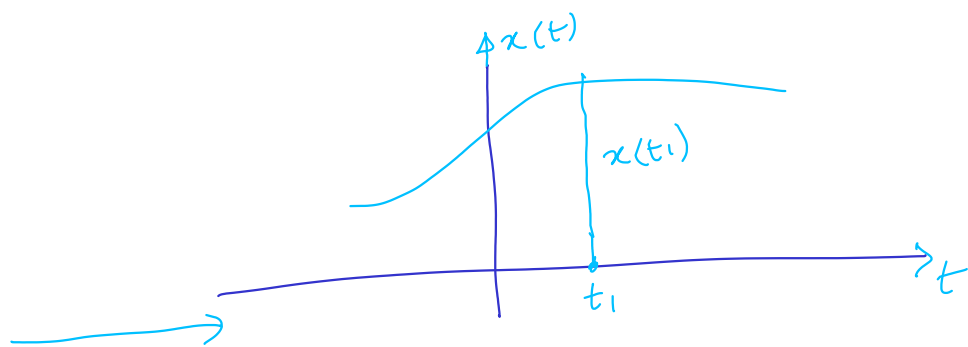


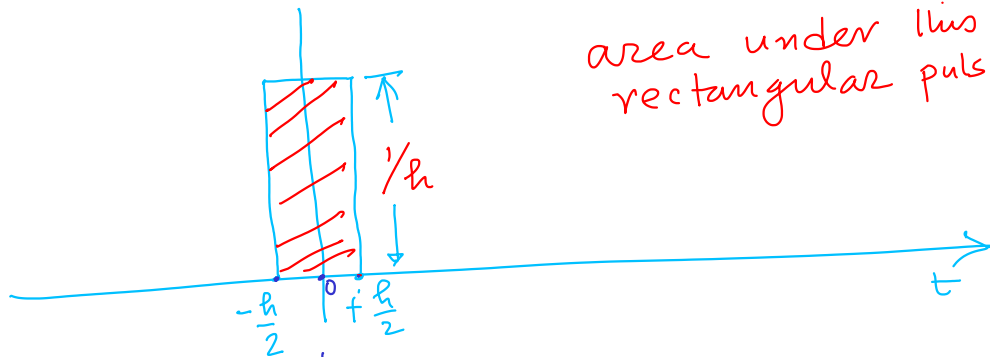


$$\frac{dy}{dt} + ay = x(t)$$

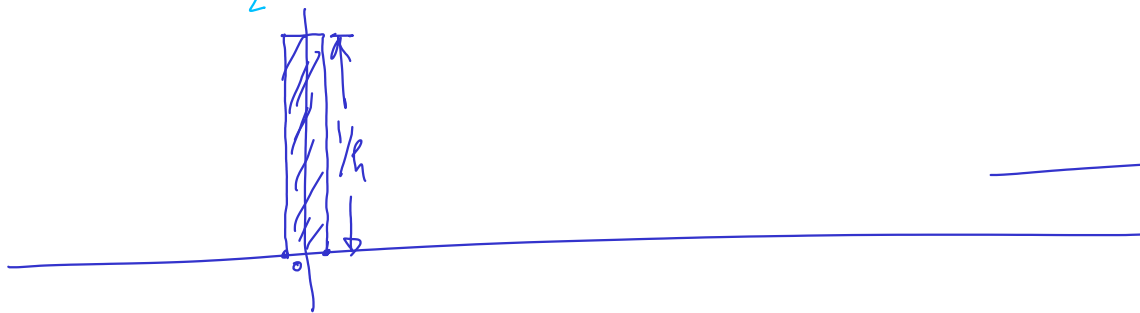
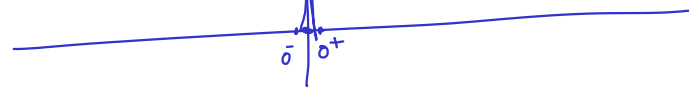
if  $x(t) = \delta(t)$   
 how to get  $y(t)$ ?

area under this rectangular pulse =  $h \times \frac{1}{h} = 1$

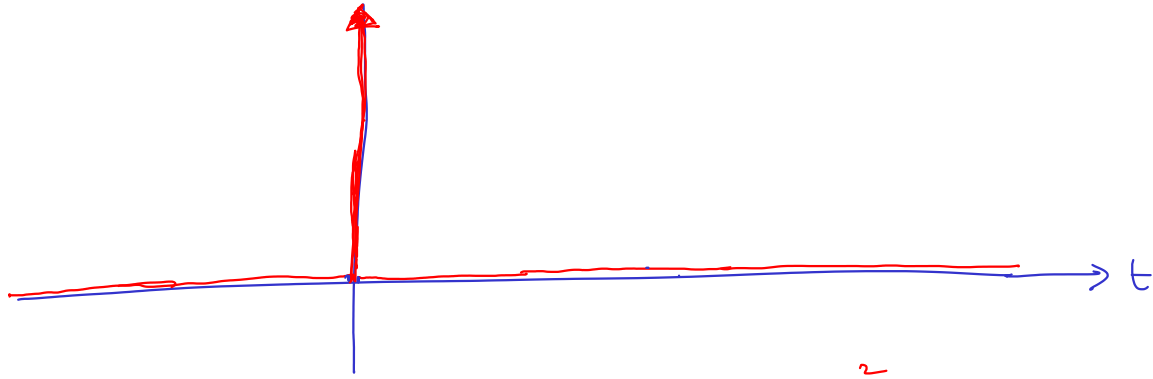
no matter whatever value you choose for 'h'



$\frac{1}{h} \rightarrow \infty$  In the limit when  $h \rightarrow 0$



$\delta(t) \rightarrow$  unit impulse function



$$\delta(t) = 0 \text{ for } t \neq 0$$

$$\delta(t) = \infty ? \text{ at } t=0$$

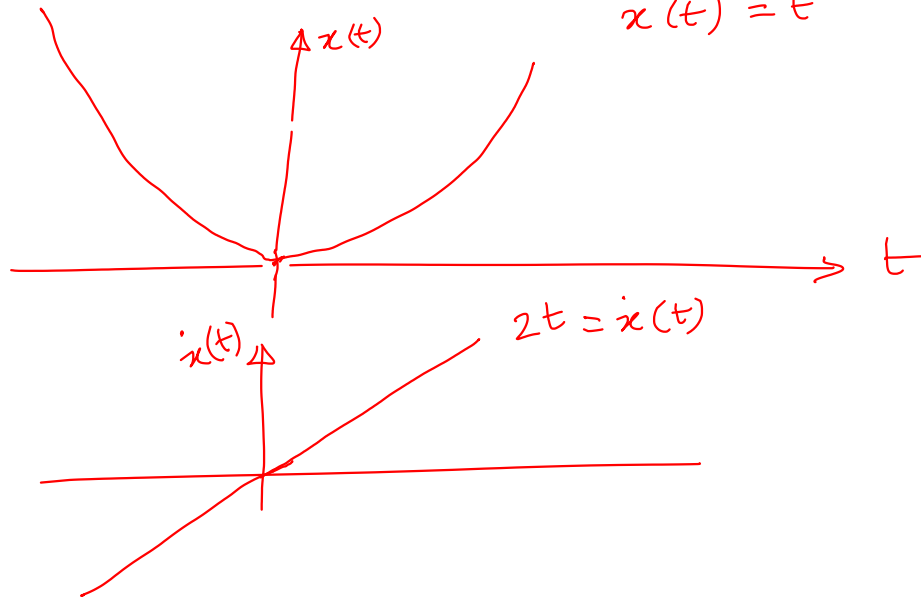
$$\int_{0^-}^{0^+} \delta(t) dt = 1$$

$$\frac{dy}{dt} + ay = \delta(t) = x(t)$$

$$y(t) = A e^{-at} + K_1 x(t) + K_2 \ddot{x}(t) + \dots \infty$$

$$K_1 \delta(t) + K_2 \delta(t) + \dots$$

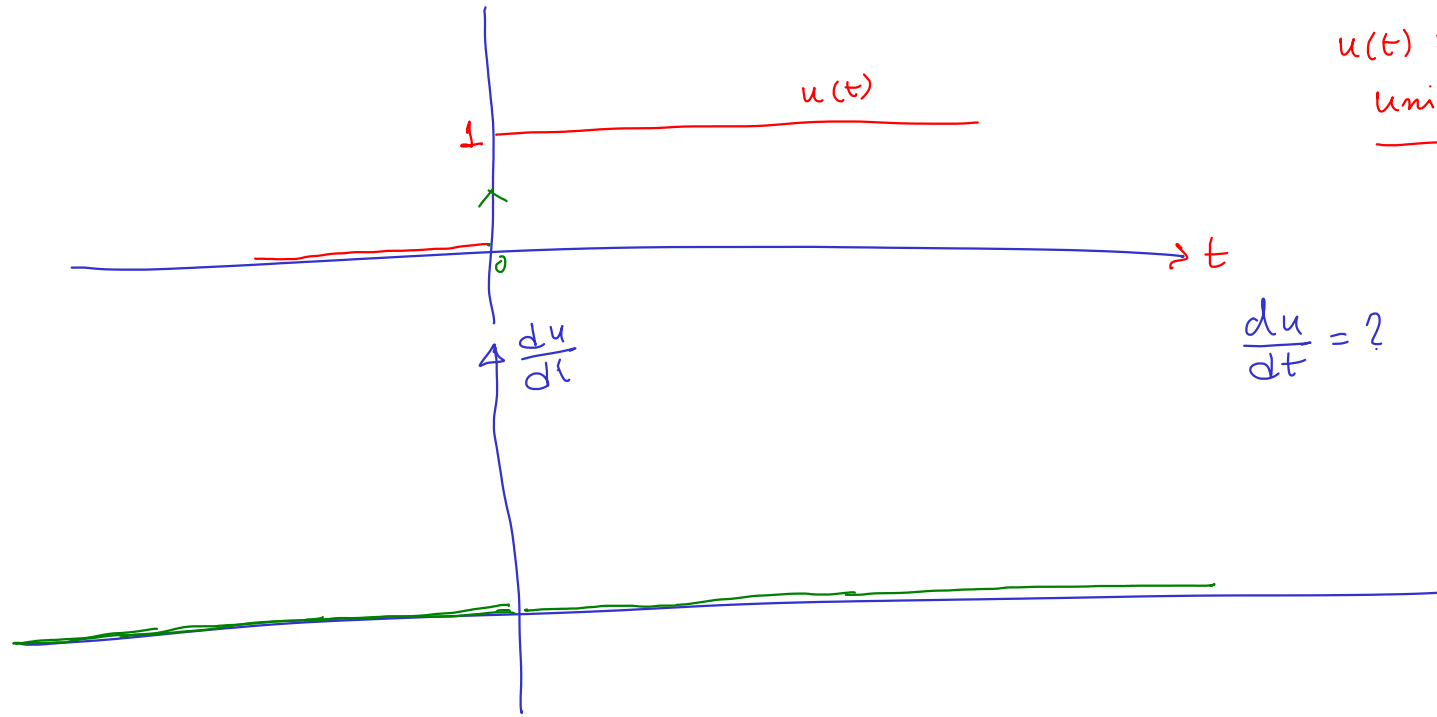
$$x(t) = t^2$$



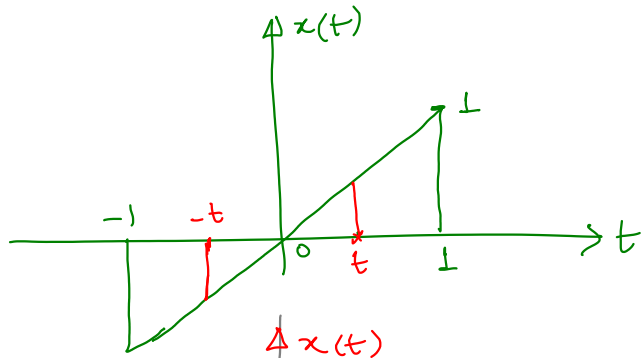
$$u(t) = 0 \quad \text{for } t < 0$$

$$u(t) = 1 \quad \text{for } t > 0$$

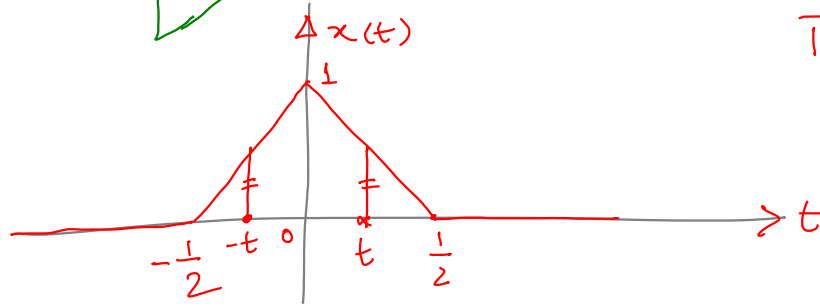
unit step f.?



$$\frac{du}{dt} = ?$$

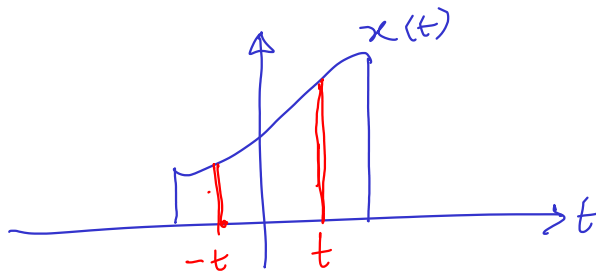


$\therefore \boxed{x(t) = -x(-t)}$  odd f.n.



This  $x(t)$  is an even f.n.  
why? because

$\boxed{x(t) = x(-t)}$



This  $x(t)$  is neither odd nor even.

Any general  $f^n$   $x(t)$  can be expressed as sum of an even  $f^n$  and an odd  $f^n$ .

$$* x(t) = x_{\text{even}}(t) + x_{\text{odd}}(t) \text{ --- (1)}$$

$$x_{\text{odd}}(-t) = -x_{\text{odd}}(t)$$

$t \rightarrow -t$

$$\therefore x(-t) = x_{\text{even}}(-t) + x_{\text{odd}}(-t)$$

$$\text{or } * x(-t) = x_{\text{even}}(t) - x_{\text{odd}}(t) \text{ --- (2)}$$

$$\textcircled{1} + \textcircled{2}$$

gives you

$$x_{\text{even}}(t) = \frac{x(t) + x(-t)}{2}$$

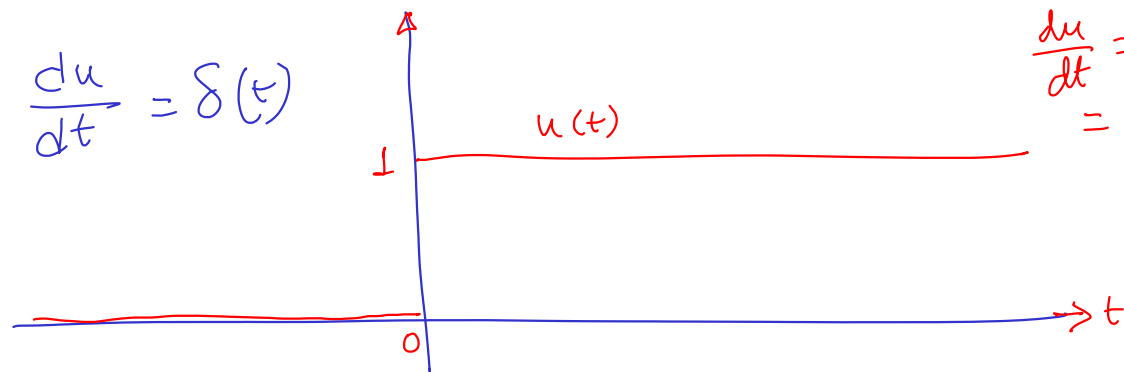
$$\textcircled{1} - \textcircled{2}$$



and

$$x_{\text{odd}}(t) = \frac{x(t) - x(-t)}{2}$$

$$\frac{du}{dt} = \delta(t)$$



$$\frac{du}{dt} = ?$$

$$= \frac{du_{\text{odd}}}{dt}$$

$$u_{\text{even}}(t) = \frac{u(t) + u(-t)}{2}$$

$$u_{\text{even}}(t) = \frac{1}{2}$$

$$u_{\text{odd}}(t) = \frac{u(t) - u(-t)}{2}$$

=

$$u(t) = u_{\text{even}}(t) + u_{\text{odd}}(t)$$

$$\therefore \frac{du}{dt} = \frac{d}{dt}(u_{\text{even}}) + \frac{d}{dt}(u_{\text{odd}})$$

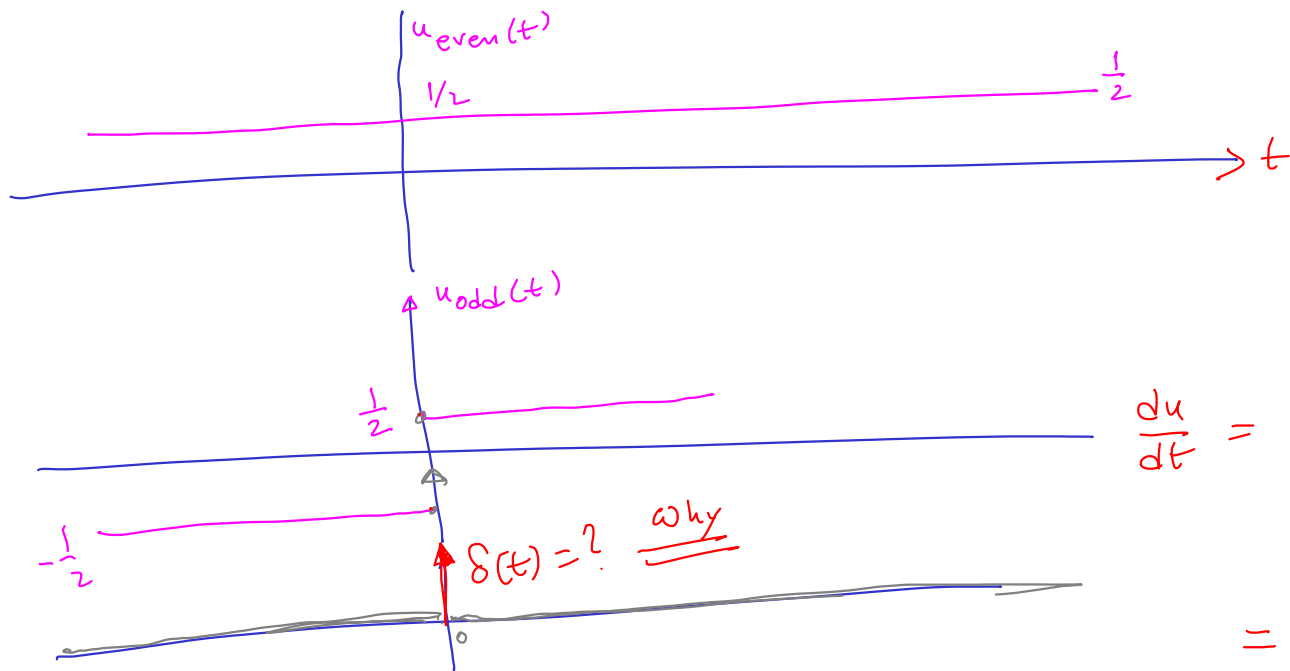
$$\text{if this is true} = 0 + \frac{d}{dt}(u_{\text{odd}})$$

$$\frac{du}{dt} = \frac{d}{dt}(u_{\text{odd}}(t)) = \delta(t) ?$$

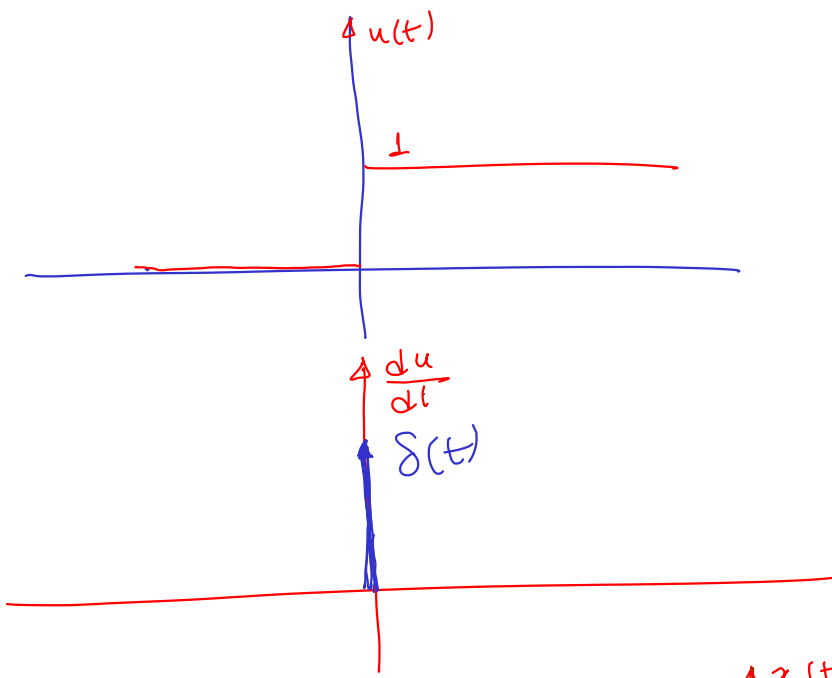
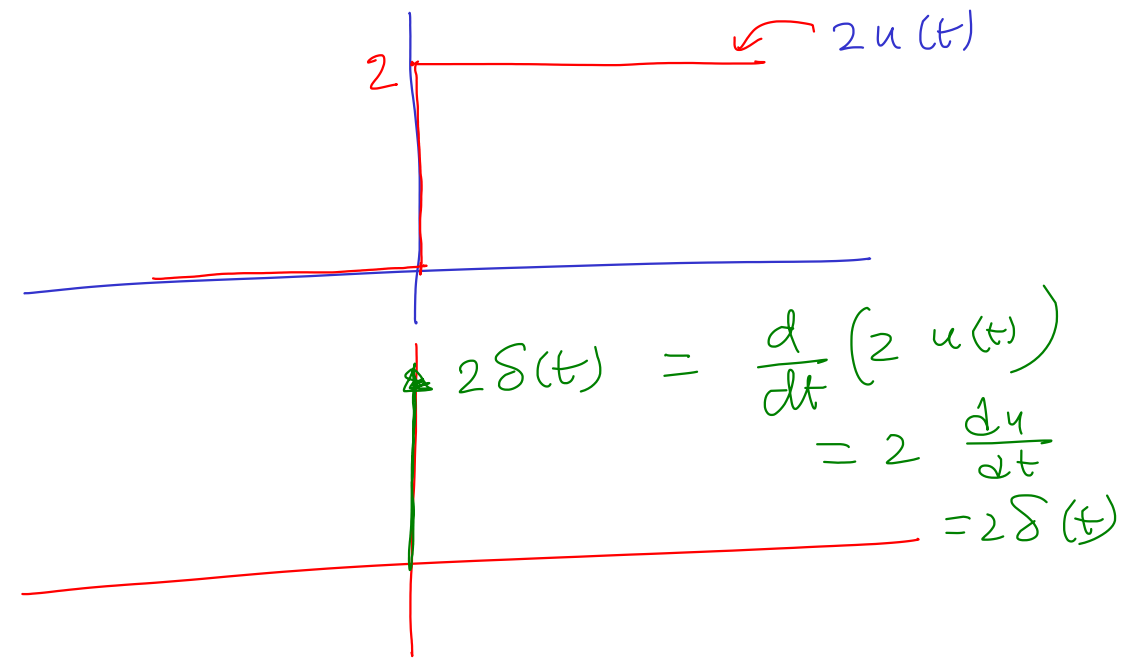
$$\int_{0^-}^{0^+} d(u_{\text{odd}}(t)) = \int_{0^-}^{0^+} \delta(t) dt$$

$$= u_{\text{odd}}(0^+) - u_{\text{odd}}(0^-) = 1$$

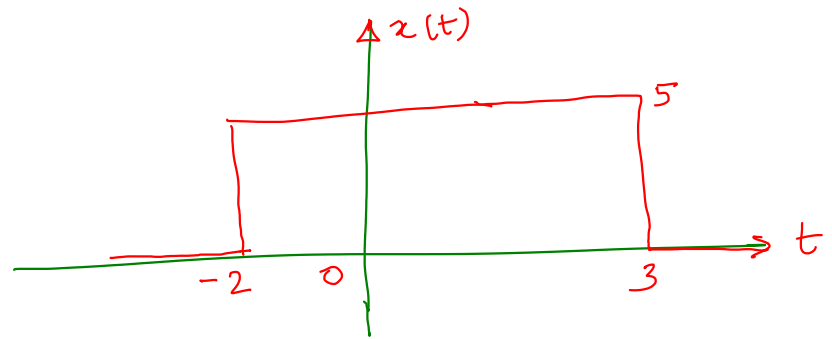
$$\frac{1}{2} - (-\frac{1}{2}) = 1 = 1$$

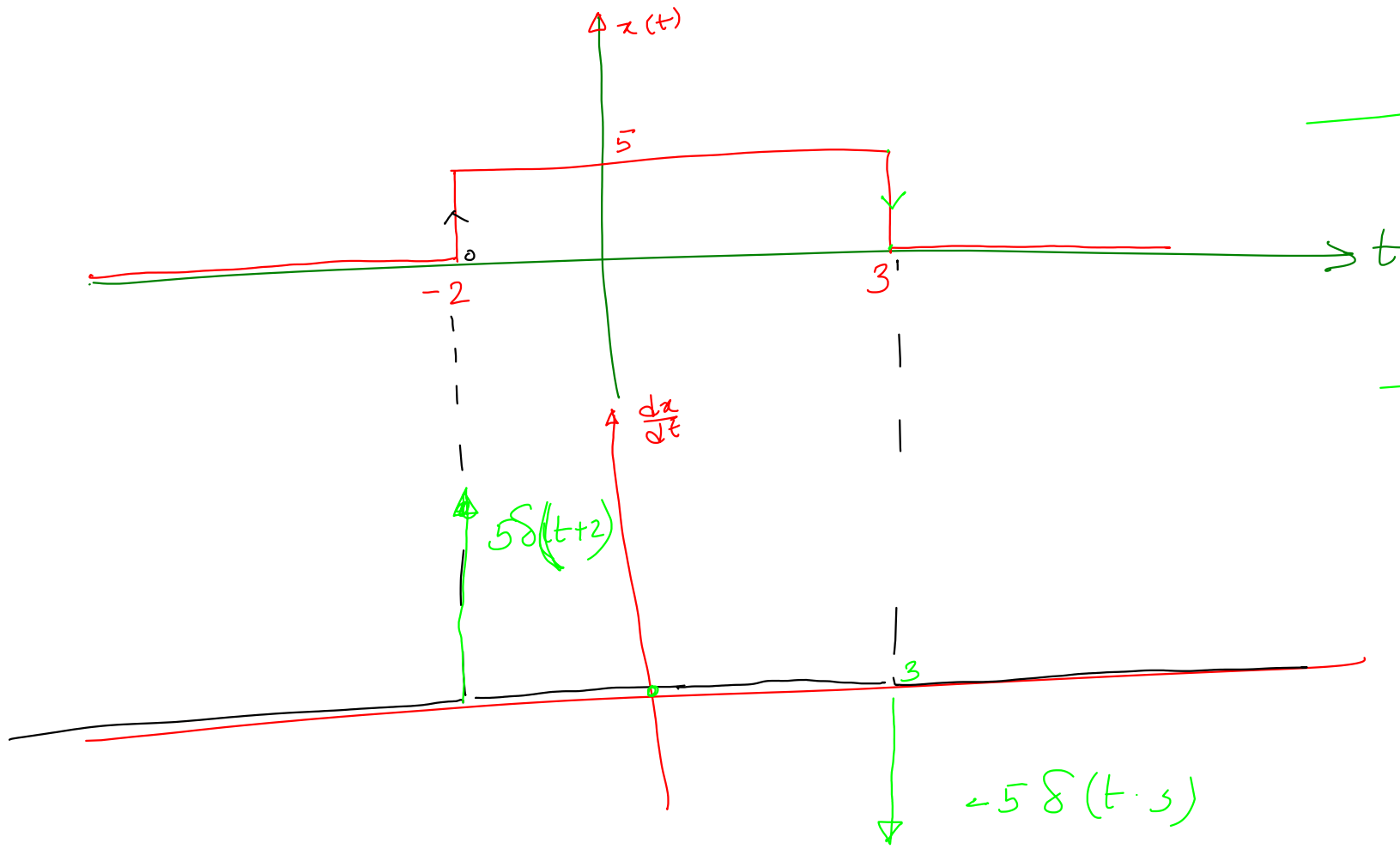


$$\delta(t) = ? \quad \text{why}$$

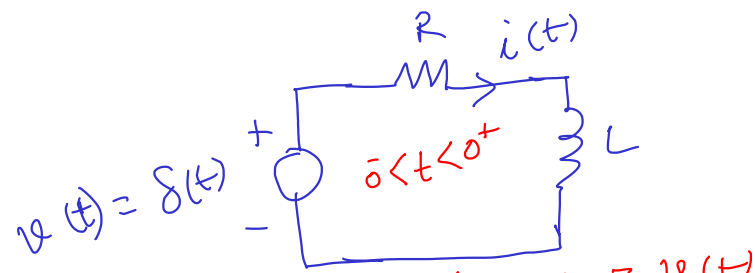
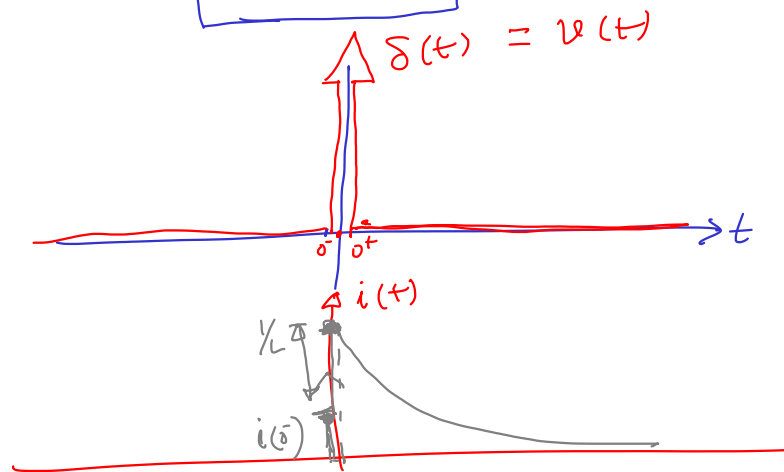


$$\begin{aligned} 2\delta(t) &= \frac{d}{dt} (2u(t)) \\ &= 2 \frac{du}{dt} \\ &= 2\delta(t) \end{aligned}$$

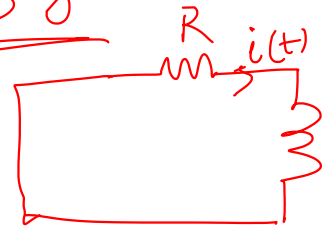




To find the impulse response



Circuit for  $t > 0^+$



$i(t) = i(0^+) e^{-\frac{R}{L}t}$

Let the  $i(0^-)$  be the initial current in the inductor.

$$\int_{0^-}^{0^+} \delta(t) dt = 1$$

$$Ri + L \frac{di}{dt} = \delta(t) \quad ?$$

$$\int_{0^-}^{0^+} Ri dt + \int_{0^-}^{0^+} L di = \int_{0^-}^{0^+} \delta(t) dt = 1$$

$$\text{or } R \int_{0^-}^{0^+} i dt + L \int_{0^-}^{0^+} di = 1$$

$$+ L [i(0^+) - i(0^-)] = 1$$

$$\text{or } i(0^+) - i(0^-) = \frac{1}{L}$$

$$\text{or } i(0^+) = i(0^-) + \frac{1}{L}$$

$$\begin{aligned} \int_{0^-}^{0^+} F dt &= \int_{0^-}^{0^+} m \frac{dv}{dt} dt \\ &= m [v]_{0^-}^{0^+} = m \Delta v \end{aligned}$$

$i(\bar{o})$  ✗

$$R i + L \frac{di}{dt} = \delta(t)$$

~~$= 0$  for  $t > 0$~~

$\frac{dy}{dt}$  

X  $y = ?$   
 $\frac{dy}{dt} = ?$   
 $\frac{d^2y}{dt^2} = ?$   
which of these will have step change?

$$\frac{d^2y}{dt^2} + a \frac{dy}{dt} + by = \delta(t)$$

$= 0$  for  $t > 0$

$y(\bar{o})$  ✓ ✗  
 $\frac{dy}{dt}(\bar{o})$  ✓ ✓

$$\frac{d^2y}{dt^2} dt + a \frac{dy}{dt} dt + by dt = \delta(t) \cdot dt$$

$$\text{or } \frac{d}{dt} \left( \frac{dy}{dt} \right) dt + a \frac{dy}{dt} dt + by dt = \delta(t) dt$$
$$\text{or } \int_{0^-}^{0^+} d\left(\frac{dy}{dt}\right) + a \int_{0^-}^{0^+} dy + b \int_{0^-}^{0^+} y dt = \int_{0^-}^{0^+} \delta(t) dt$$

$y(0^+) = y(0^-)$  ✓

$$\left\{ \frac{dy}{dt}(0^+) - \frac{dy}{dt}(0^-) \right\} + a \{ y(0^+) - y(0^-) \} + 0 = 1$$

$$\frac{dy}{dt}(0^+) - \frac{dy}{dt}(0^-) = 1$$

will have a step change

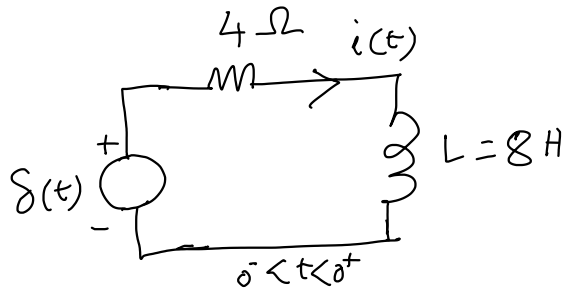
$$\frac{dy}{dt}(0^+) = 1 + \frac{dy}{dt}(0^-)$$

for  $t > 0^+$

$$\frac{d^2y}{dt^2} + a \frac{dy}{dt} + by = 0$$

$y(0^+) = y(0^-)$  ✗  
 $\frac{dy}{dt}(0^+) = 1 + \frac{dy}{dt}(0^-)$

# Example-1



$$4i + L \frac{di}{dt} = \delta(t)$$

for  $t > 0^+$

$$4i + 8 \frac{di}{dt} = 0$$

I must know  $i(0^+)$  ✓

$$i(0^+) = i(0^-) + \frac{1}{L}$$

$$i(0^+) = 1 + \frac{1}{8} = 1.125 \text{ A}$$

$$\therefore A = 1.125$$

$$\frac{di}{dt} + \frac{1}{2}i = 0$$

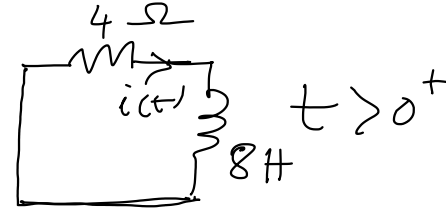
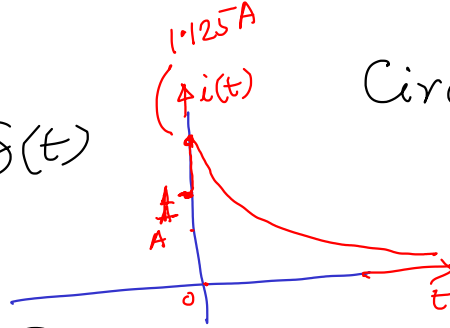
$$i(t) = A e^{-\frac{1}{2}t}$$

$$\text{at } t = 0^+ = 1.125$$

$$i(t) = ? \text{ for } t > 0 \text{ or } t > 0^+$$

$$i(0^-) = 1 \text{ A}$$

Circuit at  $t > 0^+$



$$i(0^-) \rightarrow i(0^+)$$

$$1 \text{ A} \rightarrow \underline{1.125}$$

This step jump will take place at  $t = \underline{0}$

$$\therefore i(t) = 1.125 e^{-\frac{1}{2}t}$$



$$\frac{d^2 v}{dt^2} + 5 \frac{dv}{dt} + 6v = \delta(t)$$

Solve for  $v(t)$  for  $\underline{t > 0^+}$

B.C given  $v(0^-)$  ✓  
 $\frac{dv}{dt}(0^-)$  ✓ } are known

which will  
 have step jump?  
 it is  $\left(\frac{dv}{dt}\right) \uparrow$

$$\frac{d}{dt} \left( \frac{dv}{dt} \right) + 5 \frac{dv}{dt} + 6v = \delta(t)$$

$$\int_{0^-}^{0^+} d\left(\frac{dv}{dt}\right) + 5 \int_{0^-}^{0^+} dv + 6 \int_{0^-}^{0^+} v dt = \int_{0^-}^{0^+} \delta(t) dt$$

$$\left\{ \frac{dv}{dt}(0^+) - \frac{dv}{dt}(0^-) \right\} + 5(v(0^+) - v(0^-)) + 6 \int_{0^-}^{0^+} v dt = 1$$

$\downarrow$   
 $v(0^+) = v(0^-)$

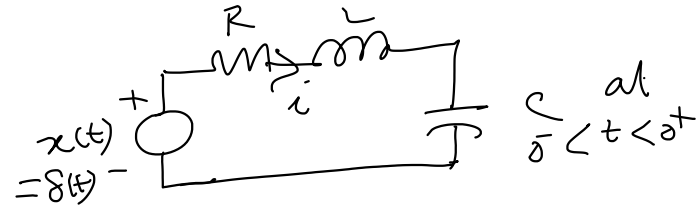
$$\frac{dv}{dt}(0^+) - \frac{dv}{dt}(0^-) = 1$$

Series R, L, C

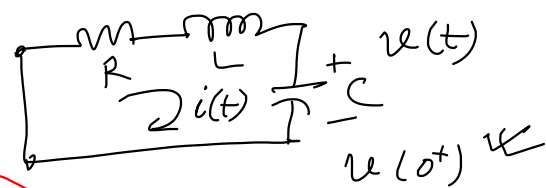
$$R C \frac{dv}{dt} + L \frac{d}{dt} \left( C \frac{dv}{dt} \right) + v = x(t)$$

$$L C \frac{d^2 v}{dt^2} + R C \frac{dv}{dt} + v = x(t)$$

$$\frac{d^2 v}{dt^2} + \frac{R}{L} \frac{dv}{dt} + \frac{1}{LC} v = x(t)$$

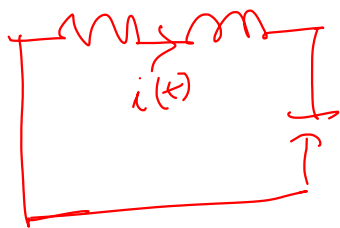
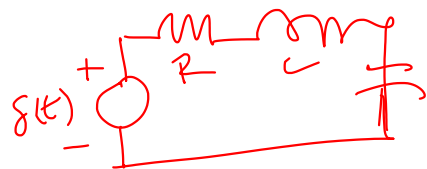


for  $t > 0^+$

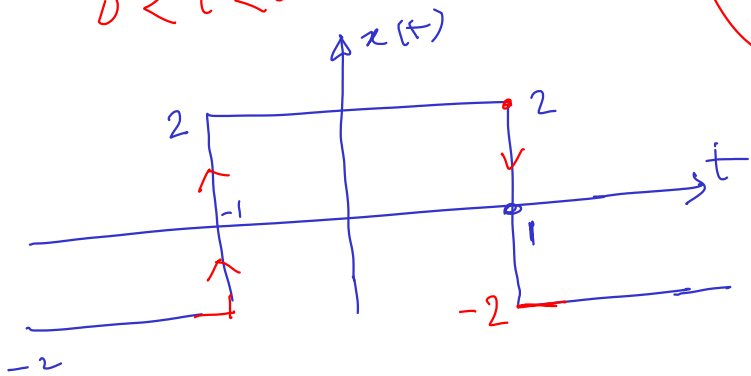


$$\frac{dv}{dt}(0^+) = 1 + \left( \frac{dv}{dt}(0^-) \right)$$

for  $t > 0^+$



$0^- < t < 0^+$



$$\frac{d^2 v}{dt^2} + 5 \frac{dv}{dt} + 6v = 0$$

Ch. equ<sup>n</sup>:  $(m+3)(m+2) = 0$

$$m_1, m_2 = -3, -2$$

$$v(t) = A e^{-3t} + B e^{-2t}$$

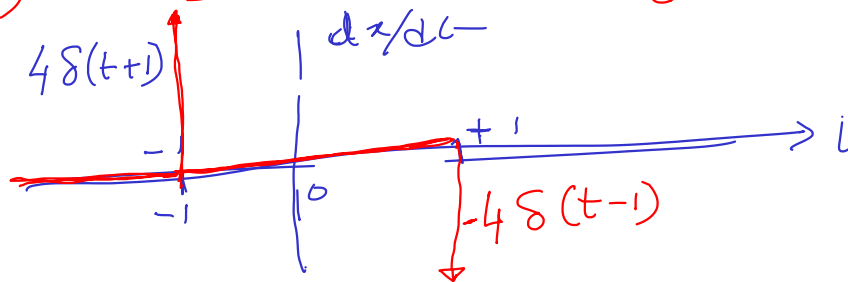
valid for  $t > 0^+$

$$v(0^+) = v(0^-) = A + B$$

$$\frac{dv}{dt} = -3A e^{-3t} - 2B e^{-2t}$$

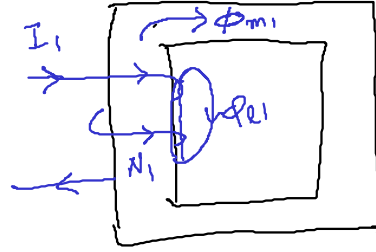
Solve for A & B

$$\frac{dv}{dt}(0^+) = -3A - 2B = 1 + \frac{dv}{dt}(0^-)$$



# Self & Mutual Inductances : coupled coils

Dec-37



$\Phi_{t1}$  = total flux created by coil 1 when it carries  $I_1$ .



$$\Phi_{t1} = \Phi_{m1} + \Phi_{e1}$$

$\uparrow$   
 mutual flux in the core

$\rightarrow$  leakage flux

$$\Phi_{e1} = \bar{K}_1 \Phi_{t1}$$

$\bar{K}_1$  is small

Self inductance of coil 1

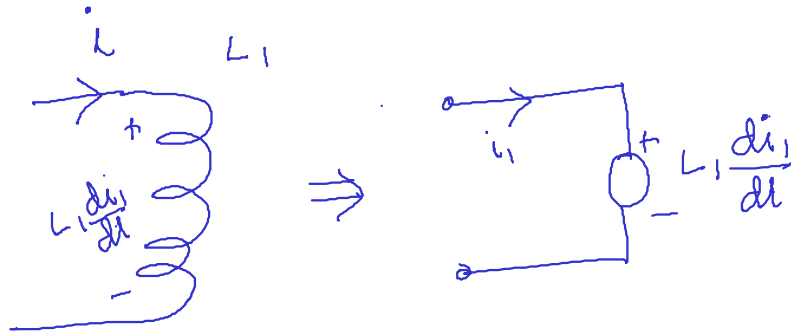
$$L_1 = \frac{N_1 \Phi_{t1}}{I_1}$$

$$= \frac{N_1 \Phi_{m1} + N_1 \Phi_{e1}}{I_1}$$

$$L_1 = \frac{N_1 \Phi_{m1}}{I_1} + \frac{N_1 \Phi_{e1}}{I_1} = L_{m1} + L_{e1}$$

$\circ$   
 $L_1 = L_m + L_e$

$\rightarrow$  magnetising inductance  
 $\rightarrow$  leakage inductance (small)  
 $L_{m1} \gg L_{e1}$

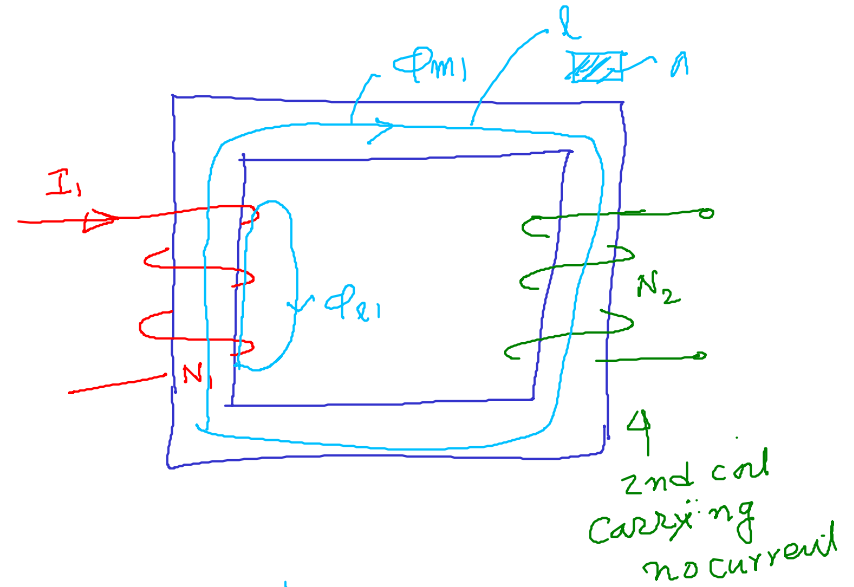


Mutual inductance

$$M_{21} = \frac{\text{flux linkage with the 2nd coil}}{I_1}$$

$$M_{21} = \frac{N_2 \Phi_{m1}}{I_1} = \frac{N_2}{I_1} \frac{N_1 I_1}{\mathcal{R}}$$

$$M_{21} = \frac{N_1 N_2}{\mathcal{R}} = M_{12}$$



Note

$$H_1 = \frac{N_1 I_1}{l}$$

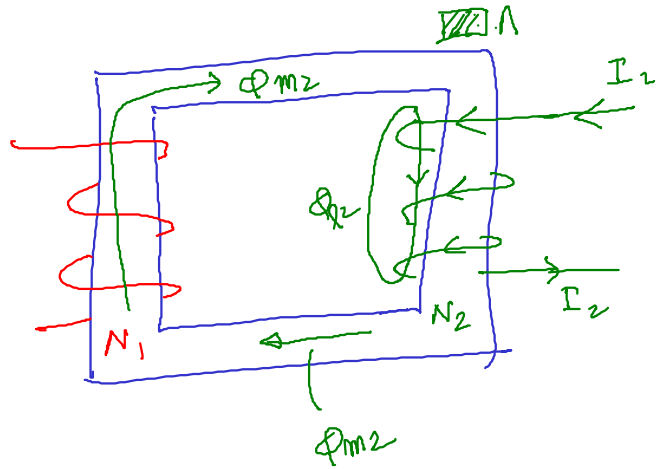
$$B_{m1} = \mu_0 \mu_r \frac{N_1 I_1}{l}$$

$$\Phi_{m1} = B_{m1} A$$

$$= \mu_0 \mu_r \frac{N_1 I_1}{l} A = \frac{N_1 I_1}{\left( \frac{l}{\mu_0 \mu_r A} \right)}$$

$$\Phi_{m1} = \frac{N_1 I_1}{\mathcal{R}} \quad \text{Reluctance}$$

Now excite The 2nd coil alone with  $I_2$



$$\Phi_{t2} = \Phi_{m2} + \Phi_{l2}$$

$$L_2 = \frac{N_2 \Phi_{t2}}{I_2} = \frac{N_2 \Phi_{m2}}{I_2} + \frac{N_2 \Phi_{l2}}{I_2}$$

$$L_2 = L_{m2} + L_{l2}$$

$$L_{m2} \gg L_{l2}$$

$M =$  The mutual inductance

$$M_{12} = \frac{N_1 \Phi_{m2}}{I_2}$$

$$= \frac{N_1}{I_2} \times \frac{\mu_0 \mu_r}{\ell} \frac{N_2 I_2}{\ell} A$$

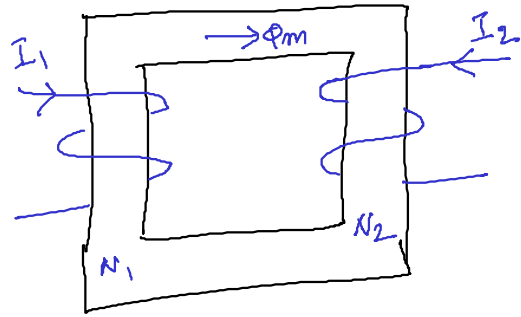
$$= \frac{N_1 N_2}{\ell} \frac{\mu_0 \mu_r A}{\ell} = \frac{N_1 N_2}{\left( \frac{\ell}{\mu_0 \mu_r A} \right)} \quad \text{Reluctance}$$

$$M_{12} = \frac{N_1 N_2}{\mathcal{R}} = M_{21} = M$$

$$\Phi_{m2} = B_{m2} A$$

$$= \mu_0 \mu_r H A$$

$$= \mu_0 \mu_r \frac{N_2 I_2}{\ell} A$$



If  $I_1$  &  $I_2$  are constants

$$M_{12} = M_{21} = M$$

$$L_1 = \frac{\Phi_{L1} N_1}{I_1}$$

$$= \frac{(\Phi_{m1} + \Phi_{L1}) N_1}{I_1}$$

$$L_1 = L_{m1} + L_{L1}$$

$$L_2 = \frac{\Phi_{L2} N_2}{I_2}$$

$$= \frac{(\Phi_{m2} + \Phi_{L2}) N_2}{I_2}$$

$$= L_{m2} + L_{L2}$$

$$N_1 \Phi_{L1} = L_{L1} I_1 = N_1 \Phi_{L1}$$

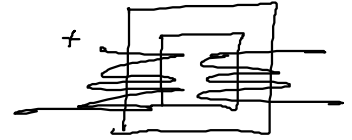
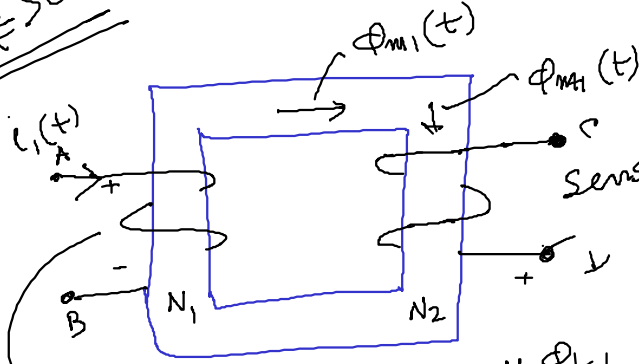
$$\textcircled{M I_1} = N_2 \Phi_{m1}$$

$$\frac{N_2 \Phi_{m1}}{I_1} = M$$

# When coils carry time varying current

$\frac{di_1}{dt} > 0$

$\phi_{m1}$  &  $i_1$  in phase



Sense of the coils are known

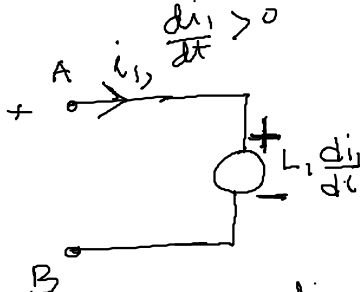
$$N_2 \frac{d}{dt} (\phi_{m1}(t)) = \frac{d}{dt} (N_2 \phi_{m1})$$

$$= \frac{d}{dt} (M i_1) = M \frac{di_1}{dt}$$

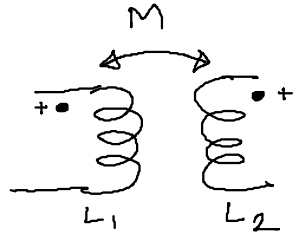
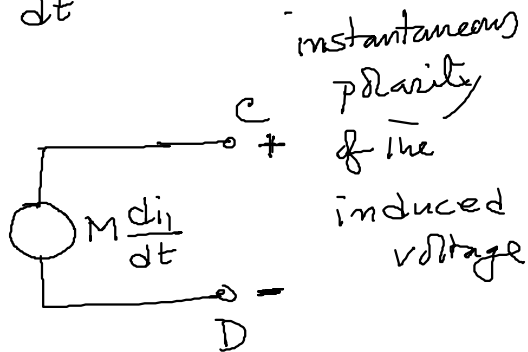
$$e_1 = \frac{d}{dt} (L_1 i_1)$$

$$= L_1 \frac{di_1}{dt}$$

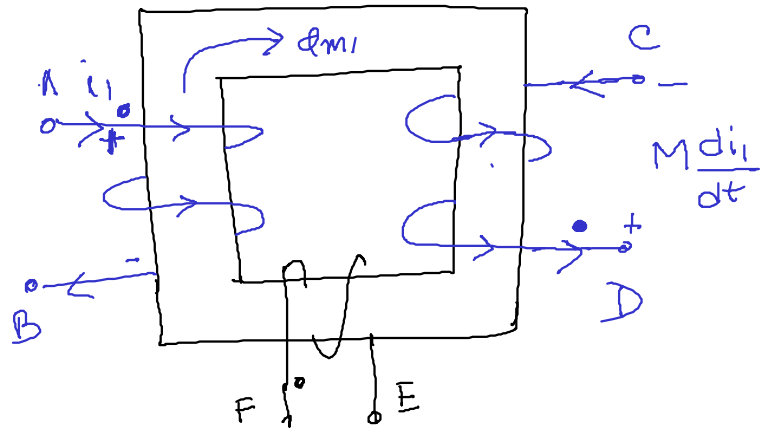
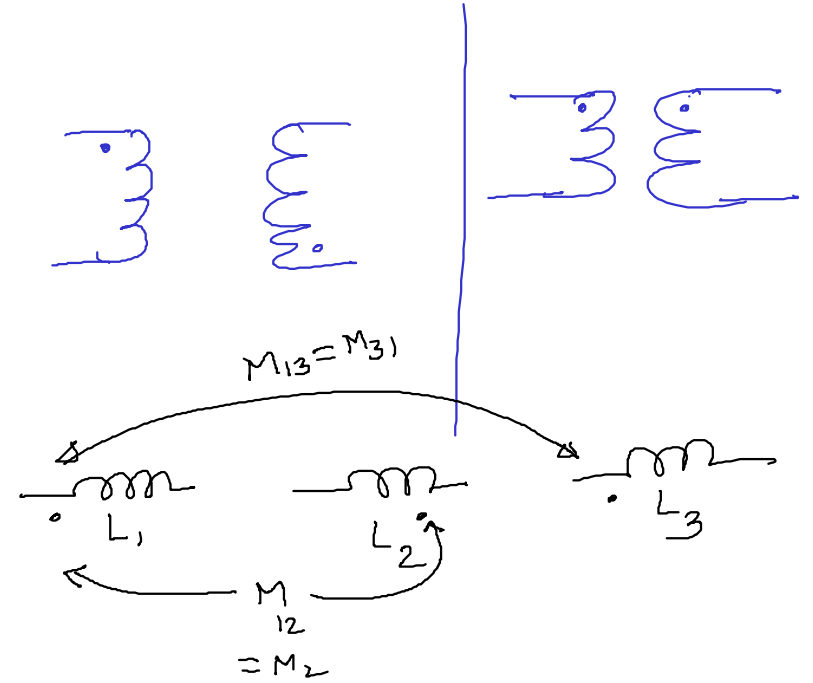
Dot Convention



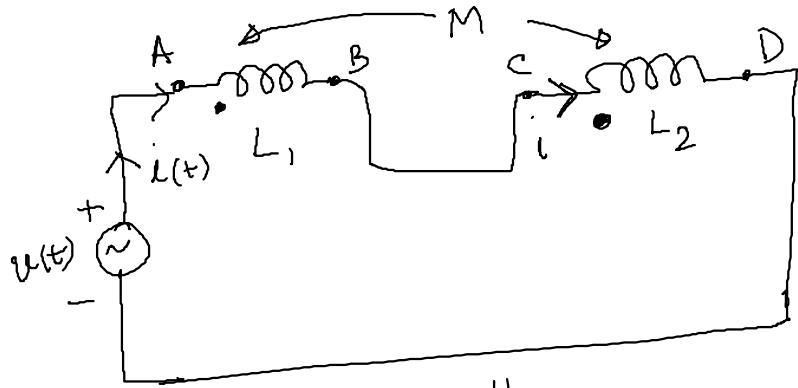
$$V_{AB} = L_1 \frac{di_1}{dt}$$



instantaneous polarity of the induced voltage

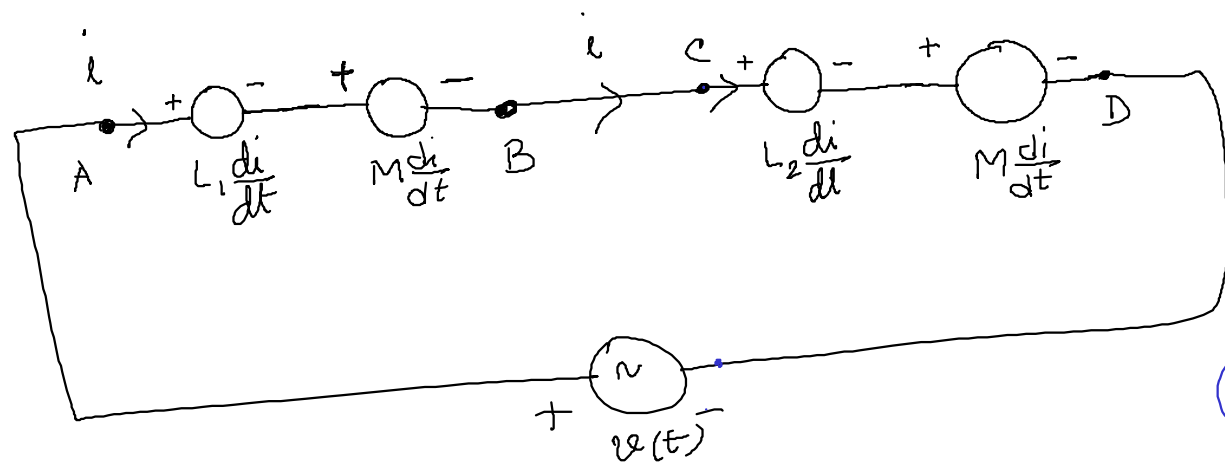


lec-39



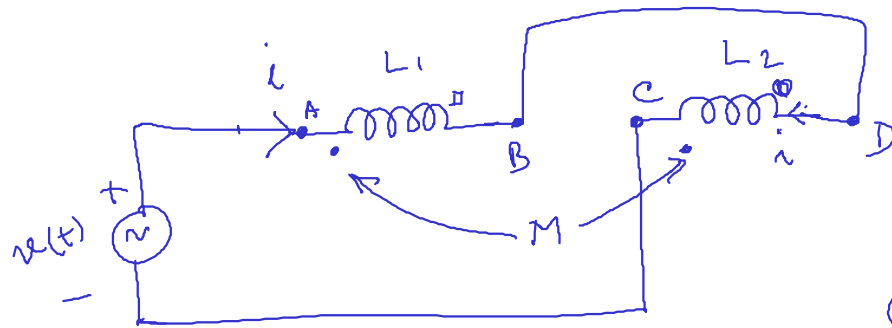
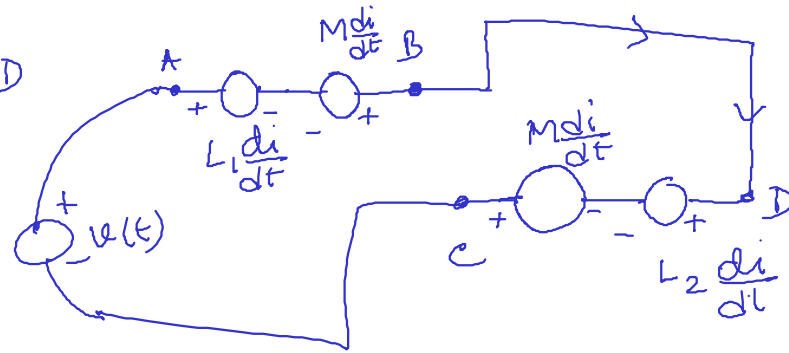
$$v(t) = \left( L_1 \frac{di}{dt} + M \frac{di}{dt} \right)_{AB} + \left( L_2 \frac{di}{dt} + M \frac{di}{dt} \right)_{CD}$$

$$= (L_1 + L_2 + 2M) \frac{di}{dt}$$



$$v(t) = (L_1 + L_2 + 2M) \frac{di}{dt}$$

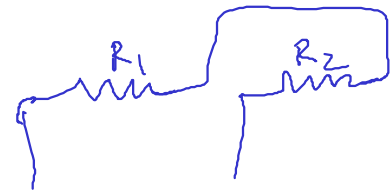
$$\therefore L_{eq} = L_1 + L_2 + 2M$$

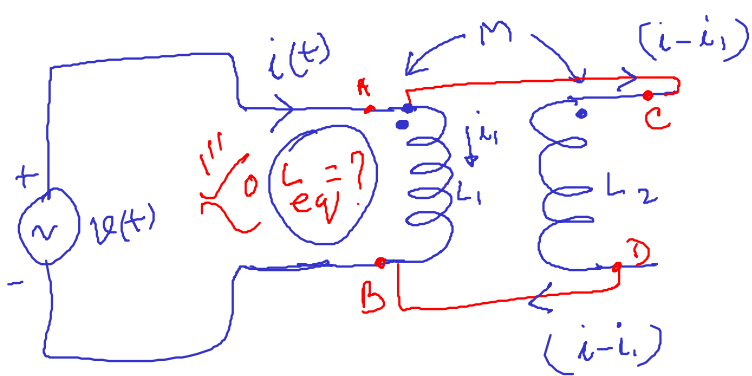
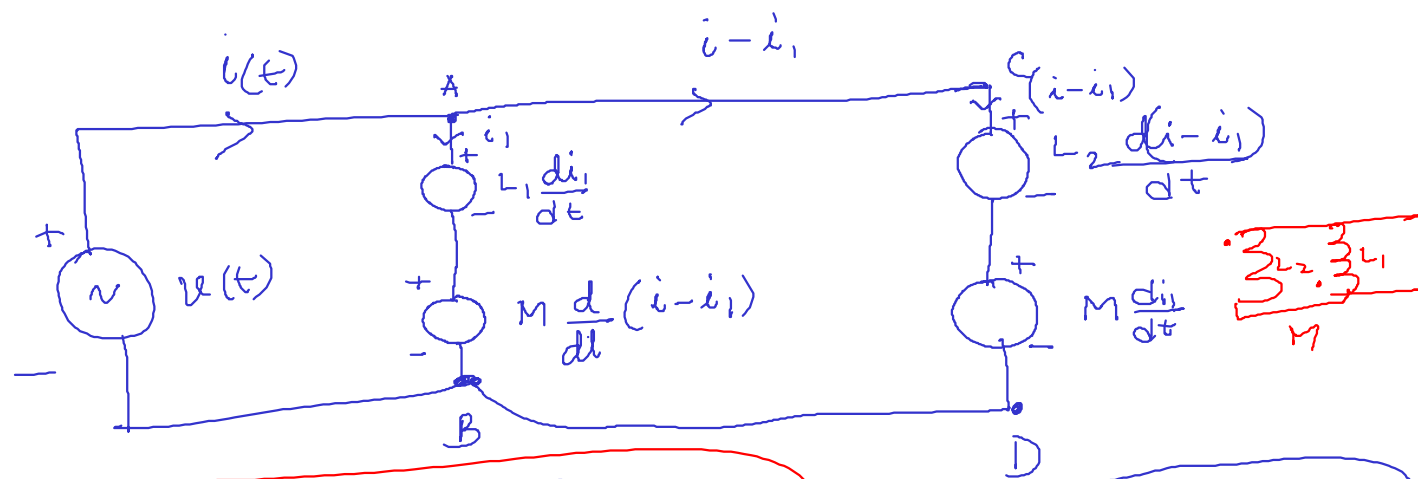


$$v(t) - L_1 \frac{di}{dt} + M \frac{di}{dt} - L_2 \frac{di}{dt} + M \frac{di}{dt} = 0$$

$$\therefore v(t) = (L_1 + L_2 - 2M) \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 - 2M$$





$$v(t) = \left( \text{Constant} \right) \frac{di}{dt}$$

$L_{eq}$

$$v(t) = L_1 \frac{di_1}{dt} + M \frac{d(i-i_1)}{dt}$$

try to eliminate  $i_1$

also  $v(t) = L_2 \frac{d(i-i_1)}{dt} + M \frac{di_1}{dt}$

Equating the RHS

$$L_1 \frac{di_1}{dt} + M \frac{d(i-i_1)}{dt} = L_2 \frac{d(i-i_1)}{dt} + M \frac{di_1}{dt}$$

$$\left( L_1 \frac{di_1}{dt} - M \frac{di_1}{dt} + L_2 \frac{di_1}{dt} - M \frac{di_1}{dt} \right) = L_2 \frac{di}{dt} - M \frac{di}{dt}$$

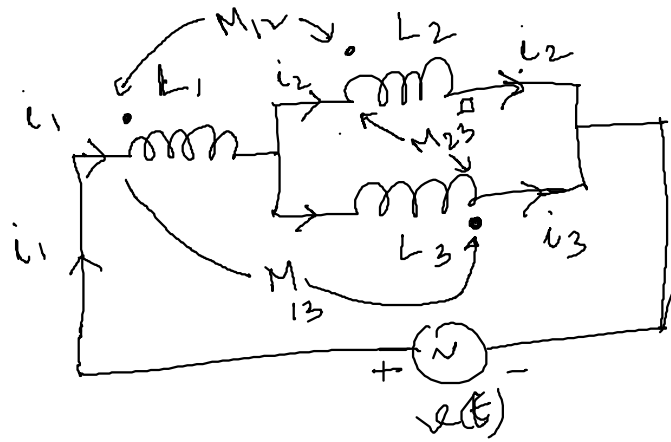
$$\text{or } (L_1 + L_2 - 2M) \frac{di_1}{dt} = (L_2 - M) \frac{di}{dt} \quad \therefore \frac{di_1}{dt} = \left( \frac{L_2 - M}{L_1 + L_2 - 2M} \right) \frac{di}{dt}$$

$$v(t) = \frac{L_1(L_2 - M)}{(L_1 + L_2 - M)} \frac{di}{dt}$$

$$+ M \frac{di}{dt} - M \frac{di}{dt}$$

$L_{eq} = ?$

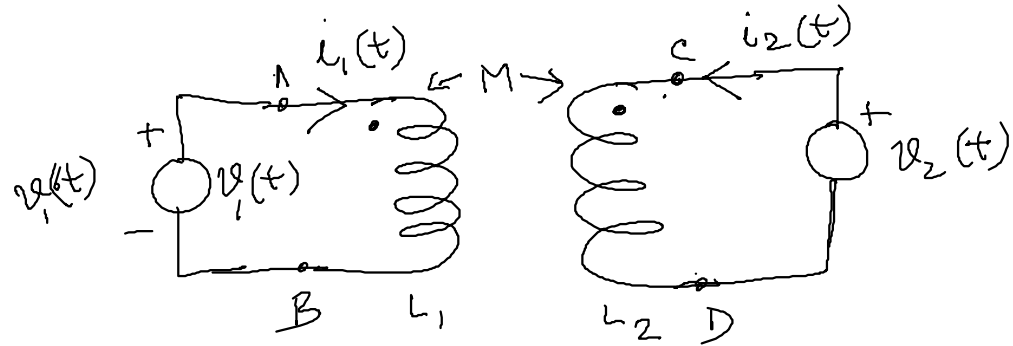
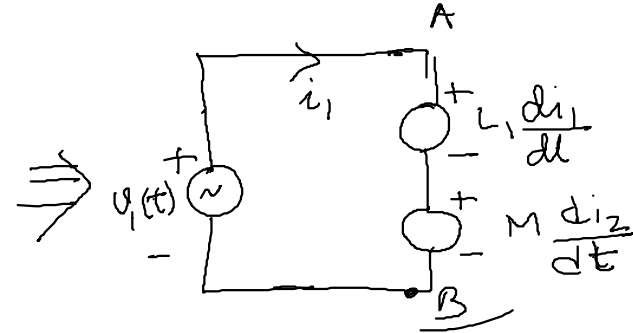
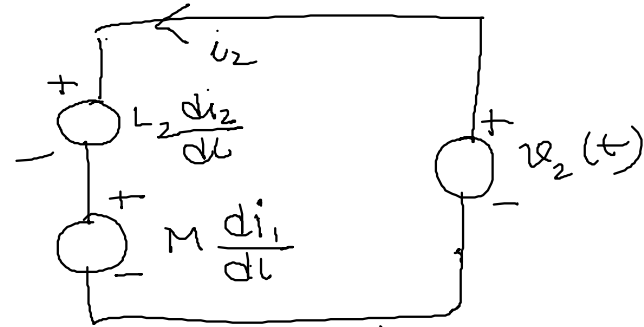
lec 40



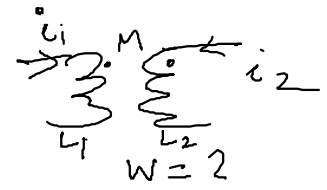
$$\begin{aligned} & L_2 \frac{di_2}{dt} + M_{12} \frac{di_1}{dt} + M_{23} \frac{d(-i_3)}{dt} \\ & + L_3 \frac{d}{dt}(-i_3) + M_{13} \frac{di_1}{dt} \\ & + M_{23} \frac{di_2}{dt} \end{aligned}$$

$$\begin{aligned} & \left( + L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} - M_{13} \frac{di_3}{dt} \right) \\ & + \left( L_2 \frac{di_2}{dt} + M_{12} \frac{di_1}{dt} - M_{23} \frac{di_3}{dt} \right) = v(t) \end{aligned}$$
$$\begin{aligned} & L_2 \frac{di_2}{dt} + M_{12} \frac{di_1}{dt} - M_{23} \frac{di_3}{dt} \\ & - L_3 \frac{di_3}{dt} + M_{13} \frac{di_1}{dt} + M_{23} \frac{di_2}{dt} = 0 \end{aligned}$$

# Energy stored in mutually coupled coils



$$i_1 \rightarrow \left\{ \begin{array}{l} L_1 \\ L_2 \end{array} \right. \quad W = \frac{1}{2} L i^2$$



Instantaneous power absorbed by AB:-

$$p_{AB}(t) = \left( L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \right) i_1$$

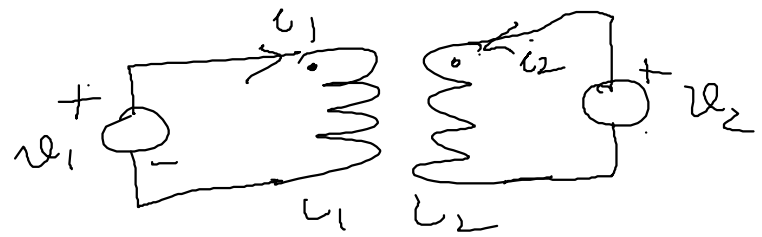
$$p_{CD}(t) = \left( L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \right) i_2$$

To find instantaneous power absorbed

$$p(t) = p_{AB} + p_{CD} = \left( L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \right) i_1 + \left( L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \right) i_2$$

total Elemental energy supplied to the coupled coils  
 ∴ Energy

$$dW = p(t) dt$$



$$= i_1 \left( L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \right) dt + i_2 \left( L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \right) dt$$

$$= L_1 i_1 \frac{di_1}{dt} dt + M i_1 di_2 + i_2 L_2 \frac{di_2}{dt} dt + i_2 M di_1 \quad M d(i_1 i_2)$$

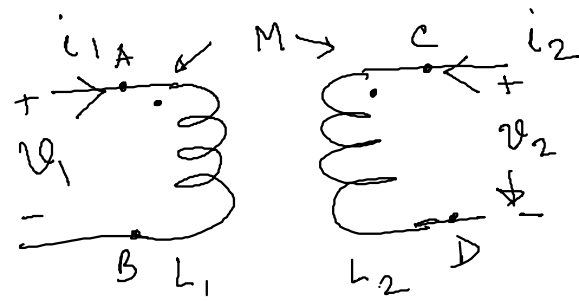
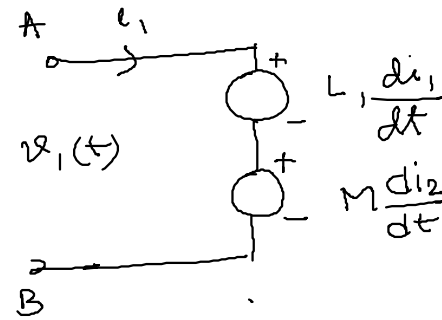
$$dW = L_1 i_1 di_1 + L_2 i_2 di_2 + M (i_1 di_2 + i_2 di_1)$$

total Energy absorbed by the system if  $i_1$  and  $i_2$  are brought to the final levels  $I_1$  &  $I_2$  from zero values.

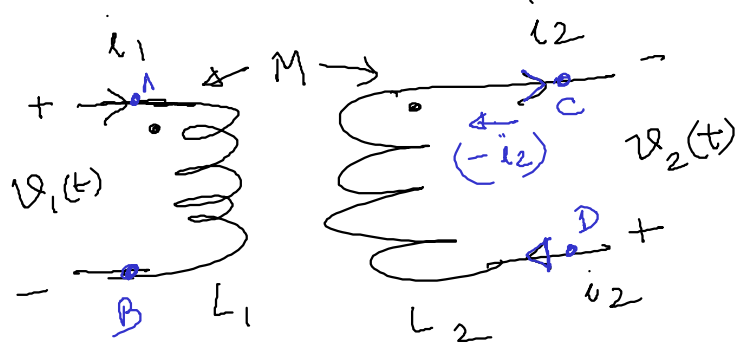
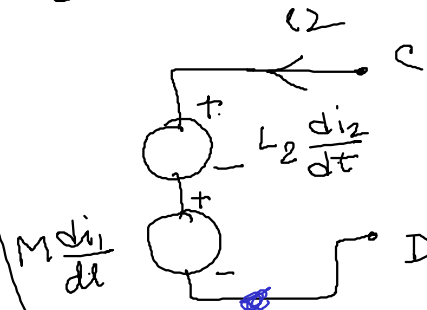
$$W = L_1 \int_0^{I_1} i_1 di_1 + L_2 \int_0^{I_2} i_2 di_2 + M \int_0^{I_1 I_2} d(i_1 i_2) = \left( \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2 \right)$$

$$W = \frac{1}{2} L_1 \dot{i}_1^2 + \frac{1}{2} L_2 \dot{i}_2^2 + M \dot{i}_1 \dot{i}_2$$

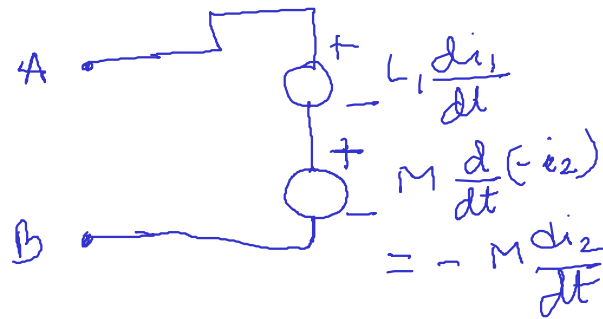
$$L_1 \frac{di_1}{dt} dt$$



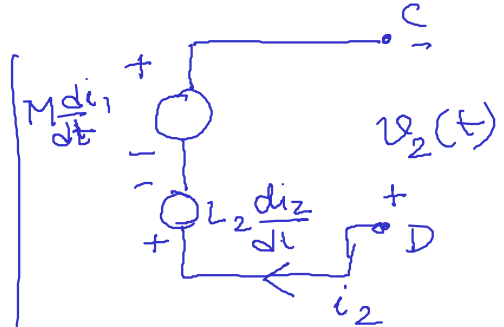
$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

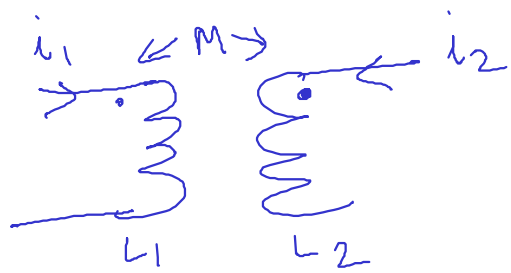


$$v_2(t) = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$



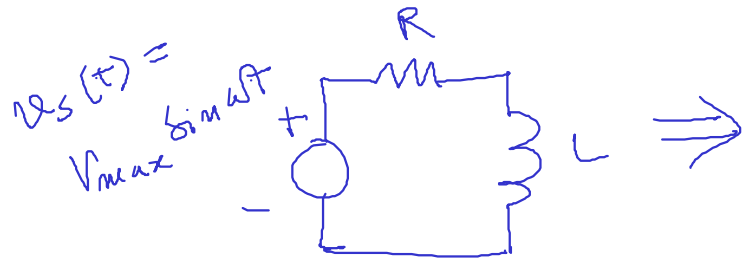
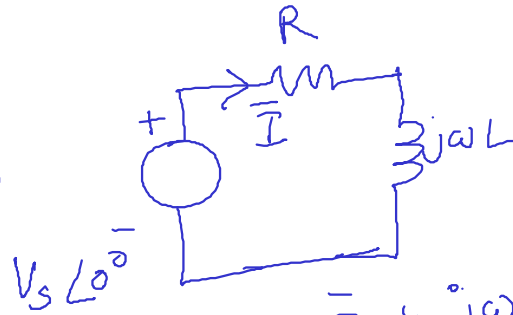
$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$





$$W = \frac{1}{2} L i_1^2 + \frac{1}{2} L i_2^2 + M i_1 i_2$$

What if we are interested to know steady state current when the excitations (or input) are sinusoidal in nature (i.e.,  $\sin \omega t$ )

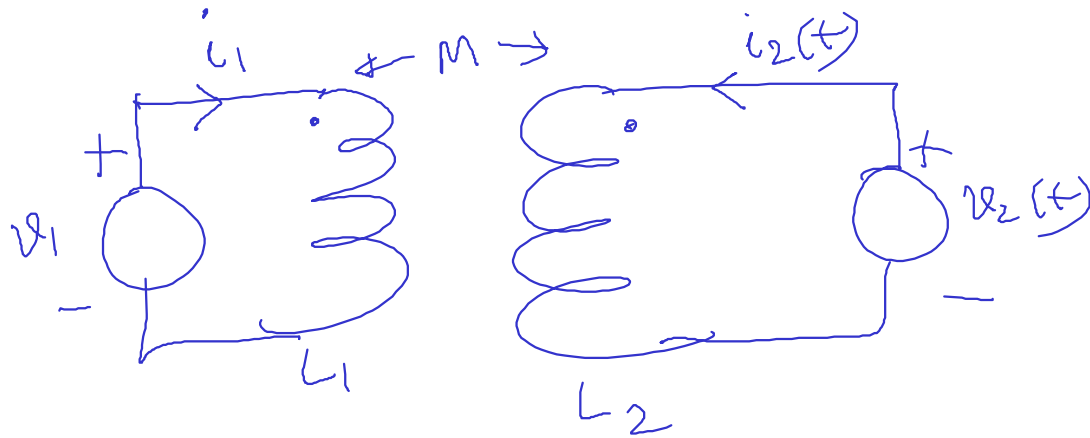
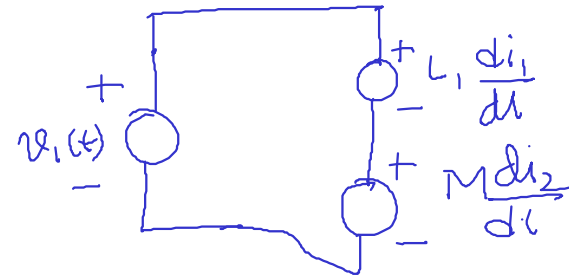


$$R i + L \frac{di}{dt} = v_s(t)$$

$$\frac{d}{dt} \equiv j\omega$$

$$R \bar{I} + j\omega L \bar{I} = \bar{V}_s$$

phasor



$$\frac{d}{dt} \equiv j\omega$$

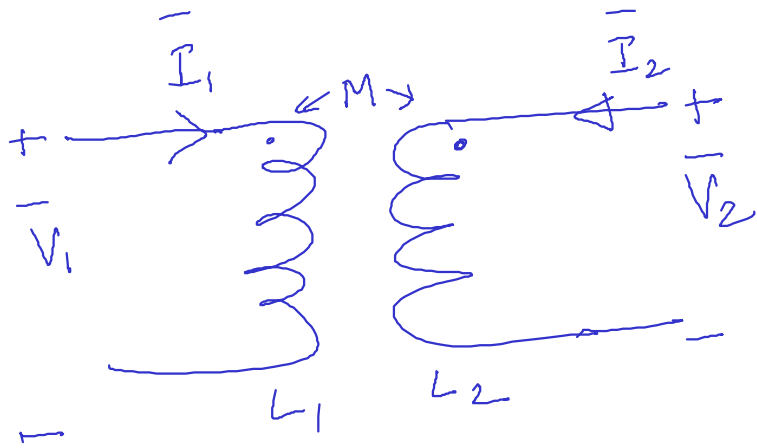
$$v_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2(t) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

if input sinusoidal then  
Replace  $\frac{d}{dt} \equiv j\omega$  & quantities in phasors.

$$\bar{V}_1 = j\omega L_1 \bar{I}_1 + j\omega M \bar{I}_2$$

$$\bar{V}_2 = j\omega L_2 \bar{I}_2 + j\omega M \bar{I}_1$$

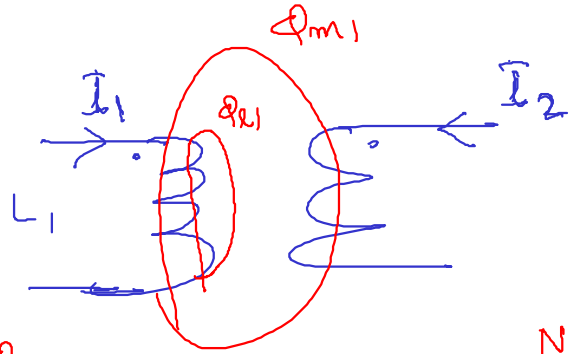


$$\bar{V}_1 = j\omega L_1 \bar{I}_1 + j\omega M \bar{I}_2$$

$$\bar{V}_2 = j\omega L_2 \bar{I}_2 + j\omega M \bar{I}_1$$

$$M = \frac{N_1 \Phi_{m2}}{I_2} = \frac{N_1}{I_2} (\Phi_{t2} - \bar{K}_2 \Phi_{t2})$$

$$= \frac{N_1 \Phi_{t2} (1 - \bar{K}_2)}{I_2}$$



$$L_1 = \frac{N_1 \Phi_{t1}}{I_1} = \frac{N_1}{I_1} (\Phi_{m1} + \Phi_{l1})$$

or

$$\Phi_{l1} = \bar{K}_1 \Phi_{t1}$$

$\bar{K}_1$  is a small number

$$\Phi_{l2} = \bar{K}_2 \Phi_{t2}$$

$$M = \frac{N_2 \Phi_{m1}}{I_1} = \frac{N_2 \Phi_{m2}}{I_2}$$

$$M = \frac{N_2}{I_1} (\Phi_{t1} - \Phi_{l1}) = \frac{N_2}{I_1} (1 - \bar{K}_1) \Phi_{t1}$$

$$L_1 L_2 = \frac{N_1 \Phi_{t1}}{I_1} \times \frac{N_2 \Phi_{t2}}{I_2}$$

$$= \frac{N_1 N_2}{I_1 I_2} \Phi_{t1} \Phi_{t2}$$

$$L_1 = \frac{N_1 \Phi_{t1}}{I_1} \quad L_2 = \frac{N_2 \Phi_{t2}}{I_2}$$

$$L_1 L_2 = \frac{N_1 N_2}{I_1 I_2} (\Phi_{m1} + \Phi_{l1}) (\Phi_{m2} + \Phi_{l2})$$

$$M = \frac{N_1 \Phi_{t2} (1 - \bar{K}_2)}{I_2} = \frac{N_2 \Phi_{t1} (1 - \bar{K}_1)}{I_1}$$

$$\Phi_{l1} = \bar{K}_1 \Phi_{t1}$$

$$\Phi_{l2} = \bar{K}_2 \Phi_{t2}$$

$$M^2 = \frac{N_1 N_2 (1 - \bar{K}_2)(1 - \bar{K}_1) \Phi_{t1} \Phi_{t2}}{I_1 I_2}$$

$$= (1 - \bar{K}_2)(1 - \bar{K}_1) \frac{N_1 \Phi_{t1}}{I_1} \cdot \frac{N_2 \Phi_{t2}}{I_2}$$

$$= \underbrace{(1 - \bar{K}_2)(1 - \bar{K}_1)}_{K^2} L_1 L_2$$

$$M^2 = K^2 L_1 L_2$$

$$\text{or } M = K \sqrt{L_1 L_2}$$

where  $K$  is called  
the coeff of coupling