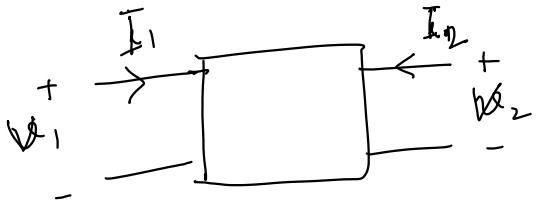


$$V_1 = z_{11} I_1 + z_{12} I_2$$
$$V_2 = z_{21} I_1 + z_{22} I_2$$

$z_{12} = z_{21}$ for Reciprocity

Lec: 76

If $z_{11} = z_{22}$ → symmetric

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$I_2 = y_{21} V_1 + y_{22} V_2$$

$\left.\frac{V_1}{I_1}\right|_{I_2=0}$:

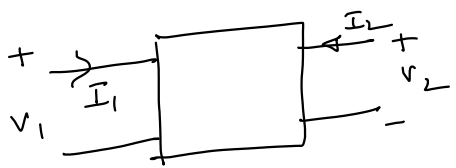
$$0 = y_{11} V_1 + y_{12} V_2$$
$$V_2 = -\frac{y_{21}}{y_{22}} V_1$$
$$I_1 = y_{11} V_1 + y_{12} V_2$$
$$\text{or } I_1 = y_{11} V_1 - \frac{y_{21} y_{12}}{y_{22}} V_1$$
$$= \frac{(y_{11} y_{22} - y_{21} y_{12})}{y_{22}} V_1$$
$$\therefore \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{y_{22}}{y_{11} y_{22} - y_{21} y_{12}}$$

$\left.\frac{V_2}{I_2}\right|_{I_1=0}$:

$$I_2 = y_{21} V_1 + y_{22} V_2$$
$$= y_{21}\left(-\frac{y_{12}}{y_{11}}\right) V_2 + y_{22} V_2$$
$$= \left(y_{22} - \frac{y_{21} y_{12}}{y_{11}}\right) V_2$$
$$\therefore \left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{y_{11}}{y_{11} y_{22} - y_{21} y_{12}}$$

$y_{11} = y_{22}$

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$



$$\left.\frac{V_1}{I_1}\right|_{I_2=0} \quad \& \quad \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

$$\left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$0 = h_{21} I_1 + h_{22} V_2$$
$$\therefore V_2 = -\frac{h_{21}}{h_{22}} I_1$$
$$V_1 = h_{11} I_1 - \frac{h_{12} h_{21}}{h_{22}} I_1$$

$$\therefore \left.\frac{V_1}{I}\right|_{I_2=0} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} \quad \cdots (1)$$

$$\left.\frac{V_2}{I_2}\right|_{I_1=0}$$

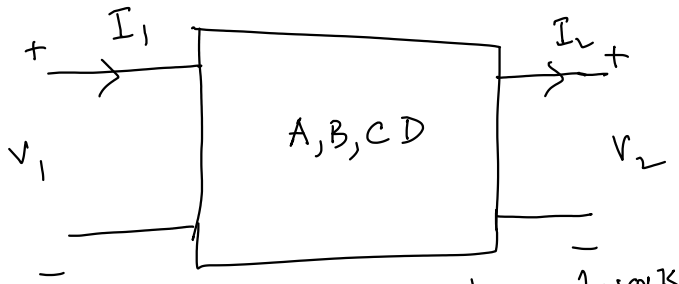
$$V_1 = h_{11} \times 0 + h_{12} V_2 = h_{12} V_2$$
$$I_2 = h_{21} \times 0 + h_{22} V_2 = h_{22} V_2$$

$$\left.\frac{V_2}{I_2}\right|_{I_1=0} = \frac{1}{h_{22}}$$

$$\therefore \frac{1}{h_{22}} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}$$

$$\text{or} \quad h_{11} h_{22} - h_{12} h_{21} = 1$$

$$\text{or} \quad \det \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = 1$$



$$V_1 = A V_2 + B I_2$$
$$I_1 = C V_2 + D I_2$$

If this Two-port network is to be symmetric

$$\left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_2}{-I_2}\right|_{I_1=0}$$

$$\left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$V_1 = A V_2$

$I_1 = C V_2$

$$\therefore \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{A}{C} \quad \text{---} (1)$$

$$\left.\frac{V_2}{-I_2}\right|_{I_1=0}$$

$V_1 = A V_2 + B I_2$

$\rightarrow 0 = C V_2 + D I_2$

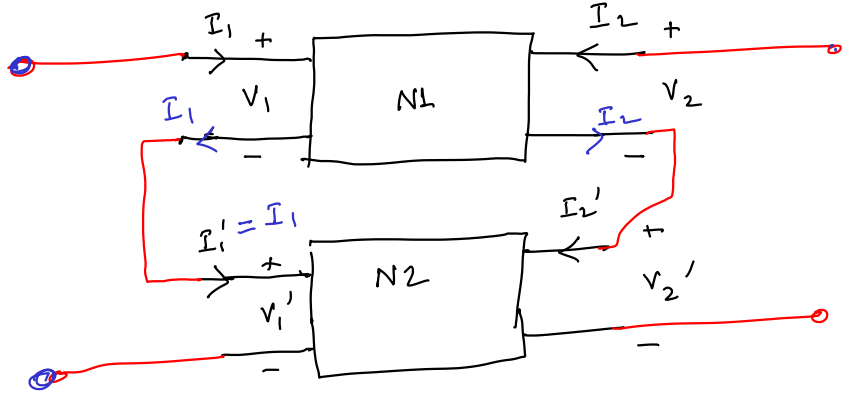
$\therefore C V_2 = -D I_2$

or $$\frac{V_2}{-I_2} = \frac{D}{C}$$

$\therefore \frac{A}{C} = \frac{D}{C}$

or $A = D$

Series Connection of Two port Networks



$I_1' = I_1$

and $I_2' = I_2$

for N1 :-

$V_1 = z_{11} I_1 + z_{12} I_2$ --- (1)

$V_2 = z_{21} I_1 + z_{22} I_2$ -- (2) *

for N2 :-

$V_1' = z_{11}' I_1' + z_{12}' I_2'$ -- (3)

$V_2' = z_{21}' I_1' + z_{22}' I_2'$ -- (4)

(3) & (4)

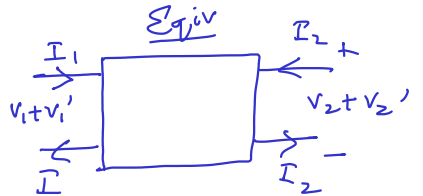
$V_1' = z_{11}' I_1 + z_{12}' I_2$ -- (5)

$V_2' = z_{21}' I_1 + z_{22}' I_2$ -- (6) *

Add (1) + (5)

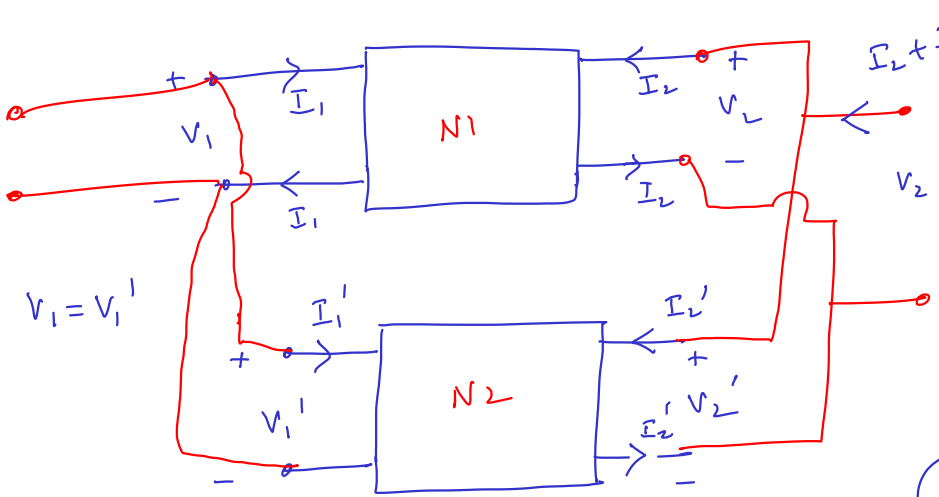
$V_1 + V_1' = (z_{11} + z_{11}') I_1 + (z_{12} + z_{12}') I_2$

Add (2) + (6) $V_2 + V_2' = (z_{21} + z_{21}') I_1 + (z_{22} + z_{22}') I_2$



over all imp. matrix is

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} + \begin{bmatrix} z_{11}' & z_{12}' \\ z_{21}' & z_{22}' \end{bmatrix} = \begin{bmatrix} z_{11}+z_{11}' & z_{12}+z_{12}' \\ z_{21}+z_{21}' & z_{22}+z_{22}' \end{bmatrix}$$



Connect input ports in parallel & output ports in parallel

$V_1' = V_1$

for N_1

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—(1)}$$

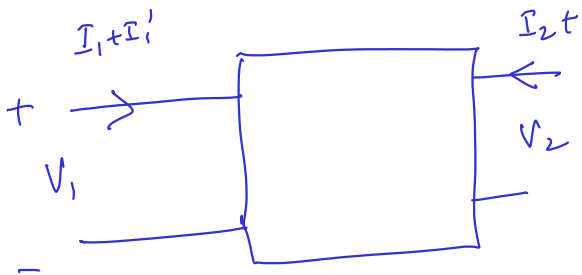
$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—(2)}$$

for N_2

$$I_1' = Y_{11}' V_1 + Y_{12}' V_2 \quad \text{—(3)}$$

$$I_2' = Y_{21}' V_1 + Y_{22}' V_2 \quad \text{—(4)}$$

$\therefore$ Equiv



Add (1) + (3)

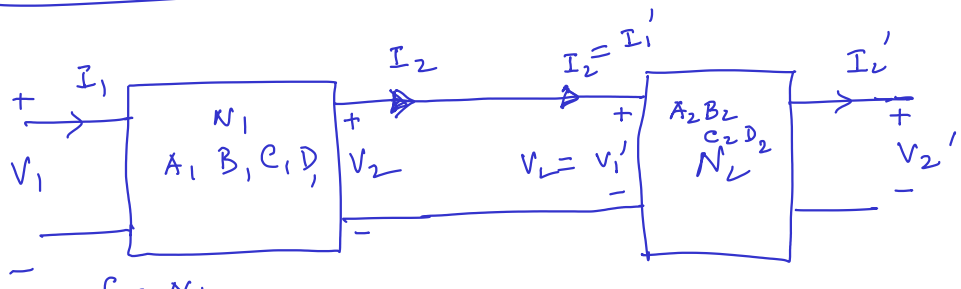
$$I_1 + I_1' = (Y_{11} + Y_{11}') V_1 + (Y_{12} + Y_{12}') V_2$$

Add (2) + (4)

$$I_2 + I_2' = (Y_{21} + Y_{21}') V_1 + (Y_{22} + Y_{22}') V_2$$

$$\therefore [Y_{eq}] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} + \begin{bmatrix} Y_{11}' & Y_{12}' \\ Y_{21}' & Y_{22}' \end{bmatrix}$$

Two Port Network in Cascade



for N_1

$$V_1 = A_1 V_2 + B_1 I_2$$

$$I_1 = C_1 V_2 + D_1 I_2$$

or $$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$V_2 = V_1' = A_2 V_2' + B_2 I_2'$$

$$I_2 = I_1' = C_2 V_2' + D_2 I_2'$$

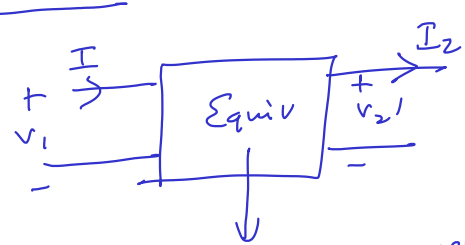
$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ I_2' \end{bmatrix}$$

or $$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ I_2' \end{bmatrix}$$

or $$= \begin{bmatrix} A_{eq} & B_{eq} \\ C_{eq} & D_{eq} \end{bmatrix}$$

Represent Each two-Network in terms of A-B-C-D parameters

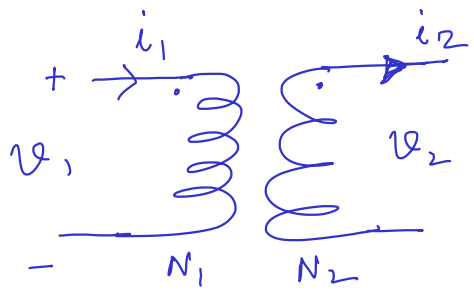
Equiv.



A B C D parameters of this equiv Two port network

$$= \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \times \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

How to represent an ideal Transformer as a two port network



$$\frac{v_2}{v_1} = \frac{N_2}{N_1} \quad \therefore \quad N_2 i_2 = N_1 i_1$$

$$i_2 = \frac{N_1}{N_2} i_1$$

$$v_2 = \frac{N_2}{N_1} v_1 = \frac{v_1}{a}$$

Let $\frac{N_1}{N_2} = a$

$$\therefore \quad v_2 = \frac{1}{a} v_1$$

$$i_2 = a\, i_1$$

$$v_1 = a v_2$$

$$i_1 = \frac{i_2}{a}$$

$$[h] = \begin{bmatrix} 0 & a \\ a & 0 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$v_1 = h_{11} i_1 + h_{12} v_2 = a v_2$$

$$i_2 = h_{21} i_1 + h_{22} v_2 = a i_1$$

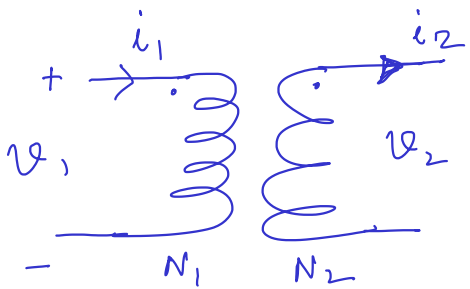
$$v_1 = z_{11} i_1 + z_{12} i_2$$

$$v_2 = z_{21} i_1 + z_{22} i_2$$

$$i_1 = y_{11} v_1 + y_{12} v_2$$

$$i_2 = y_{21} v_1 + y_{22} v_2$$

How to represent an ideal Transformer as a two port network



$a = \frac{N_1}{N_2}$

$$\frac{v_2}{v_1} = \frac{N_2}{N_1}$$

$$N_2 i_2 = N_1 i_1$$

$$i_2 = \frac{N_1}{N_2} i_1$$

$\therefore$

$$v_2 = \frac{N_2}{N_1} v_1 = \frac{v_1}{a}$$

Let $\frac{N_1}{N_2} = a$

$\therefore$

$$v_2 = \frac{1}{a} v_1$$

$$i_2 = a\, i_1$$

$$v_1 = a\, v_2$$

$$i_1 = \frac{i_2}{a}$$

$$[h] = \begin{bmatrix} 0 & a \\ a & 0 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$v_1 = h_{11} i_1 + h_{12} v_2 = a v_2$$

$$i_2 = h_{21} i_1 + h_{22} v_2 = a i_1$$

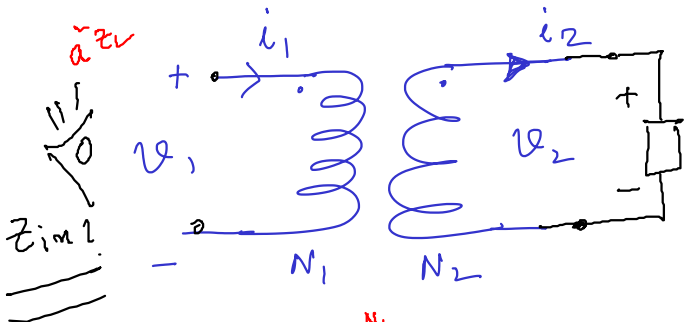
$$v_1 = z_{11} i_1 + z_{12} i_2$$

$$v_2 = z_{21} i_1 + z_{22} i_2$$

$$i_1 = y_{11} v_1 + y_{12} v_2$$

$$i_2 = y_{21} v_1 + y_{22} v_2$$

In terms of A B C D (or T parameters)



$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & 1/a \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix} \quad (ABCD)$$

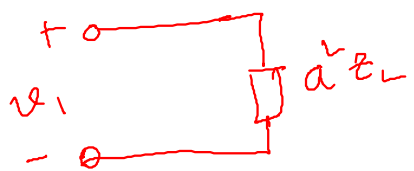
$$v_1 = a v_2 \qquad a = \frac{N_1}{N_2}$$

$$i_1 = i_2/a$$

$$\therefore \frac{v_1}{i_1} = \frac{a v_2}{i_2/a} = a^2 \frac{v_2}{i_2}$$

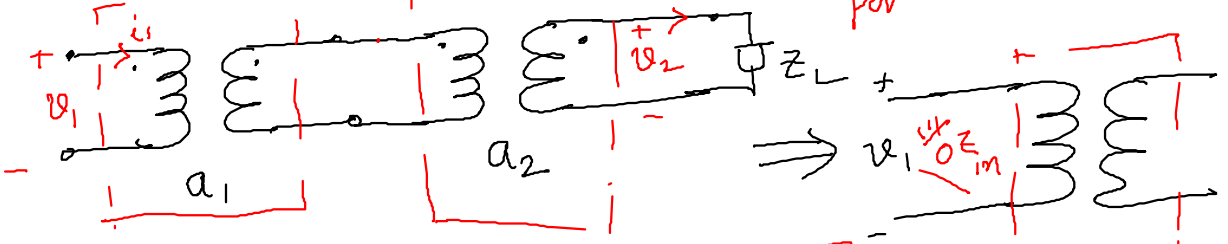
$$Z_{in} = a^2 Z_L$$

$a = \frac{N_1}{N_2}$ ⇓



Cascade

imp. seen through input por

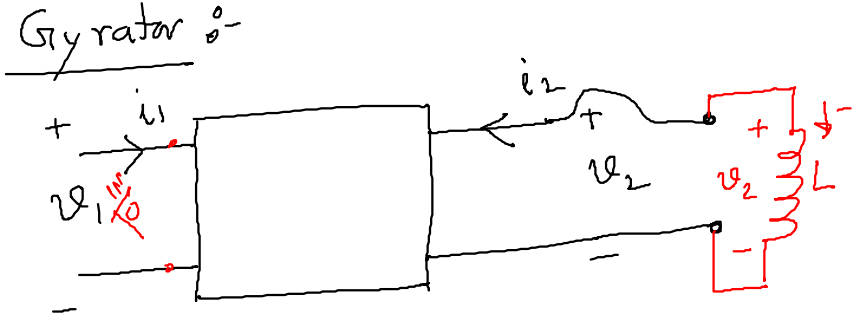


$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ 0 & 1/a_1 \end{bmatrix} \begin{bmatrix} a_2 & 0 \\ 0 & 1/a_2 \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} a_1 a_2 & 0 \\ 0 & 1/(a_1 a_2) \end{bmatrix}$$

$$a_{eq} = a_1 a_2$$

$$Z_{eq} = (a_1 a_2)^2 Z_L$$

Gyrator :-

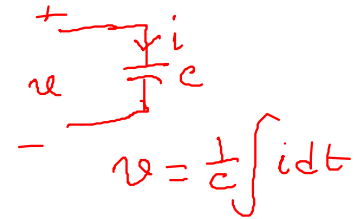


Suppose :

$v_1 = -K i_2$

$v_2 = K i_1$

K = Gyrator constant or Gyrator resistance



$v = \frac{1}{C}\int i\,dt$

$-i_2 = \frac{1}{L}\int v_2\,dt$ or $i_2 = -\frac{1}{L}\int v_2\,dt$

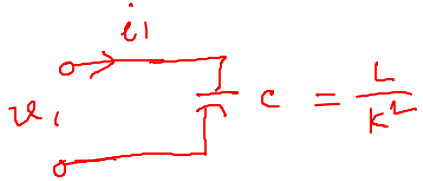
$v_1 = -K i_2$

$= (-K)\left(-\frac{1}{L}\right)\int v_2\,dt = \frac{K}{L}\int v_2\,dt$

but $v_2 = K i_1$

$v_1 = \frac{K}{L}\int K i_1\,dt = \frac{K^2}{L}\int i_1\,dt$

$v_1 = \frac{K^2}{L}\int i_1\,dt = \frac{1}{(L/K^2)}\int i_1\,dt$

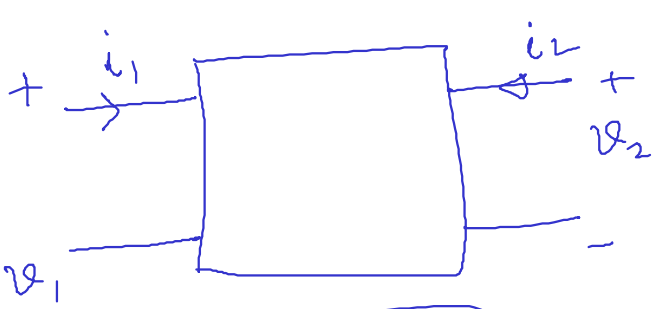


$C = \frac{L}{K^2}$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & -K \\ K & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

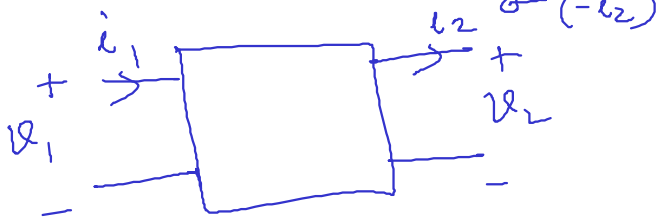
Z parameters

A, B, C, D parameter representation of a two port gyrator



$$v_1 = -K i_2$$
$$v_2 = K i_1$$

for ABCD parameters



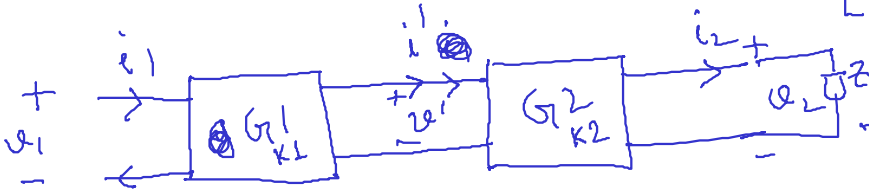
$$v_1 = K(-i_2) = K i_2$$
$$v_2 = K i_1 \Rightarrow i_1 = \frac{1}{K} v_2$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 0 & K \\ \frac{1}{K} & 0 \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} \frac{K_1}{K_2} & 0 \\ 0 & \frac{K_2}{K_1} \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

$$= \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$

where $a = \frac{K_1}{K_2}$



$$\Rightarrow Z_{eq} = \left(\frac{K_1}{K_2}\right)^2 Z_L$$

$$\begin{bmatrix} v' \\ i' \end{bmatrix} = \begin{bmatrix} 0 & K_2 \\ \frac{1}{K_2} & 0 \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix} \quad \text{for } G_2$$

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} 0 & K_1 \\ \frac{1}{K_1} & 0 \end{bmatrix} \begin{bmatrix} v' \\ i' \end{bmatrix} = \begin{bmatrix} 0 & K_1 \\ \frac{1}{K_1} & 0 \end{bmatrix} \begin{bmatrix} 0 & K_2 \\ \frac{1}{K_2} & 0 \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$