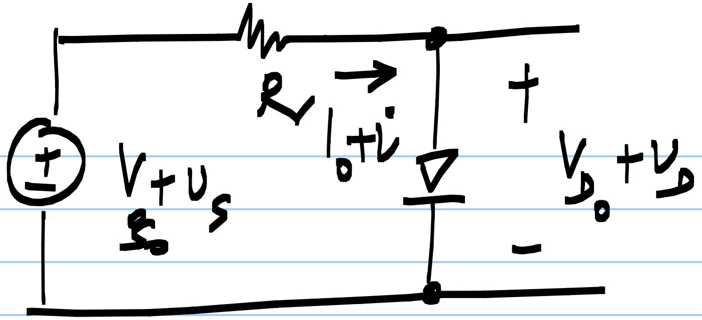
$$r = \frac{1}{f'\Big|_{v_0}}$$

Small signal $\underbrace{\qquad\qquad}_{\text{incremental resistance}}$

$$V = h(I)$$

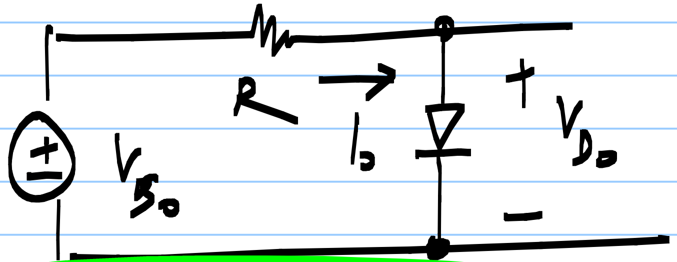
$$r = h'\Big|_{I_o} \quad ; \quad g' = \frac{1}{h'\Big|_{I_o}}$$

op. point



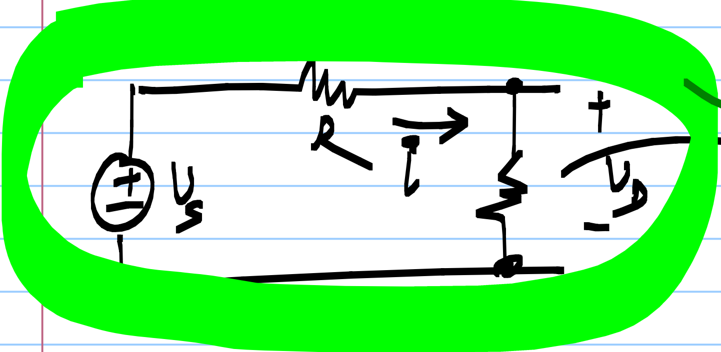
$$v_D = v_s \cdot \frac{r}{r+R}$$

$$i = \frac{v_s}{r+R}$$



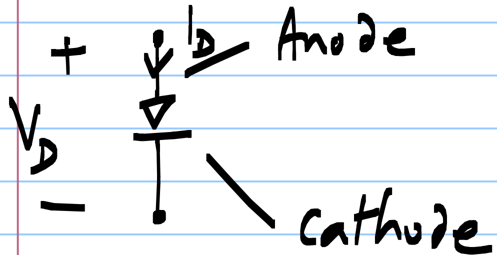
Incremental resistance at the o.p.

$$r = \frac{1}{f'\big|_{V_0}}$$



Small signal incremental eq. circuit

Diode model

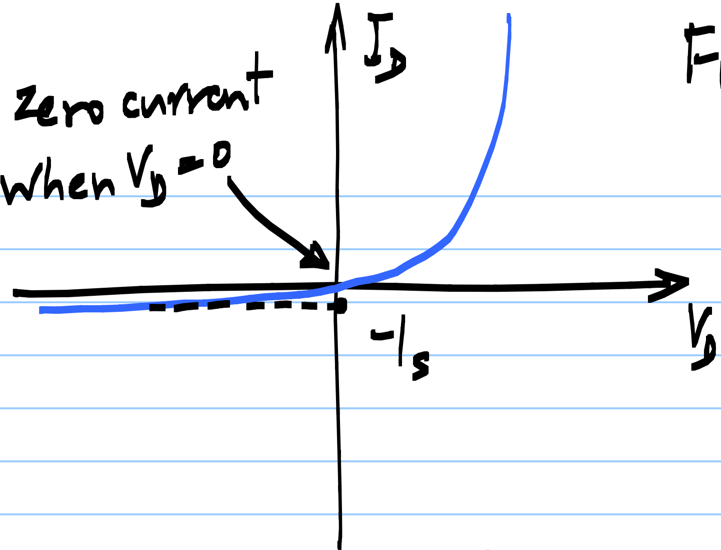


$$I_D = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$

I_s : reverse saturation current

$V_t = \frac{kT}{q}$ Thermal voltage

$\simeq 25mV$ @ 300K



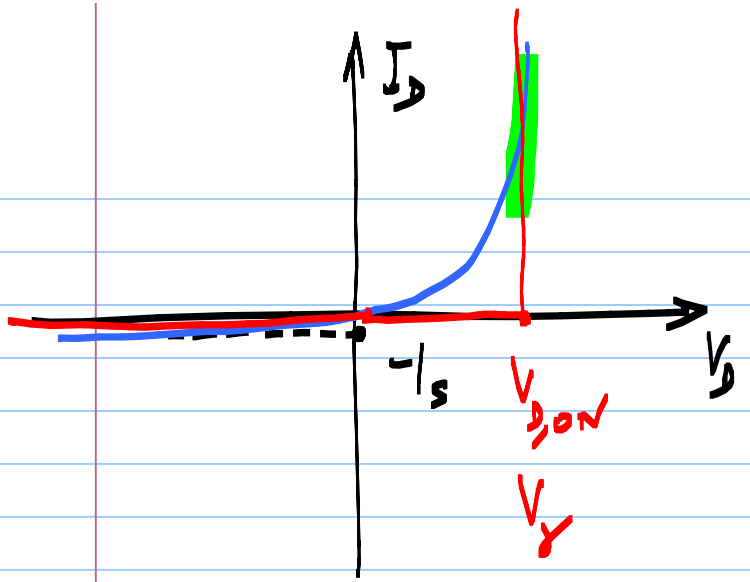
$$V_D = V_t \cdot \ln\left(\frac{I_D}{I_s}\right)$$

Forward bias

with $I_D \gg I_s$

$$\exp\left(\frac{V_D}{V_t}\right) \gg 1$$

$$I_D \simeq I_s \exp\left(\frac{V_D}{V_t}\right)$$



$|I_D| \leq I_S$ in reverse bias

$I_D \simeq 0$ in reverse bias

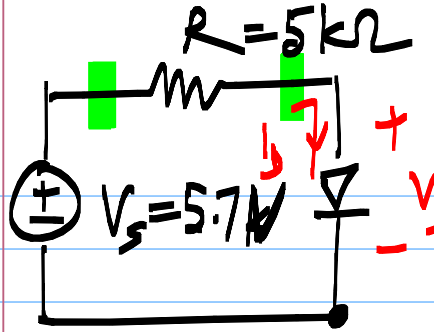
$V_D \simeq V_t \ln\left(\frac{I_D}{I_S}\right)$

V_D changes slowly with I_D,

V_D : Constant with I_D

$V_{D,ON}$: 0.7V ✓

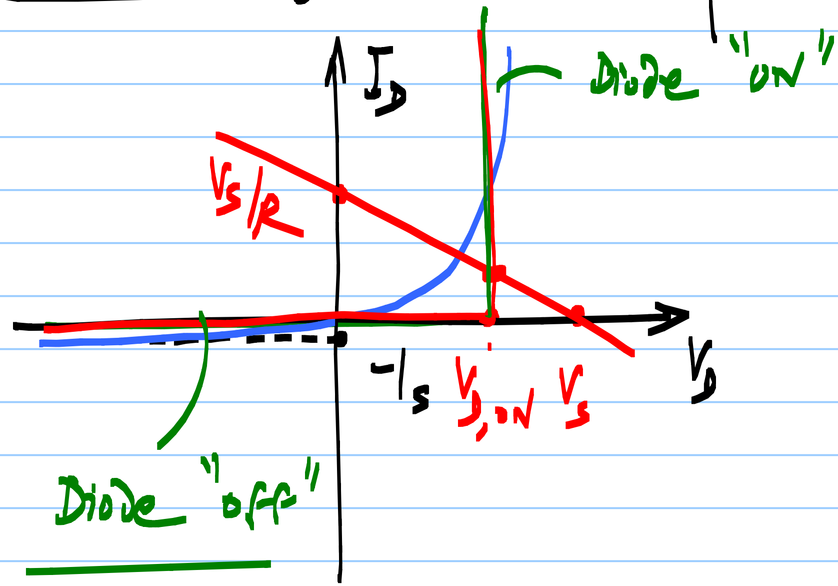
$V_{D,ON}$: 1V (high currents)



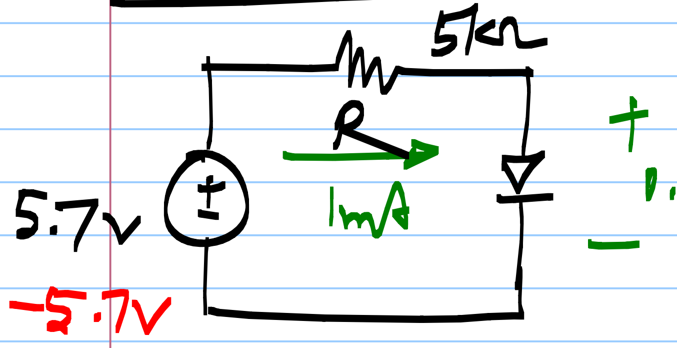
$I_D = \frac{V_s - V_D}{R}$

$V_D = V_{D,ON}$ when there is a forward current

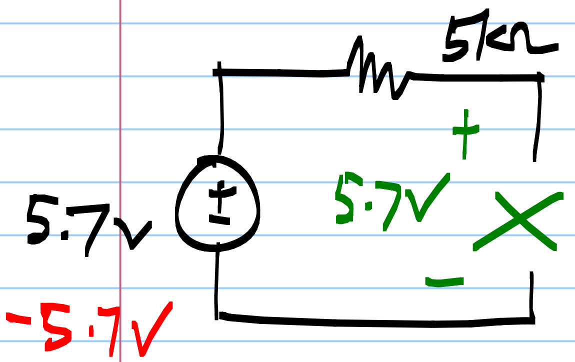
$$I = \frac{5.7V - 0.7V}{5k\Omega} = \underline{1mA}$$



Solving diode problems using the approx. characteristic



Diode ON



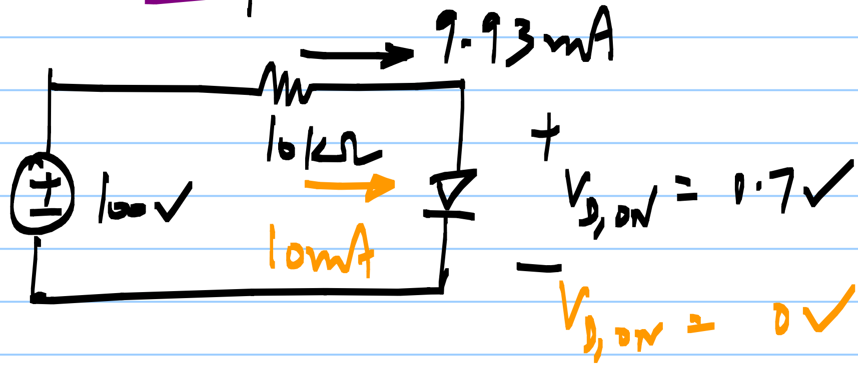
Contradictory

Diode OFF

open circuit

$V_{D,ON} \simeq 0$ Forward bias

$I_D = 0$ Reverse bias



"Large signal" characteristics of a Diode

Total ($\Rightarrow$ not "increments")

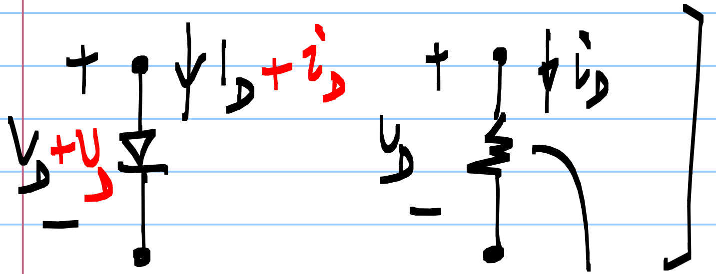
Exact: $I_D = I_S \left(\exp\left(\frac{V_D}{\eta}\right) - 1 \right)$

$$\begin{cases} I_D \simeq I_S \exp\left(\frac{V_D}{\eta}\right) & \text{Fwd bias} \\ \quad\simeq -I_S & \text{Rev bias} \end{cases}$$

$I_D = 0$ when $V_D < 0.7V$ Rev. bias

$V_D = 0 \checkmark$ $\underline{V_D = 0.7V}$ when $I_D > 0$ Fwd bias

Diode: Small signal characteristics



only incremental currents & voltages

$$I_D = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$

$$g_D = \frac{I_s}{V_t}\exp\left(\frac{V_D}{V_t}\right)\Big|_{V_D=V_{D0}} \quad \text{(op. point)}$$

$$I_{D0} = f(V_{D0})$$

$$I_{D0} + i_D = f(V_{D0} + v_D)$$

$$i_D \approx f'\Big|_{V_{D0}} \cdot v_D$$

$$r = \frac{1}{f'\Big|_{\text{op. point}}}$$

FWD. BIAS $\quad g_D = \frac{I_{D0}}{V_t}\Big|$ (I_{D0} : op. point current)

REV. BIAS $\quad g_D = 0$ (OPEN CIRCUIT)

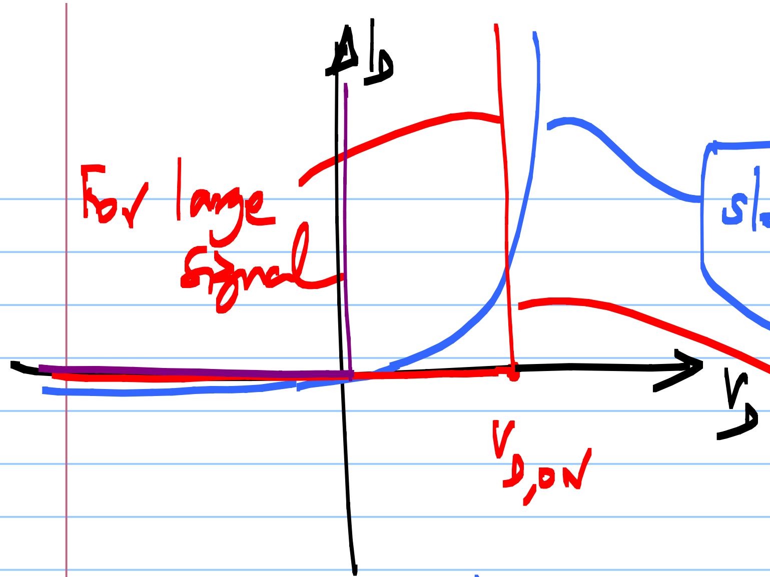
$$r = \frac{1}{g_D} = \frac{V_t}{I_{D0}}$$

$$V_t \simeq 25mV \text{ @ } 300K$$

$$r_d = \frac{V_t}{I_d} = \frac{25mV}{1mA} = \underline{25\Omega}$$

$$(I_{DQ} = 1mA)$$

$$g_d = \frac{1mA}{25mV} = 40mS$$



$g_D = f'$

$\boxed{\text{slope} \simeq \frac{I_{D0}}{V_T}}$

op. point

small signal incremental char.

slope = ∞

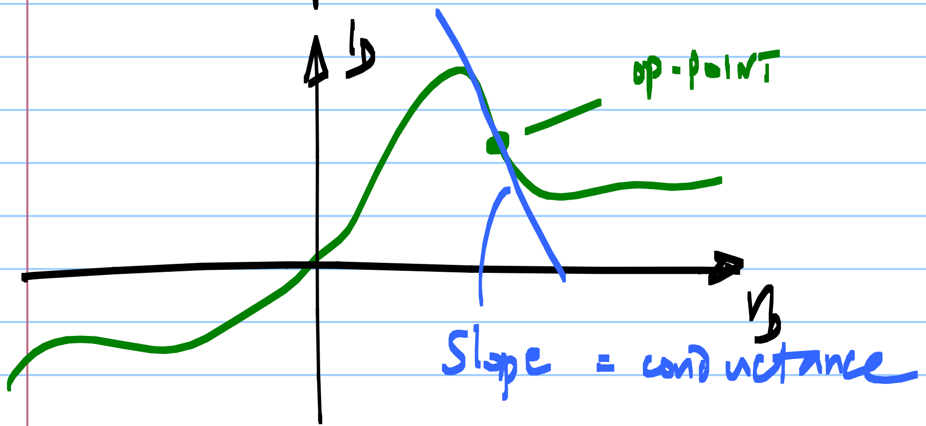
$g_D = \infty, \; r_D = 0$

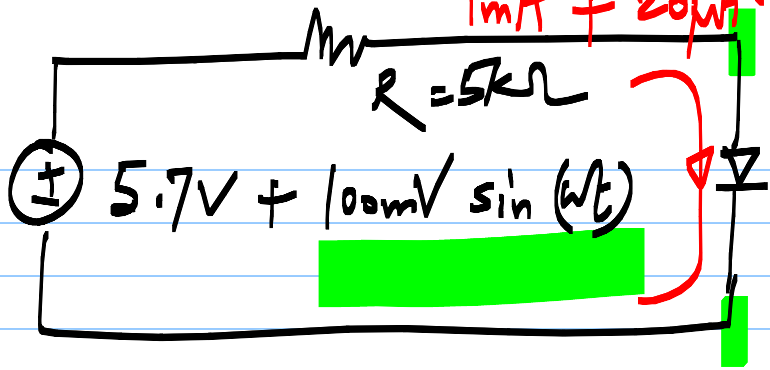
Fwd bias:

Diode: Incremental short

Rev. bias:

Incremental open

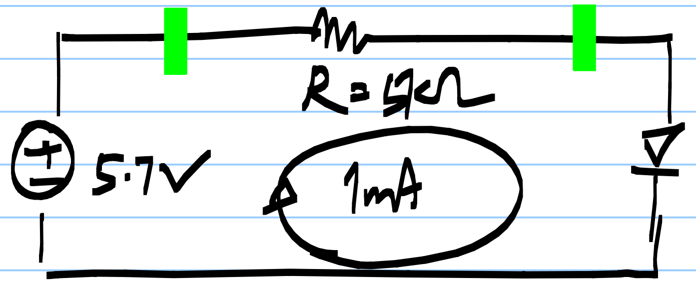




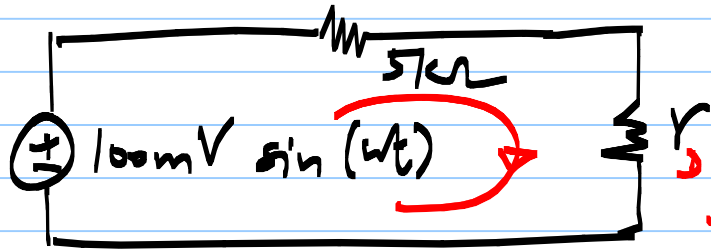
$$\frac{5.7V + 100mV \cdot \sin \omega t - V_D}{R} = I_s\left(\exp\left(\frac{V_D}{V_t}\right)-1\right)$$

$0.7V + 0.5mV \sin \omega t$

+



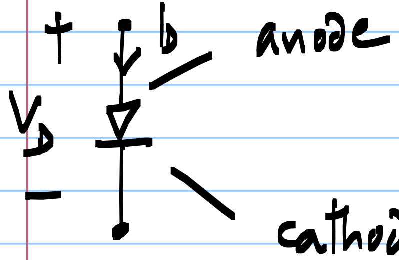
$$100mV \cdot \sin \omega t \cdot \frac{25\Omega}{25\Omega + 5k\Omega}$$

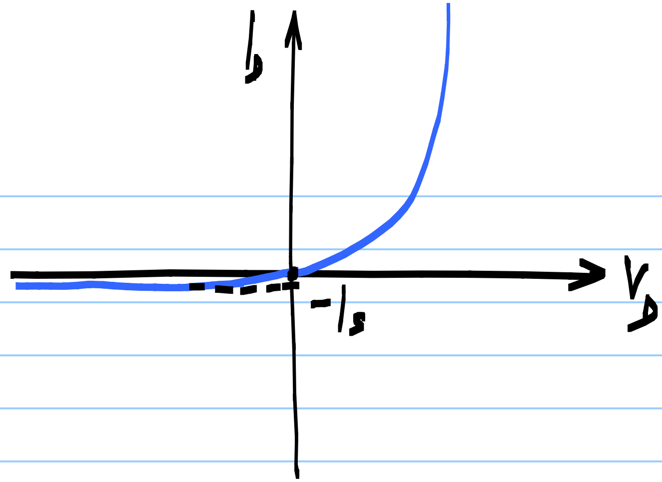


$$r_D = 25\Omega\left(\frac{V_t}{I_D}\right) \simeq 0.5mV \cdot \sin \omega t$$

$$\frac{100mV \sin \omega t}{25\Omega + 5k\Omega} = 20\mu A \cdot \sin \omega t$$

Large signal model of a diode

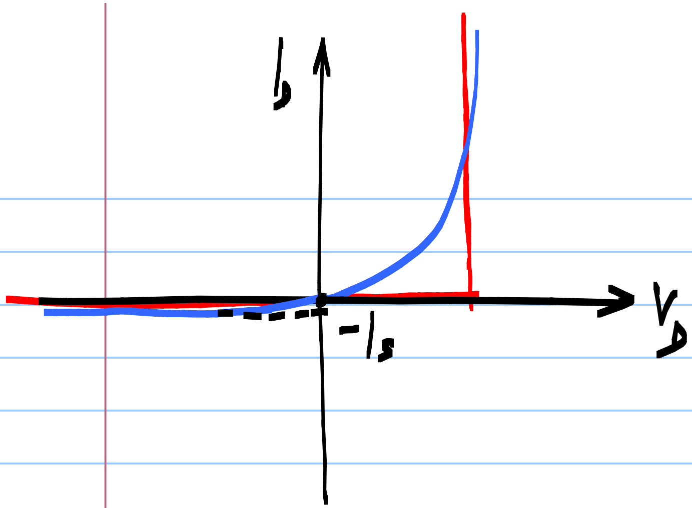




$$I_D = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$

I_s : reverse saturation current

$V_t = \frac{kT}{q}$: Thermal voltage $\simeq$ 25mV @ 300K



$$I_D = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$

$V_D \gg V_t$ $\quad I_D \simeq I_s \exp\left(\frac{V_D}{V_t}\right)$

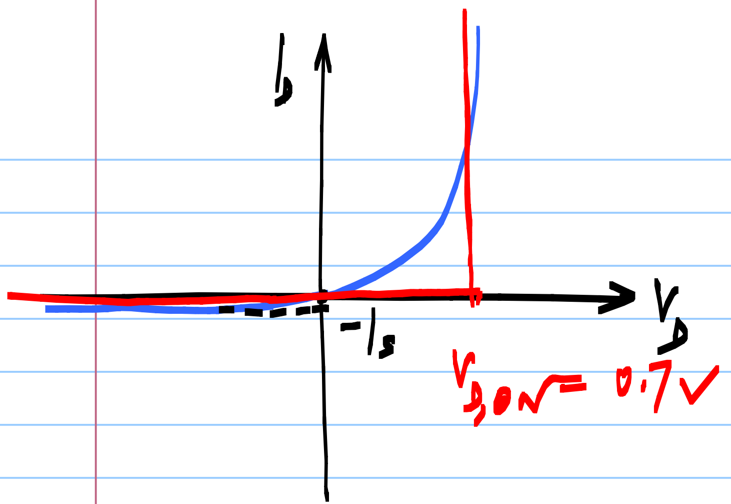
$V_D < 0 \Rightarrow I_D = 0$

$$V_D = V_t \cdot \ln\left(\frac{I_D}{I_s}\right)$$

$I_D \rightarrow 2I_D$

$$V_t \ln\left(\frac{2I_D}{I_s}\right) = V_t \ln\left(\frac{I_D}{I_s}\right) + \underbrace{V_t \cdot \ln(2)}_{17.5\,mV}$$

Diode voltage constant when there is a current in the fwd Direction

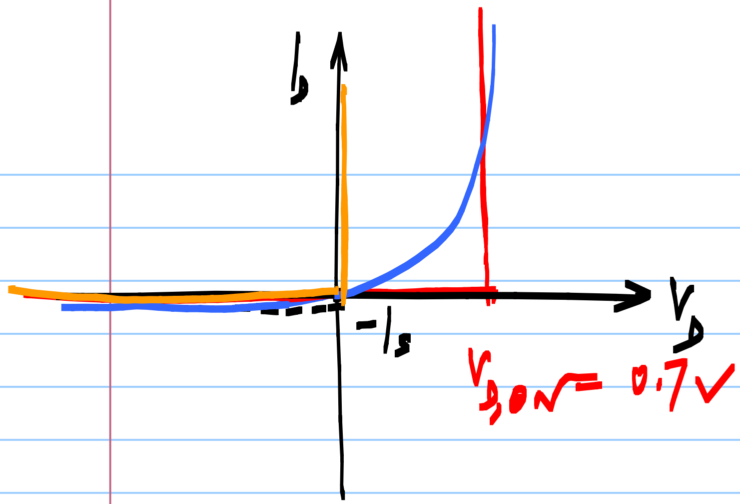


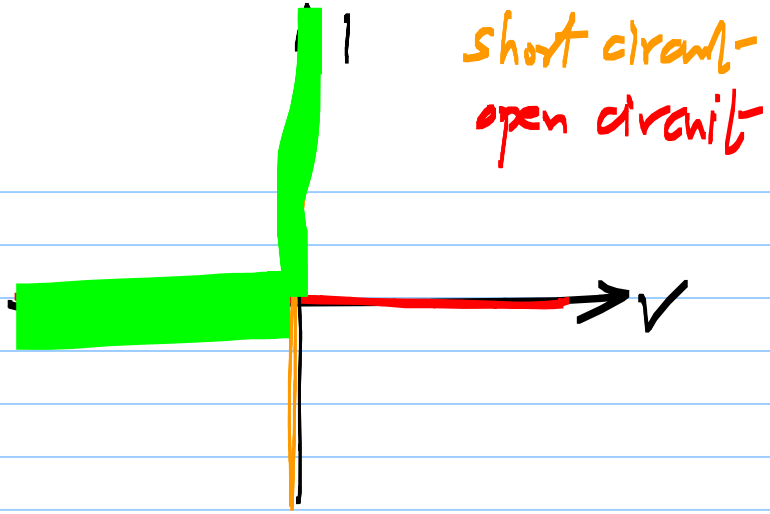
<u>Forward bias:</u> (substantial I_D flowing in the fwd direction)
(Diode ON)

$V_D = V_{D,ON} = 0.7V$

~~Reverse bias:~~
(Diode OFF)

$\underline{\underset{V_{D,ON}}{V_D < 0.7V} \Rightarrow I_D = 0}$





we approximate $V_{D,ON} \simeq 0V$

"Exact": $I_D = I_s\left(\exp\left(\frac{V_D}{V_T}\right) - 1\right)$

(1) : $I_D = I_s \exp\left(\frac{V_D}{V_T}\right)$ $\quad V_D \gg V_T$

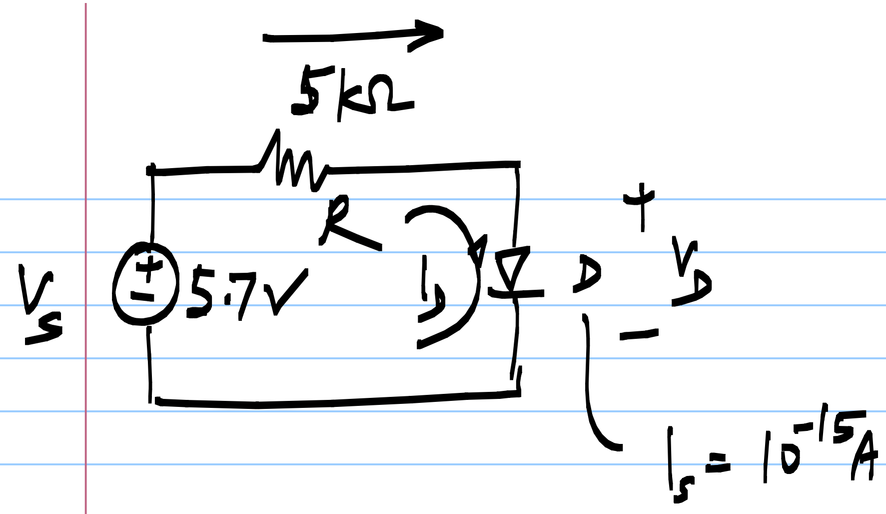
$= 0$ $\quad V_D < 0$

(2) If $I_D > 0$, $V_D = V_{D,ON} = 0.7V$ (ON)

If $V_D < V_{D,ON}$, $I_D = 0$ (OFF)

(3) If $I_D > 0$, $V_D = 0$ (ON)

If $V_D < 0$, $I_D = 0$ (OFF)



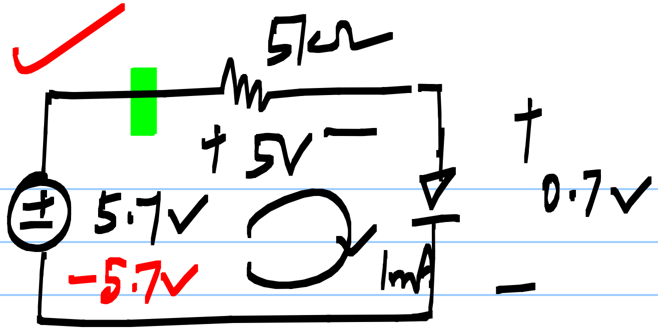
$$\frac{V_s - V_D}{R} = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right)$$

$$V_D = 0.72V$$

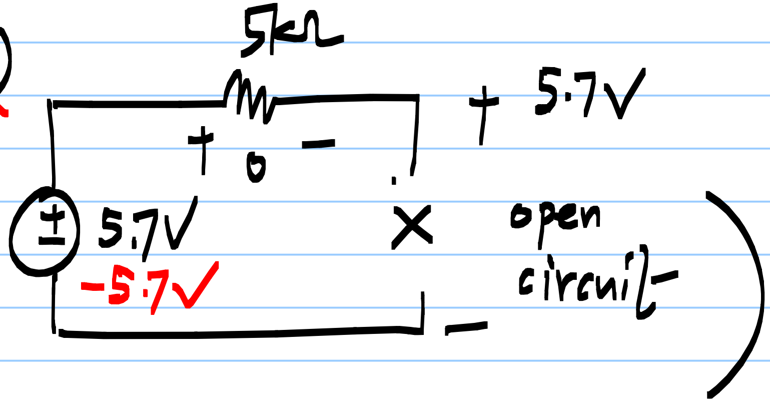
$V_D = V_{D,ON}$ if $I_D > 0$

$I_D = 0$ if $V_D < V_{D,ON}$

(ON)



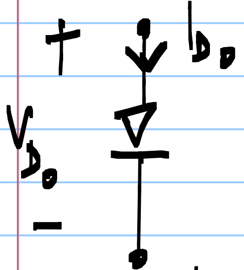
~~(OFF)~~

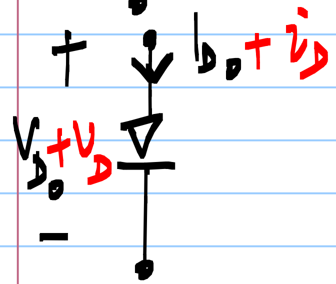


Contradiction

circuit with N diodes, 2^N possibilities

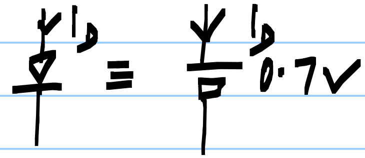
$I_D = f(V_D)$





I_{D0}, V_{D0}: operating point of the diode

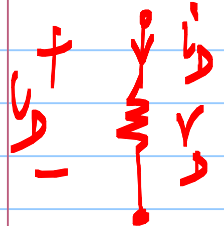
$V_{D0} = 0.7V$; I_{D0}: circuit analysis



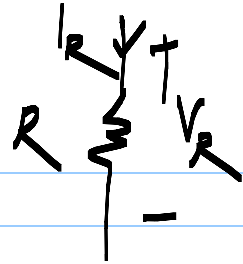
(if $I_D > 0$)

i_D, v_D: increments over the op. point

$$r_D = \frac{1}{f'|_{op}} \; ; \; g_D = f'|_{op}$$



: small signal incremental resistance



$I_R = h(V_R)$

$I_R = \frac{V_R}{R}$

$g_R = h'| = \frac{1}{R} = c$

$r_R = R$

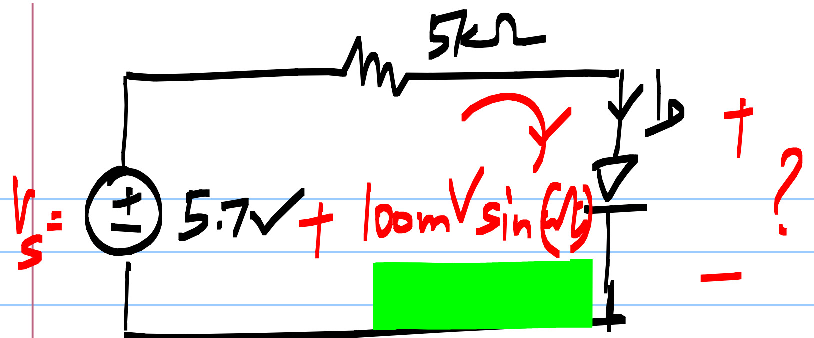
$$I_D = I_s\left(\exp\left(\frac{V_D}{V_t}\right) - 1\right) \simeq I_s \exp\left(\frac{V_D}{V_t}\right) \qquad V_D \gg V_t$$

$$g_D = \left.\frac{\partial I_D}{\partial V_D}\right|_{op} = \left.\frac{I_s}{V_t} \cdot \exp\left(\frac{V_D}{V_t}\right)\right|_{op} = \frac{I_{D_0}}{V_t} \quad \text{(op. point current)}$$

$$r_D = \frac{1}{g_D} = \frac{V_t}{I_{D_0}}$$

$$I_{D_0} = 1mA \qquad g_D = \frac{1mA}{25mV} = 40mS \, ; \; r_D = 25\,\Omega$$

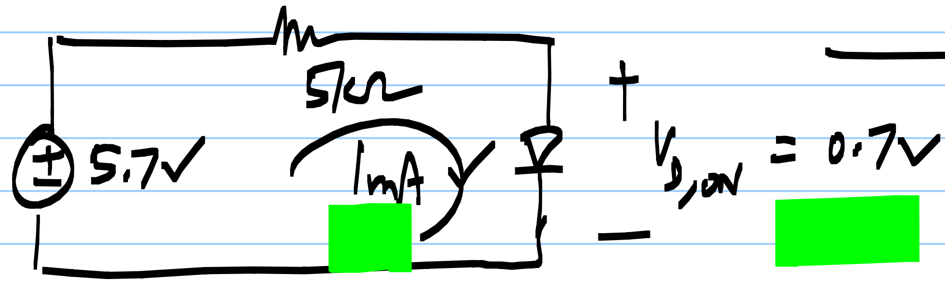
(room temp.)



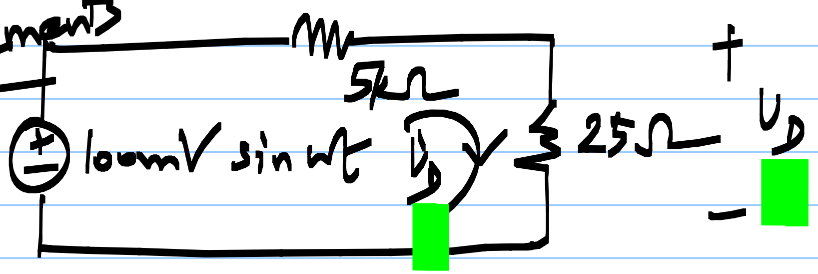
$$\frac{V_s - V_D}{R} = I_s\left(\exp\left(\frac{V_D}{V_T}\right) - 1\right)$$

$$\frac{5.7V + 100mV \sin \omega t - V_D}{5k\Omega} = 10^{-15}\left(\exp\left(\frac{V_D}{V_T}\right) - 1\right)$$

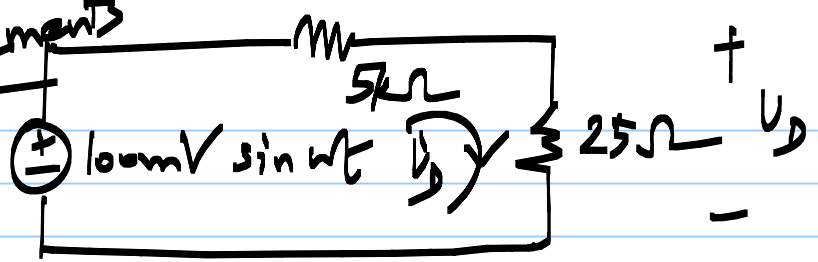
op. point



increments



increments



$$v_D = \underline{100mV \cdot \sin \omega t} \cdot \frac{25\Omega}{5k\Omega + 25\Omega} = 0.5mV \sin \omega t$$

$$i_D = \frac{100mV \sin \omega t}{5k\Omega + 25\Omega} = 20\mu A \cdot \sin \omega t$$