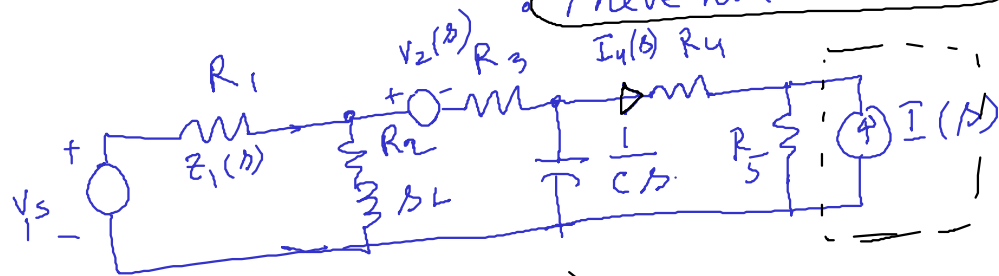
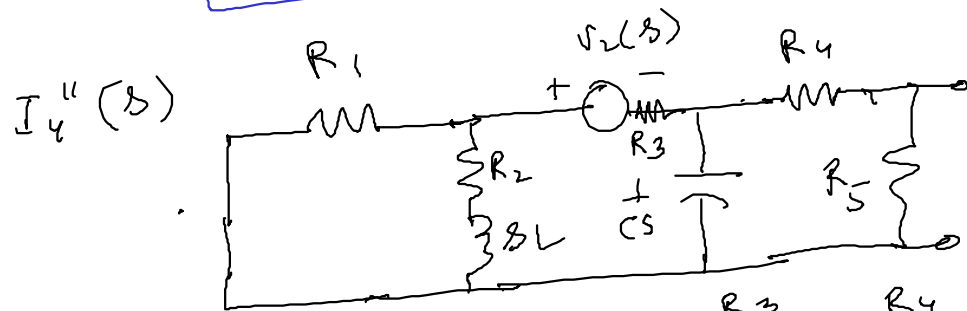
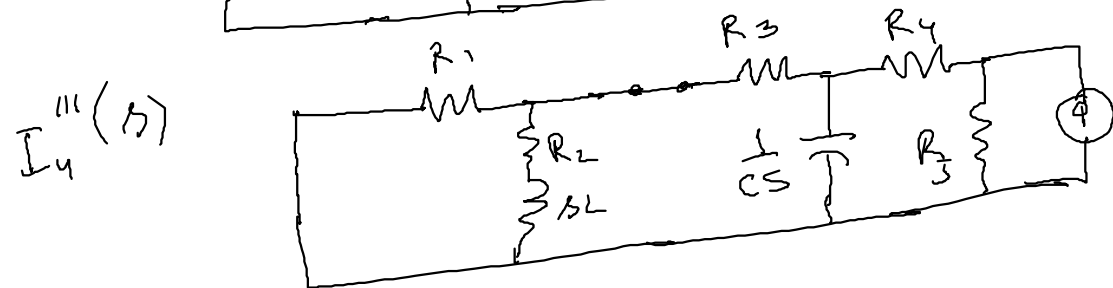
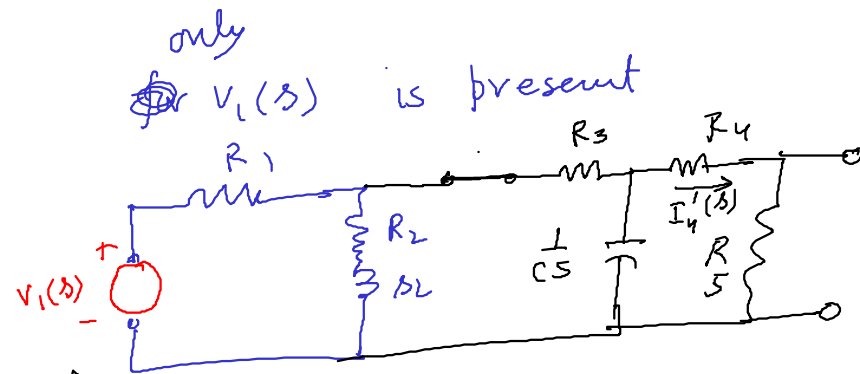


Network Theorems :- • Superposition Theorem ✓

Thevenin Theorem



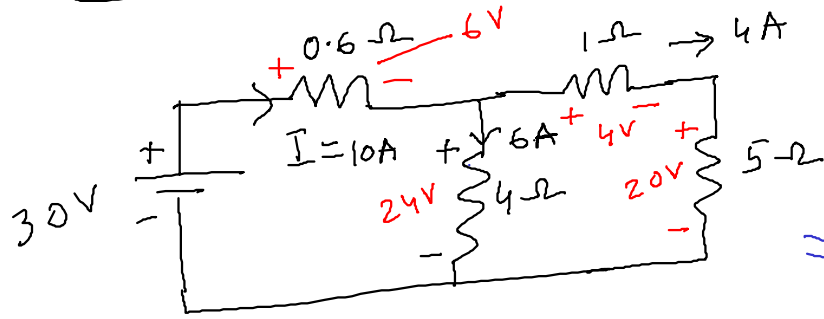
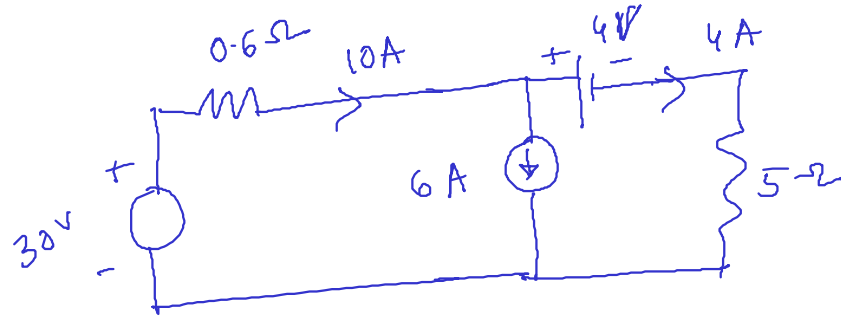
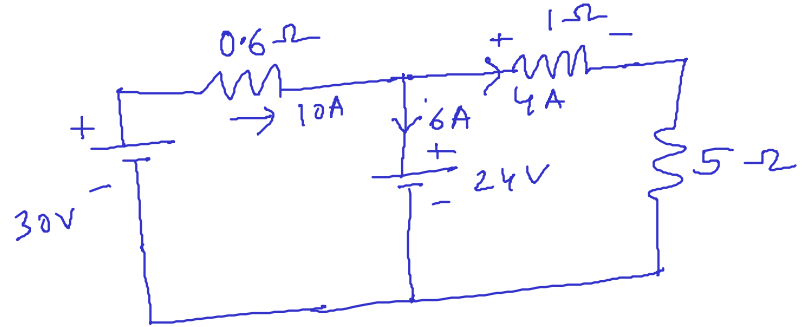
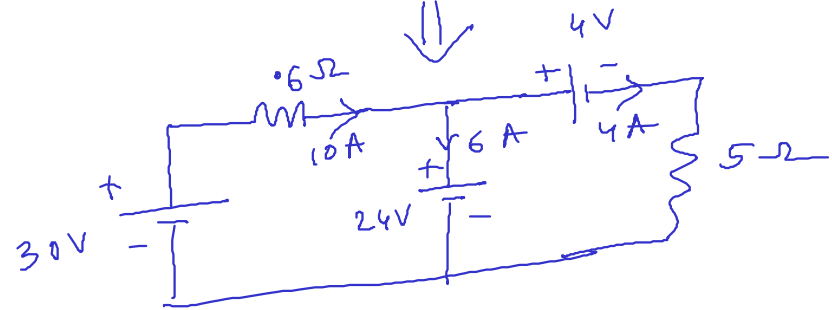
$I_4'(s)$ due to $V_1(s)$ alone
 $I_4''(s)$ due to $V_2(s)$ alone
 $I_4'''(s)$ due to $I(s)$ alone.

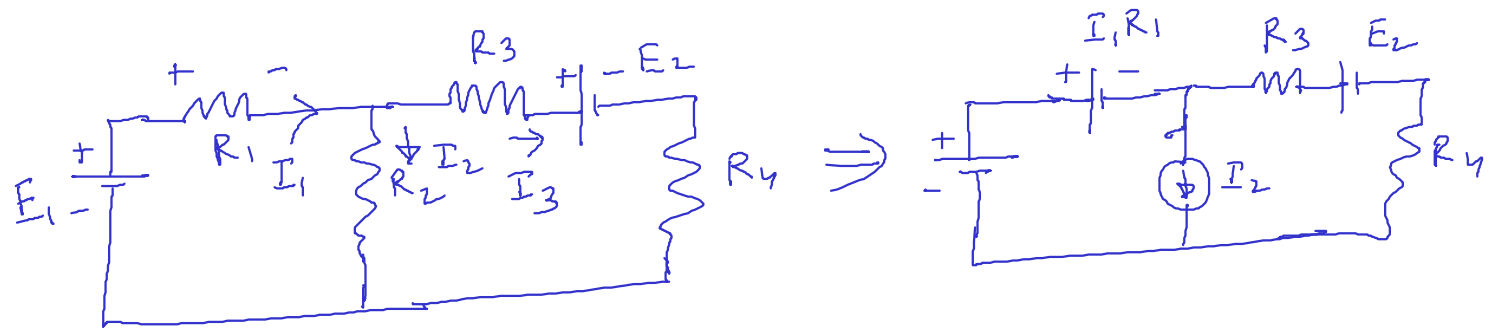
$$I_4(s) = I_4'(s) + I_4''(s) + I_4'''(s)$$

Substitution Theorem:

$$10 \times \frac{4}{10} = 4A$$

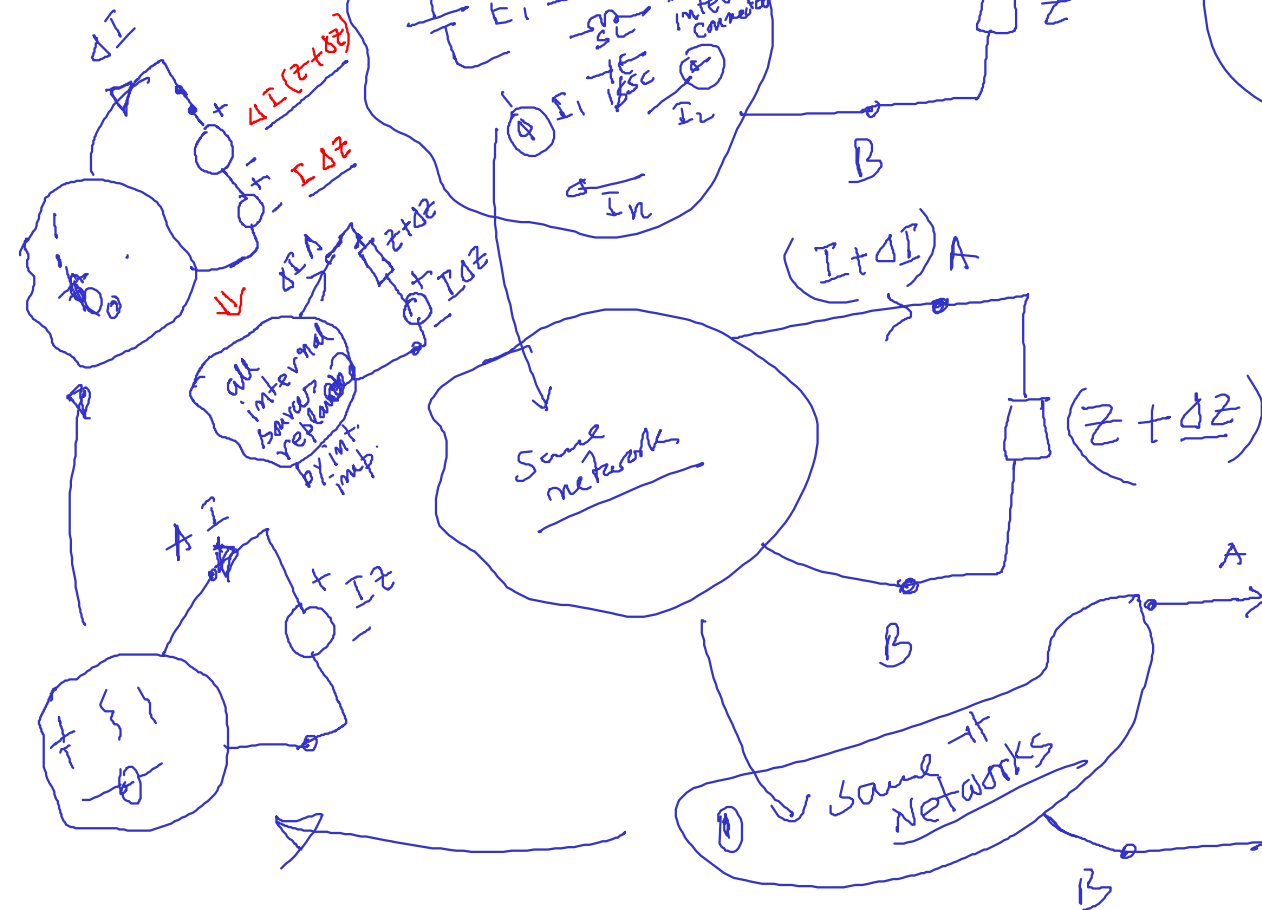
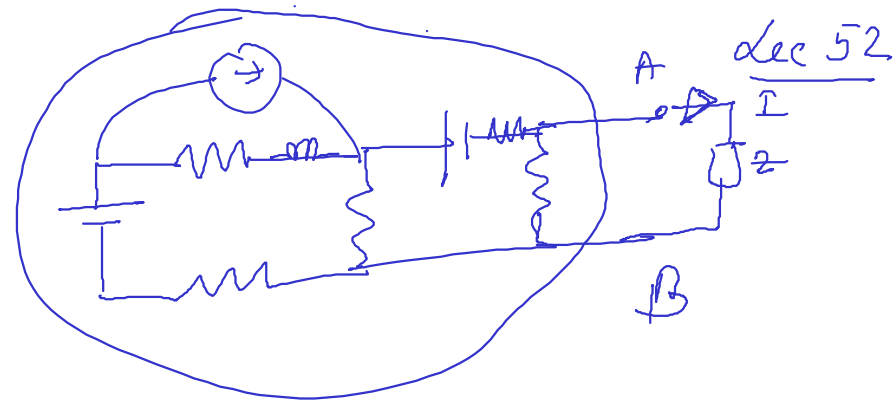
$$I = \frac{30}{0.6 + \frac{4 \times 6}{10}} = \frac{30}{3} = 10A$$



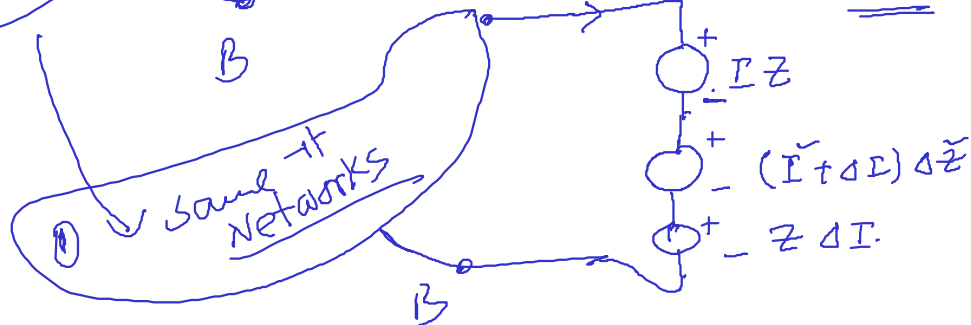


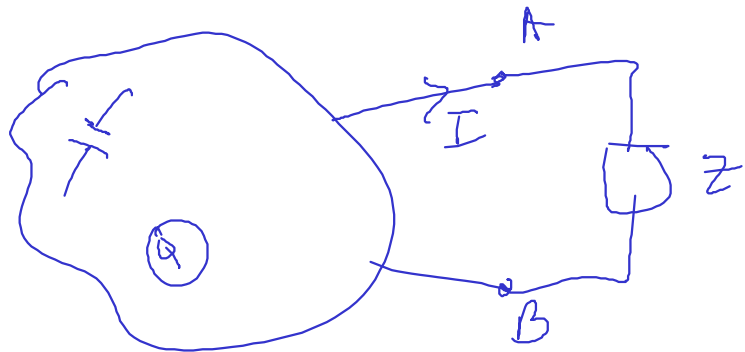
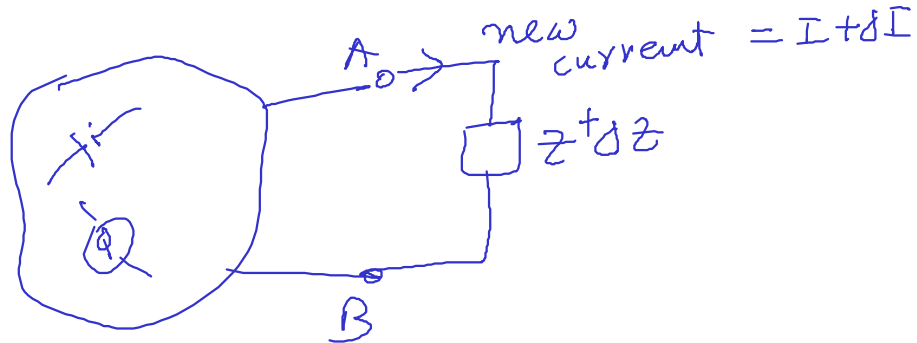
Linear Networks

$$(I + \Delta I) \Delta Z + Z \Delta I = \Delta I (Z + \Delta Z) + I \Delta Z$$

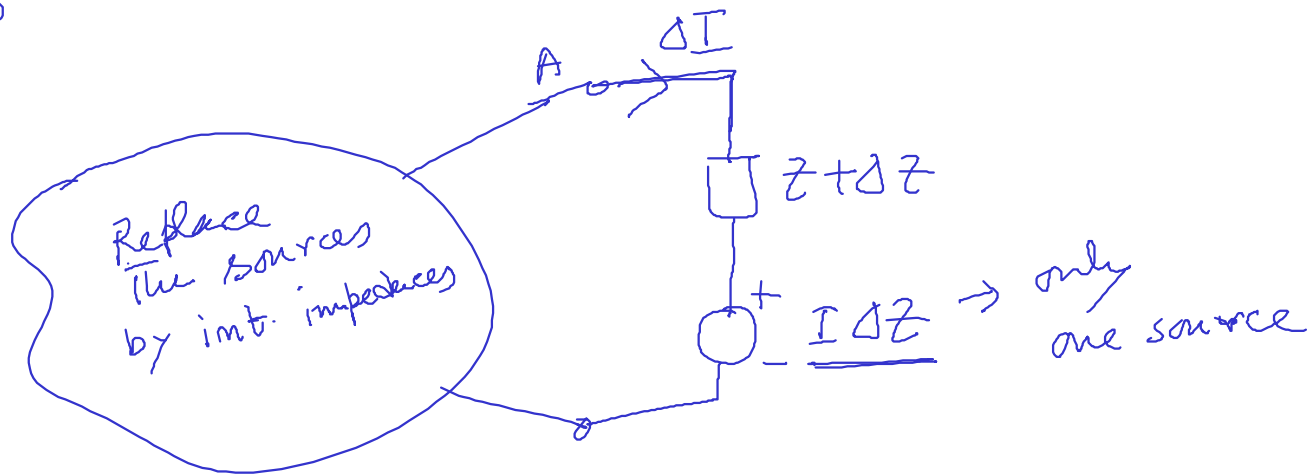


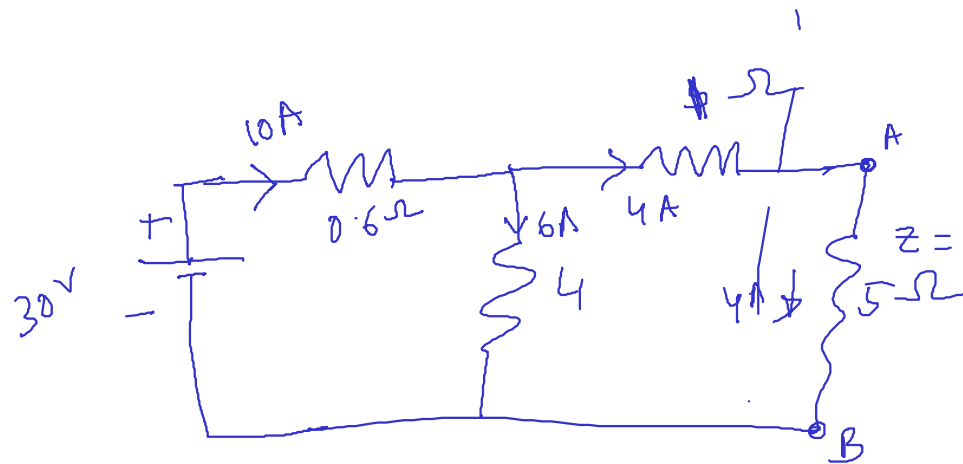
$$\begin{aligned} V_{AB} &= (Z + \Delta Z)(I + \Delta I) \\ &= (Z + \Delta Z)I + (Z + \Delta Z)\Delta I \\ &= IZ + I\Delta Z + (Z + \Delta Z)\Delta I \\ &= \underline{IZ} + (I + \Delta I)\Delta Z + Z\Delta I \\ &= \underline{IZ} \end{aligned}$$





actual current
thru $Z + \Delta Z$
 $= I + \Delta I$





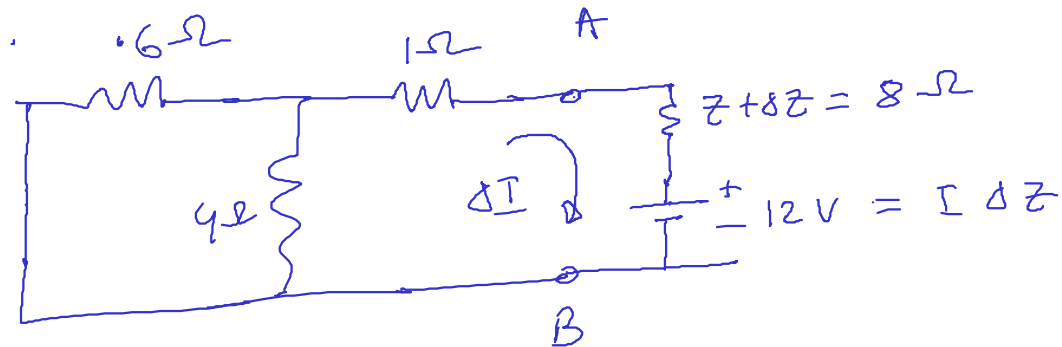
$$I = 4A \quad \checkmark$$

$$Z = 5\Omega$$

de 5Ω imp is changed

to 8Ω

$$\Delta Z = \underline{\underline{3\Omega}} \quad I \Delta Z = \underline{\underline{12V}}$$



$$\underline{\underline{\Delta I}} = \underline{\underline{-1.26A}}$$

$$4 + \Delta I$$

$$4 - 1.26A$$

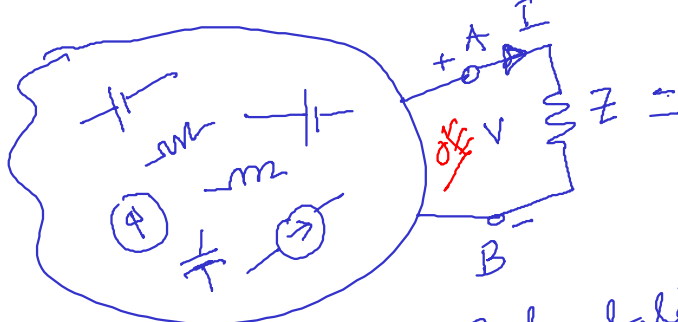
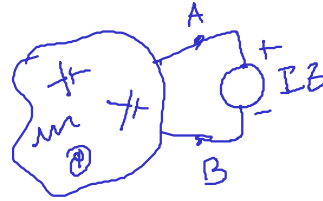
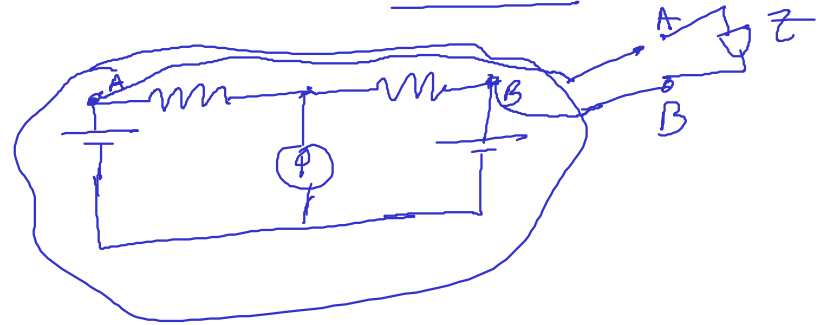
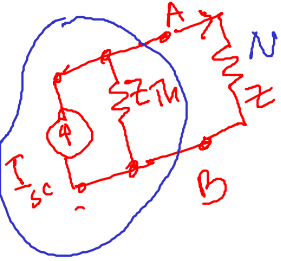
$$= \underline{\underline{2.74A}} \quad \checkmark$$

Network Theorems : Theremin Theorem Norton's Theorem

lec-5.3

$$I = \frac{I_{sc}}{1 + \frac{Z}{Z_{Th}}}$$

$$I = \frac{Z_{Th}}{(Z + Z_{Th})} I_{sc}$$



Norton's Theorem :

Calculate I by applying Superposition Theorem.

① get current (I_{sc}) due to all internal sources of the network

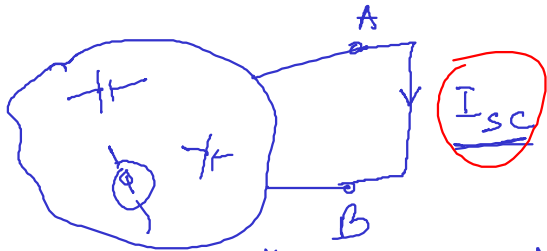
② Consider only the external source I_Z to be present and all internal sources replaced by internal impedances. $\therefore$ total current

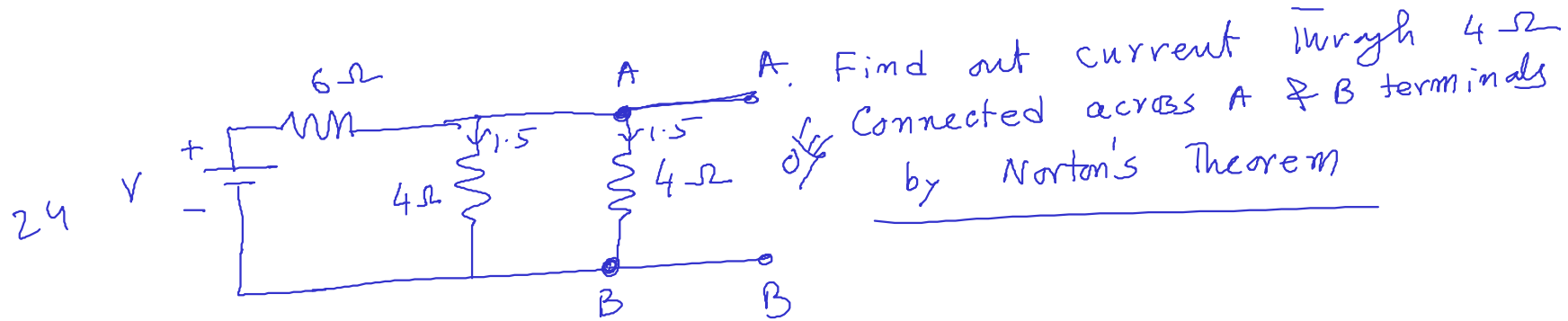
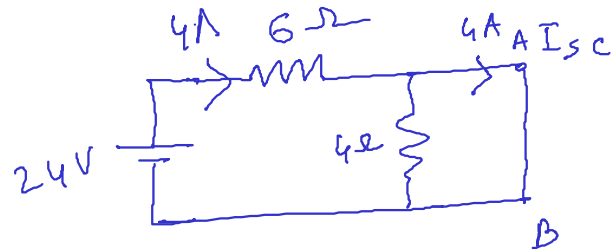
$$\therefore I' = - \frac{I_Z}{Z_{Th}}$$

$$I \left(1 + \frac{Z}{Z_{Th}} \right) = I_{sc}$$

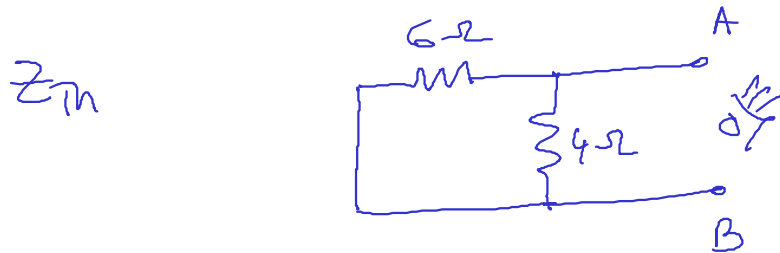
$$I = I_{sc} - \frac{I_Z Z}{Z_{Th}}$$

Let Z_{Th} = imp. of the network with no sources looking from A & B



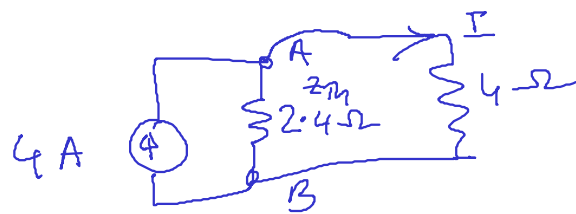


$$\therefore I_{sc} = 4A$$



$$Z_{Th} = \frac{6 \times 4}{10} = 2.4\Omega$$

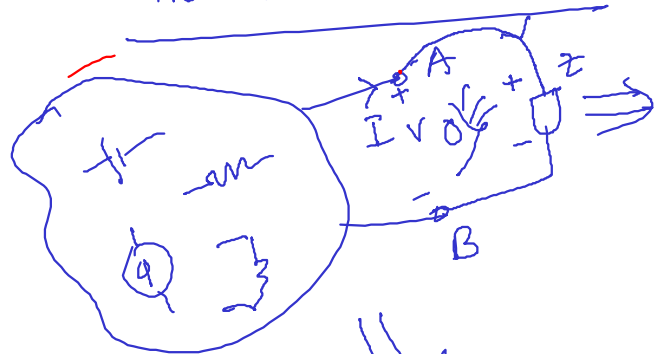
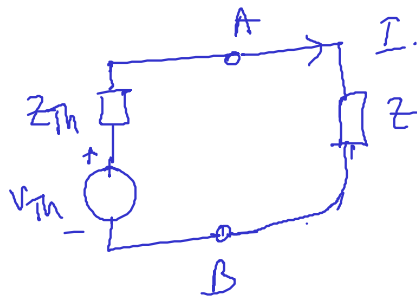
Norton's equivalent ckt. across A & B



$$I = \frac{4 \times 2.4}{6.4} = 1.5A \checkmark$$

Thevenin Network

Dec-54

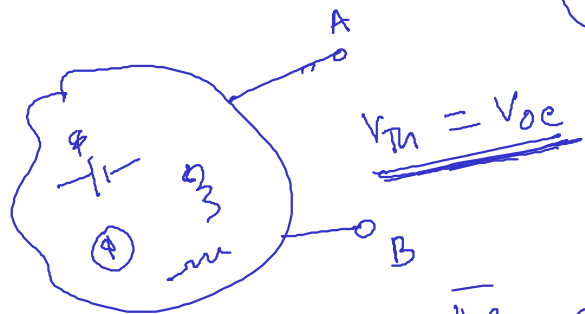
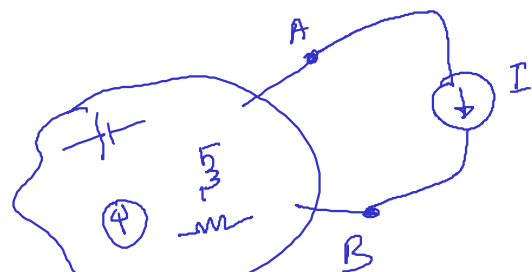


$$I = \frac{V_{TH}}{Z + Z_{TH}}$$

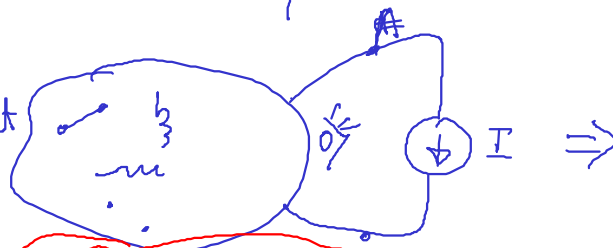
Calculate V_{AB} by superposition theorem.

① Calculate V_{AB} due to all the internal source

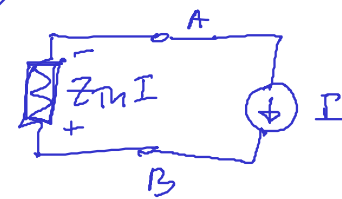
② Only the external current source (I) is present and all the internal sources replaced by the internal impedance



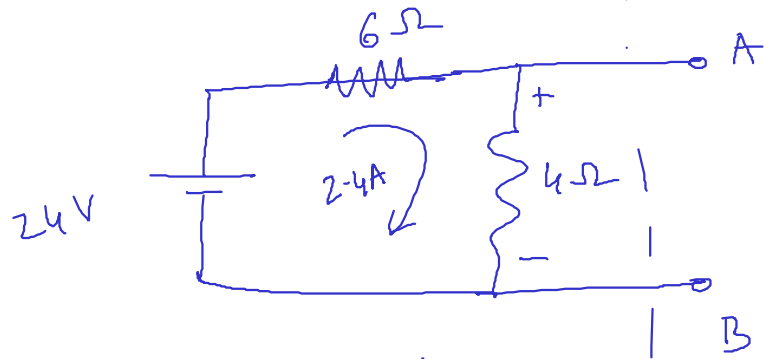
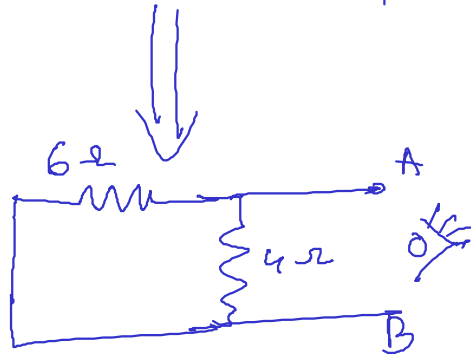
$\therefore V_{AB}$ When all the sources are present



$$IZ = V_{TH} - Z_{TH}I \rightarrow I = \frac{V_{TH}}{Z + Z_{TH}}$$



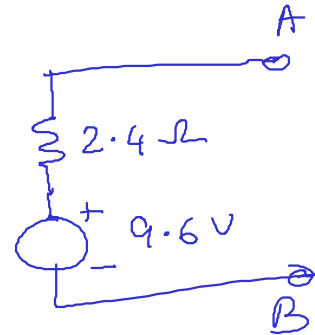
$$V_{AB} = -Z_{TH}I$$



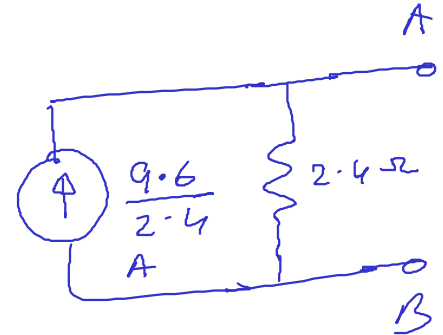
find ~~the~~ Thev. Equiv
of this network across AB

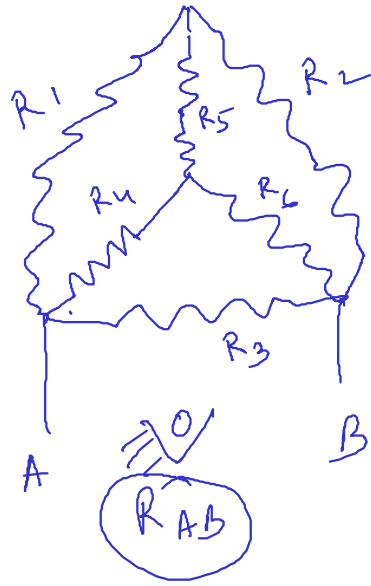
$$V_{Th} = \frac{24}{10} \times 4 = \underline{9.6V}$$

$$R_{Th} = \frac{4 \times 6}{10} = 2.4\Omega$$



$\Rightarrow$

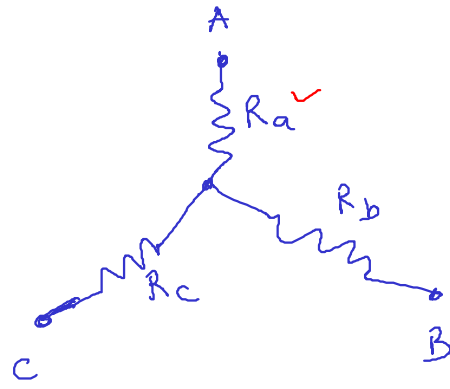




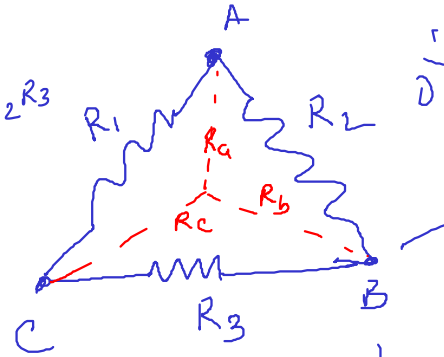
More on calculation of
 Z_{Th} or R_{Th}

$$R_{eq} = \frac{V}{I}$$

Star-Delta Transformation (useful to calculate equiv impedance)



$R_a, R_b \& R_c$
are known
Then what are R_1, R_2, R_3
or vice versa



$R_1, R_2, \& R_3$?

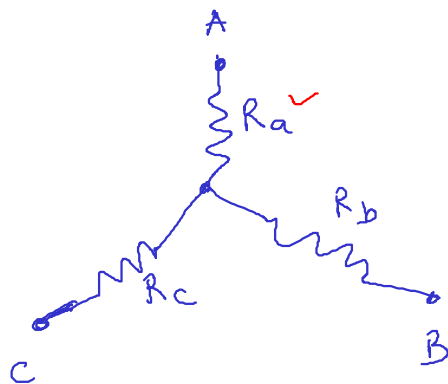
$$\begin{aligned}
 R_{AB} &= R_a + R_b = \frac{R_2 (R_1 + R_3)}{R_1 + R_2 + R_3} \quad \dots \textcircled{1} \\
 R_{BC} &= R_b + R_c = \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3} \quad \dots \textcircled{2} \\
 R_{CA} &= R_c + R_a = \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3} \quad \dots \textcircled{3}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{1} + \textcircled{2} + \textcircled{3} \\
 Z(R_a + R_b + R_c) &= \frac{Z(R_1 R_2 + R_2 R_3 + R_3 R_1)}{R_1 + R_2 + R_3} \\
 \textcircled{4} - \textcircled{1} & \dots \textcircled{4} \\
 R_c &= \frac{R_3 R_1}{R_1 + R_2 + R_3} \\
 R_b &= \frac{R_2 R_3}{R_1 + R_2 + R_3} \\
 R_a &= \frac{R_1 R_2}{R_1 + R_2 + R_3}
 \end{aligned}$$

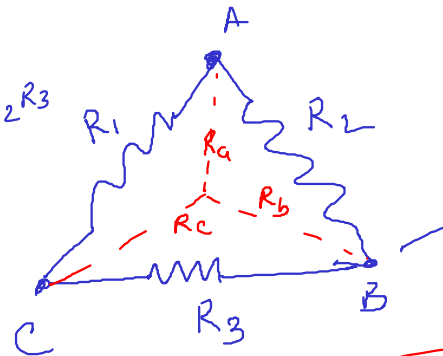
Δ to Y
conversion

Star-Delta Transformation

lec 55



R_a, R_b & R_c
are known
Then what are R_1, R_2, R_3
or vice versa



$$R_c = \frac{R_3}{R_2} R_a$$

$$\frac{R_c}{R_a} = \frac{R_3}{R_2}$$

$$\frac{R_a}{R_b} = \frac{R_1}{R_3} \quad R_b = \frac{R_3}{R_1} R_a$$

$$\frac{R_b}{R_c} = \frac{R_2}{R_1}$$

If R_1, R_2 & R_3 are known then

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_3 = \frac{R_b R_1}{R_c}$$

$$R_2 = \frac{R_b R_1}{R_c}$$

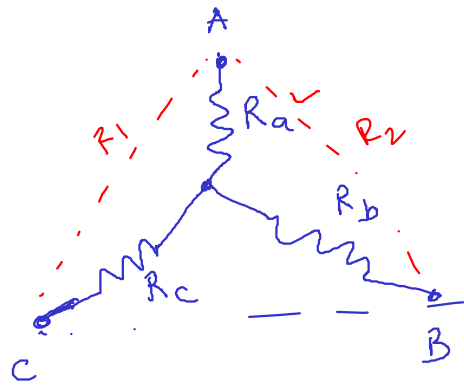
$$\& R_c = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$$\rightarrow R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3} =$$

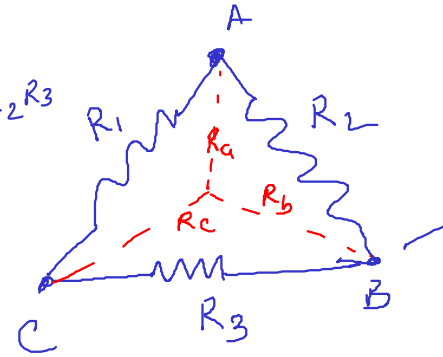
$$R_a = \frac{R_1 \frac{R_b R_1}{R_c}}{\left(R_1 + \frac{R_b R_1}{R_c} + \frac{R_b R_1}{R_a}\right)}$$

$$= \frac{R_1 \frac{R_b}{R_c}}{\left(1 + \frac{R_b}{R_c} + \frac{R_b}{R_a}\right)} = R_a$$

$$R_1 = \frac{R_c R_a}{R_b} \left(1 + \frac{R_b}{R_c} + \frac{R_b}{R_a}\right) = \frac{R_c R_a}{R_b} + R_a + R_c$$



R_a, R_b & R_c
are known
Then what are R_1, R_2, R_3
or vice versa



Δ to Δ

$$\Delta \text{ to } \Delta$$

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

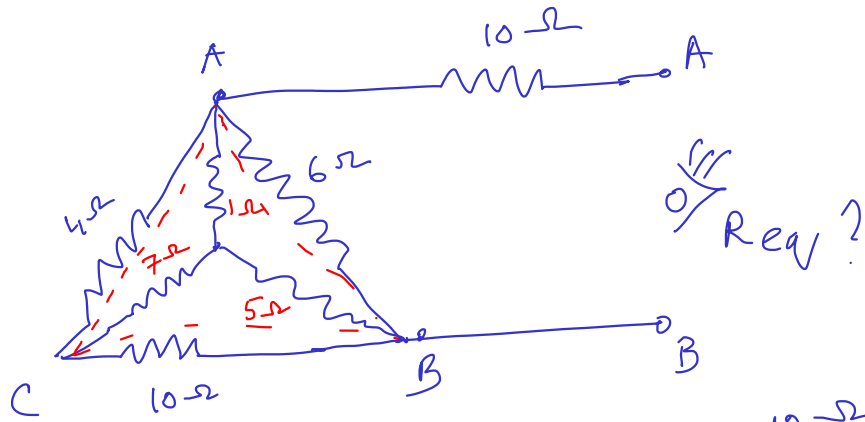
$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

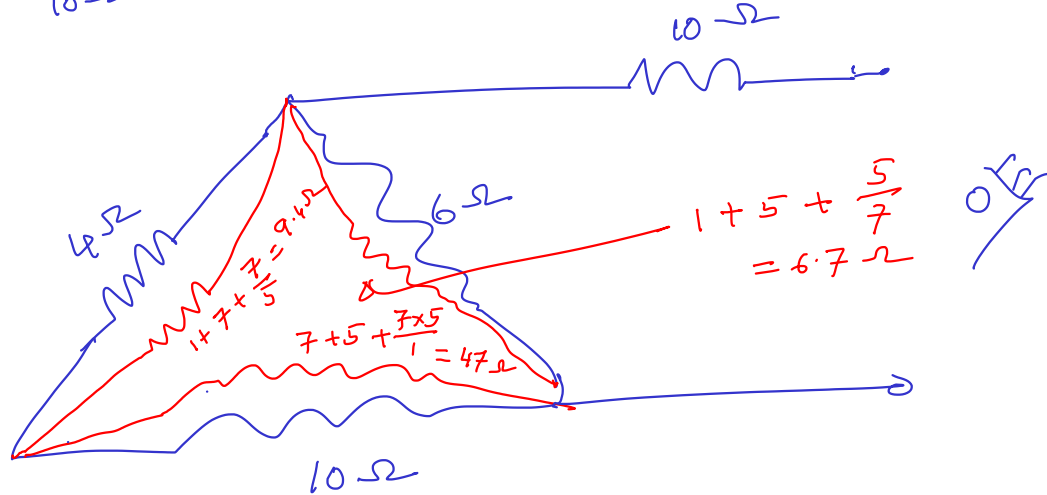
$$R_1 = R_a + R_c + \frac{R_a R_c}{R_b}$$

$$R_2 = R_b + R_a + \frac{R_b R_a}{R_c}$$

$$R_3 = R_b + R_c + \frac{R_b R_c}{R_a}$$



Change Δ connected
i.e., 5Ω & 7Ω
in equivalent Δ



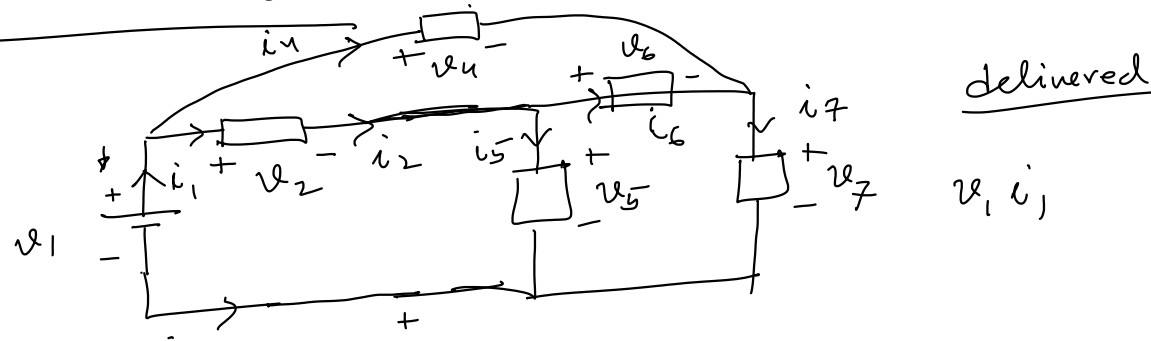
$10 +$

$$1 + 5 + \frac{5}{7} = 6.7\Omega$$

$$7 + 5 + \frac{7 \times 5}{1} = 47\Omega$$

$$1 + 7 + \frac{7}{5} = 9.1\Omega$$

Tellegen's Theorem :-



$$v_1 i_1 = v_2 i_2 + v_4 i_4 + v_5 i_5 + v_6 i_6 + v_7 i_7$$

or

$$\boxed{v_1 (-i_1)} + v_2 i_2 + v_4 i_4 + v_5 i_5 + v_6 i_6 + v_7 i_7 = 0$$

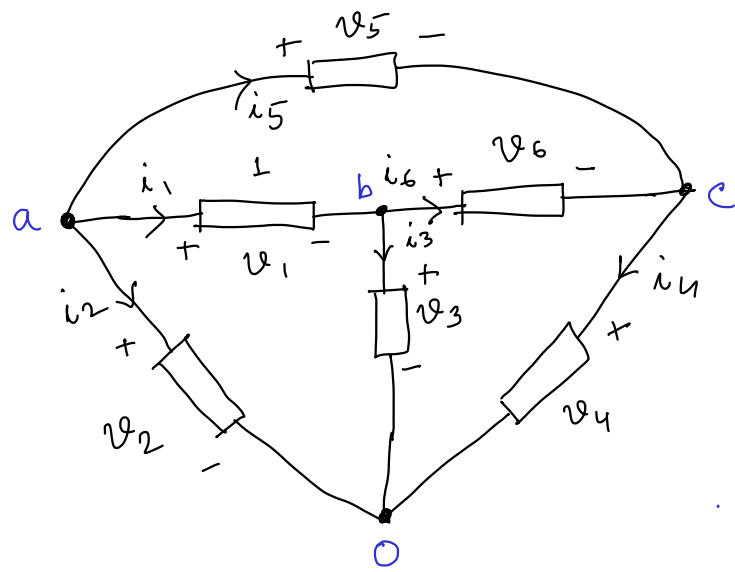
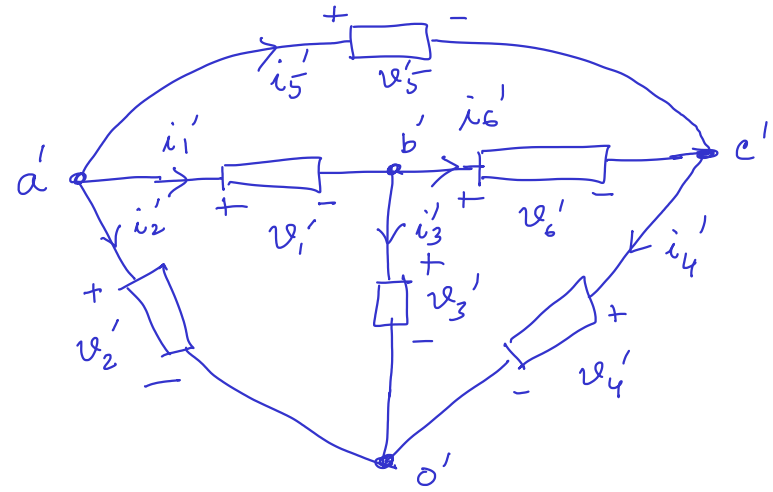
~~power~~ absorbed
by v_1

Based Power delivered by each element

$$v_1 i_1 + v_2 (-i_2) + v_4 (-i_4) + v_5 (-i_5) + v_6 (-i_6) + v_7 (-i_7) = 0$$

Absorbed:

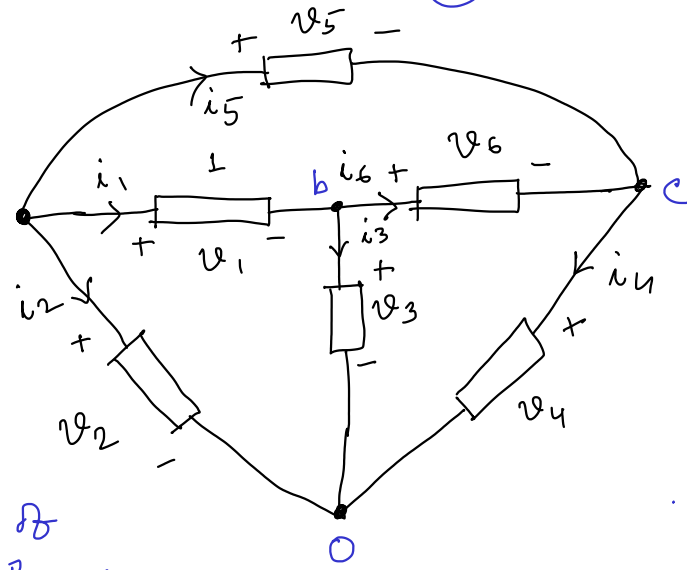
$$\begin{aligned} &v_2 i_2 \\ &+ \\ &v_4 i_4 \\ &+ \\ &v_5 i_5 \\ &+ \\ &v_6 i_6 \\ &+ \\ &v_7 i_7 \end{aligned}$$



Topology of
These two networks
are identical.

Tellegen Theorem

(A)



Sum of Power absorbed by each element = 0

$$v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 + v_5 i_5 + v_6 i_6 = 0$$

Tellegen Th:-

Also:-

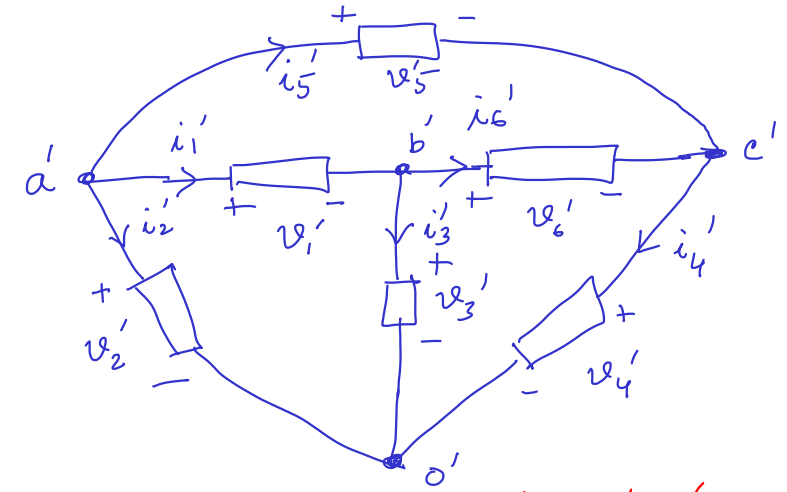
$$v_1 i_1' + v_2 i_2' + v_3 i_3' + v_4 i_4' + v_5 i_5' + v_6 i_6' = 0$$

$$v_1' i_1 + v_2' i_2 + v_3' i_3 + v_4' i_4 + v_5' i_5 + v_6' i_6 = 0$$

Same topology

(B)

dec 56



$$v_1' i_1' + v_2' i_2' + v_3' i_3' + v_4' i_4' + v_5' i_5' + v_6' i_6' = 0$$

Proof of Tellegen Th.

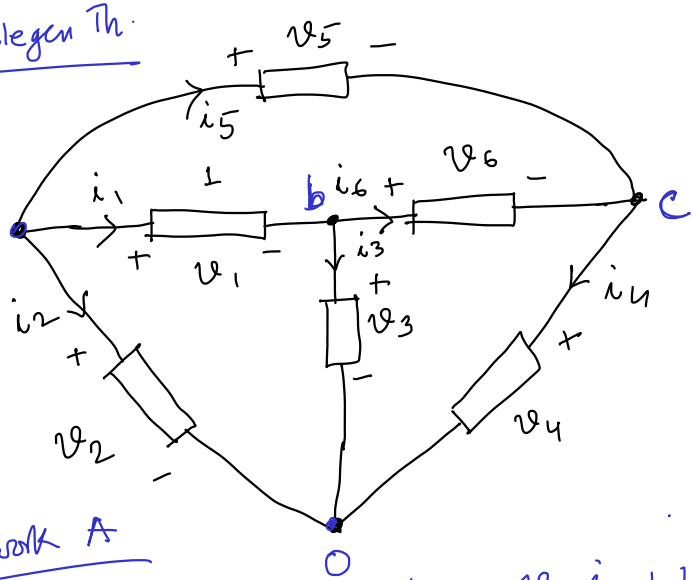
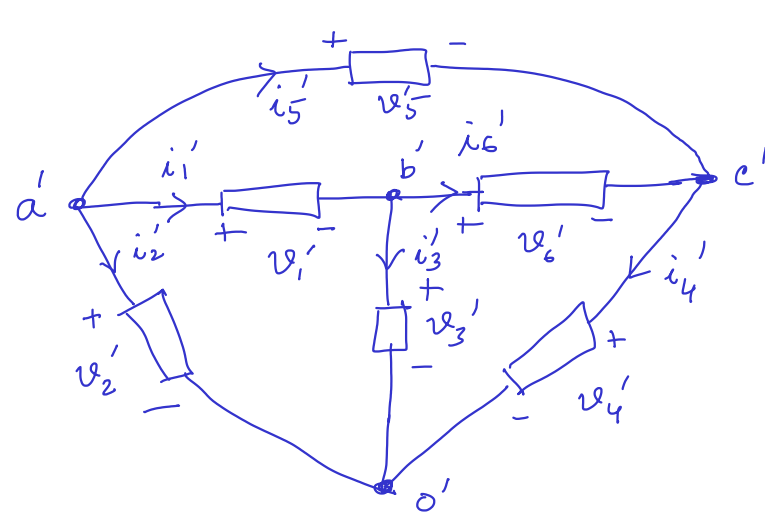
KCL at node

a) $i_1 + i_2 + i_5 = 0$

b) $i_3 + i_6 - i_1 = 0$

c) $i_5 + i_6 - i_4 = 0$

$i_4 - i_5 - i_6 = 0$



In network A

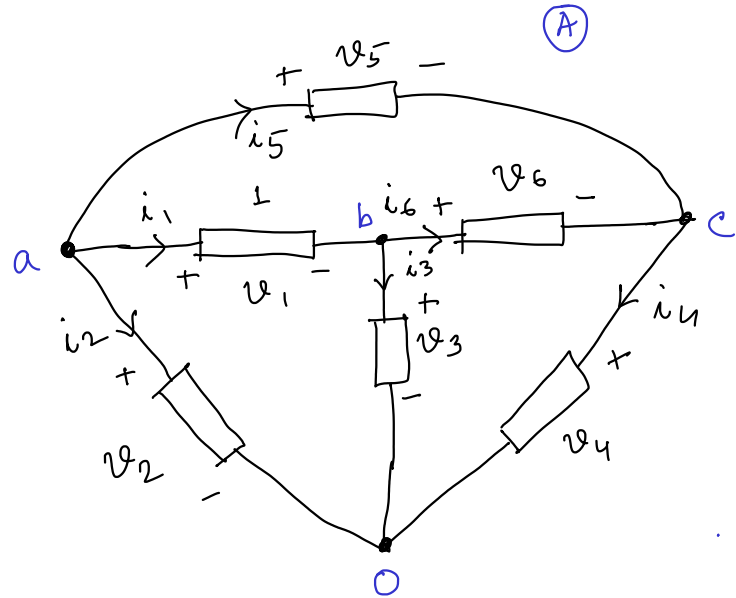
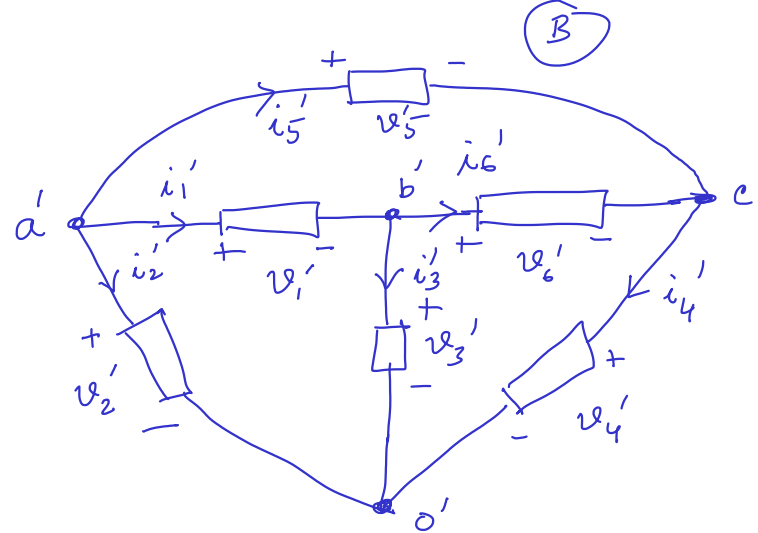
$$v_1 i_1 + v_2 i_2 + v_3 i_3 + v_4 i_4 + v_5 i_5 + v_6 i_6$$

$$\text{or } (v_{ao} - v_{bo}) i_1 + v_{ao} i_2 + v_{bo} i_3 + v_{co} i_4 + (v_{ao} - v_{co}) i_5 + (v_{bo} - v_{co}) i_6$$

$$\text{or } v_{ao} (i_1 + i_2 + i_5) + v_{bo} (-i_1 + i_3 + i_6) + v_{co} (i_4 - i_5 - i_6) = 0$$

In network B :- $v'_{ao} (i'_1 + i'_2 + i'_5) + v'_{bo} (-i'_1 + i'_3 + i'_6) + v'_{co} (i'_4 - i'_5 - i'_6) = 0$

Q.E.D.

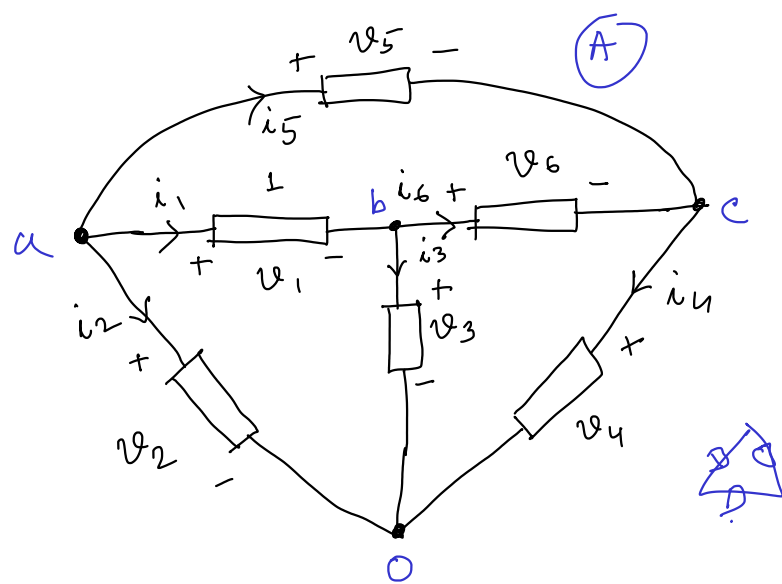
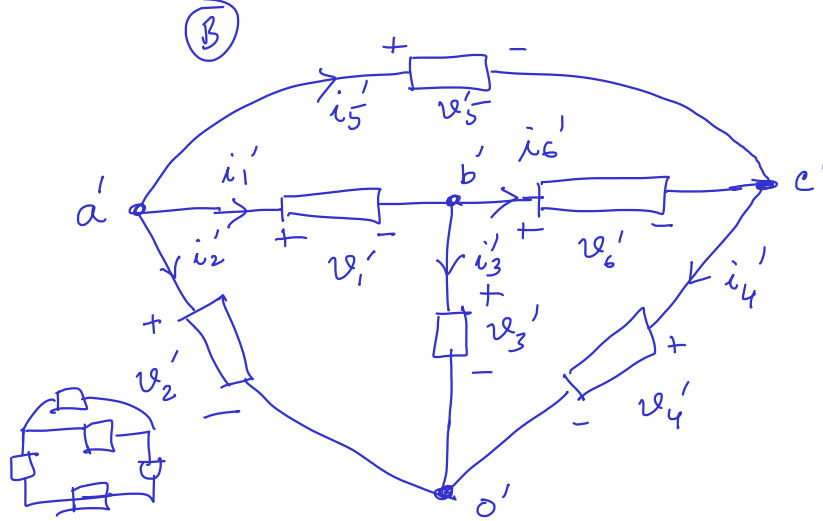


KCL at a'
 $i_1' + i_2' + i_5' = 0$
 KCL at b'
 $i_3' + i_6' - i_1' = 0$
 KCL at c'
 $i_4' - i_5' - i_6' = 0$

$$\rightarrow v_1 i_1' + v_2 i_2' + v_3 i_3' + v_4 i_4' + v_5 i_5' + v_6 i_6'$$

$$\sim \begin{cases} (v_{ao} - v_{bo}) i_1' + v_{ao} i_2' + v_{bo} i_3' + v_{co} i_4' + (v_{ao} - v_{co}) i_5' + (v_{bo} - v_{co}) i_6' \\ v_{ao} (i_1' + i_2' + i_5') + v_{bo} (-i_1' + i_3' + i_6') + v_{co} (i_4' - i_5' - i_6') \end{cases}$$

$$\rightarrow = v_{ao} \times 0 + v_{bo} \times 0 + v_{co} \times 0 = 0$$



Also note

$$v'_1 i_1 + v'_2 i_2 + v'_3 i_3 + v'_4 i_4 + v'_5 i_5 + v'_6 i_6$$

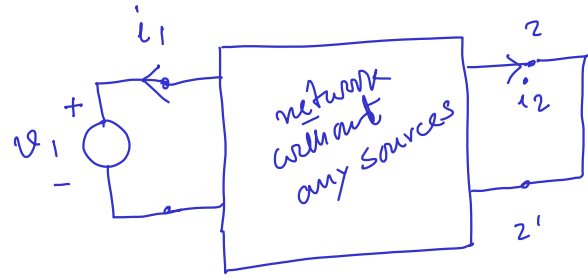
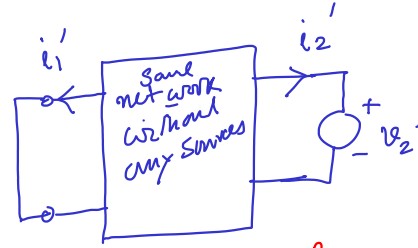
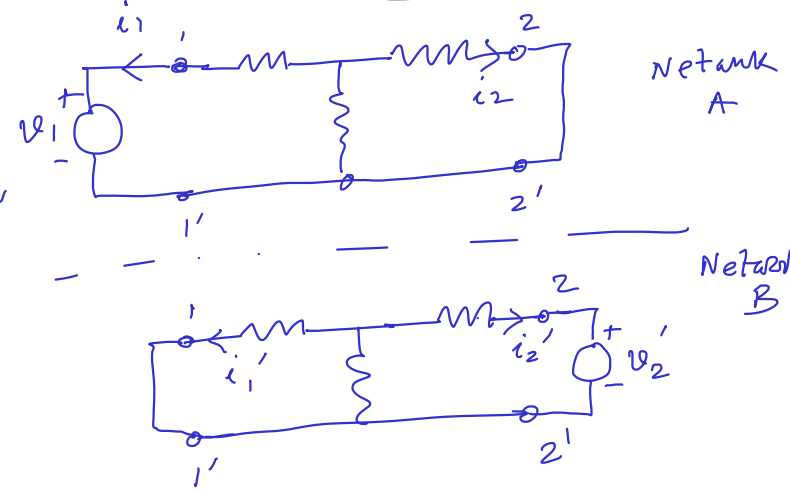
$$= (v_{a'o'} - v_{b'o'}) i_1 + v_{a'o'} i_2 + v_{b'o'} i_3 + v_{c'o'} i_4 + (v_{a'o'} - v_{c'o'}) i_5 + (v_{b'o'} - v_{c'o'}) i_6$$

$$= v_{a'o'} \underbrace{(i_1 + i_2 + i_5)}_{\text{KCL at node a} = 0} + v_{b'o'} \underbrace{(-i_1 + i_3 + i_6)}_{\text{KCL at node b of net n}} + v_{c'o'} \underbrace{(i_4 - i_5 - i_6)}_{\text{KCL at node c of net-A}}$$

Reciprocity Theorem:-

Network-B

Network A



Apply Tellegen's Theorem to network A & B

$$v_1 i_1' + 0 \times i_2' + \sum_K v_K i_K' = 0 \times i_1 + v_2' i_2 + \sum_K v_K' i_K$$

$$\text{or } v_1 i_1' + \sum_K (R_K i_K) i_K' = v_2' i_2 + \sum_K (R_K i_K') i_K$$

$$\text{or } \boxed{v_1 i_1' = v_2' i_2} \quad \therefore$$

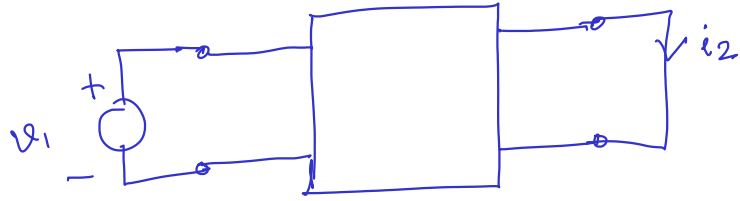
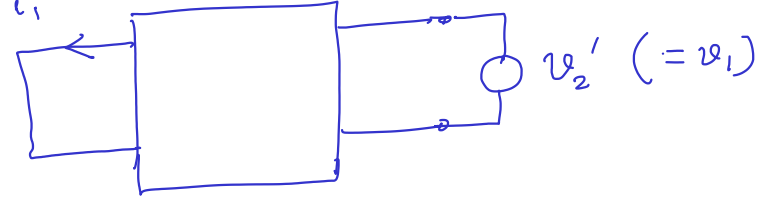
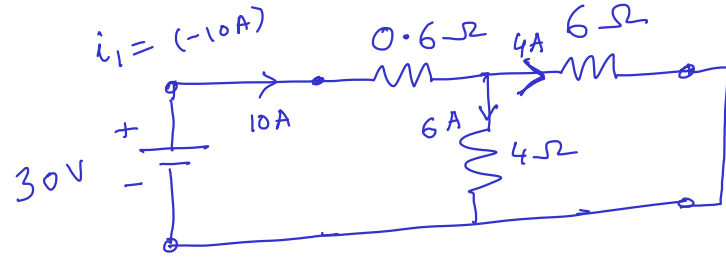
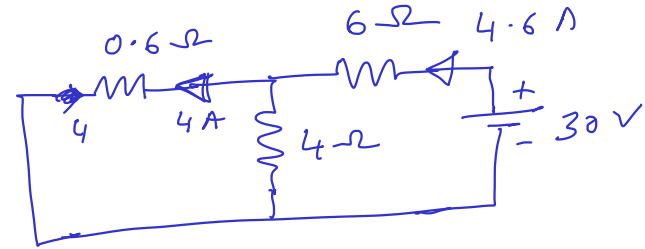
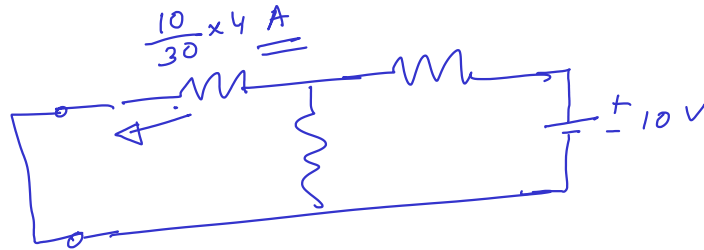
$$\boxed{\frac{i_2}{i_1'} = \frac{v_1}{v_2'}}$$

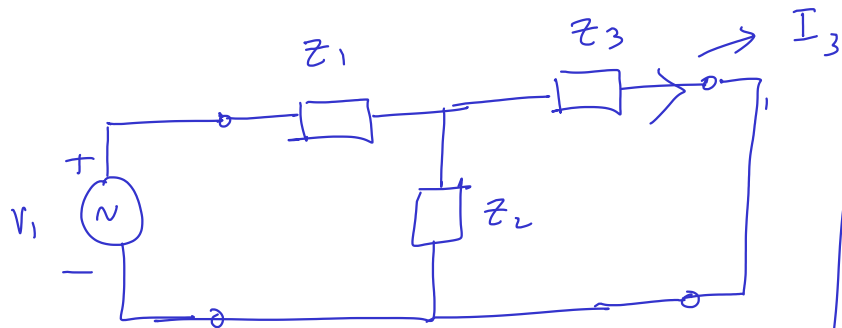
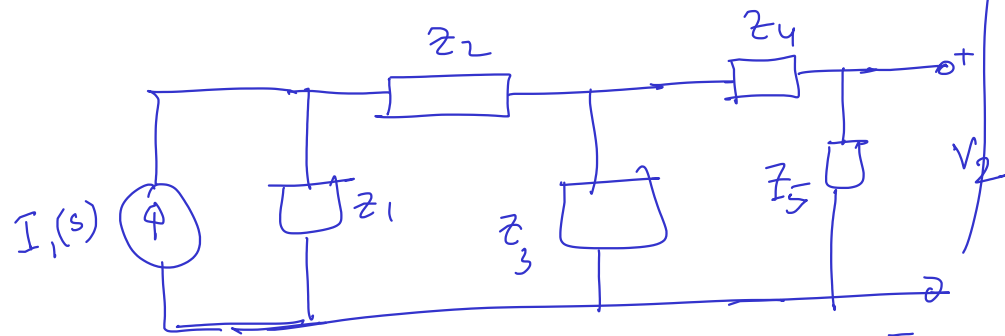
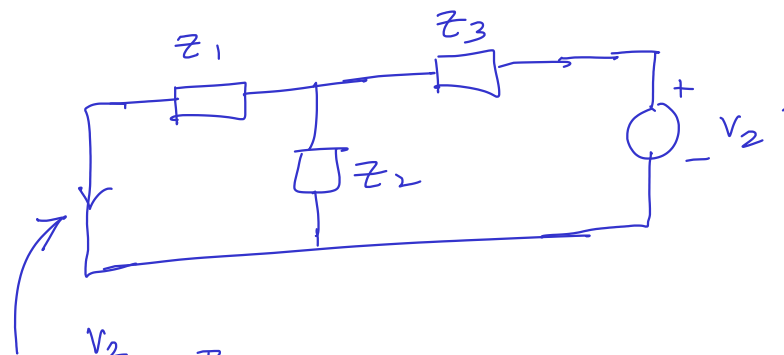
$$\boxed{i_1' = \frac{v_2'}{v_1} i_2}$$

$$\text{or } \boxed{i_2 = \frac{v_1}{v_2'} i_1'}$$

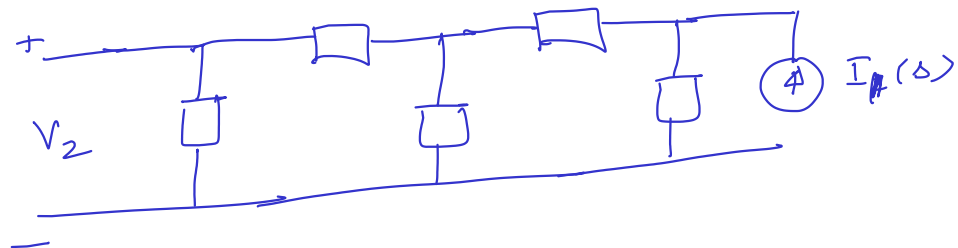
If $v_1 = v_2'$ then $i_1' = i_2$

$i_1' = i_2$
 $i_1' = \frac{v_2'}{v_1} i_2$



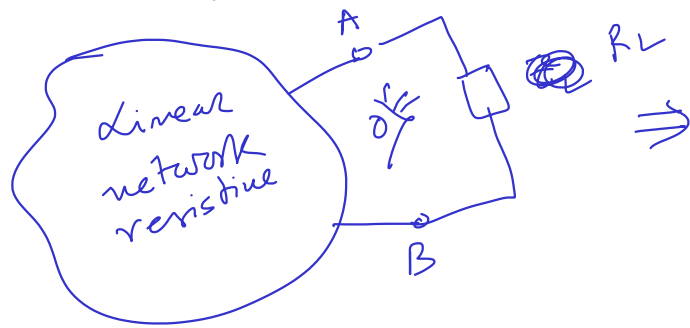
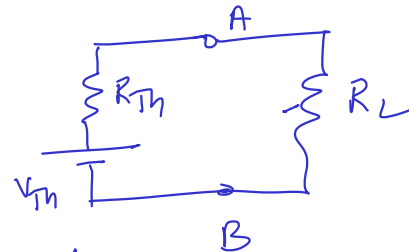


$$\frac{V_2}{V_1} \times I_3$$

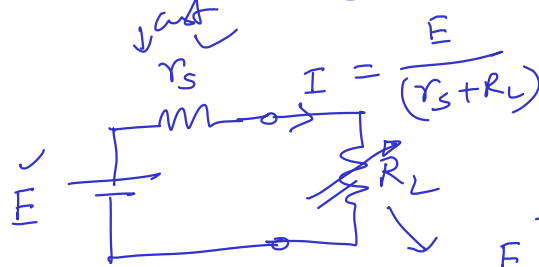


Maximum Power Transfer Theorem :-

dec-58



In D.C ckt:-

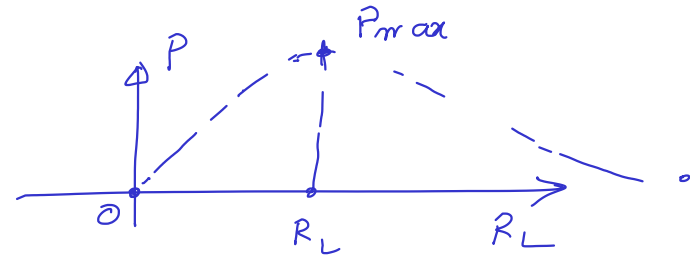


$$P = \frac{E^2 R_L}{(r_s + R_L)^2}$$

$$P = \frac{E^2 R_L}{(r_s + R_L)^2}$$

$$= \frac{E^2 R_L}{r_s^2 + R_L^2 + 2r_s R_L} = \frac{E^2}{\left(\frac{r_s^2}{R_L} + R_L + 2r_s\right)}$$

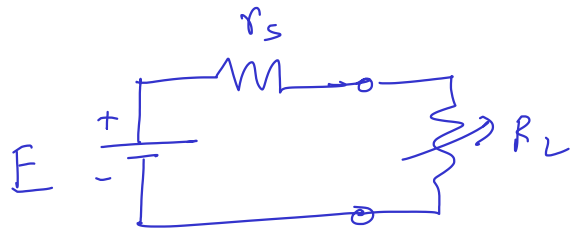
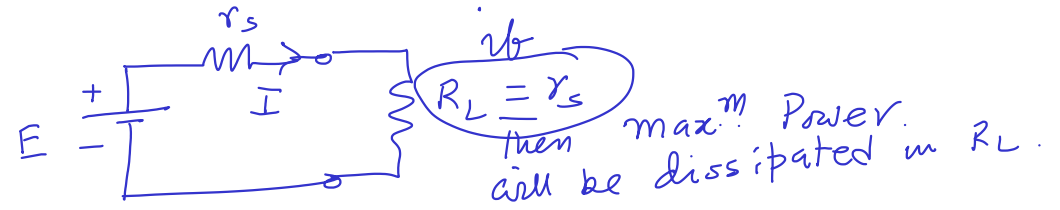
$R_L^2 = r_s^2 \Rightarrow \boxed{R_L = r_s}$



So P will be max^m if $\frac{r_s^2}{R_L} + R_L + 2r_s$ is min^m.

$$\frac{d}{dR_L} \left(\frac{r_s^2}{R_L} + R_L + 2r_s \right) = 0$$

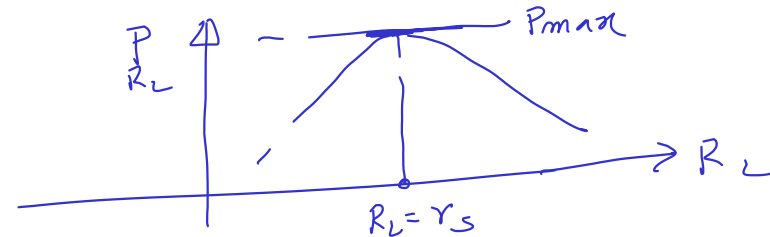
$$-\frac{r_s^2}{R_L^2} + 1 = 0$$



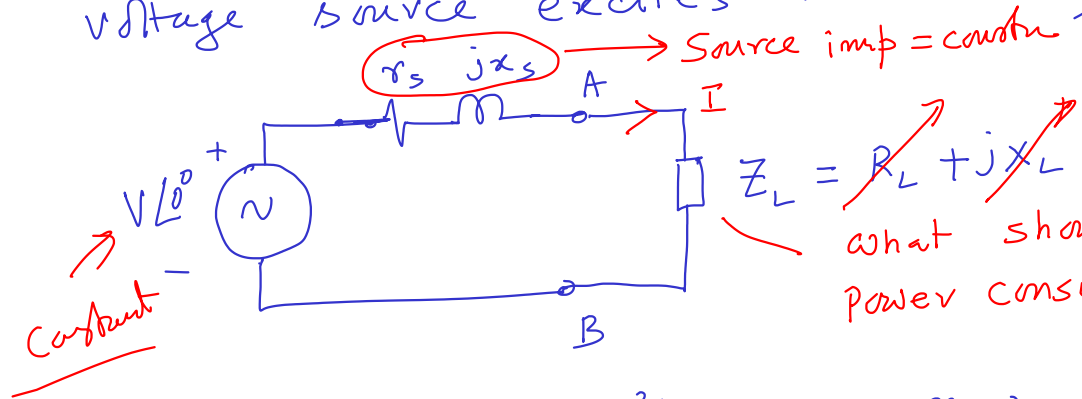
Efficiency under max^m Power condition.

$$\eta = \frac{\text{Power in } R_L (= r_s)}{\text{total power delivered by the source}}$$

$$= \frac{I^2 r_s (= R_L = r_s)}{I^2 r_s + I^2 r_s} = \frac{1}{2} = \underline{\underline{50\%}}$$



Maximum Power in A.C ~~is~~ circuit (sinusoidal voltage source excites the circuit)



what should be Z_L so that Real power consumed in Z_L is maximum?

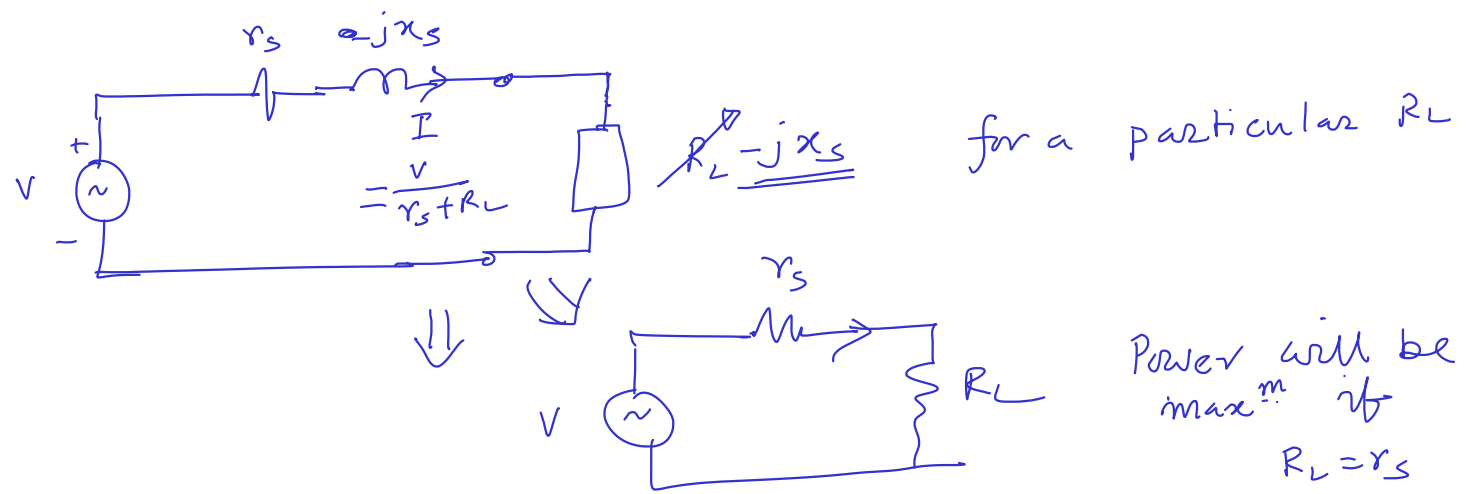
Now $P = I^2 R_L$ now $I = \frac{V}{\sqrt{\{(r_s + R_L)^2 + (x_s + \cancel{x_L})^2\}}}$

$$P = \frac{V^2 \cancel{R_L}}{(r_s + \cancel{R_L})^2 + (x_s + \cancel{x_L})^2}$$

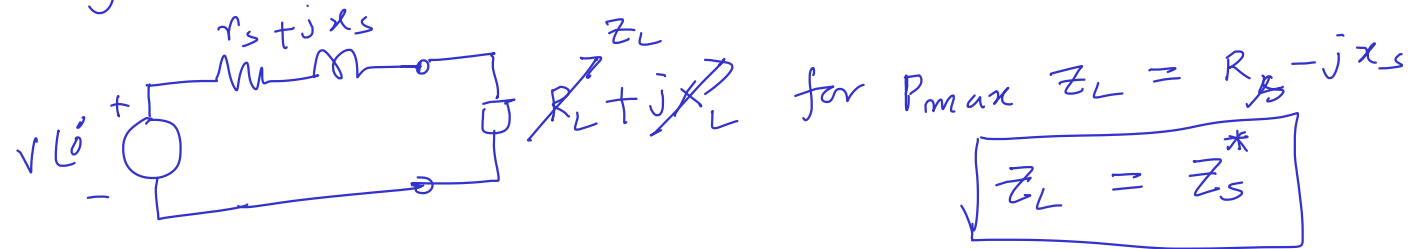
Let R_L be fixed at some particular value & X_L is only varied

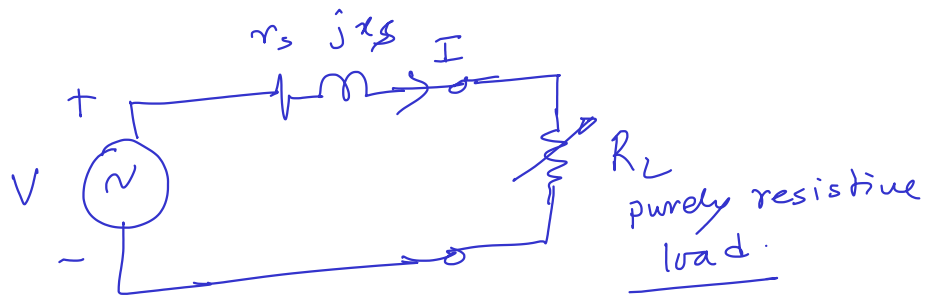
Then Power in R_L $\therefore P = I^2 R_L$
 So P now will be max^m if I is maximum
 if $x_L = -x_s$

Now



final conclusion is that





what should be R_L
for max^m power
delivered to R_L

P will be max^m when denominator is min^m.

$$I = \frac{V}{\sqrt{(r_s + R_L)^2 + x_s^2}}$$

$$P = I^2 R_L = \frac{V^2 R_L}{(r_s + R_L)^2 + x_s^2}$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L} \left(\frac{V^2 R_L}{(r_s + R_L)^2 + x_s^2} \right) = 0$$

$$\Rightarrow \frac{V^2 (r_s + R_L)^2 + x_s^2 - 2R_L(r_s + R_L)}{(r_s + R_L)^2 + x_s^2} = 0$$

$$\Rightarrow (r_s + R_L)^2 + x_s^2 - 2R_L(r_s + R_L) = 0$$

$$\Rightarrow r_s^2 + 2r_s R_L + R_L^2 + x_s^2 - 2r_s R_L - 2R_L^2 = 0$$

$$\Rightarrow r_s^2 + x_s^2 - R_L^2 = 0$$

$$\Rightarrow R_L^2 = r_s^2 + x_s^2$$

$$\Rightarrow R_L = \sqrt{r_s^2 + x_s^2}$$

$$R_L = |Z_s|$$

Diagram illustrating the power transfer condition. It shows a voltage source V connected to a load R_L in series with a reactance x_s . The condition for maximum power transfer is when the load resistance R_L equals the magnitude of the source impedance $|Z_s|$.