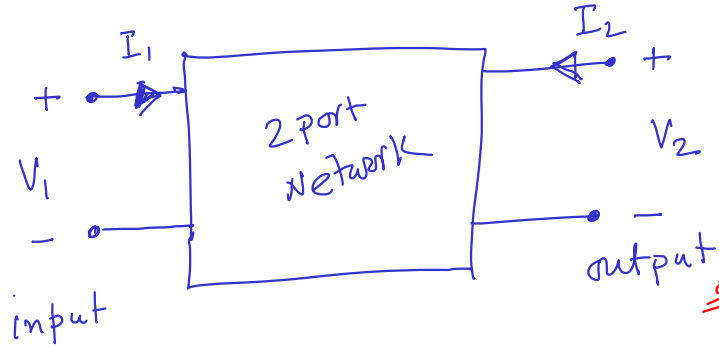


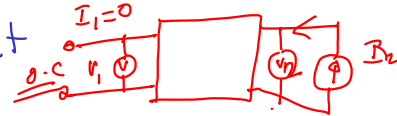
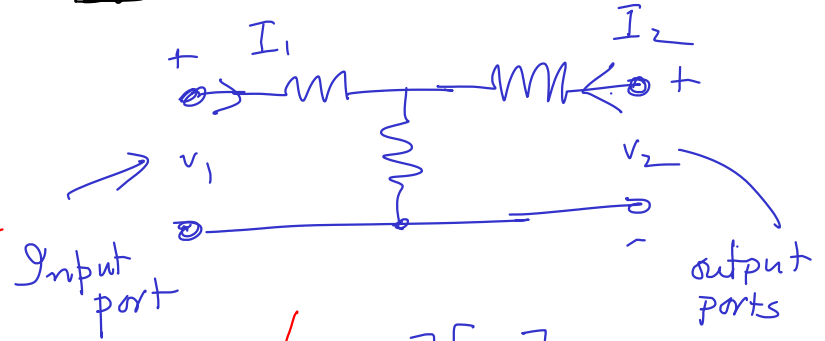
Two Port Network :-

Lec :- 71



$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}$$

$$Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$



Z-parameters

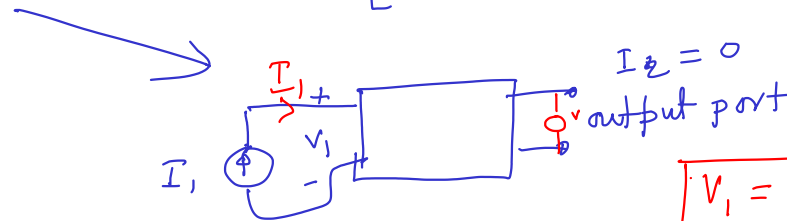
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0}$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$$

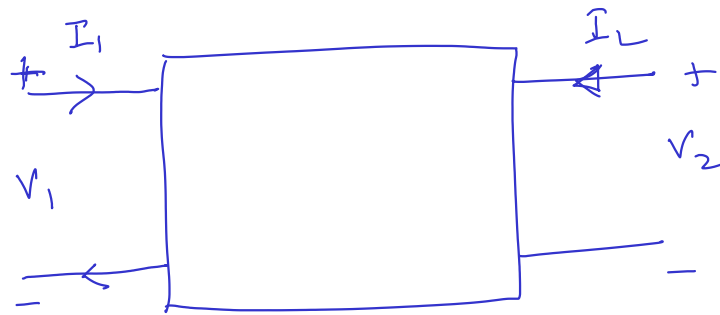


$$I_2 = 0$$

$$V_1 = Z_{11} I_1 + 0$$

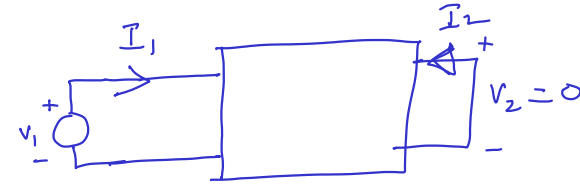
$$V_2 = Z_{21} I_1 + 0$$

$$\begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned}$$



$$\begin{cases} I_1 = Y_{11} V_1 + Y_{12} V_2 \\ I_2 = Y_{21} V_1 + Y_{22} V_2 \end{cases}$$

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}$$



$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}$$

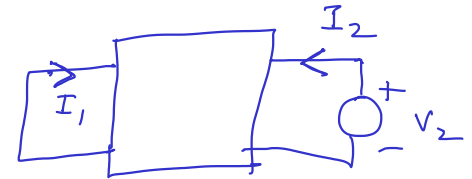
$$\begin{aligned} I_1 &= Y_{11} V_1 + 0 \\ I_2 &= Y_{21} V_1 + 0 \end{aligned}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

↑
Y-parameters

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

$$Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

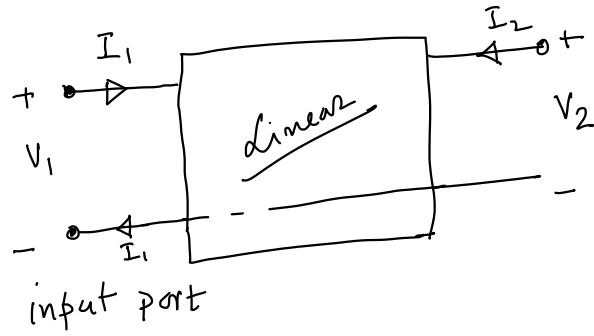
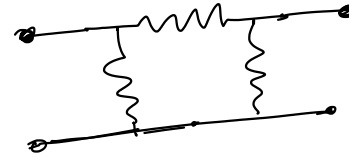
$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

=

$$\begin{aligned} I_1 &= 0 + Y_{12} V_2 \\ I_2 &= 0 + Y_{22} V_2 \end{aligned}$$

Two port network

dec-72



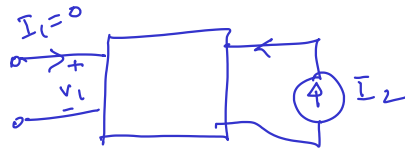
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

Transfer impedance

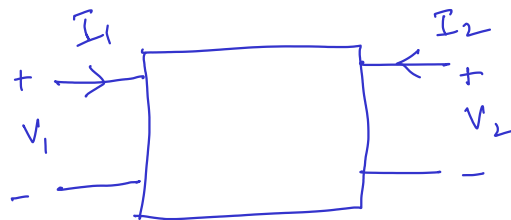


$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

driving point impedance



$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$



$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$



$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{cases}$$

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

= driving point admittance

$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$

= transfer admittance.

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{(ad-bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{\begin{pmatrix} Z_{11}Z_{22} & -Z_{12}Z_{21} \end{pmatrix}} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$Y_{22} = \frac{Z_{11}}{(Z_{11}Z_{22} - Z_{12}Z_{21})}$$

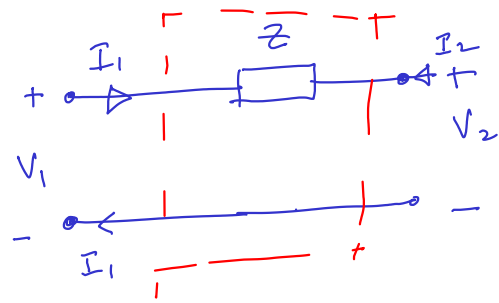
$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} ; Y_{11} = \frac{Z_{22}}{(Z_{11}Z_{22} - Z_{12}Z_{21})}$$

$$Y_{12} = -\frac{Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}}$$

$$Y_{21} = \frac{-Z_{21}}{(Z_{11}Z_{22} - Z_{12}Z_{21})} \rightarrow$$

$$Y_{11} \neq \frac{1}{Z_{11}}$$

$$Z_{12} \neq 0$$



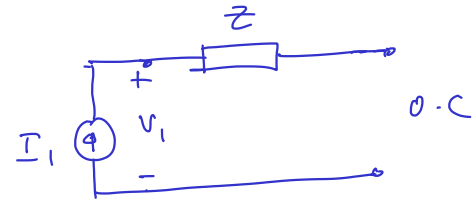
Trying find
~~To find~~ Z parameters

$$V_1 = z_{11} I_1 + z_{12} I_2$$

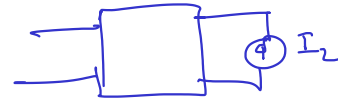
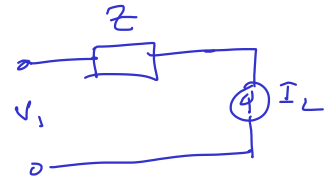
$$V_2 = z_{21} I_1 + z_{22} I_2$$

Z matrix does not
exist

$$z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0} \rightarrow \infty$$



$$z_{12} = \frac{V_1}{I_2} \Big|_{I_1}$$



$$I_1 = \frac{V_1 - V_2}{Z}$$

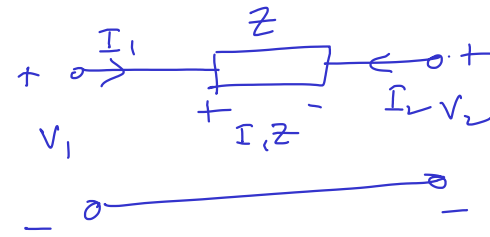
$$I_2 = \frac{V_2 - V_1}{Z} = -I_1$$

$$V_1 - V_2 = Z I_1$$

or

Does Y parameter exist

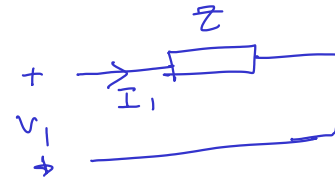
for



$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$



$$V_1 = I_1 Z$$

$$\text{or } \frac{I_1}{V_1} = \frac{1}{Z} = Y_{11}$$

$$V_1 = V_2 + I_1 Z$$

$$\text{or } I_1 Z = V_1 - V_2$$

$$\text{or } \begin{cases} I_1 = \frac{1}{Z} V_1 - \frac{1}{Z} V_2 \\ I_2 = -\frac{1}{Z} V_1 + \frac{1}{Z} V_2 \end{cases}$$

$$I_2 = \frac{V_2 - V_1}{Z_1}$$

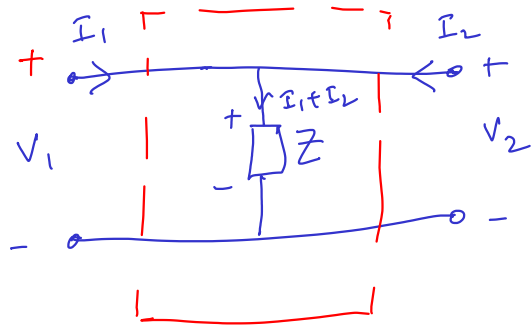
$$[Z] = [Y]^{-1}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{Z} & -\frac{1}{Z} \\ -\frac{1}{Z} & \frac{1}{Z} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[Y] \rightarrow \det^{\uparrow} = 0$$

lec 73

Trying to find out z-parameters



$$V_1 = (I_1 + I_2)Z = ZI_1 + ZI_2$$

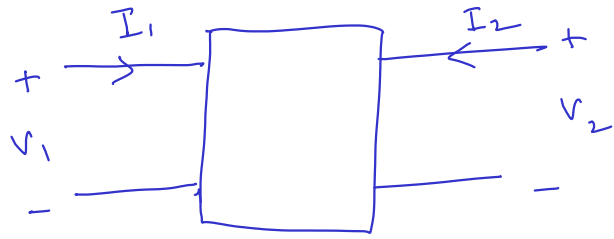
$$V_2 = (I_1 + I_2)Z = ZI_1 + ZI_2$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z & Z \\ Z & Z \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

↑
det = 0

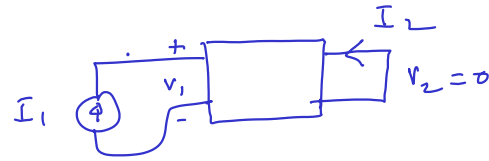
Hybrid parameters of Two port network $[h]$



$$V_1 = \underline{h}_{11} I_1 + \underline{h}_{12} V_2$$

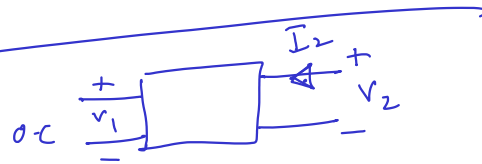
$$I_2 = \underline{h}_{21} I_1 + \underline{h}_{22} V_2$$

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2=0} \rightarrow (\Omega)$$



$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2=0} \rightarrow \text{dimensionless}$$

$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1=0}$$



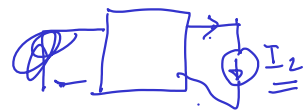
$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1=0} \rightarrow \mathcal{V}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

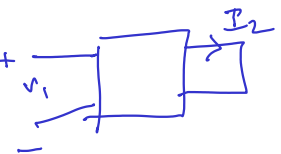
$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$= \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$B = \frac{V_1}{I_2} \Big|_{V_2=0}$$



$$\frac{1}{B} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

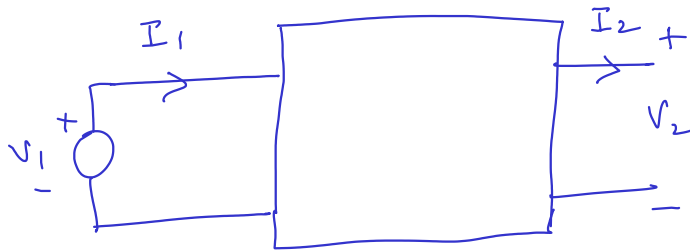


$$D = \frac{I_1}{I_2} \Big|_{V_2=0}$$

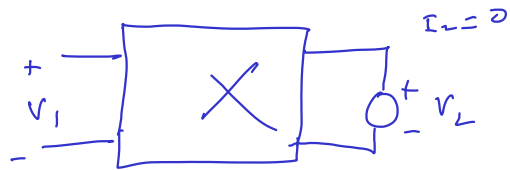
$$\frac{1}{D} = \frac{I_2}{I_1} \Big|_{V_2=0}$$



ABCD parameters [Transmiss paraters]
T-parameters

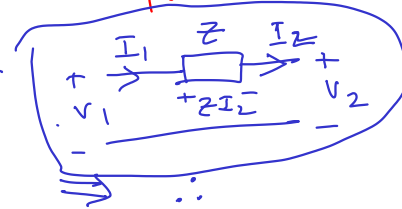


$$\begin{aligned} V_1 &= AV_2 + BI_2 \\ I_1 &= CV_2 + DI_2 \end{aligned}$$



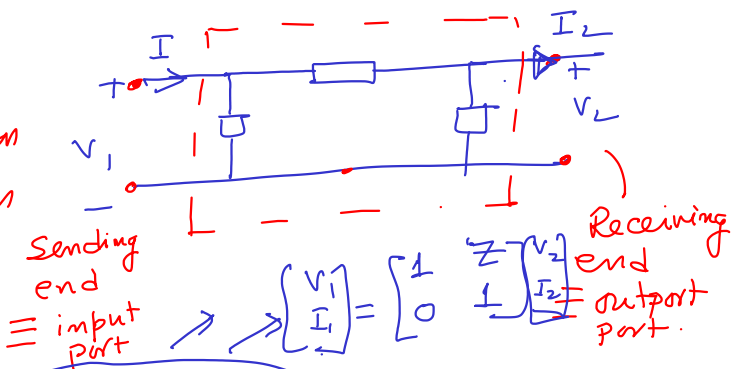
note
change of convention
current direction
in the output
port.

$$\begin{aligned} V_1 &= V_2 + ZI_2 \\ I_1 &= I_2 \end{aligned}$$

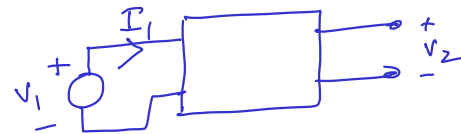


$$C = \frac{I_1}{V_2} \Big|_{I_2=0}$$

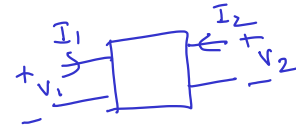
$$\frac{1}{C} = \frac{V_2}{I_1} \Big|_{I_2=0}$$



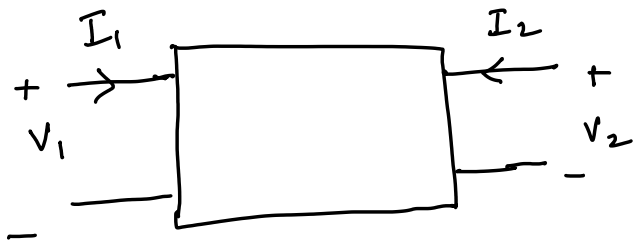
$$\frac{1}{A} = \frac{V_2}{V_1} \Big|_{I_2=0}$$



Receiving
end
output
port.



for z , y and h parameters



$$\begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned} \quad \& \quad \begin{aligned} I_1 &= y_{11} V_1 + y_{12} V_2 \\ I_2 &= y_{21} V_1 + y_{22} V_2 \end{aligned}$$

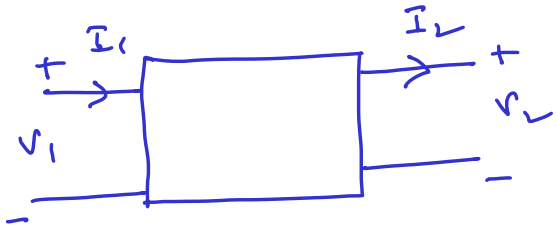
$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$\&$

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$



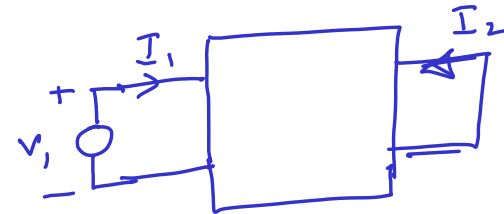
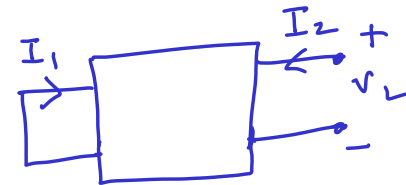
$$V_1 = A V_2 + B I_2$$

$$I_1 = C V_2 + D I_2$$

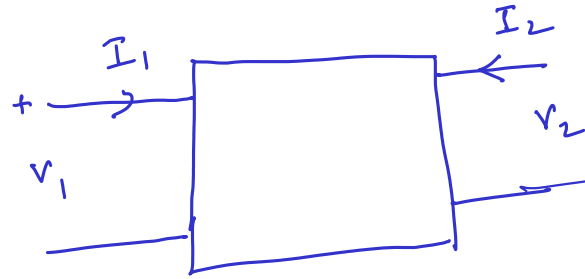
Theory of Reciprocity:

If Reciprocal then

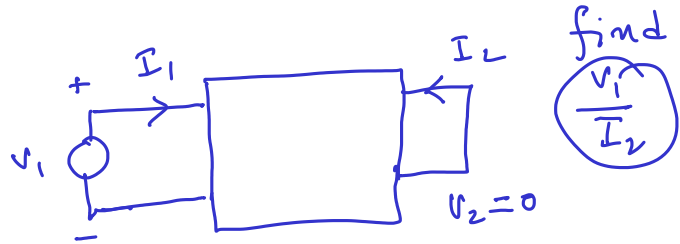
$$\frac{V_1}{I_2} = \frac{V_2}{I_1}$$



To find out ~~parameters~~ ^{when} a two-port network is reciprocal
Condition



$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 \end{aligned}$$



find $\frac{V_1}{I_2}$

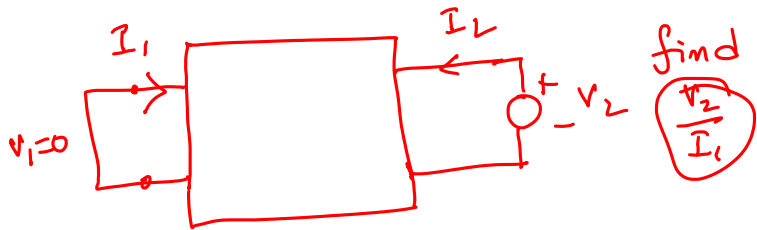
$$V_1 = Z_{11}I_1 + Z_{12}I_2 \Rightarrow \frac{V_1}{I_2} = Z_{11}\frac{I_1}{I_2} + Z_{12}$$

$$0 = Z_{21}I_1 + Z_{22}I_2 \therefore \frac{I_1}{I_2} = -\frac{Z_{22}}{Z_{21}}$$

$$\frac{V_1}{I_2} = -\frac{Z_{11}Z_{22}}{Z_{21}} + Z_{12}$$

$$\begin{aligned} \frac{V_1}{I_2} &= \frac{V_2}{I_1} \\ -\frac{Z_{11}Z_{22}}{Z_{21}} + Z_{12} &= Z_{21} - \frac{Z_{11}Z_{22}}{Z_{12}} \\ \text{or } \frac{Z_{12}Z_{21} - Z_{11}Z_{22}}{Z_{21}} &= \frac{Z_{12}Z_{21} - Z_{11}Z_{22}}{Z_{12}} \end{aligned}$$

Reciprocal two port network
 $Z_{12} = Z_{21}$



find $\frac{V_2}{I_1}$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 \therefore \frac{V_2}{I_1} = Z_{21} + Z_{22}\frac{I_2}{I_1}$$

$$0 = Z_{11}I_1 + Z_{12}I_2 \therefore$$

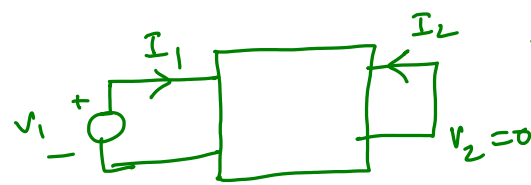
$$\frac{I_2}{I_1} = -\frac{Z_{11}}{Z_{12}}$$

$$\frac{V_2}{I_1} = Z_{21} - \frac{Z_{11}Z_{22}}{Z_{12}}$$

Condition for a 2 port network to be Reciprocal in terms of $[Y]$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

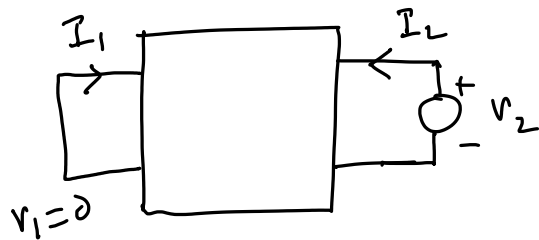
$$I_2 = Y_{21} V_1 + Y_{22} V_2$$



find $\left(\frac{V_1}{I_2}\right)$

$$I_1 = Y_{11} V_1$$

$$I_2 = Y_{21} V_1 \rightarrow \boxed{\frac{V_1}{I_2} = \frac{1}{Y_{21}}}$$



find $\left(\frac{V_2}{I_1}\right)$

$$I_1 = Y_{12} V_2$$

$$\therefore \frac{V_2}{I_1} = \frac{1}{Y_{12}}$$

if Reciprocal.

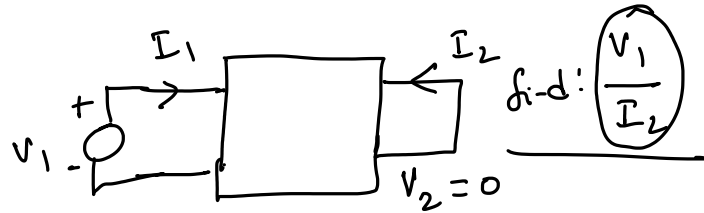
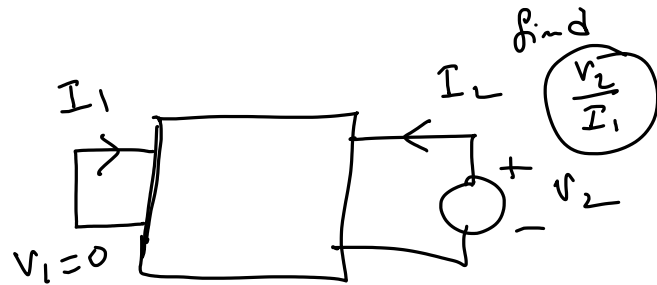
$$\frac{1}{Y_{12}} = \frac{1}{Y_{21}}$$

$$\text{or } Y_{12} = Y_{21}$$

Condition for Reciprocity in terms of h parameters

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$



$$\begin{aligned} V_1 &= h_{11} I_1 \\ I_2 &= h_{21} I_1 \end{aligned} \Rightarrow \boxed{\frac{V_1}{I_2} = \frac{h_{11}}{h_{21}}}$$

||

$$0 = h_{11} I_1 + h_{12} V_2 \therefore \boxed{\frac{V_2}{I_1} = -\frac{h_{11}}{h_{12}}}$$

$$\therefore \text{Condition: } -\frac{h_{11}}{h_{21}} = -\frac{h_{11}}{h_{12}}$$

$$\text{or } \boxed{h_{12} = -h_{21}}$$

$z \rightarrow z_{12} = z_{21}$
 $y \rightarrow y_{12} = y_{21}$
 $h \rightarrow h_{12} = -h_{21}$

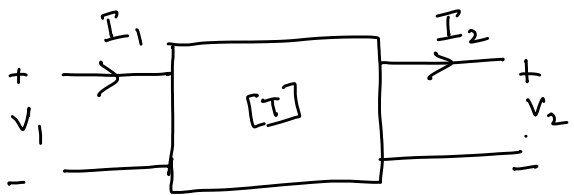
Condition of Reciprocity of a two-port network when represented as (A, B, C, D) parameters

Dec-75

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

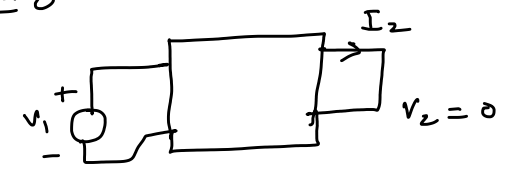
transmission line parameters.
 $[T] =$

$$\det \begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$$



$$\begin{aligned} V_1 &= AV_2 + BI_2 \\ I_1 &= CV_2 + DI_2 \end{aligned}$$

find $\left(\frac{V_1}{I_2} \right)$ with $V_2 = 0$

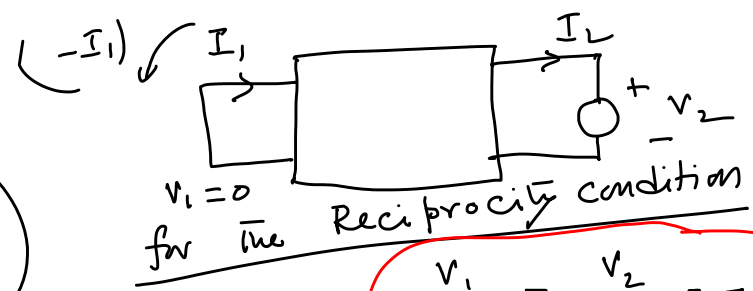


$$V_1 = BI_2$$

$$\frac{V_1}{I_2} = B$$

$$B = -\frac{B}{(BC-AD)} = \frac{B}{(AD-BC)}$$

$$AD - BC = 1$$



$$\frac{V_2}{(-I_1)}$$

Reciprocity condition for the

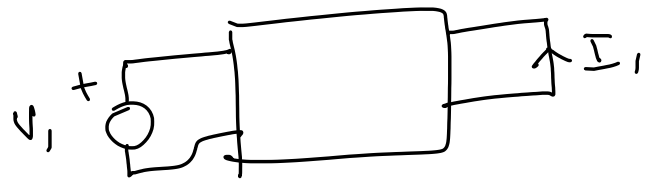
$$\frac{V_1}{I_2} = \frac{V_2}{(-I_1)} = -\frac{V_2}{I_1}$$

$$0 = AV_2 + BI_2 \rightarrow I_2 = -\frac{A}{B}V_2$$

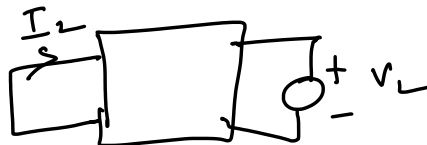
$$I_1 = CV_2 + DI_2$$

$$\therefore I_1 = CV_2 - \frac{A}{B}V_2 = \frac{(BC-AD)}{B}V_2$$

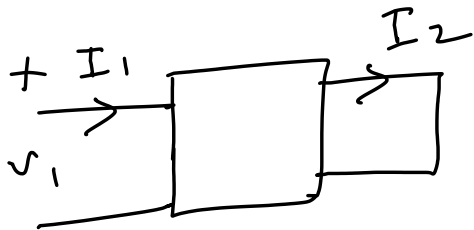
$$\therefore \frac{V_2}{I_1} = \frac{B}{(BC-AD)}$$



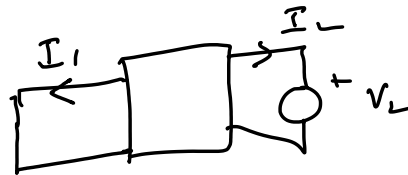
$$\frac{V_1}{I_1}$$



$$\frac{V_2}{I_2}$$



$$\frac{V_1}{I_2}$$



Suppose a Two port network is Reciprocal $\Rightarrow \underline{z_{12} = z_{21}}$
 If represented in terms of Z-parameters

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{12} I_1 + z_{22} I_2$$

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_1 = z_{11} I_1 - z_{12} I_1 + z_{12} I_1 + z_{12} I_2$$

$$V_1 = (z_{11} - z_{12}) I_1 + z_{12} (I_1 + I_2) \quad \text{--- (1)}$$

$$V_2 = \underbrace{z_{22} I_2 - z_{12} I_2} + z_{12} I_2 + z_{12} I_1$$

$$V_2 = (z_{22} - z_{12}) I_2 + z_{12} (I_1 + I_2) \quad \text{--- (2)}$$

