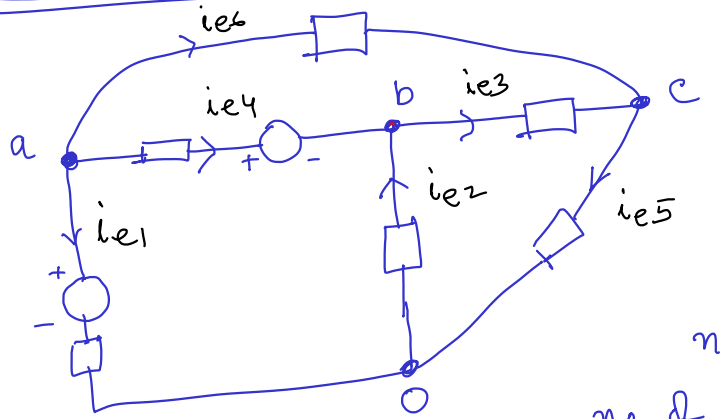
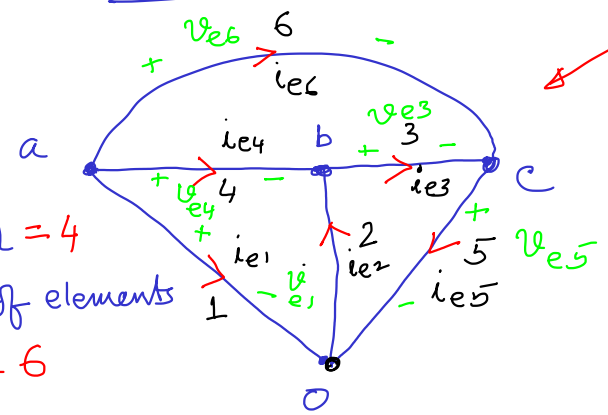
Graph Theory applied to Network Problem :-



Graph of the Network



no. of nodes = n = 4

no. of branches = no. of elements

= e = 6

Incidence Matrix [A]

Write KCL equations at nodes a, b, c and o

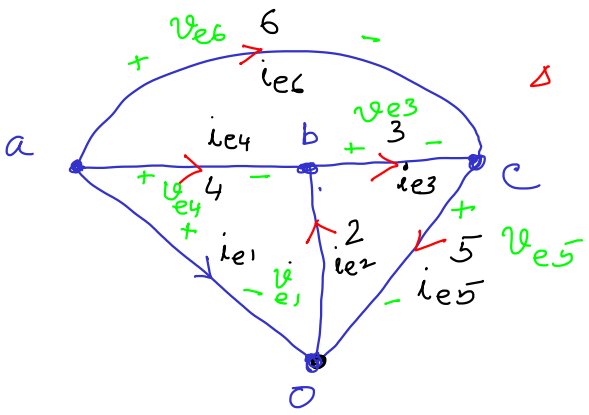
at node a : $i_{e1} + i_{e4} + i_{e6} = 0$ — (1)

.... b : $i_{e3} - i_{e4} - i_{e2} = 0$ — (2)

at node c : $i_{e5} - i_{e3} - i_{e6} = 0$ — (3)

at node o : $i_{e2} - i_{e1} - i_{e5} = 0$ → no new equation

Independent Eqn.



KCL at – node a : $i_{e1} + i_{e4} + i_{e6} = 0$ — (1)

.... b : $i_{e3} - i_{e4} - i_{e2} = 0$ — (2)

at node c : $i_{e5} - i_{e3} - i_{e6} = 0$ — (3)

at node o : $i_{e2} - i_{e1} - i_{e5} = 0 \rightarrow$ (4) → not new eqn.

$[A]$

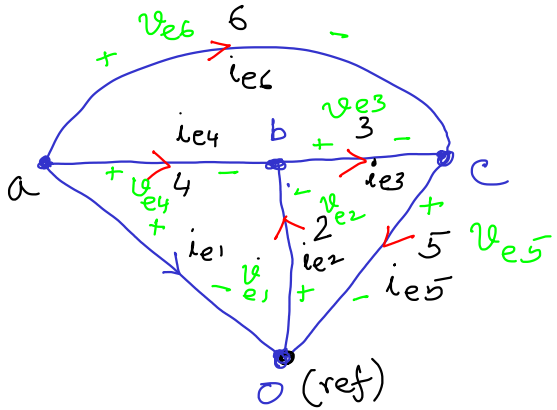
$$\begin{array}{c} \\ a \\ \rightarrow b \\ c \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} i_{e1} \\ i_{e2} \\ i_{e3} \\ i_{e4} \\ i_{e5} \\ i_{e6} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \\ 0 \end{bmatrix}$$

$\Rightarrow$ reduced incidence Matrix

$$[A][i_e] = [0]$$

— (1)

Lec-60



If n = total nodes = 4

e = no. of edges = 6

$$v_{e1} = v_{ao}$$

$$v_{e2} = v_{ob} = -v_{bo}$$

$$v_{e3} = v_{bo} - v_{co}$$

$$v_{e4} = v_{ao} - v_{bo}$$

$$v_{e5} = v_{co}$$

$$v_{e6} = v_{ao} - v_{co}$$

$$[A]\,[i_e] = [0]$$

$(n-1)\times e$ $\quad$ $(e\times 1)$

$$\begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & -1 \end{bmatrix} \end{array} \begin{bmatrix} i_{e1} \\ i_{e2} \\ i_{e3} \\ i_{e4} \\ i_{e5} \\ i_{e6} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \leftarrow e\times 1$$

Try to express the element voltages in terms of node voltages (v_{ao}, v_{bo}, v_{co})

$$\begin{bmatrix} v_{e1} \\ v_{e2} \\ v_{e3} \\ v_{e4} \\ v_{e5} \\ v_{e6} \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} v_{ao} & v_{bo} & v_{co} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \end{array} \begin{bmatrix} v_{ao} \\ v_{bo} \\ v_{co} \end{bmatrix}$$

$e\times 1$ $\qquad$ $e\times(n-1)$

$\rightarrow [A]^T$

or $[v_e] = [A]^T [v_N]$ $\quad$ (v_N: column vector, $(n-1)\times 1$)

$(e\times 1)$ $\qquad$ $e\times(n-1)$

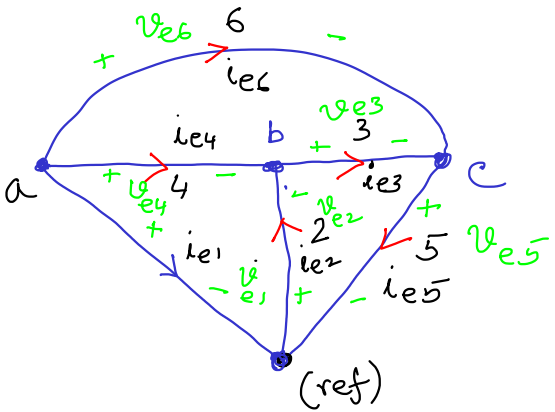
(Reduced order) [A] matrix

$$[A]\,[i_e] = [0]$$
$$[A]^T\,[v_N] = [v_e]$$

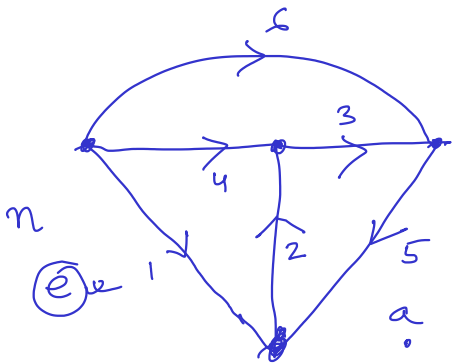
Solving ckt by nodal analysis method these equns to be used.

Tie set Matrix [B]

Tree of a graph.



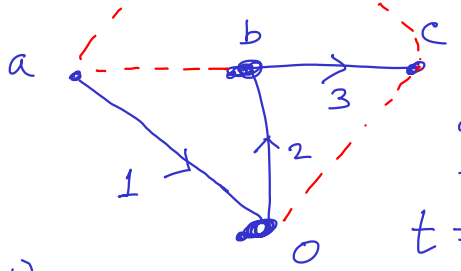
Graph.



Tree

co-tree

links



$(n-1)$

no. of twigs

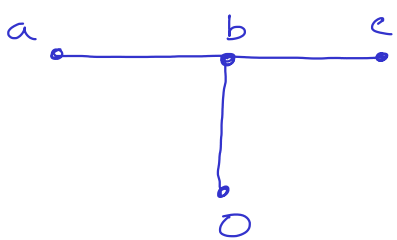
$t = (n-1)$

$e - (n-1)$

branches of a co-tree = links

branches of a tree = twigs.

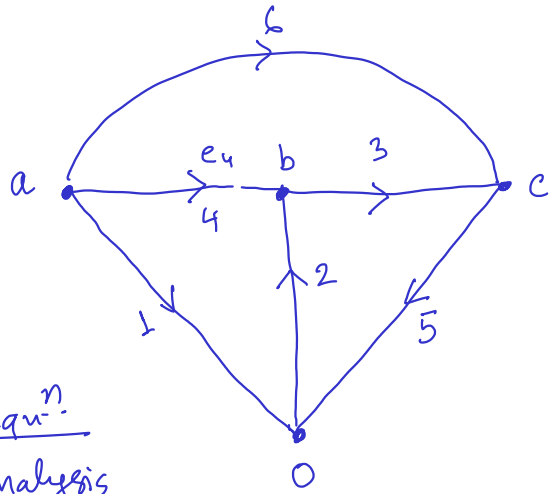
$(n-1)$

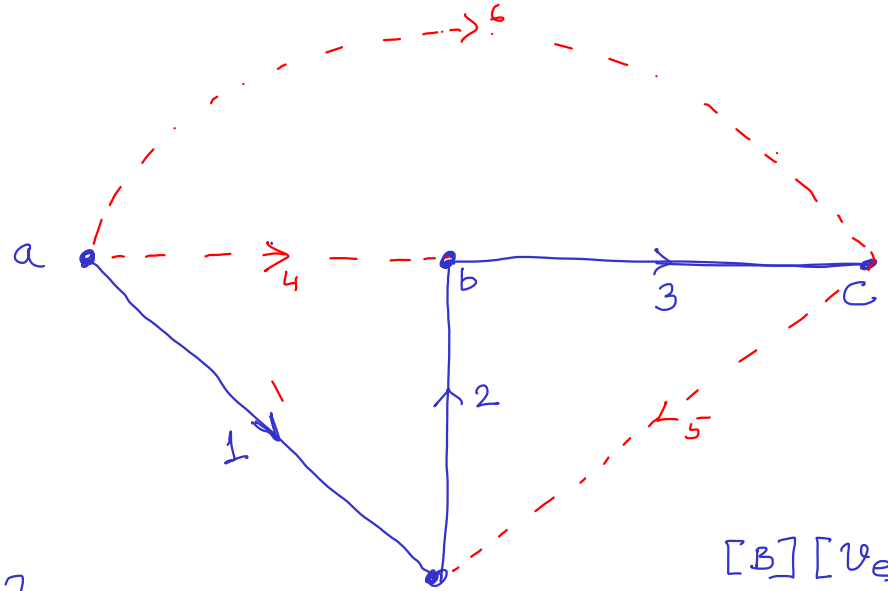


Lec - 61

— Tree (twigs)

--- Co-tree (links)





KVL Equⁿ.

Loop analysis

L4 : $e_4 - e_2 - e_1 = 0$

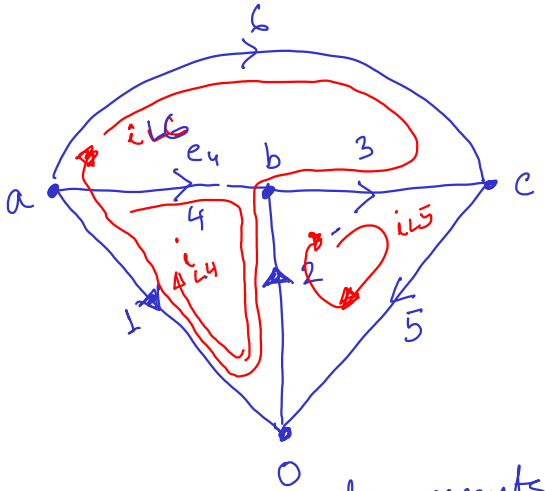
L5 : $e_5 + e_2 + e_3 = 0$

L6 : $e_6 - e_3 - e_2 - e_1 = 0$

In matrix form

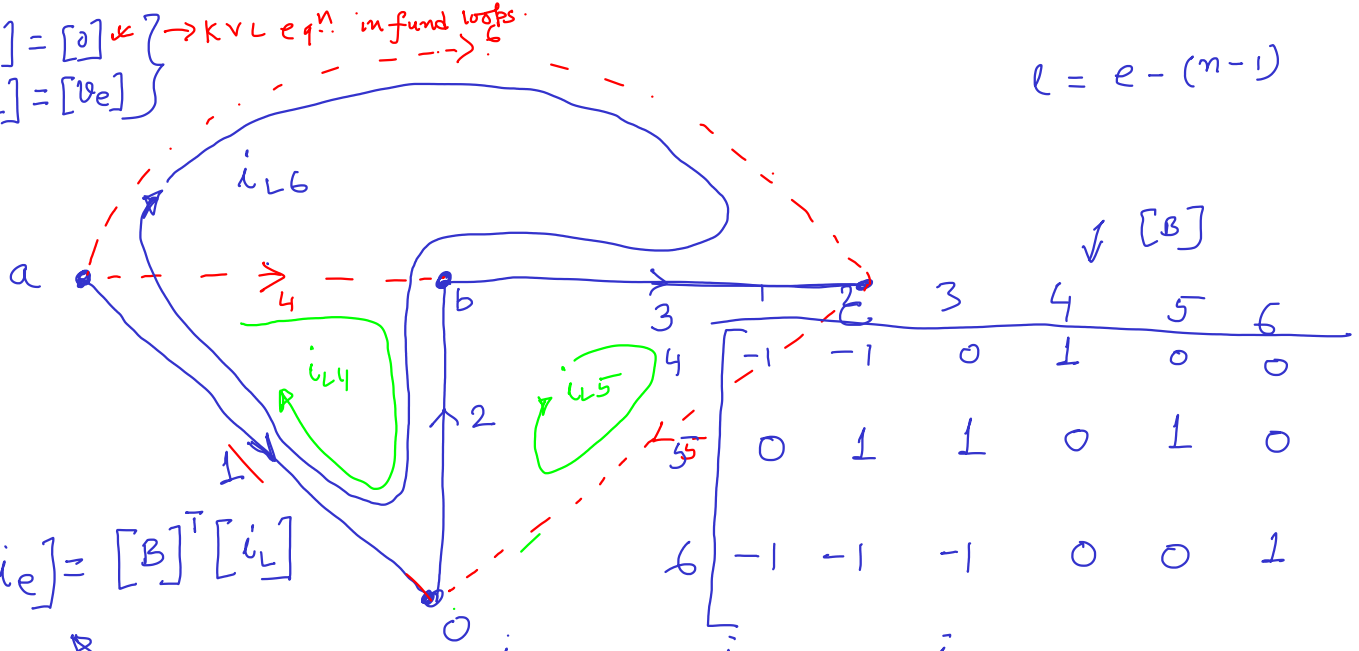
$$[B][v_e] = [0] \quad \leftarrow \text{KVL}$$

$$\begin{array}{c} \\ L4 \\ L5 \\ L6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} v_{e1} \\ v_{e2} \\ v_{e3} \\ v_{e4} \\ v_{e5} \\ v_{e6} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$



$[B][v_e] = [0]$ → KVL eqⁿ in fund loops
$[B]^T [i_L] = [v_e]$

$l = e - (n-1)$



Expressing element currents in terms of loop currents.

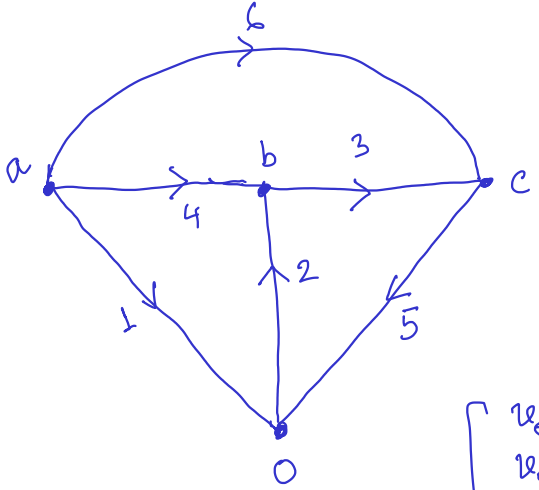
$[i_e] = [B]^T [i_L]$

$$\begin{cases} i_{e1} = -i_{L4} - i_{L6} \\ i_{e2} = i_{L5} - i_{L4} - i_{L6} \\ i_{e3} = i_{L5} - i_{L6} \\ i_{e4} = i_{L4} \\ i_{e5} = i_{L5} \\ i_{e6} = i_{L6} \end{cases}$$

$$\begin{bmatrix} i_{e1} \\ i_{e2} \\ i_{e3} \\ i_{e4} \\ i_{e5} \\ i_{e6} \end{bmatrix} = \begin{bmatrix} i_{L4} & i_{L5} & i_{L6} \\ -1 & 0 & -1 \\ -1 & 1 & -1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{L4} \\ i_{L5} \\ i_{L6} \end{bmatrix}$$

$e \times 6$ $\qquad e \times l$

Nodal Method



Incidence Matrix $[A]$ =

	1	2	3	4	5	6
a	1	0	0	1	0	1
b	0	-1	1	-1	0	0
c	0	0	-1	0	1	-1

$$[A][i_e] = 0$$

$$[v_e] = [A]^T [v_N]$$

$$v_N = \begin{bmatrix} v_{a0} \\ v_{b0} \\ v_{c0} \end{bmatrix}$$

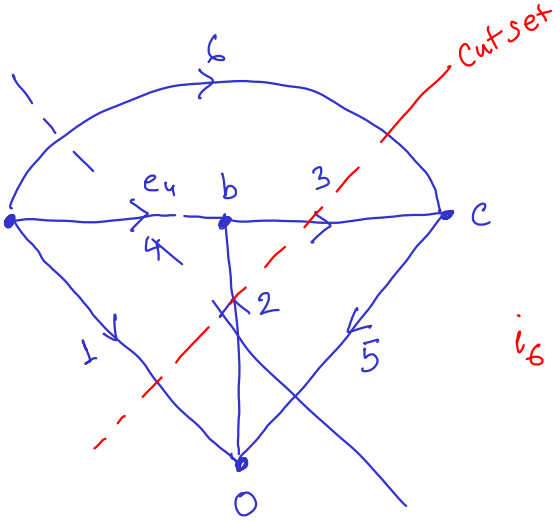
$[v_e]$

$$\begin{bmatrix} v_{e1} \\ v_{e2} \\ v_{e3} \\ v_{e4} \\ v_{e5} \\ v_{e6} \end{bmatrix}$$

Tie Set Matrix $[B]$:

$$[B][v_e] = [0]$$ → KVL equn's in fundamental loops.

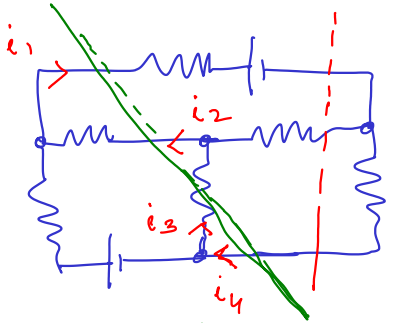
$$[B]^T [i_L] = [i_e]$$



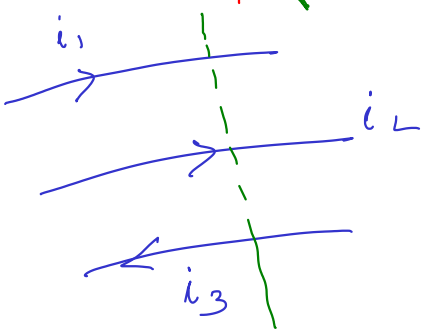
Cut set Matrix [Q]

KCL not at nodes
but at some line (cut set)

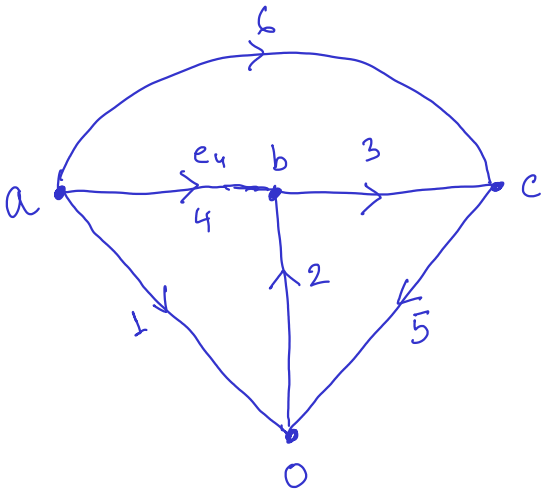
$i_6 + i_3 - i_2 + i_1 = 0$



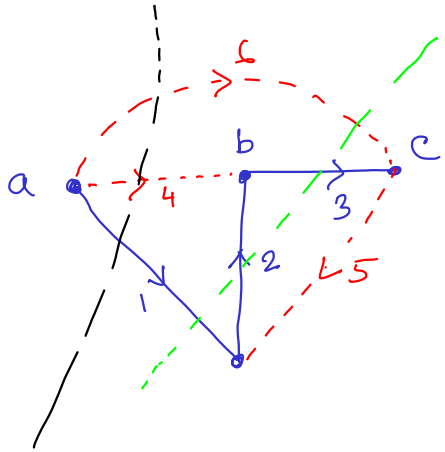
$i_1 - i_2 + i_3 - i_4 = 0$



$i_1 + i_2 - i_3 = 0$



$\Rightarrow$



not a fundamental cut-set

— (1), 2, 3 are twigs.

---- 4, 5, 6 are links.

Fundamental cut-set is one which will involve one and only one twig.

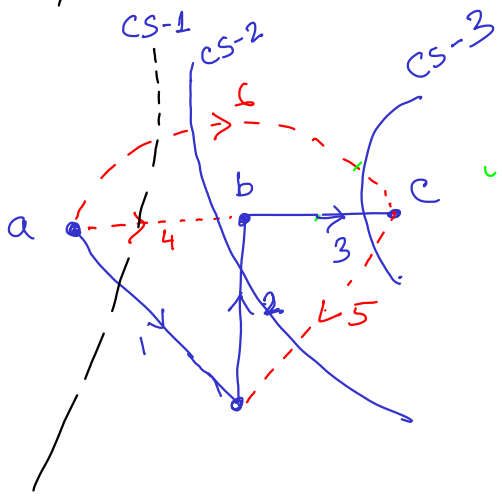
Write down KCL at the cut-set lines

KCL at cutsets:

CS-1: $i_{e1} + i_{e4} + i_{e6} = 0$

CS-2: $i_{e2} + i_{e4} + i_{e6} - i_{e5} = 0$ ← connection

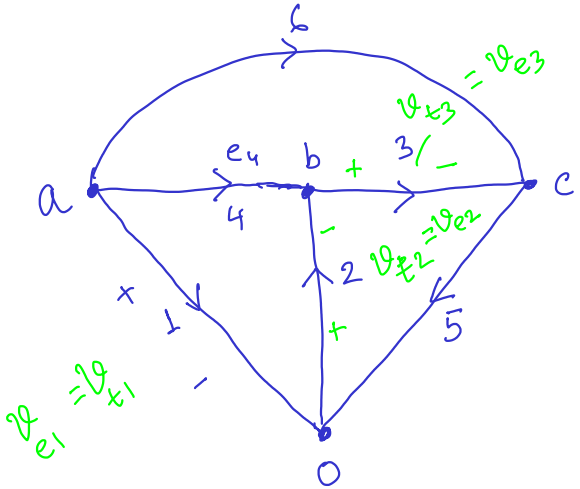
CS-3: $i_{e3} + i_{e6} - i_{e5} = 0$

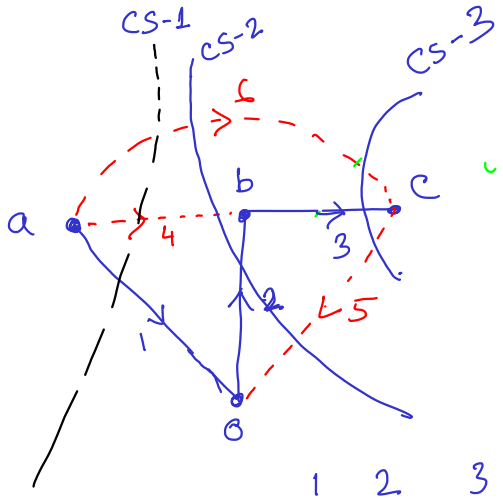


[Q]

$t = n-1$

Suppose v_{t1}, v_{t2} & v_{t3} are known





— (1), 2, 3 are twigs.

---- 4, 5, 6 are links.

Fundamental cut-set is one which will involve one and only one twig.

KCL at cutsets:

cs-1: $i_{e1} + i_{e4} + i_{e6} = 0$

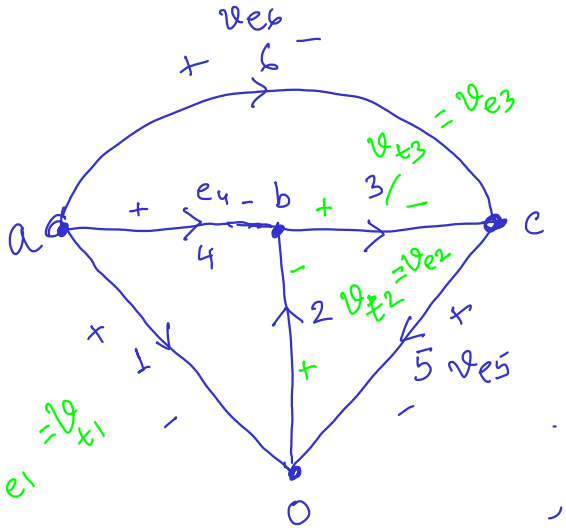
cs-2: $i_{e2} + i_{e4} + i_{e6} - i_{e5} = 0$ ← correction

cs-3: $i_{e3} + i_{e6} - i_{e5} = 0$

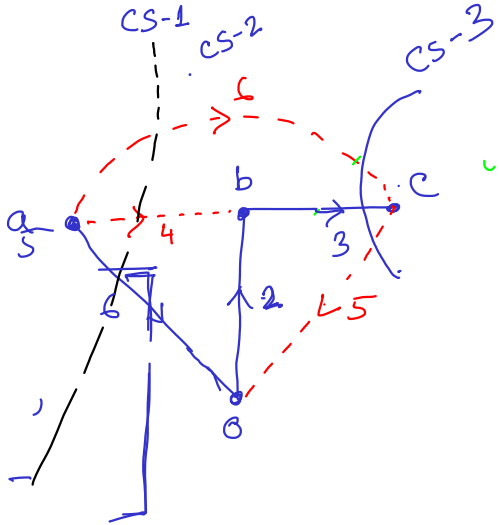
$$\begin{array}{c} \\ \text{cs-1} \\ \text{cs-2} \\ \text{cs-3} \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} i_{e1} \\ i_{e2} \\ i_{e3} \\ i_{e4} \\ i_{e5} \\ i_{e6} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[Q]_{t \times e} [i_e]_{e \times 1} = [0]_{t \times 1}$$

[Q]



Suppose v_{t1}, v_{t2} & v_{t3} are known



KCL at cutsets :

CS-1 : $i_{e1} + i_{e4} + i_{e6} = 0$

CS-2 : $i_{e2} + i_{e4} + i_{e6} = 0$ (correction: $-i_{e5}$)

S-3 : $i_{e3} + i_{e6} - i_{e5} = 0$

$$[Q][i_e] = 0$$

$$[Q]^T [v_t] = [v_e]$$

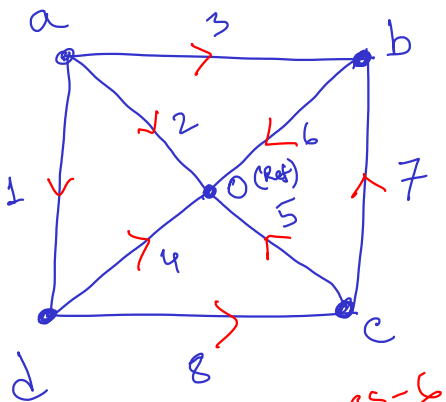
Relate element voltages with twig voltages.

$$v_{e1} = v_{t1}$$
$$v_{e2} = v_{t2}$$
$$v_{e3} = v_{t3}$$
$$v_{e4} = v_{t1} + v_{t2}$$
$$v_{e5} = -v_{t2} - v_{t3}$$
$$v_{e6} = v_{t1} + v_{t2} + v_{t3}$$

$$[A][i_e] = [0]$$
$$[A]^T [v_N] = [v_e]$$

$$\begin{bmatrix} v_{e1} \\ v_{e2} \\ v_{e3} \\ v_{e4} \\ v_{e5} \\ v_{e6} \end{bmatrix} = - \begin{bmatrix} t1 & t2 & t3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_{t1} \\ v_{t2} \\ v_{t3} \end{bmatrix} \quad [v_t]$$

$$[Q]^T [v_t]$$



$n = 5$

$e = 8$

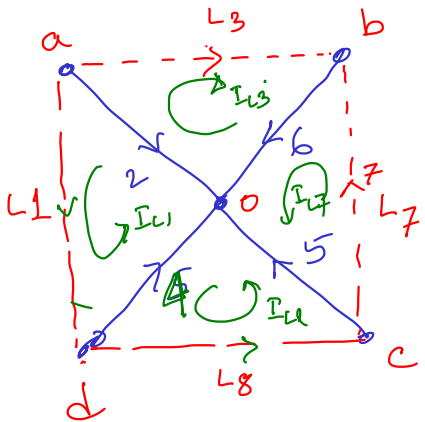
$[B][V_e] = 0$

$[A] =$

	1	2	3	4	5	6	7	8
a	1	1	1	0	0	0	0	0
b	0	0	−1	0	0	1	−1	0
c	0	0	0	0	1	0	1	−1
d	−1	0	0	1	0	0	0	1

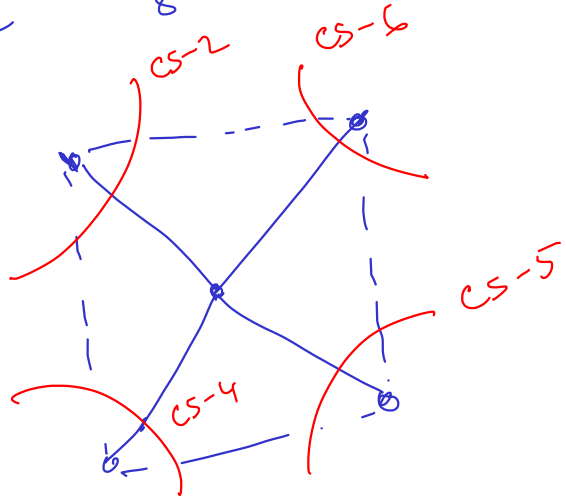
KCL

Take This Tree

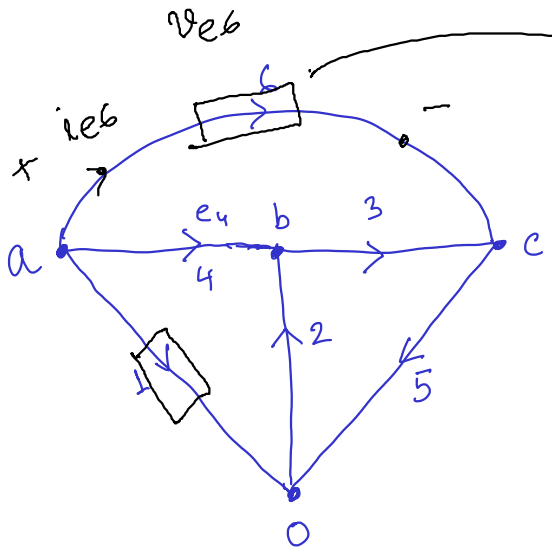


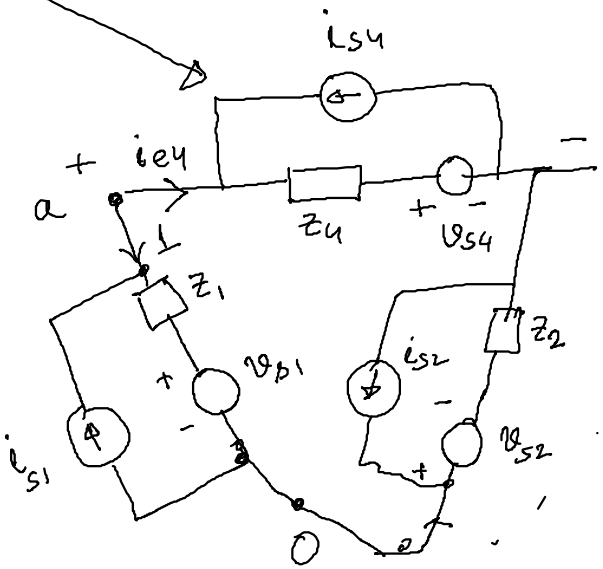
$[B] =$

	1	2	3	4	5	6	7	8
L3:	0	−1	1	0	0	1	0	0
L1:	1	−1	0	1	0	0	0	0
L7:	0	0	0	0	−1	1	1	0
L8	0	0	0	−1	1	0	0	1



write $[Q]$ matrix





etc

incidence matrix

$\rightarrow [A][i_e] = [0]$ — KCL

$[A]^T [v_N] = [v_e]$ — Nodal

Tie-set $[B][v_e] = [0]$ — KVL

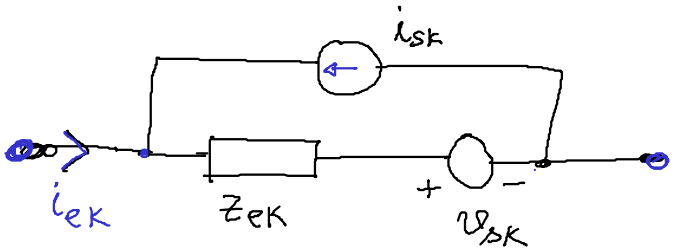
$[B]^T [i_L] = [i_e]$ — Loop analysis

Cut-set $[Q][i_e] = [0]$ — KCL at fund. cutsets

$[Q]^T [v_t] = [v_e]$

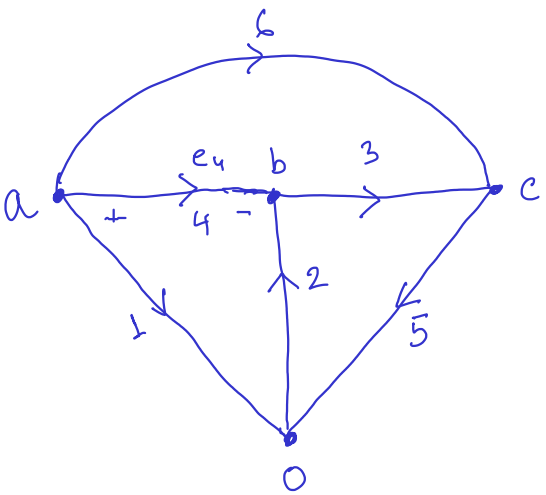
Want to solve a network problem by loop analysis

k^{th} element looks like:



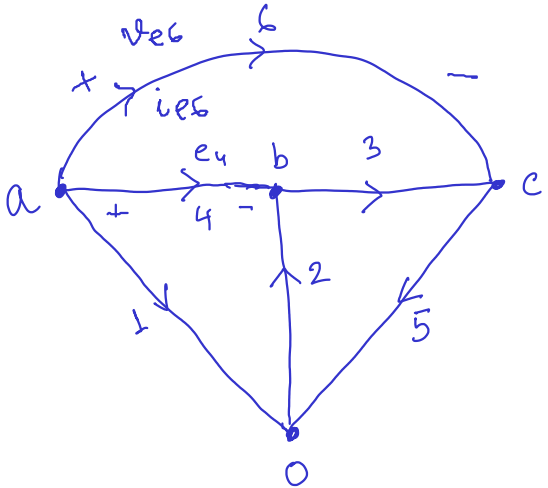
i_{sk} → source current in the k^{th} element

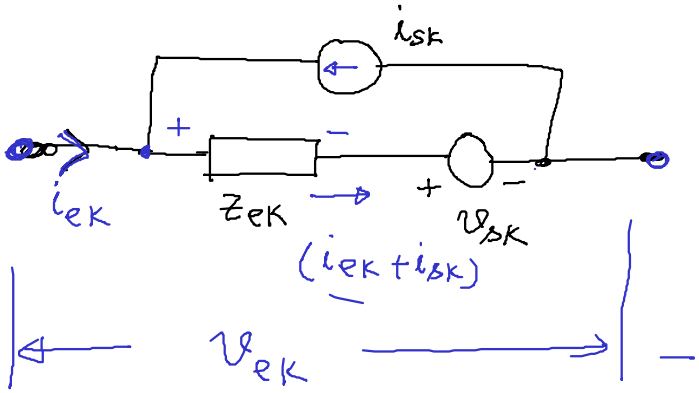
v_{sk} → source voltage in k^{th} element



Want to solve a network problem by loop analysis

k^{th} element looks like:





i_{sk} → source current in the k^{th} element

v_{sk} → source voltage in k^{th} element

$$v_{ek} = z_{ek}(i_{ek}+i_{sk}) + v_{sk} = z_{ek}\, i_{ek} + z_{ek}\, i_{sk} + v_{sk}$$

$$v_{e1} = z_{e1}\, i_{e1} + z_{e1}\, i_{s1} + v_{s1}$$
$$v_{e2} = z_{e2}\, i_{e2} + z_{e2}\, i_{s2} + v_{s2}$$
$$v_{e3} = z_{e3}\, i_{e3} + z_{e3}\, i_{s3} + v_{s3}$$

$$v_{e4} = z_{e4}\, i_{e4} + z_{e4}\, i_{s4} + v_{s4}$$
$$v_{e5} = z_{e5}\, i_{e5} + z_{e5}\, i_{s5} + v_{s5}$$
$$v_{e6} = z_{e6}\, i_{e6} + z_{e6}\, i_{s6} + v_{s6}$$

$$v_{ek} = z_{ek}(i_{ek} + i_{sk}) + v_{sk} = z_{ek} i_{ek} + z_{ek} i_{sk} + v_{sk}$$

$$v_{e1} = z_{e1} i_{e1} + z_{e1} i_{s1} + v_{s1}$$
$$v_{e2} = z_{e2} i_{e2} + z_{e2} i_{s2} + v_{s2}$$
$$v_{e3} = z_{e3} i_{e3} + z_{e3} i_{s3} + v_{s3}$$
$$v_{e4} = z_{e4} i_{e4} + z_{e4} i_{s4} + v_{s4}$$
$$v_{e5} = z_{e5} i_{e5} + z_{e5} i_{s5} + v_{s5}$$
$$v_{e6} = z_{e6} i_{e6} + z_{e6} i_{s6} + v_{s6}$$

$$[v_e] = [z_e][i_e] + [z_e][i_s] + [v_s]$$

$$\begin{bmatrix} v_{e1} \\ v_{e2} \\ v_{e3} \\ v_{e4} \\ v_{e5} \\ v_{e6} \end{bmatrix} = \begin{bmatrix} z_{e1} & & & & & \\ & z_{e2} & & & & \\ & & z_{e3} & & & \\ & & & z_{e4} & & \\ & & & & z_{e5} & \\ & & & & & z_{e6} \end{bmatrix} \begin{bmatrix} i_{e1} \\ i_{e2} \\ i_{e3} \\ i_{e4} \\ i_{e5} \\ i_{e6} \end{bmatrix} + \begin{bmatrix} z_{e1} & & & & & \\ & z_{e2} & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & z_{e6} \end{bmatrix} \begin{bmatrix} i_{s1} \\ i_{s2} \\ i_{s3} \\ i_{s4} \\ i_{s5} \\ i_{s6} \end{bmatrix} + \begin{bmatrix} v_{s1} \\ v_{s2} \\ v_{s3} \\ v_{s4} \\ v_{s5} \\ v_{s6} \end{bmatrix}$$

$$= \quad =$$

$$[v_e] = [z_e][i_e] + [z_e][i_s] + [v_s]$$

$[B][v_e] = 0$

KVL ↙ loop current

$[B]^T [i_L] = [i_e]$

Premultiply by $[B]$

$$[B][v_e] = [B][z_e][i_e] + [B][z_e][i_s] + [B][v_s] = 0$$

or $$[B][z_e]\underline{[i_e]} = -[B][z_e][i_s] - [B][v_s]$$

or $$\underline{[B][z_e][B]^T}[i_L] = -[B][z_e][i_s] - [B][v_s]$$

↖ $[Z]$ = loop imp. matrix -

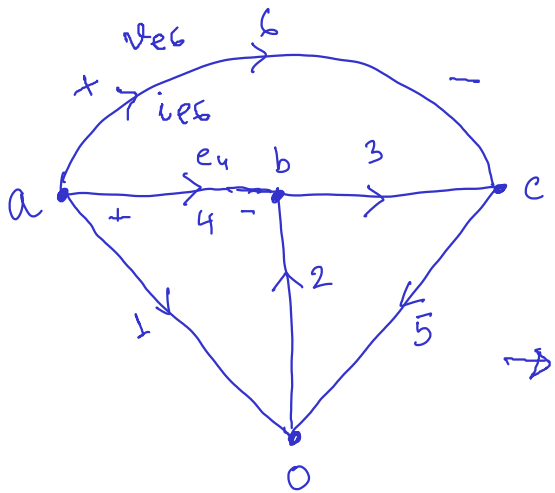
$$[i_L] = -[Z]^{-1}[B][z_e][i_s] - [Z]^{-1}[B][v_s]$$

$[i_e] = [B]^T [i_L]$ Then $i_{z_1} = i_{e_1} + i_{s_1}$ | $[i_{zk}] = [i_e] + [i_s]$

Want to solve a network problem by ~~for~~ Nodal analysis

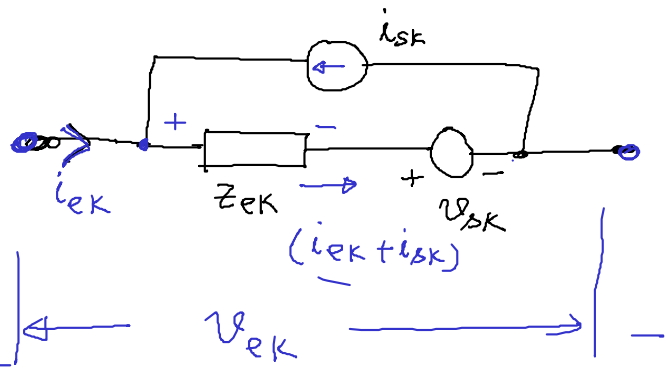
dec 65

k^{th} element looks like:



$$[A][i_e] = [0]$$

$$[A]^T [v_N] = [v_e]$$



i_{sk} → source current in the k^{th} element

v_{sk} → source voltage in k^{th} element

$$i_{ee} = [i_e] + [i_s]$$

$$v_{ek} = z_{ek} i_{ek} + z_{ek} i_{sk} + v_{sk}$$

$$z_{ek} i_{ek} = v_{ek} - z_{ek} i_{sk} - v_{sk}$$

$$i_{ek} = \frac{1}{z_{ek}} v_{ek} - i_{sk} - \frac{v_{sk}}{z_{ek}} = y_{ek} v_{ek} - i_{sk} - y_{ek} v_{ek}$$

$$v_{ek} = z_{ek}(i_{ek} + i_{sk}) + v_{sk} = z_{ek} i_{ek} + z_{ek} i_{sk} + v_{sk}$$

$$[A][i_e] = [0]$$
$$[A]^T [v_N] = [v_e]$$

After getting $[v_N]$

$$i_{ek} = \frac{1}{z_{ek}} v_{ek} - i_{sk} - \frac{v_{sk}}{z_{ek}} = y_{ek} v_{ek} - i_{sk} - y_{ek} v_{sk}$$

or, $i_{ek} = y_{ek} v_{ek} - i_{sk} - y_{ek} v_{sk}$

$$[Y] = \begin{bmatrix} y_{e1} & & & \\ & y_{e2} & & \\ & & \ddots & \\ & & & y_{e6} \end{bmatrix}$$

$$[v_e] = [A]^T [v_N]$$

$$[i_e] = [Y][v_e] - [i_s] - [Y][v_s]$$

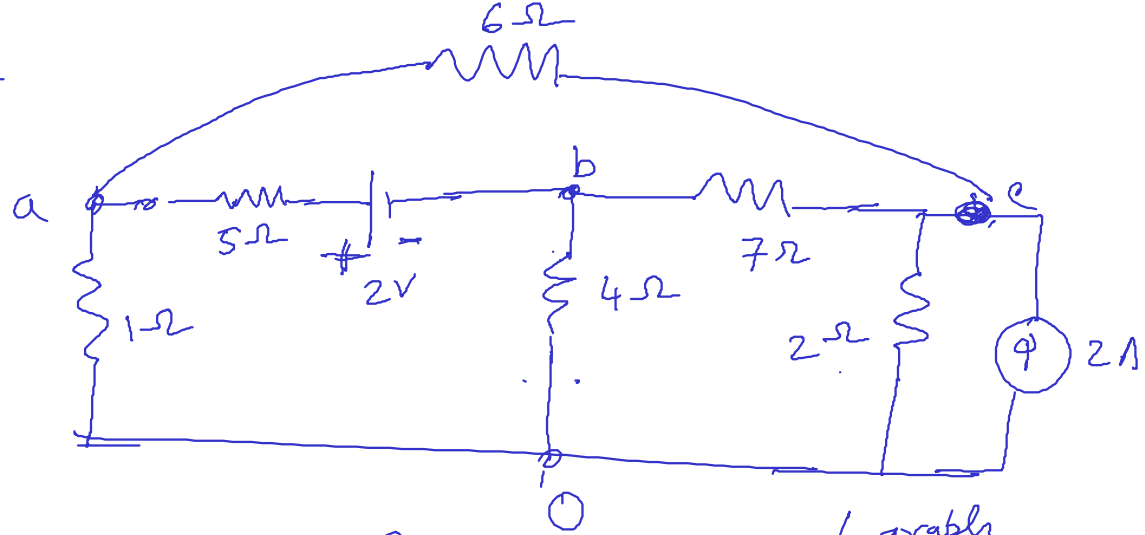
$$[A][i_e] = [A][Y][v_e] - A[i_s] - [A][Y][v_e] = [0]$$

$$= [0] \quad \therefore [A][Y][v_e] = [A][i_s] + [A][Y][v_s]$$

Node Admittance matrix $[Y]$

$$[A][Y][A]^T [v_N] = [A][i_s] + [A][Y][v_s]$$

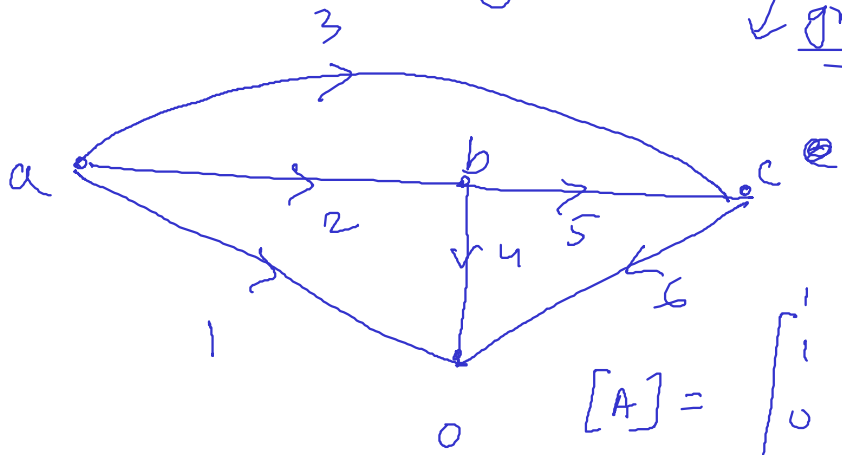
Example



$$[Y] = \begin{bmatrix} 1 & 1/5 & 1/6 & 1/4 & 1/7 & 1/2 \end{bmatrix}$$

$$[Z_e] = \begin{bmatrix} 1 & & & & & \\ & 5 & & & & \\ & & 6 & & & \\ & & & 4 & & \\ & & & & 7 & \\ & & & & & 2 \end{bmatrix}$$

↓ graph



$$[v_s] = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad [i_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$$

$$[A] = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} i_{e1} \\ i_{e2} \\ i_{e3} \\ i_{e4} \\ i_{e5} \\ i_{e6} \end{bmatrix}$$