

✓ Solve diff eqⁿ in time domain

✓ Phasor — Algebraic eqⁿ

Laplace Transform Technique

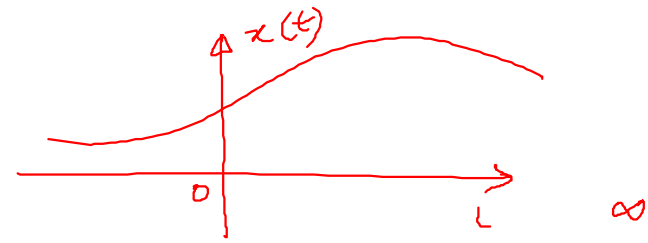
time domain $\xrightarrow{\text{phasor}}$ Solve algebraic eqⁿ for output

Go back to time domain to represent output in domain.

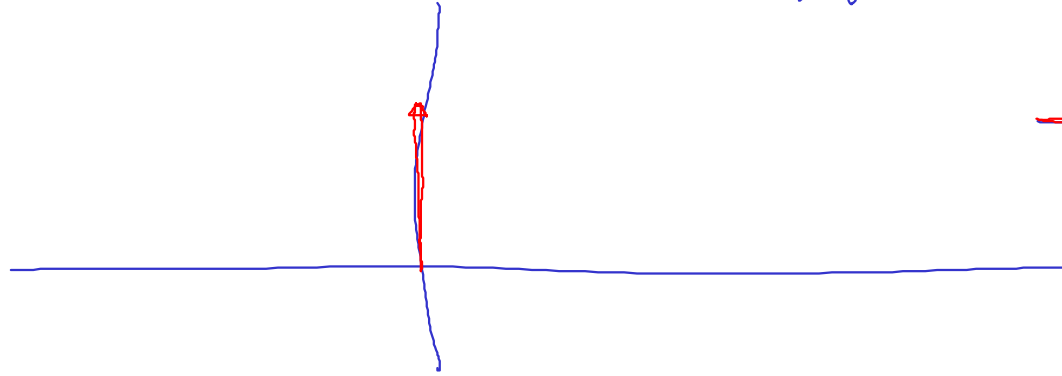
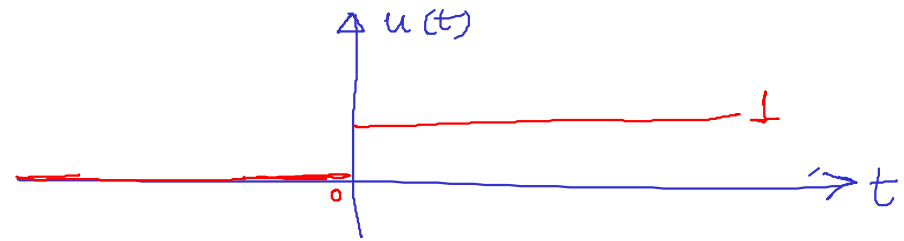
Signals :-

$x(t)$

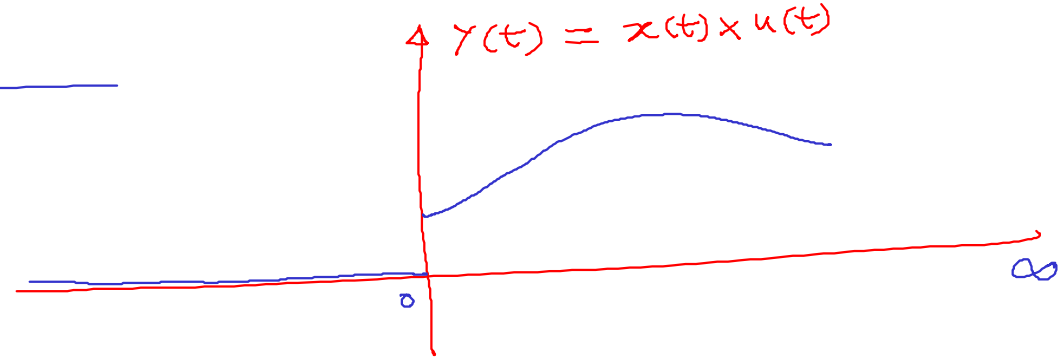
$$x(t) = 0 \text{ for } t < 0 \quad -\infty$$
$$= x(t) \text{ for } t > 0$$

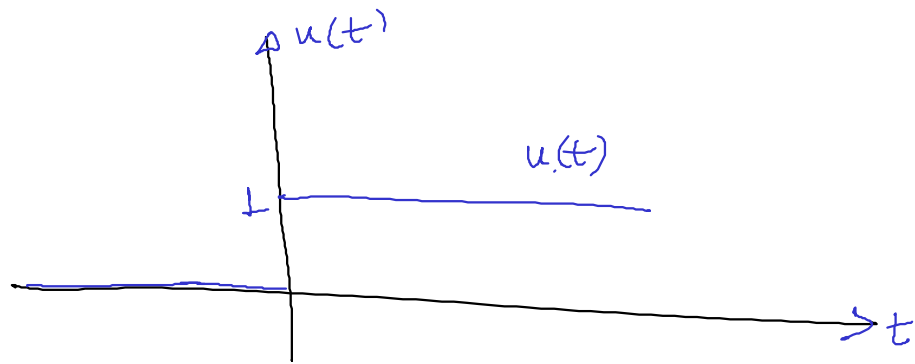
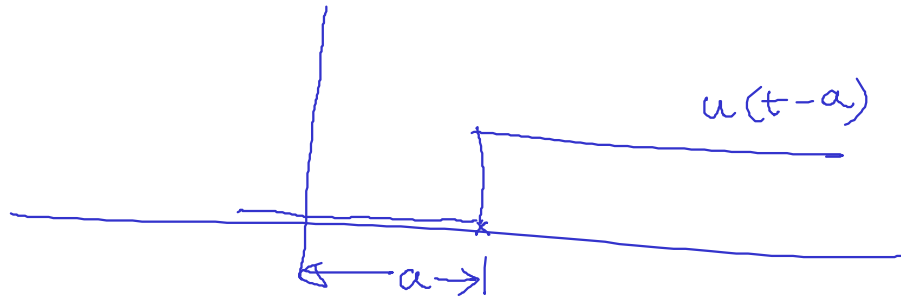
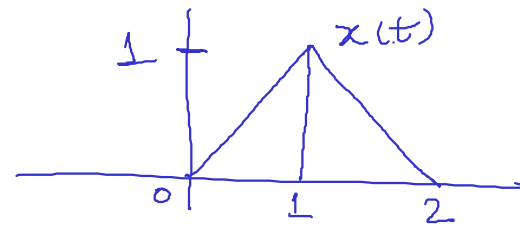
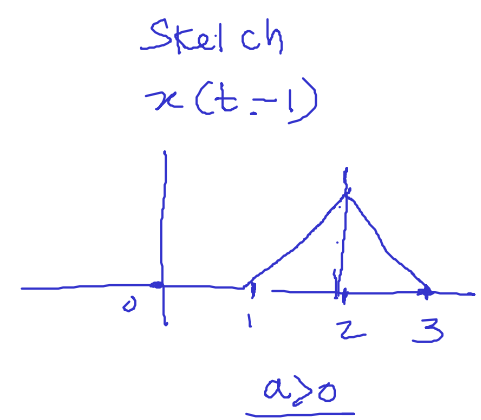


$u(t)$



$$y(t) = x(t) \times u(t)$$





$$u(t-a) = \begin{cases} 1 & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases}$$

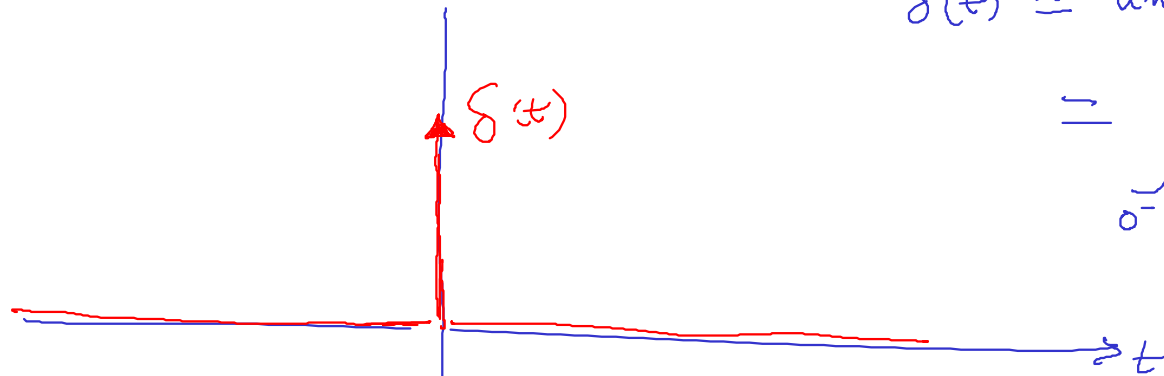
$$u(t-2) = 1 \text{ for } t \geq 2$$

$\delta(t) = \text{undefined} (\rightarrow \infty) \text{ at } t=0$

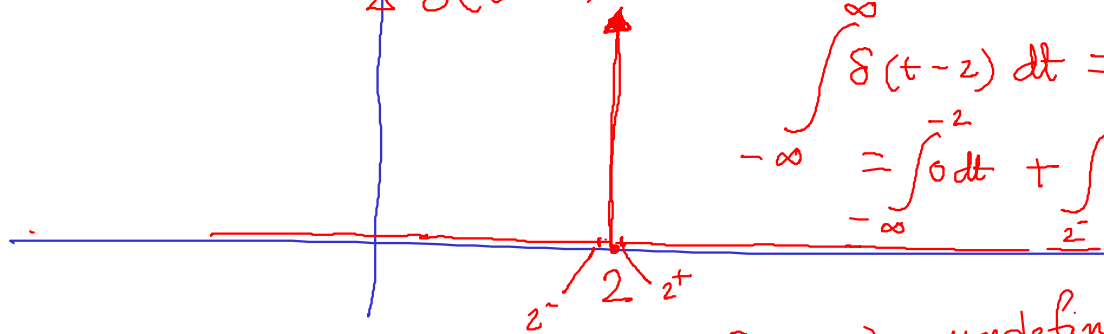
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$$\int_{0^-}^{0^+} \delta(t) dt = 1 = \int_{-\infty}^{\infty} \delta(t) dt = 1$$

and $\delta(t) = 0$ for $t \neq 0$



$\delta(t-2)$



Also note:-

$$\int_{-\infty}^{\infty} \delta(t-2) dt = 1$$

$$= \int_{-\infty}^{-2} 0 dt + \int_{2^-}^{2^+} \delta(t-2) dt + \int_{2^+}^{\infty} 0 dt = 1$$

$\delta(t-2) = \text{undefined}$
at $t=2$

$\delta(t-2) = 0$ for $t \neq 2$

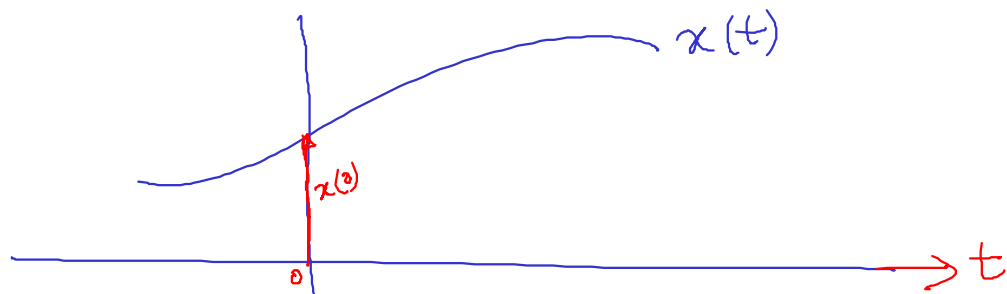
$$\int_{2^-}^{2^+} \delta(t-2) dt = 1$$

$$= \int_{-\infty}^{0^-} \delta(t) dt + \int_{0^-}^{0^+} \delta(t) dt + \int_{0^+}^{\infty} \delta(t) dt$$

$$= 0 + 1 + 0 = 1$$

$$\int_{0^-}^{0^+} \delta(t) dt = 1$$

What happens if a function $x(t)$ is multiplied with an impulse $\delta(t)$?



$$x(t) \delta(t) = ?$$

$$= 0 \text{ for } t \neq 0$$

$$\int_{t=-\infty}^{\infty} x(t) \delta(t) dt = x(0)$$

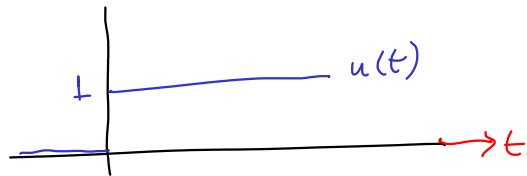
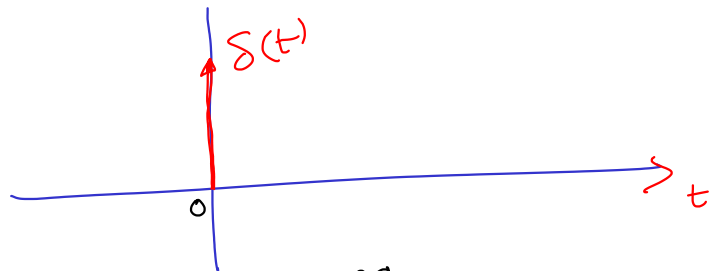
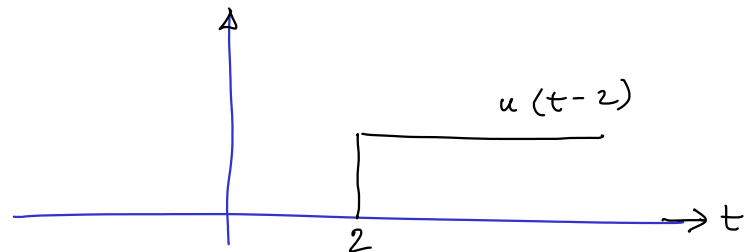
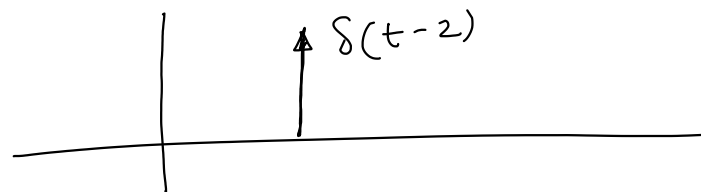
$$= \int_{-\infty}^{0^-} x(t) \times 0 dt + \int_{0^-}^{0^+} x(t) \delta(t) dt + \int_{0^+}^{\infty} x(t) \times 0 dt$$

$$\int_{0^-}^{0^+} \delta(t) dt = 1$$

$$\int_{0^-}^{0^+} x(t) \delta(t) dt = \int_{0^-}^{0^+} x(0) \delta(t) dt$$

$$= x(0) \int_{0^-}^{0^+} \delta(t) dt = x(0)$$

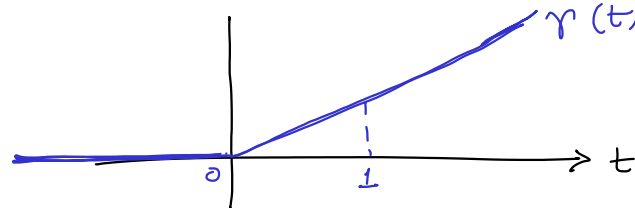
$$\int_{-\infty}^{\infty} x(t) \delta(t) dt = x(0)$$



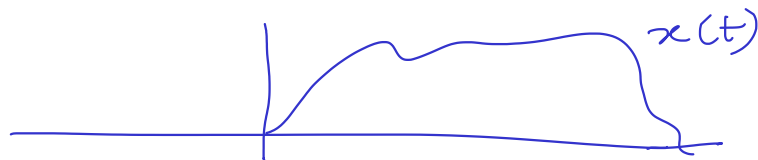
$$\int_{-\infty}^{\infty} x(t) \delta(t) dt = x(0)$$

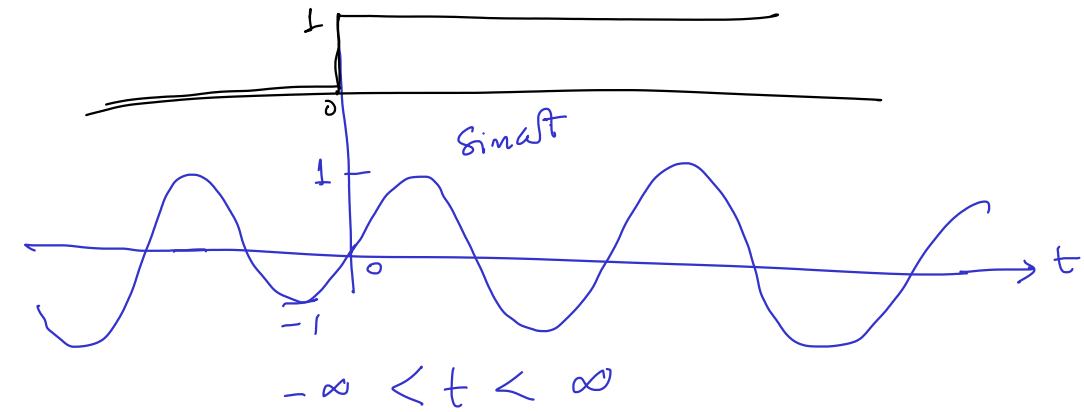
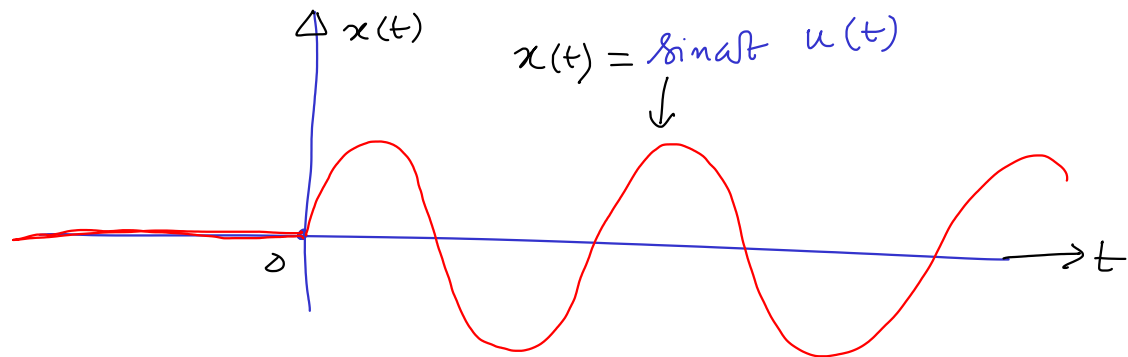
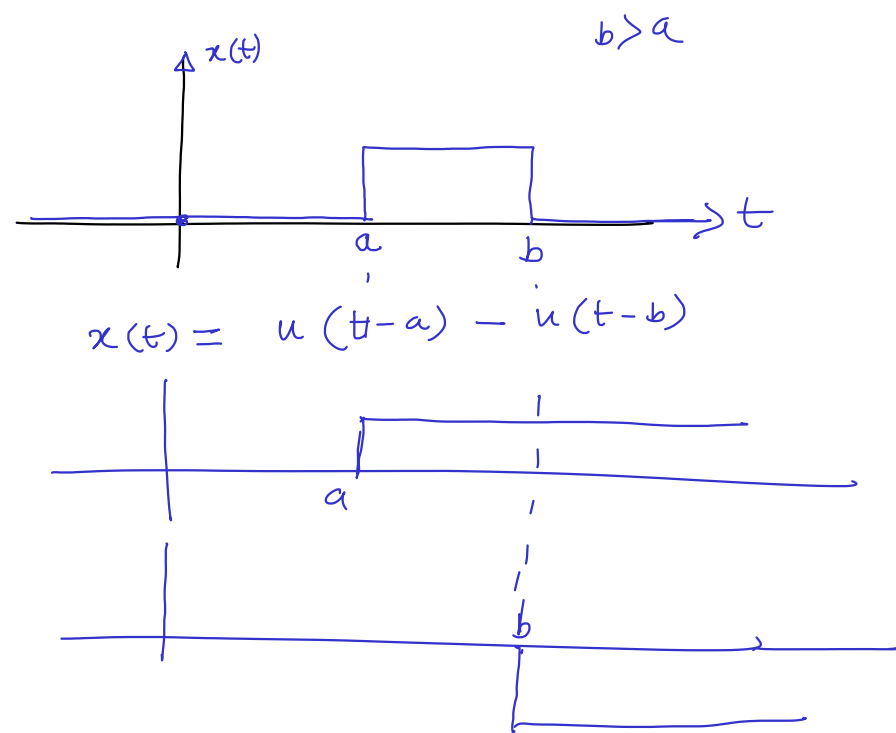
$$\int_{-\infty}^{\infty} x(t) \delta(t-2) dt = x(2)$$

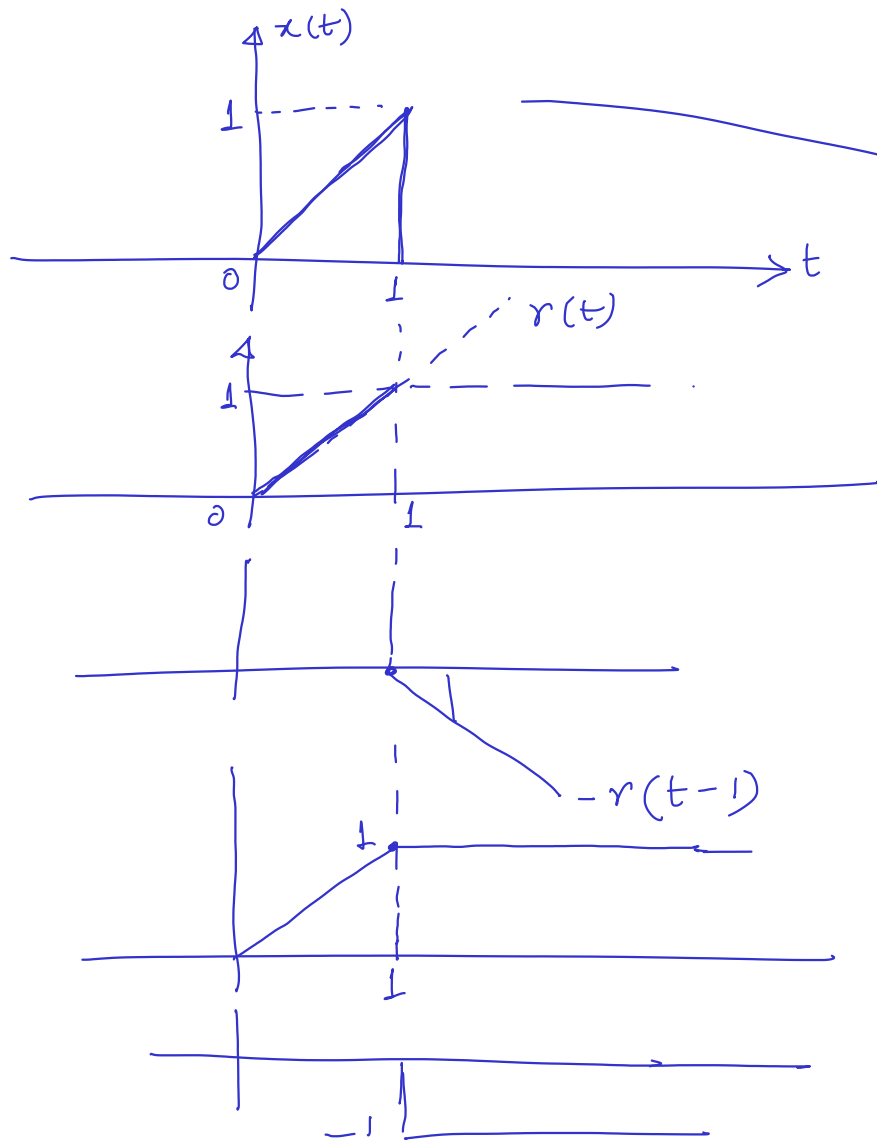
unit Ramp function



$$r(t) = \begin{cases} t & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$$

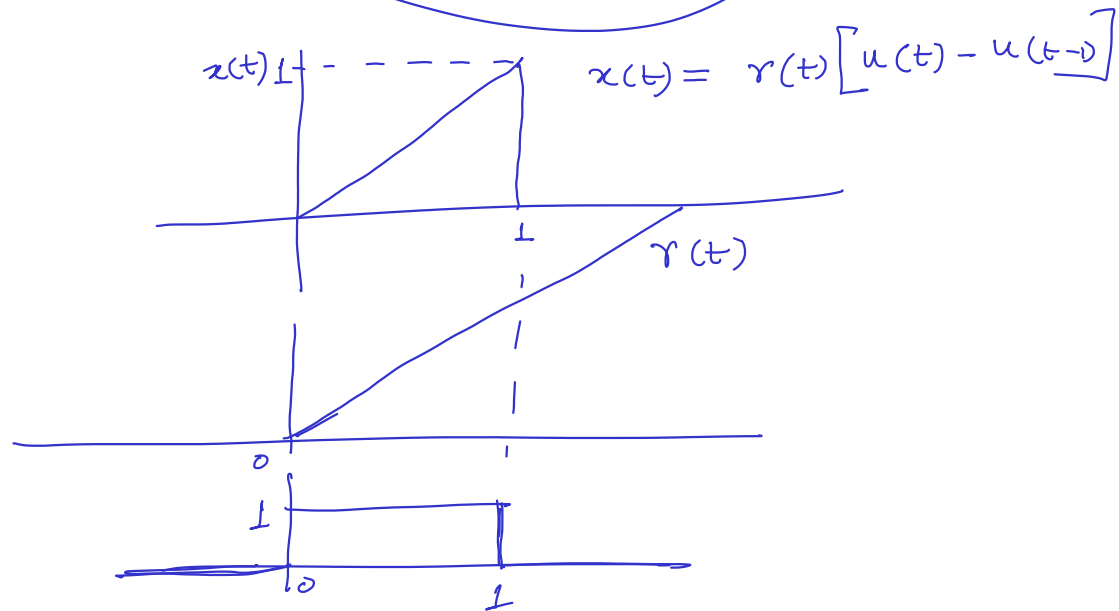






$$\rightarrow x(t) = r(t) - r(t-1) - u(t-1)$$

$$\begin{aligned} x(t) &= 0 & \text{for } t < 0 \\ &= t & \text{for } 0 \leq t \leq 1 \\ &= 0 & \text{for } t > 1 \end{aligned}$$



Laplace Transform:-

$$\mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} e^{-st} x(t) dt = X(s)$$

if $x(t) = 0$ for $t < 0$
 $= x(t)$ for $t \geq 0$

Causal signal exist for $t \geq 0$
 $= 0$ for $t < 0$

$$\boxed{\mathcal{L}\{x(t)\} = \int_0^{\infty} e^{-st} x(t) dt = X(s)}$$

$$\mathcal{L}\{x(t)u(t)\} = \int_0^{\infty} e^{-st} x(t) \underline{u(t)} dt = \int_0^{\infty} e^{-st} x(t) dt$$

e^{at} dimension
of $a \Rightarrow \underline{\underline{\frac{1}{\text{sec}}}}$

$s = \sigma + j\omega \equiv$ complex
freq.

Imag. axis
Re axis

Find out $\mathcal{L}[u(t)] = ?$

$$\mathcal{L}[u(t)] = \int_0^{\infty} e^{-st} u(t) dt$$

$$= \left. \frac{e^{-st}}{(-s)} \right|_0^{\infty}$$

$$= \int_0^{\infty} e^{-st} dt$$

$$\left. \frac{e^{-st}}{-s} \right|_{t=\infty}$$

$$- \left. \frac{e^{-st}}{(-s)} \right|_{t=0}$$

$$e^{-st} = ? \text{ as } t \rightarrow \infty$$

$$e^{-(\sigma + j\omega)t} = e^{-\sigma t} \cdot e^{-j\omega t}$$

$$= 0 - \frac{1}{(-s)}$$

$$= \frac{1}{s}$$

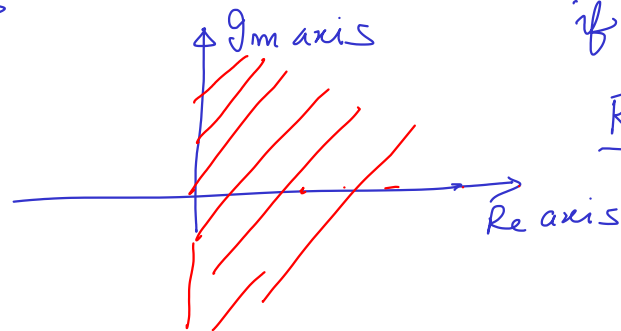
$$|e^{j\omega t}| = 1$$

$$e^{-\sigma t} \rightarrow 0 \text{ as } t \rightarrow \infty$$

if $\sigma > 0$

ROC

$$\mathcal{L}\{u(t)\} = \frac{1}{s} \quad \text{if } \sigma > 0$$



$$\mathcal{L} \left\{ u(t-a) \right\} = \int_0^{\infty} e^{-st} u(t-a) dt = \int_a^{\infty} e^{-st} dt$$

$$\underline{\underline{a > 0}}$$

$$= \left. \frac{e^{-st}}{-s} \right|_a^{\infty} = \left. \frac{e^{-st}}{-s} \right|_{t=\infty} - \left. \frac{e^{-st}}{(-s)} \right|_{t=a}$$

$$u(t-a) = 1 \quad t > a$$

$$= 0 \quad t < a$$

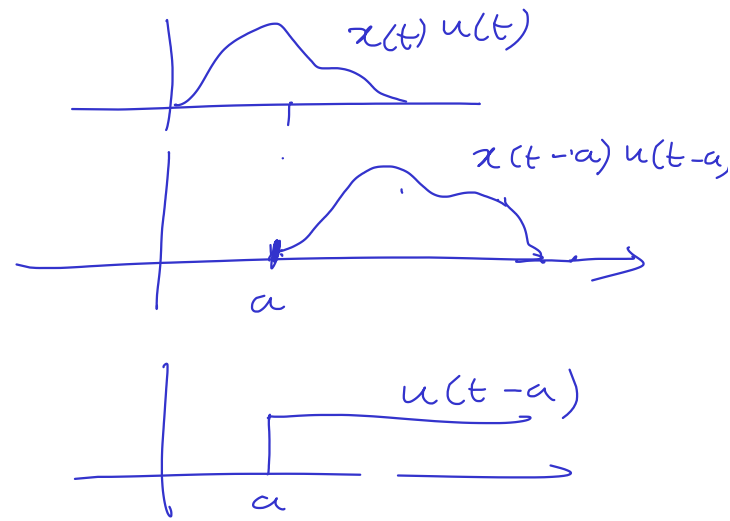
$$\mathcal{L} \left\{ u(t-a) \right\} = \frac{e^{-as}}{s}$$

$$\mathcal{L} \left\{ u(t) \right\} = \frac{1}{s}$$

$$= 0 + \frac{e^{-as}}{s}$$

Suppose $\mathcal{L} \left\{ \begin{matrix} x(t) \\ u(t) \end{matrix} \right\} = X(s)$ known

Then $\mathcal{L} \left\{ \begin{matrix} x(t-a) \\ u(t-a) \end{matrix} \right\} = ?$



$$\mathcal{L} \left\{ \begin{matrix} x(t-a) \\ u(t-a) \end{matrix} \right\} = \int_0^{\infty} e^{-st} x(t-a) u(t-a) dt$$

$$= \int_a^{\infty} e^{-st} x(t-a) dt$$

put $t-a=z$ $dt=dz$ & $t=z+a$

$$\begin{aligned} &= \int_{z=0}^{\infty} e^{-s(a+z)} x(z) dz = e^{-as} \int_{z=0}^{\infty} e^{-sz} x(z) dz \\ &= e^{-as} \int_{t=0}^{\infty} e^{-st} x(t) dt = e^{-as} X(s) \end{aligned}$$

if $\mathcal{L} \left\{ \underbrace{x(t)}_{u(t)} \right\} = X(s)$

$$\mathcal{L} \left\{ x(t-a) u(t-a) \right\} = e^{-as} X(s)$$

$$\mathcal{L} [u(t)] = \frac{1}{s}$$

$$\therefore \mathcal{L} [u(t-a)] = \frac{e^{-as}}{s}$$

Laplace Transform of standard signals

& Some properties

suppose $x(t) \xleftrightarrow{u(t)} X(s)$ $\int_0^{\infty} x(t) e^{-st} dt = X(s)$

What is $\mathcal{L} \{ e^{at} x(t) u(t) \} = ?$

$$\mathcal{L} \{ e^{at} x(t) u(t) \} = \int_0^{\infty} e^{-st} \{ e^{at} x(t) \} dt = \int_0^{\infty} e^{-(s-a)t} x(t) dt$$

$$\mathcal{L} \{ u(t) \} = \frac{1}{s} = X(s-a)$$

$$\mathcal{L} \{ e^{at} u(t) \} = \frac{1}{s-a}$$

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$$\mathcal{L} \{ u(t) \} = \frac{1}{s}$$

$$\mathcal{L} \left\{ \begin{matrix} x(t) \\ u(t) \end{matrix} \right\} = X(s)$$

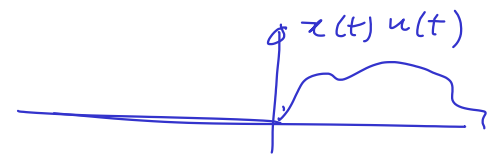
$$\mathcal{L} \{ x(t-a) u(t-a) \} = e^{-as} X(s)$$

$$\begin{matrix} x(t) \\ u(t) \end{matrix} \leftrightarrow X(s)$$

$$\begin{matrix} e^{at} x(t) \\ u(t) \end{matrix} \leftrightarrow X(s-a)$$

$$\text{If } \mathcal{L}\{x(t)u(t)\} = X(s)$$

$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = ?$$



$$\mathcal{L}\left\{\frac{dx}{dt}\right\} = \int_0^{\infty} e^{-st} \left(\frac{dx}{dt}\right) dt = \int_0^{\infty} \left(\frac{dx}{dt}\right) e^{-st} dt$$

$\swarrow$ 1st f.
 $\nwarrow$ 2nd f.

$$= \left. e^{-st} x(t) \right|_0^{\infty} - \int_0^{\infty} (-s) e^{-st} x(t) dt$$

$$= \left. e^{-st} x(t) \right|_{t=\infty} - \left. e^{-st} x(t) \right|_{t=0} + s \int_0^{\infty} e^{-st} x(t) dt$$

$$= \underbrace{e^{-st}}_{\downarrow 0} \cdot e^{-j\omega t}$$

$$\boxed{\mathcal{L}\left\{\frac{dx}{dt}\right\} = 0 - x(0) + sX(s) = sX(s) - \underline{x(0)}}$$

$$\mathcal{L} \left\{ \begin{matrix} x(t) \\ xu(t) \end{matrix} \right\} = X(s)$$

$$\boxed{\mathcal{L} \left\{ \frac{dx}{dt} \right\} = sX(s) - x(0)}$$

$$\mathcal{L} \left[\frac{d^3 x}{dt^3} \right]$$

$$= s^3 X(s) - s^2 x(0) - s \dot{x}(0) - \ddot{x}(0)$$

Then $\mathcal{L} \left[\frac{d^2 x}{dt^2} \right] = ?$

$$\begin{aligned} \mathcal{L} \left[\frac{d}{dt} \left(\frac{dx}{dt} \right) \right] &= s \mathcal{L} \left[\frac{dx}{dt} \right] - \left. \frac{dx}{dt} \right|_{t=0} \\ &= s \mathcal{L} \left[\frac{dx}{dt} \right] - \dot{x}(0) \\ &= s \left[sX(s) - x(0) \right] - \dot{x}(0) \end{aligned}$$

$$\boxed{\mathcal{L} \left[\frac{d^2 x}{dt^2} \right] = s^2 X(s) - s x(0) - \dot{x}(0)}$$

$$\mathcal{L}\{x(t) u(t)\} = X(s) \checkmark \text{ known}$$

What is $\mathcal{L}\left\{\int_0^t x(\tau) d\tau\right\} = ?$

Let $\int_0^t x(\tau) d\tau = g(t)$
 Then $\frac{dg}{dt} = x(t)$

$$\mathcal{L}\{g(t)\} = \int_0^\infty e^{-st} g(t) dt = \int_0^\infty \underset{\substack{\uparrow \\ \text{1st f.}^\text{th}}}{g(t)} \underset{\substack{\rightarrow \\ \text{2nd f.}^\text{th}}}{e^{-st}} dt$$

$$= g(t) \frac{e^{-st}}{(-s)} \bigg|_0^\infty - \int_0^\infty \left(\frac{dg}{dt}\right) \frac{e^{-st}}{(-s)} dt$$

$$g(t) = \int_0^t x(\tau) d\tau$$

$$g(0) = \int_0^0 x(\tau) d\tau = 0$$

$$= \frac{g(t) e^{-st}}{(-s)} \bigg|_{t=0}^{t=\infty} - \frac{g(0)}{(-s)} + \frac{1}{s} \int_0^\infty x(t) e^{-st} dt = \frac{X(s)}{s}$$

$$\mathcal{L} \left\{ \int_0^t x(\tau) d\tau \right\} = \frac{X(s)}{s}$$

where $X(s) = \int_0^{\infty} e^{-st} x(t) dt$
 $= \mathcal{L} \{ x(t) \}$

If $\mathcal{L} \{ x(t) u(t) \} = X(s)$

$$\mathcal{L} \{ t x(t) u(t) \} = ?$$

we know

$$X(s) = \int_0^{\infty} e^{-st} x(t) dt$$

differentiate both sides
wrt 's'

$$\frac{dX}{ds} = \int_0^{\infty} (-t) e^{-st} x(t) dt = - \int_0^{\infty} e^{-st} t x(t) dt$$

$$\text{or } \int_0^{\infty} e^{-st} \{ t x(t) \} dt = - \frac{dX(s)}{ds}$$

$$\mathcal{L} \{ t^2 x(t) \} = ?$$

$$\text{if } \mathcal{L} \{ x(t) \} = X(s).$$

$$\mathcal{L} \{ t x(t) \} = - \frac{dX}{ds}.$$

$$\mathcal{L} \left\{ t \left(\underbrace{t x(t)} \right) \right\} = - \frac{d}{ds} \left[- \frac{dX(s)}{ds} \right]$$

$$\mathcal{L} \{ t^2 x(t) u(t) \} = \frac{d^2 X(s)}{ds^2}$$

$$\mathcal{L} \left\{ \frac{dx}{dt} \right\} = s X(s) - \underline{x(0)}$$

$$\mathcal{L} [t u(t)] = - \frac{d}{ds} \left(\frac{1}{s} \right) = \frac{1}{s^2}$$

$$\mathcal{L}^{-1} \left[\frac{1}{(s+a)^2} \right] = t e^{-at} u(t)$$

$$s = \sigma + j\omega$$

$$\mathcal{L}[u(t)] = \frac{1}{s}$$

$$\mathcal{L}[e^{at} u(t)] = \frac{1}{s-a}$$

$$\mathcal{L}[x(t)] = \int_0^{\infty} e^{-st} x(t) dt = X(s)$$

$$\mathcal{L}[\delta(t)] = \int_{0^-}^{0^+} e^{-st} \delta(t) dt = 1$$

$$\therefore \mathcal{L}[\delta(t)] = 1$$

$$\mathcal{L}[\sin \omega t u(t)] = ?$$

$$\mathcal{L}[\cos \omega t u(t)] = ?$$

$$\mathcal{L}[e^{j\omega t} u(t)] = \frac{1}{s - j\omega}$$

$$\text{or } \mathcal{L}[\cos \omega t + j \sin \omega t u(t)] = \frac{1}{s - j\omega} = \frac{s + j\omega}{s^2 + \omega^2} = \frac{s}{s^2 + \omega^2} + j \frac{\omega}{s^2 + \omega^2}$$

$$\mathcal{L}[\cos \omega t u(t)] + j \mathcal{L}[\sin \omega t u(t)] = \frac{s}{s^2 + \omega^2} + j \frac{\omega}{s^2 + \omega^2}$$

$$\therefore \mathcal{L}[\cos \omega t u(t)] = \frac{s}{s^2 + \omega^2}$$

$$\text{and } \mathcal{L}[\sin \omega t u(t)] = \frac{\omega}{s^2 + \omega^2}$$

$$\mathcal{L}[\sin \omega t u(t)] = \frac{\omega}{s^2 + \omega^2} \quad \checkmark$$

$$\mathcal{L} \left[e^{at} \sin \omega t u(t) \right]$$

We know

$$\mathcal{L} \left[\sin \omega t u(t) \right] = \frac{\omega}{s^2 + \omega^2}$$

$$x(t) \longleftrightarrow X(s)$$

$$X(s) = \int_0^{\infty} e^{-st} x(t) dt$$

Inverse L.T

$$\mathcal{L}^{-1} [X(s)] = x(t)$$

$$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} ds$$

$$\text{Then } \mathcal{L} \left[e^{at} \sin \omega t u(t) \right] = \frac{\omega}{(s-a)^2 + \omega^2}$$

$$\mathcal{L} \left[e^{at} \cos \omega t u(t) \right] = \frac{(s-a)}{(s-a)^2 + \omega^2}$$

→ This equⁿ will not be necessary to get Laplace Inverse

$$\mathcal{L}[u(t)] = \frac{1}{s}$$

$$\mathcal{L}[e^{5t}u(t)] = \frac{1}{s-5}$$

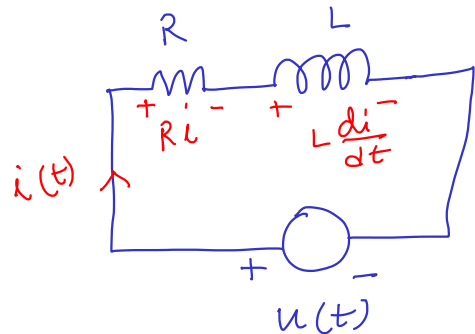
$$\therefore \mathcal{L}^{-1}\left[\frac{1}{s}\right] = u(t)$$

$$\therefore \mathcal{L}^{-1}\left[\frac{1}{s-5}\right] = e^{5t}u(t)$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2 + a^2}$$

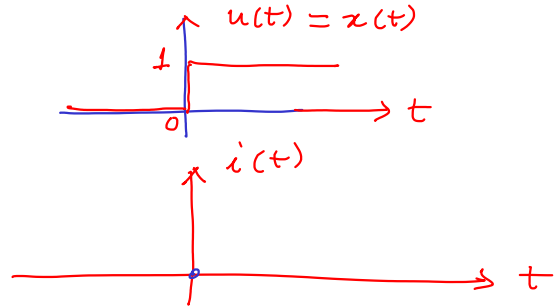
$$\begin{aligned}\mathcal{L}^{-1}\left[\frac{2}{s^2+4}\right] &= \mathcal{L}^{-1}\left[\frac{2}{s^2+2^2}\right] \\ &= \sin 2t u(t)\end{aligned}$$

$$\begin{aligned}\mathcal{L}^{-1}\left(\frac{1}{s^2+9}\right) &= \mathcal{L}^{-1}\frac{1}{s^2+3^2} \\ &= \mathcal{L}^{-1}\frac{\frac{1}{3} \times 3}{s^2+3^2} \\ &= \frac{1}{3} \sin 3t u(t)\end{aligned}$$



$$L \frac{di}{dt} + Ri = u(t)$$

Take L.T of both the sides.



Let

$$\mathcal{L}[i(t)] = \tilde{I}(s)$$

$$i(t) = ?$$

$$L \mathcal{L}\left[\frac{di}{dt}\right] + R \mathcal{L}[i] = \mathcal{L}[u(t)]$$

$$L [s \tilde{I}(s) - i(0^-)] + R \tilde{I}(s) = \frac{1}{s}$$

$$\text{or } (sL + R) \tilde{I}(s) - L i(0^-) = \frac{1}{s}$$

$$\text{suppose } i(0^-) = 0 \checkmark$$

$$(R + sL) \tilde{I}(s) = \frac{1}{s}$$

Algebraic Equⁿ

$$\therefore \tilde{I}(s) = \frac{1}{s(R + sL)}$$

$$i(t) = \mathcal{L}^{-1}[\tilde{I}(s)]$$

$$I(s) = \frac{1}{s(R + sL)} = \frac{1}{L} \left[\frac{1}{s(s + \frac{R}{L})} \right] = \frac{A}{s} + \frac{B}{s + \frac{R}{L}}$$

$$= \frac{1}{L} \frac{L}{R} \left[\frac{(s + \frac{R}{L}) - s}{s(s + \frac{R}{L})} \right] = \frac{1}{R} \left[\frac{1}{s} - \frac{1}{s + \frac{R}{L}} \right]$$

$$I(s) = \frac{1}{R} \frac{1}{s} - \frac{1}{R} \frac{1}{(s + \frac{R}{L})}$$

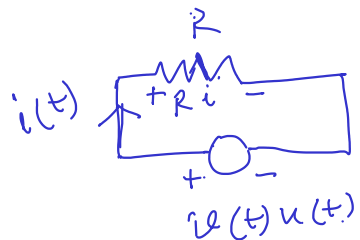
$$i(t) = \mathcal{L}^{-1}[I(s)] = \frac{1}{R} \mathcal{L}^{-1}\left(\frac{1}{s}\right) - \frac{1}{R} \mathcal{L}^{-1}\left\{\frac{1}{(s + \frac{R}{L})}\right\}$$

$$= \frac{1}{R} u(t) - \frac{1}{R} e^{-\frac{R}{L}t} u(t) = \frac{1}{R} \left(1 - e^{-\frac{R}{L}t}\right) u(t) = i(t)$$

$$\mathcal{L}[u(t)] = \frac{1}{s}$$

$$\mathcal{L}[e^{-at} u(t)] = \frac{1}{s+a}$$

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$$i(t) = \frac{v(t) u(t)}{R}$$

$$I(s) = \frac{1}{R} V(s) = \frac{V(s)}{R}$$

$$I(s) = \frac{V(s)}{R} = \frac{\text{L.T of voltage}}{\text{impedance}} =$$

in s-domain

Resistive element $\rightarrow R$
(R)

Inductive element $\rightarrow sL$
(L)

Capacitive element $\rightarrow \frac{1}{Cs}$
(C)

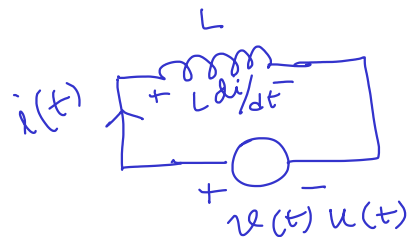
impedance

in s-domain

$$V(s) = R I(s)$$

$$V(s) = (sL) I(s)$$

$$V(s) = \left(\frac{1}{Cs}\right) I(s)$$



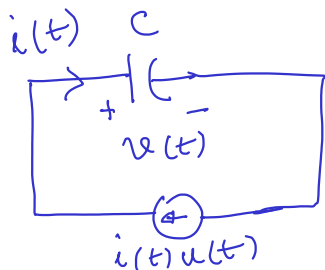
$$v(t) u(t) = L \frac{di}{dt}$$

$$\text{L.T} \quad V(s) = L (s I(s) - \underline{i(0^-)})$$

$$\text{If } i(0^-) = 0$$

$$V(s) = sL I(s) \quad \text{or}$$

$$I(s) = \frac{V(s)}{sL} = \frac{\text{L.T of voltage}}{\text{impedance}}$$



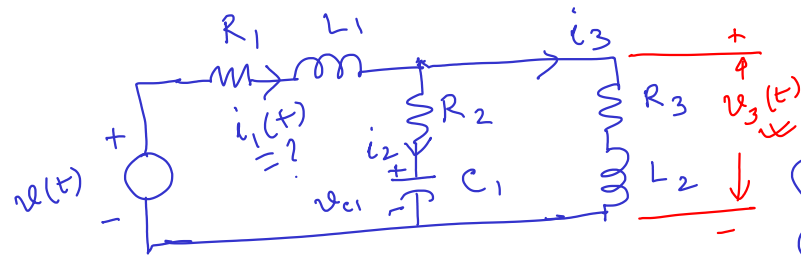
$$C \frac{dv}{dt} = i(t) u(t)$$

$$C [s V(s) - v(0^-)] = I(s)$$

$$\text{If } v(0^-) = 0$$

$$C s V(s) = I(s)$$

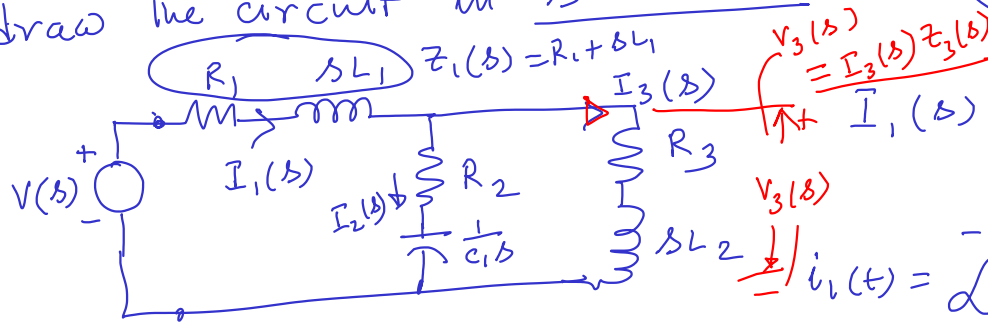
$$I(s) = \frac{V(s)}{(1/Cs)} \rightarrow \text{impedance}$$



in time domain

Suppose all initial conditions of energy storing elements are relaxed (initial conditions zeros)

Redraw the circuit in s-domain



To find $i_1(t)$: ① Form a differential eqnⁿ in time domain involving $i_1(t)$

- ② Take L.T of both the sides.
- ③ Get an expression for $I_1(s)$
- ④ Take Laplace inverse of $I_1(s)$ to obtain $i_1(t)$.

$$V = IR$$

$$I_3(s) = I_1(s) \frac{R_2 + \frac{1}{sC_1}}{(R_2 + R_3 + \frac{1}{sC_1} + sL_2)}$$

$V(s) = I(s) Z(s)$

$$\begin{aligned} i_1(0^-) &= 0 & v_{C1}(0^-) &= 0 \\ i_3(0^-) &= 0 \end{aligned}$$

$$v_3(s) = I_3(s) Z_3(s)$$

$$v_3(s)$$

$$i_1(t) = \mathcal{L}^{-1} [I_1(s)]$$

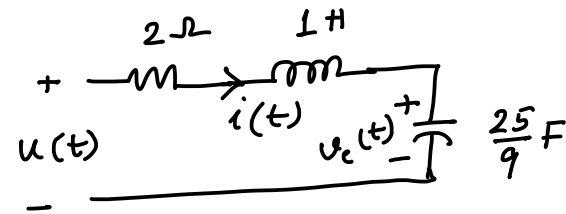
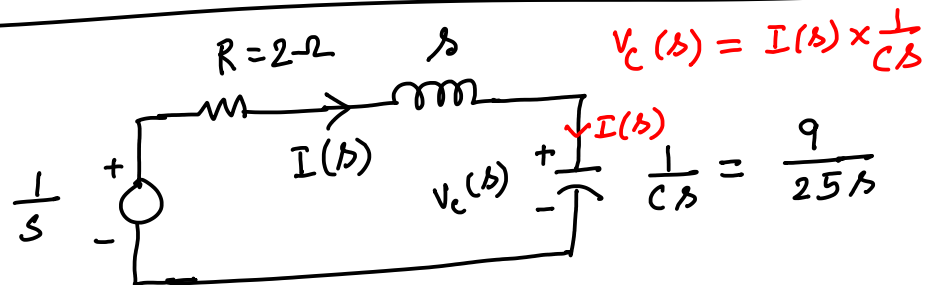
$$I_1(s) = \frac{V(s)}{R_1 + sL_1 + \frac{(R_2 + \frac{1}{sC_1})(R_3 + sL_2)}{(R_2 + R_3 + \frac{1}{sC_1} + sL_2)}}$$

Network Analysis with Laplace Transform :-

lec 47

$$R = 2\Omega ; L = H \quad \text{and} \quad C = \frac{25}{9} F$$

$$i(0^-) = 0 \quad v_c(0^-) = 0$$



$$v_c(s) = I(s) \times \frac{1}{Cs}$$

$$I(s) = \frac{1}{2s + s^2 + \frac{9}{25}} =$$

$$I(s) = \frac{1}{\left(s + \frac{9}{5}\right) \left(s + \frac{1}{5}\right)}$$

$$Z(s) = 2 + s + \frac{9}{25s}$$

$$I(s) = \frac{V(s)}{Z(s)} = \frac{1/s}{2 + s + \frac{9}{25s}}$$

$$\frac{1}{(s+1)^2 - \frac{16}{25}} = \frac{1}{(s+1)^2 - \left(\frac{4}{5}\right)^2}$$

$$= \frac{A}{s + \frac{9}{5}} + \frac{B}{s + \frac{1}{5}} \quad \text{Partial fraction}$$

$$I(s) = \frac{1}{(s + \frac{9}{5})(s + \frac{1}{5})} = \frac{A}{s + \frac{9}{5}} + \frac{B}{s + \frac{1}{5}}$$

or

$$s = -\frac{9}{5}$$

$$s = -\frac{1}{5}$$

$$I(s) =$$

$$1 = (s + \frac{1}{5})A + (s + \frac{9}{5})B$$

$$1 = (-\frac{9}{5} + \frac{1}{5})A + 0 = -\frac{8}{5}A = 1 \Rightarrow A = -\frac{5}{8}$$

$$1 = 0 \times A + (-\frac{1}{5} + \frac{9}{5})B = \frac{8}{5}B = 1 \therefore B = \frac{5}{8}$$

$$I(s) = \frac{-5/8}{s + \frac{9}{5}} + \frac{5/8}{s + \frac{1}{5}}$$

$$\therefore i(t) = -\frac{5}{8} e^{-\frac{9}{5}t} u(t) + \frac{5}{8} e^{-\frac{1}{5}t} u(t)$$

$$V_c(t) = \frac{1}{C} \int_0^t i(t) dt$$

identity

$$V_c(s) = I(s) \frac{1}{Cs} = I(s) \frac{9}{25s} = \frac{9}{25} \frac{I(s)}{s}$$

$$V_c(s) = \frac{9}{25} \left[\frac{1}{s(s + \frac{9}{5})(s + \frac{1}{5})} \right] = \frac{9}{25} \left[\frac{A}{s} + \frac{B}{s + \frac{9}{5}} + \frac{D}{s + \frac{1}{5}} \right]$$

$$\frac{1}{s(s + \frac{9}{5})(s + \frac{1}{5})} = \frac{A}{s} + \frac{B}{s + \frac{9}{5}} + \frac{D}{s + \frac{1}{5}}$$

$p = -\frac{1}{5}$
 $1 = 0 + 0 + D(-\frac{1}{5}) \times (-\frac{1}{5} + \frac{9}{5})$
 $1 = -D \times \frac{8}{25} \therefore D = -\frac{25}{8}$

$$1 = A(s + \frac{9}{5})(s + \frac{1}{5}) + s(s + \frac{1}{5})B + Ds(s + \frac{9}{5})$$

$s=0$: $1 = A \frac{9}{5} \times \frac{1}{5} + 0 + 0 \therefore A = \frac{25}{9}$

$s = -\frac{9}{5}$: $1 = 0 + (-\frac{9}{5})(-\frac{9}{5} + \frac{1}{5})B + 0 = (-\frac{9}{5}) \times (-\frac{8}{5})B = 1$
 $\therefore B = \frac{25}{9 \times 8} = \frac{25}{72}$

$$\therefore V_c(s) = \frac{9}{25} \left[\frac{25/9}{s} + \frac{25/72}{(s + \frac{9}{5})} - \frac{25/8}{(s + \frac{1}{5})} \right]$$

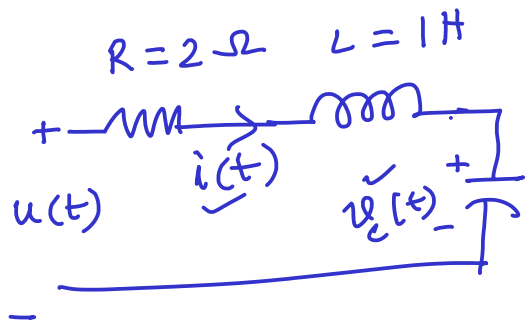
$$V_c(s) = \frac{1}{s} + \frac{1}{8} \frac{1}{(s + \frac{9}{5})} - \frac{9}{8} \frac{1}{s + \frac{1}{5}}$$

$$\therefore V_c(t) = u(t) + \frac{1}{8} e^{-\frac{9}{5}t} u(t) - \frac{9}{8} e^{-\frac{1}{5}t} u(t)$$

$$V_c(0) = 1 + \frac{1}{8} - \frac{9}{8} = 0$$

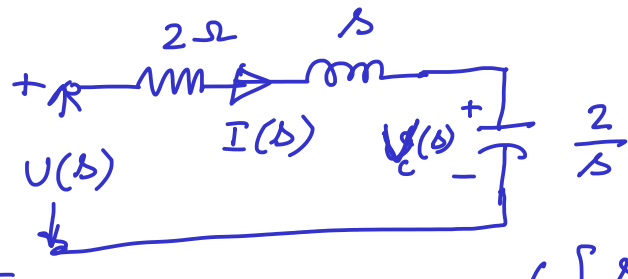
Ex 2

Initial conditions relaxed



$$C = \frac{1}{2} F$$

$\Rightarrow$



$$I(s) = \frac{U(s)}{Z(s)} = \frac{1/s}{2 + s + \frac{2}{s}}$$

$$I(s) = \frac{1}{2s + s^2 + 2} = \frac{1}{(s+1)^2 + 1^2}$$

$$\mathcal{L}[\sin at] = \frac{\omega}{s^2 + \omega^2}$$

$$\mathcal{L}[e^{at} \sin at] = \frac{\omega}{(s-a)^2 + \omega^2}$$

Taking:-

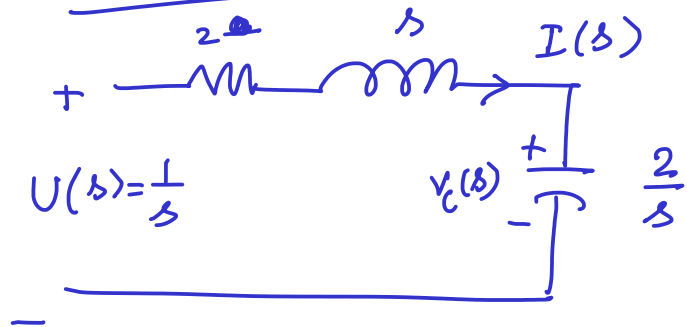
$$i(t) = e^{-t} \sin t u(t)$$

$$i(0^+) = 0$$

$$v_c(t) = 2 \int_0^t e^{-t} \sin t dt$$

Ex 2 continued.

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$$I(s) = \frac{1}{(s+1)^2 + 1}$$

$$\therefore V_L(s) = I(s) \times \frac{2}{s} = \frac{2/s}{(s+1)^2 + 1}$$

$$\text{or } V_L(s) = \frac{2}{s[(s+1)^2 + 1]} = \frac{A}{s} + \frac{Bs + D}{(s+1)^2 + 1} \quad \underline{A, B, D = ?}$$

$$\text{or } 2 = [(s+1)^2 + 1]A + (Bs + D)s$$
$$s=0 \Rightarrow 2 = 2A + 0 \quad \therefore \boxed{A=1}$$

$$2 = [(s+1)^2 + 1]A + (Bs + D)s$$

got already $A=1$

$$s=1: \quad 2 = 5 \times 1 + B + D \quad \therefore \boxed{B + D = -3}$$

$$s=-1: \quad 2 = A + (-B + D) \times (-1) = 1 + B - D$$

$$\therefore B - D = 2 - 1 = 1 \quad \text{or} \quad \boxed{B - D = 1}$$

$$2B = -2$$

$$\text{or } \boxed{B = -1}$$

$$2D = -4$$

$$\therefore \boxed{D = -2}$$

$$V_c(s) = \frac{2}{s[(s+1)^2 + 1]} = \frac{1}{s} + \frac{(-1)s - 2}{(s+1)^2 + 1}$$

$$V_c(s) = \frac{1}{s} - \frac{s+2}{(s+1)^2 + 1} = \frac{1}{s} - \frac{s+1}{(s+1)^2 + 1} - \frac{1}{(s+1)^2 + 1}$$

$$V_c(s) = \frac{1}{s} - \frac{(s+1)}{(s+1)^2 + 1^2} - \frac{1}{(s+1)^2 + 1}$$

$$\begin{aligned} \mathcal{L}^{-1}[V_c(s)] &= u(t) - \underset{u(t)}{e^{-t} \cos t} - \underset{u(t)}{e^{-t} \sin t} \\ &= [1 - e^{-t} \cos t - e^{-t} \sin t] u(t) \\ &= 1 - e^{-t} \sqrt{2} \left[\frac{1}{\sqrt{2}} \cos t + \frac{1}{\sqrt{2}} \sin t \right] \end{aligned}$$

$$V_c(t) = 1 - \sqrt{2} e^{-t} \cos(t - \pi/4)$$

$$v_c(0) = 1 - \sqrt{2} \times \frac{1}{\sqrt{2}} = 0$$



$$\mathcal{L}[\cos \omega t] = \frac{s}{s^2 + \omega^2}$$

$$\mathcal{L}[e^{-at} \cos \omega t] = \frac{(s+a)}{(s+a)^2 + \omega^2}$$

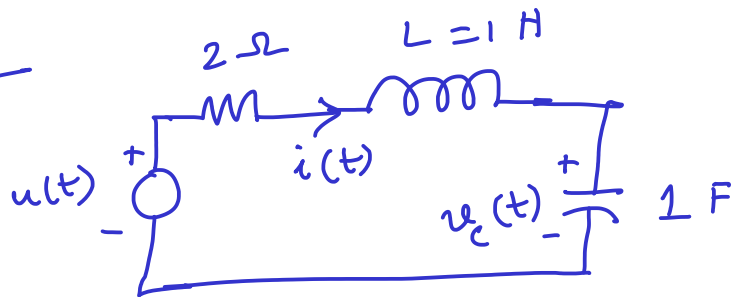
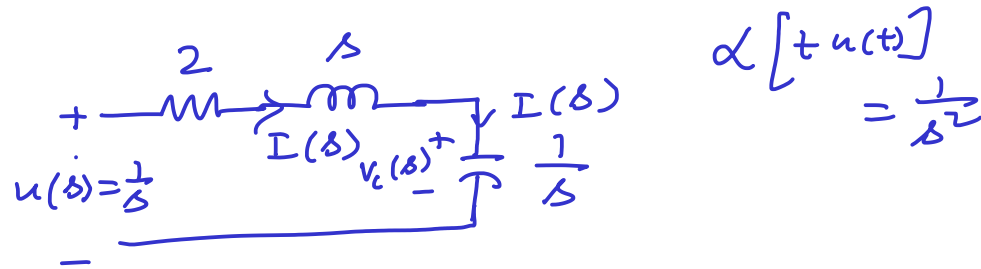
$$\mathcal{L}[e^{-at} \sin \omega t] = \frac{\omega}{(s+a)^2 + \omega^2}$$

$$= \frac{\omega}{(s+a)^2 + \omega^2}$$

$$\omega = 1$$

$$a = 1$$

Ex-3



$$\mathcal{L}[t u(t)] = \frac{1}{s^2}$$

$$I(s) = \frac{\frac{1}{s}}{2 + s + \frac{1}{s}} = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = \mathcal{L}^{-1}\left[\frac{1}{(s+1)^2}\right] = t e^{-t} u(t)$$

$$\boxed{i(t) = t e^{-t} u(t)}$$

$$i(0) = 0$$

$$\boxed{v_c(s) = \frac{1}{s(s+1)^2}}$$

$$V_c(s) = \frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{D}{(s+1)^2}$$

$$\therefore 1 = (s+1)^2 A + s(s+1)B + sD$$

$s=0:-$

$$1 = A$$

$$V_c(s) = \frac{1}{s} - \frac{1}{(s+1)} - \frac{1}{(s+1)^2}$$

$$1 = (s+1)^2 + s(s+1)B + sD$$

$s=-1:-$

$$1 = 0 + 0 - D$$

$$\therefore D = -1$$

$$u_c(t) = u(t) - e^{-t} u(t) - t e^{-t} u(t)$$

$$1 = (s+1) + s(s+1)B - s$$

$$s=+1 \quad 1 = 2^2 + 2B - 1 \quad \therefore 2B = 1 + 1 - 4 = -2$$

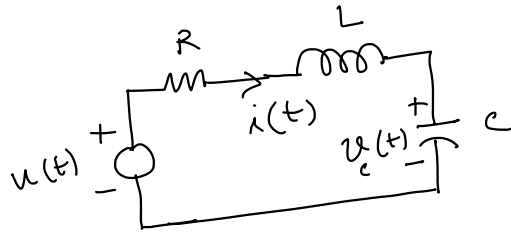
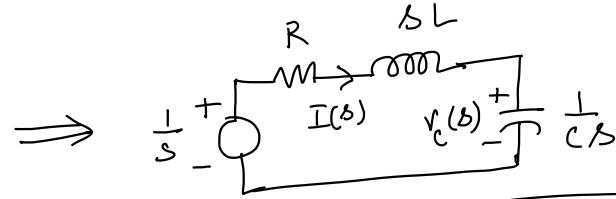
$$\therefore B = -1$$

Second order system (network) analysis

dec-49

using L.T

$$i(0^-) = 0 \text{ \& } v_c(0^-) = 0$$



$$V_c(s) = \frac{1}{sC} \times \frac{Y/s}{R + sL + \frac{1}{sC}}$$

$$I(s) = \frac{U(s)}{Z(s)} = \frac{Y/s}{R + sL + \frac{1}{sC}}$$

$$V_c(s) = \frac{1}{LC} \frac{Y/s}{(s^2 + \frac{R}{L}s + \frac{1}{LC})}$$

ch. eqⁿ

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$V_c(s) = \frac{1}{s} \frac{\frac{1}{LC} \omega_n^2}{(s^2 + \frac{R}{L}s + \frac{1}{LC})}$$

Let $\frac{1}{LC} = \omega_n^2$

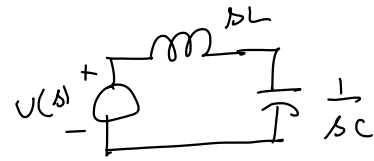
$$\frac{R}{L} = 2\zeta\omega_n \quad \left| \begin{array}{l} \zeta = \frac{R}{2} \sqrt{\frac{C}{L}} \\ \omega_n = \frac{1}{\sqrt{LC}} \end{array} \right.$$

$$V_c(s) = \frac{1}{s} \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

Case ①

$$\xi = 0$$

$$i.e., R=0$$

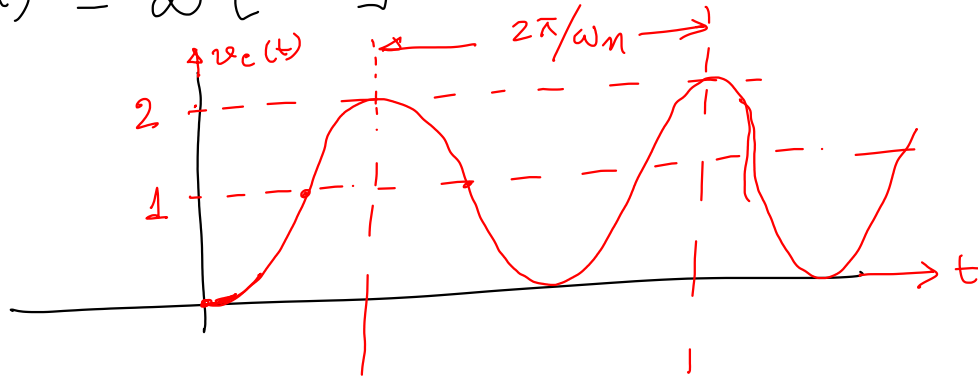


$$V_C(s) = \frac{1}{s} \frac{\omega_n^2}{(s^2 + \omega_n^2)} = \frac{1}{s} \frac{s^2 + \omega_n^2 - s^2}{(s^2 + \omega_n^2)} = \frac{1}{s} \left[1 - \frac{s^2}{s^2 + \omega_n^2} \right]$$

$$V_C(s) = \frac{1}{s} - \frac{s}{s^2 + \omega_n^2}$$

$$v_C(t) = \mathcal{L}^{-1}[V_C(s)] = u(t) - \cos \omega_n t \, u(t) = (1 - \cos \omega_n t) u(t)$$

$$i(t) = C \frac{dv_C}{dt} \quad \checkmark$$



ω_n = natural
freq. of
oscillation

Q

Case-2

$$\text{let } \xi = 1$$

$$\xi = \frac{R}{2} \sqrt{\frac{C}{L}} = 1$$

$$V_c(s) = \frac{1}{s} \frac{\omega_n^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$= \frac{\omega_n^2}{s(s^2 + 2\omega_n s + \omega_n^2)}$$

$$= \frac{\omega_n^2}{s(s + \omega_n)^2}$$

$s_1, s_2 \rightarrow -\omega_n, -\omega_n$

$$V_c(s) = \frac{\omega_n^2}{s(s + \omega_n)^2} = \frac{A}{s} + \frac{B}{(s + \omega_n)} + \frac{D}{(s + \omega_n)^2}$$

$$\therefore \omega_n^2 = (s + \omega_n)^2 A + s(s + \omega_n) B + s D$$

$$\boxed{s=0} \rightarrow$$

$$\omega_n^2 = \omega_n^2 A$$

$$\therefore \boxed{A=1}$$

$$\boxed{s=-\omega_n} \rightarrow$$

$$\omega_n^2 = -\omega_n D$$

$$\therefore \boxed{D = -\omega_n}$$

$$\boxed{B = -1}$$

$$\omega_n^2 = (s + \omega_n) \times 1 + s(s + \omega_n) B - \omega_n s$$

$$\boxed{s=1}$$

$$\omega_n^2 = (\omega_n + 1)^2 + (\omega_n + 1) B - \omega_n$$

$$\omega_n^2 = \omega_n^2 + 2\omega_n + 1 + (\omega_n + 1) B - \omega_n$$

$$(2\omega_n + 1) + (\omega_n + 1) B = 0$$

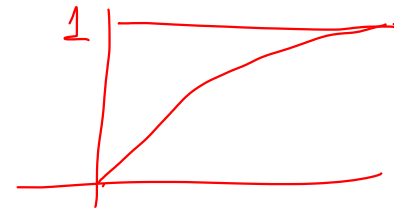
$$V_c(s) = \frac{1}{s} - \frac{1}{s + \omega_n} - \frac{\omega_n}{(s + \omega_n)^2}$$

$$v_c(t) = u(t) - e^{-\omega_n t} u(t) - \omega_n e^{-\omega_n t} t u(t)$$

Roots are
equal & real.

$$\zeta = 1$$

Critically damped
system



Case 3:

$$0 < \xi < 1$$

underdamped system.

$$V_c(s) = \frac{1}{s} \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} \frac{\omega_n^2}{(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)}$$

$$V_c(s) = \frac{1}{s} \frac{\omega_n^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{A}{s} + \frac{Bs + D}{(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$\omega_n^2 = (s^2 + 2\xi\omega_n s + \omega_n^2)A + s(Bs + D)$$

$$s=0 \Rightarrow \omega_n^2 = \omega_n^2 A \quad \therefore \boxed{A=1}$$

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ch. equⁿ.

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad +ve$$

$$(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2) = 0$$

$$(s + \xi\omega_n)^2 = -\frac{(1 - \xi^2)\omega_n^2}{+ve}$$

$$s + \xi\omega_n = \pm j\sqrt{1 - \xi^2}\omega_n$$

$$s_{1,2} = -\xi\omega_n \pm j\sqrt{1 - \xi^2}\omega_n$$

Roots will
be complex
conjugate.

$$\boxed{A=1}$$

$$\omega_n^v = (s^2 + 2\xi\omega_n s + \omega_n^2)A + s(Bs + D)$$

$$s=1$$

$$\cancel{\omega_n^v} = 1 + 2\xi\omega_n + \cancel{\omega_n^2} + B + D$$

$$\text{or } B + D = -1 - 2\xi\omega_n$$

$$2D = -4\xi\omega_n \Rightarrow \boxed{D = -2\xi\omega_n}$$

$$2B = -2 \Rightarrow \boxed{B = -1}$$

$$s=-1$$

$$\cancel{\omega_n^v} = 1 - 2\xi\omega_n + \cancel{\omega_n^2} - (-B + D)$$

$$\therefore D - B = 1 - 2\xi\omega_n$$

$$\therefore V_c(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{1}{s} - \frac{(s + \xi\omega_n) + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)}$$

$$V_c(s) = \frac{1}{s} - \frac{(s + \xi\omega_n)}{(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)} - \frac{\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)}$$

$$V_c(s) = \frac{1}{s} - \frac{(s + \xi \omega_n)}{(s + \xi \omega_n)^2 + \omega_n^2(1 - \xi^2)} - \frac{\xi \omega_n}{(s + \xi \omega_n)^2 + \omega_n^2(1 - \xi^2)}$$

$$= \frac{1}{s} - \frac{(s + \xi \omega_n)}{(s + \xi \omega_n)^2 + (\underbrace{\sqrt{1 - \xi^2} \omega_n})^2} - \frac{\xi \omega_n}{\sqrt{1 - \xi^2} \omega_n} \frac{\sqrt{1 - \xi^2} \omega_n}{(s + \xi \omega_n)^2 + (\sqrt{1 - \xi^2} \omega_n)^2}$$

$$= u(t) - e^{-\xi \omega_n t} \cos(\sqrt{1 - \xi^2} \omega_n t) - \frac{\xi}{\sqrt{1 - \xi^2}} e^{-\xi \omega_n t} \sin(\sqrt{1 - \xi^2} \omega_n t)$$

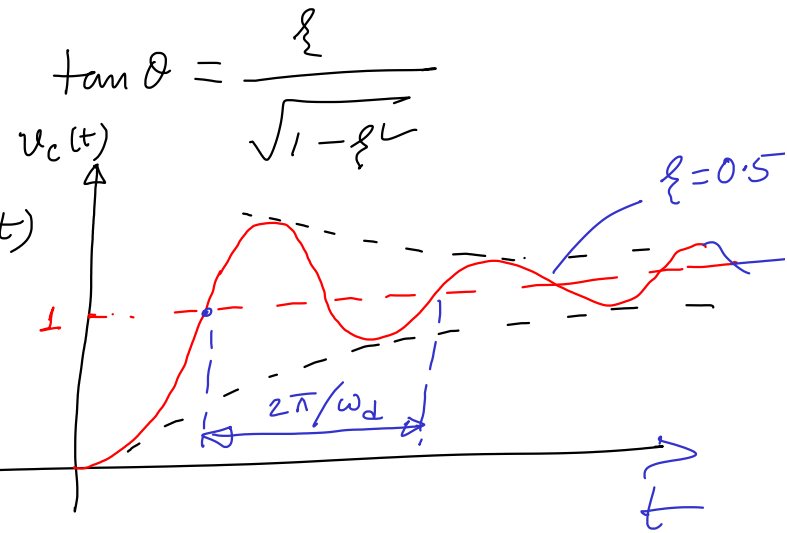
$\sqrt{1 - \xi^2} \omega_n = \omega_d = \text{freq. of damped oscillation.}$

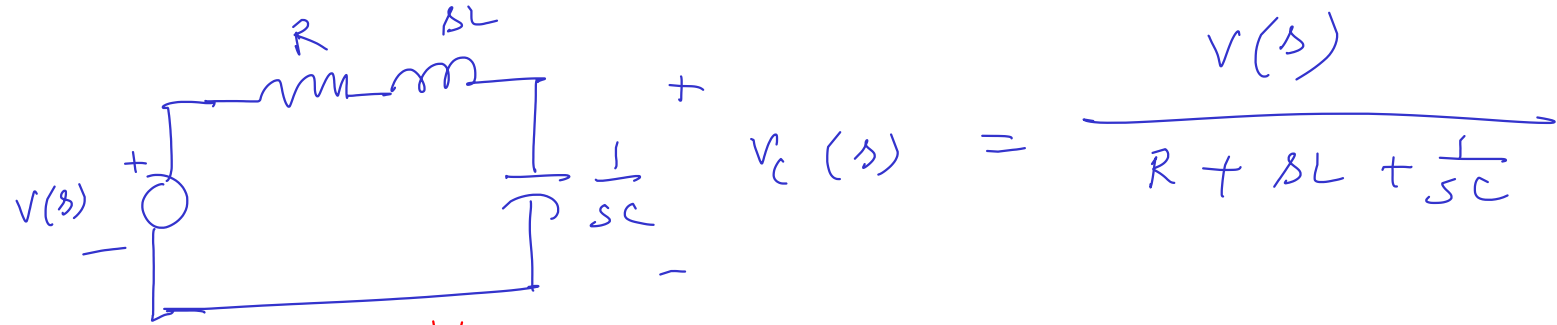
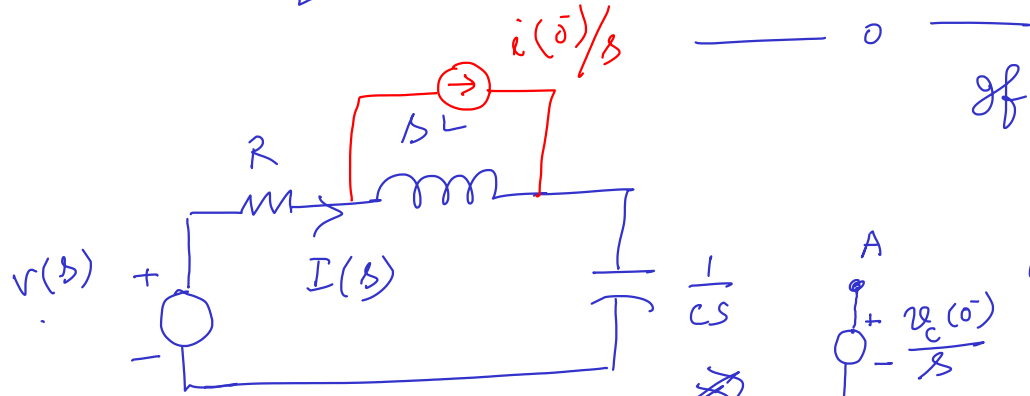
$$= u(t) - e^{-\xi \omega_n t} \cos \omega_d t - \frac{\xi}{\sqrt{1 - \xi^2}} e^{-\xi \omega_n t} \sin \omega_d t$$

$$\begin{aligned}
 v_c(t) &= u(t) - e^{-\xi \omega_n t} \cos \omega_d t - \frac{\xi}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin \omega_d t \quad 0 < \xi < 1 \\
 &= u(t) - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \left[\sqrt{1-\xi^2} \cos \omega_d t + \xi \sin \omega_d t \right] u(t)
 \end{aligned}$$

$$\begin{aligned}
 \text{define } \sqrt{1-\xi^2} &= \cos \theta \\
 \xi &= \sin \theta
 \end{aligned}$$

$$v_c(t) = u(t) - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \cos(\omega_d t - \theta) u(t)$$





if inductor has an initial current then how to solve?

