

Lec-25

$\frac{dy}{dt} + ay = x(t)$
 $(D+a)y = Ae^{st}$ if $x(t)$ can be expressed in exponential form
 s could be imaginary or complex.

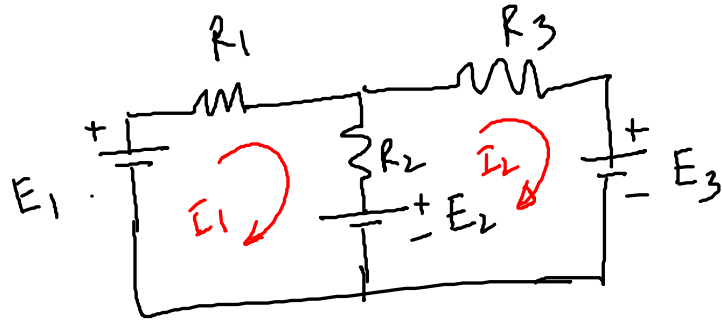
$$y_f(t) = \frac{Ae^{st}}{(D+a)} \Big|_{D=s}$$

if $x(t) = \text{constant} = A = Ae^{0t}$ $s=0$

$$\frac{d^2y}{dt^2} + a \frac{dy}{dt} + by = Ae^{st}$$

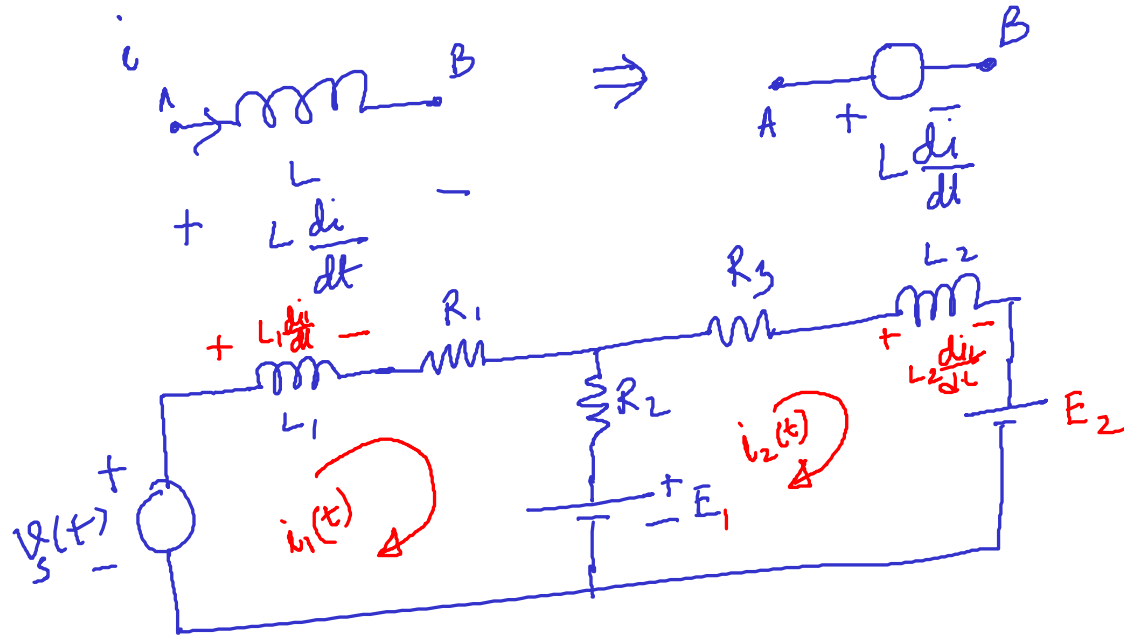
$$y_f(t) = \frac{Ae^{st}}{(D^2 + aD + b)} \Big|_{D=s}$$

$$y(t) = \underline{A_1 e^{m_1 t} + A_2 e^{m_2 t}} + y_f(t)$$



$$(R_1 + R_2) I_1 - R_2 I_2 = E_1 - E_2$$

$$-R_2 I_1 + (R_2 + R_3) I_2 = E_2 - E_3$$



Mesh-1: KVL

$$(R_1 + R_2) i_1 - R_2 i_2 = v_s(t) - E_2 - L_1 \frac{di_1}{dt}$$

Mesh-2: KVA

$$-R_2 i_1 + (R_2 + R_3) i_2 = E_1 - E_2 - L_2 \frac{di_2}{dt}$$

$$i \text{ sum} + \frac{di}{dt} \Rightarrow$$

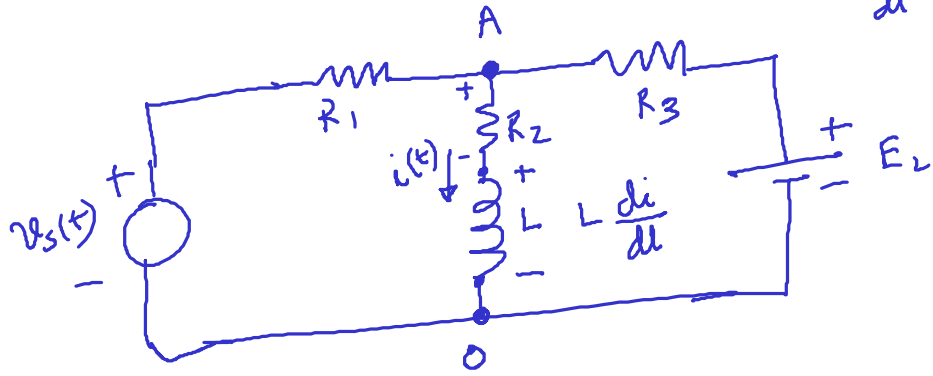


Nodal method

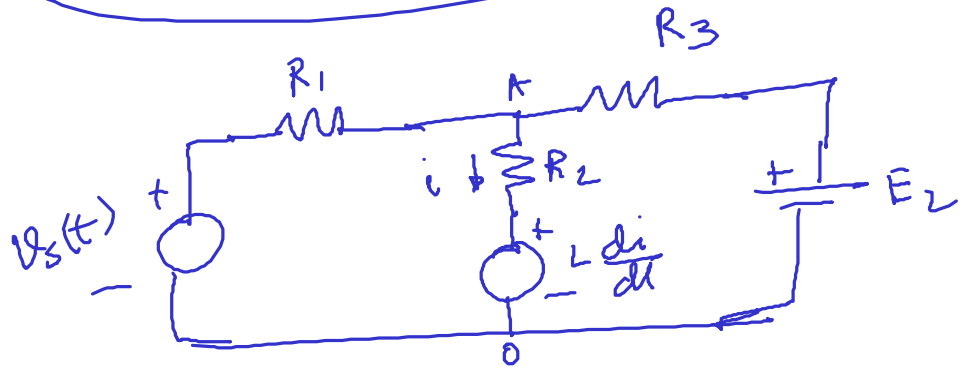
To write the KCL at node A.

$$\frac{v_{A0} - v_s(t)}{R_1} + i(t) + \frac{v_{A0} - E_2}{R_3} = 0$$

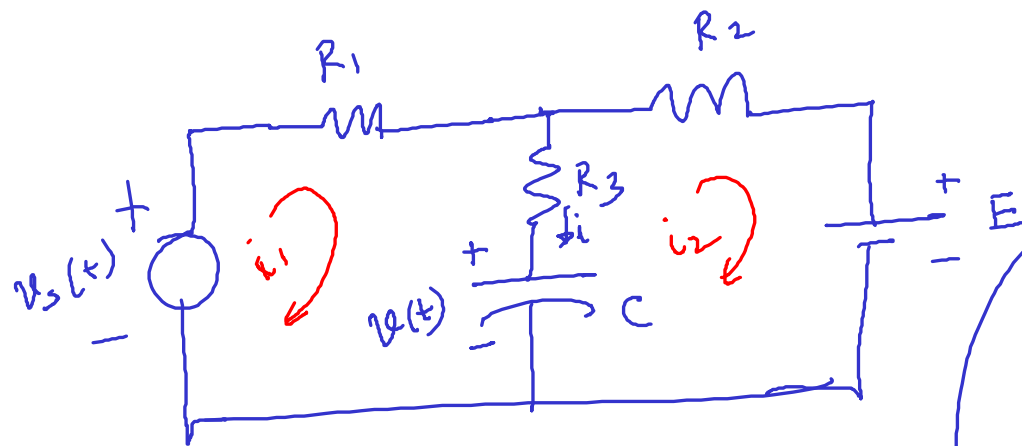
$$\text{or } \left(\frac{1}{R_1} + \frac{1}{R_3} \right) v_{A0} + i(t) = \frac{v_s(t)}{R_1} + \frac{E_2}{R_3}$$



$$v_{A0} = L \frac{di}{dt} + R_2 i$$



$$v_{A0} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{v_s}{R_1} + \frac{L \frac{di}{dt}}{R_2} + \frac{E_2}{R_3}$$



$$i = i_1 - i_2$$

mesh analysis.

$$i = C \frac{dv}{dt}$$

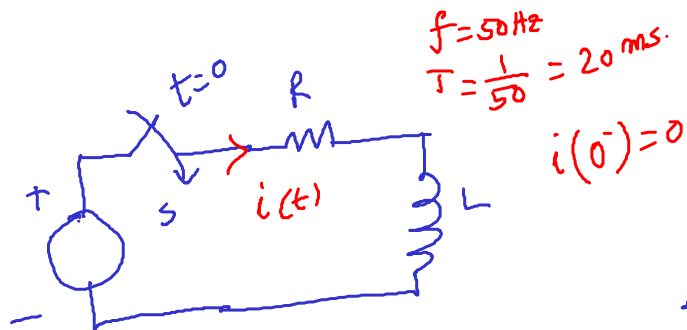
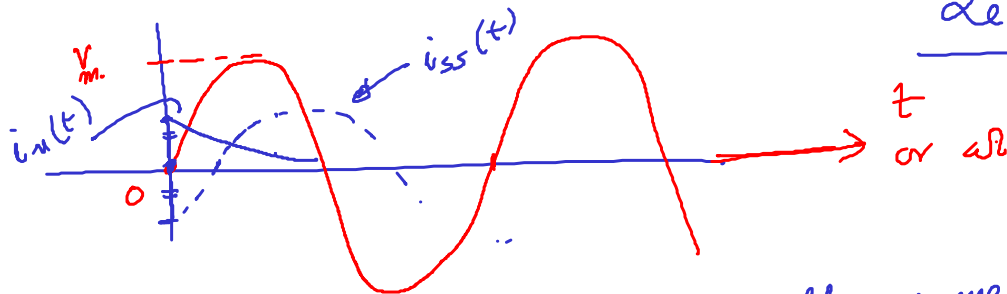
KVL in mesh 1

$$(R_1 + R_3) i_1 - R_3 i_2 = v_s(t) - v(t)$$

$$-R_3 i_1 + (R_2 + R_3) i_2 = v(t) - E$$

$$i_1 - i_2 = C \frac{dv}{dt}$$

$\therefore i_1, i_2$ & v can be
solved.

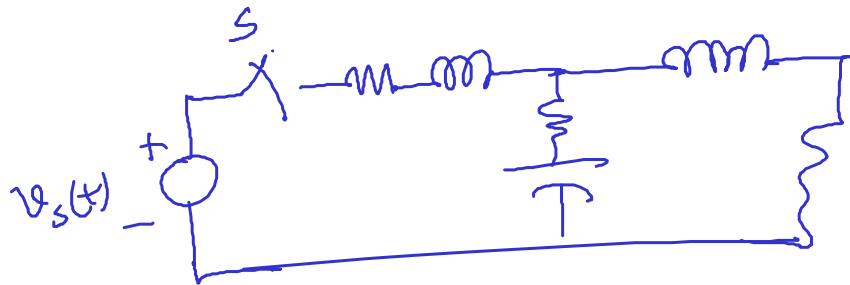


$$i(t) = -\frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin\left(-\theta - \frac{R}{L}t\right) + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \theta)$$

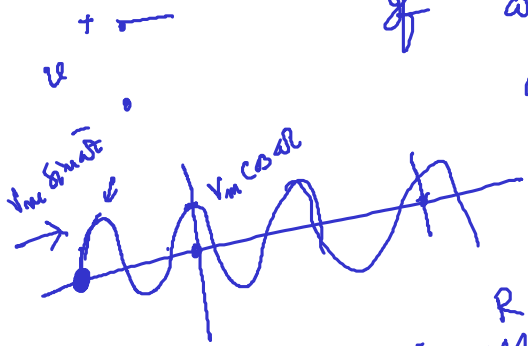
where $\tan \theta = \frac{\omega L}{R}$

So $i_m(t) \rightarrow 0$ after some time

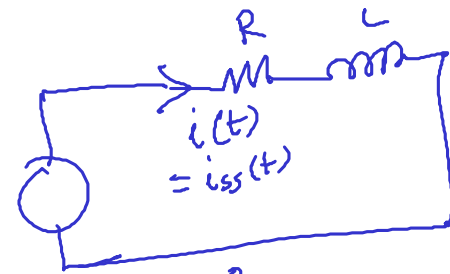
If we are interested in steady state current only then is there a simpler technique to do that.



If we are interested in steady state solution alone $i_{ss}(t) = [i_f(t)]$
 when the supply sinusoidally time varying
 $v_s(t) = V_{max} \sin(\omega t)$ $\omega = 2\pi f$
 $f = \text{supply freq.}$



Then Phasor approach is adopted

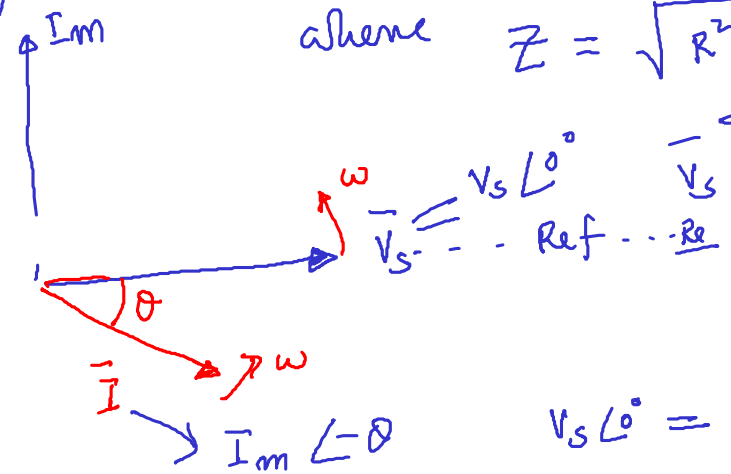
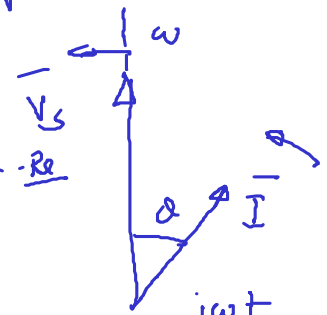


$$v_s(t) = V_m \sin \omega t$$

$$i_{ss}(t) = i(t) = \frac{V_m}{Z} \sin(\omega t - \theta)$$

where $Z = \sqrt{R^2 + \omega^2 L^2}$ $\theta = \tan^{-1} \frac{\omega L}{R}$

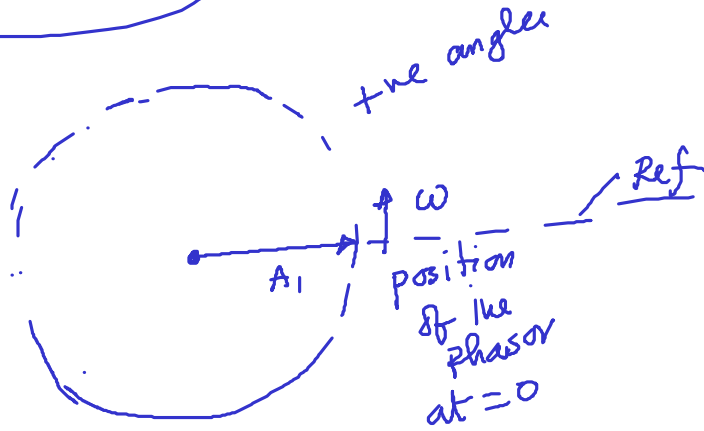
$v_s(t) = V_m \sin \omega t \Rightarrow \bar{V}_s \angle 0^\circ$
 $i(t) = \frac{V_m}{Z} \sin(\omega t - \theta) \Rightarrow \bar{I} \angle -\theta$
 $= I_m$



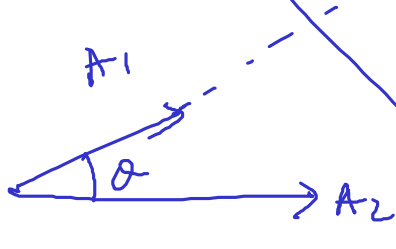
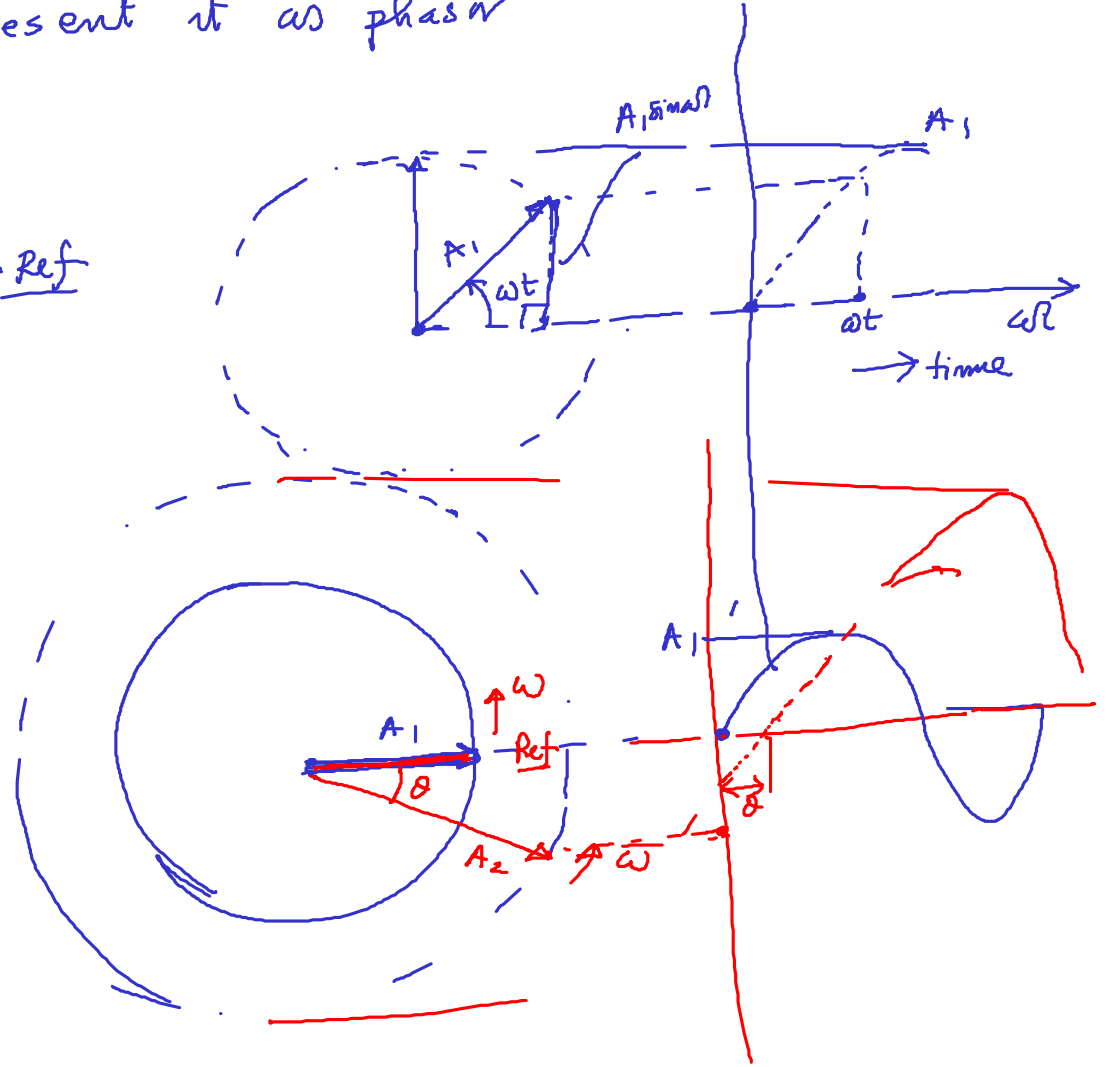
$$\bar{V}_s \angle 0^\circ = V_s e^{j\omega t}$$

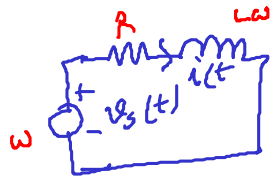
$$\bar{I} \angle -\theta = I_m e^{j(\omega t - \theta)}$$

$A_1 \sin \omega t$ how to represent it as phasor



$A_2 \sin(\omega t - \pi/6)$





Phasor Analysis of Electrical Circuit when

Dec-27

excitation or forcing f.m. is sinusoidal.

$$v_s(t) = V_m \sin \omega t$$

$$i(t) = \frac{V_m}{Z} \sin(\omega t - \theta)$$

$$I_m = \frac{V_m}{Z}$$

$$\therefore \frac{V_m}{I_m} = Z$$

out of curiosity

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

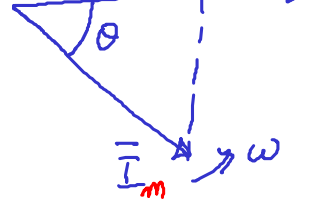
$$\tan \theta = \frac{\omega L}{R}$$

$$\cos \theta = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{R}{Z}$$

$$\sin \theta = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \frac{\omega L}{Z}$$

$$R = Z \cos \theta$$

$$\omega L = Z \sin \theta$$



Phasor

$$\bar{V}_s = V_s + j0 = V_s e^{j0} = V_s \angle 0^\circ = V_m \angle 0^\circ$$

$$\bar{I}_m = I_m \cos \theta - j I_m \sin \theta$$

$$\bar{I} = \frac{I_m}{m} e^{-j\theta} = \frac{I_m}{m} \angle -\theta$$

$$\bar{I}_s = \frac{\bar{V}_s}{Z}$$

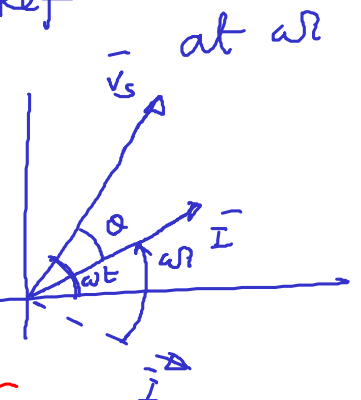
$$= Z e^{j\theta} = Z \cos \theta + j Z \sin \theta = (R + j\omega L) = \frac{\bar{V}_s}{\bar{I}_s} = \text{constant}$$

$$= \bar{Z}$$

voltage ph. $\bar{V}_s$

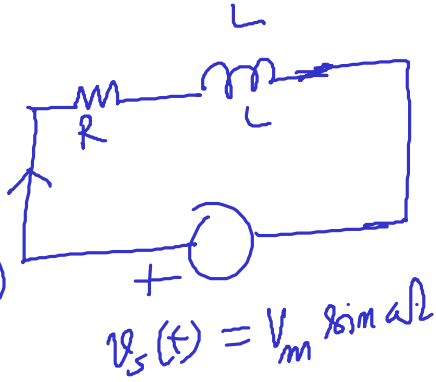
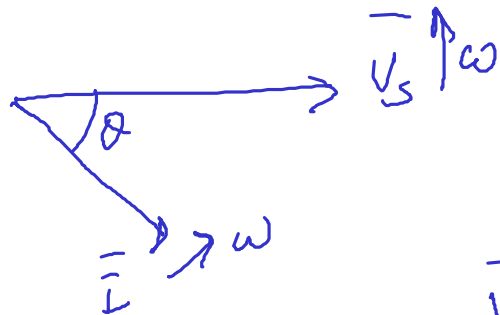
current ph. $\bar{I}_s$

$$\frac{\bar{V}_s}{\bar{I}_s} = \frac{V_m \angle 0^\circ}{I_m \angle -\theta} = \frac{V_m e^{j0}}{I_m e^{-j\theta}} = \left(\frac{V_m}{I_m} \right) e^{j(\theta)}$$



$$i(t) = I_m \sin(\omega t - \theta)$$

$$= \frac{V_m}{Z} \sin(\omega t - \theta)$$



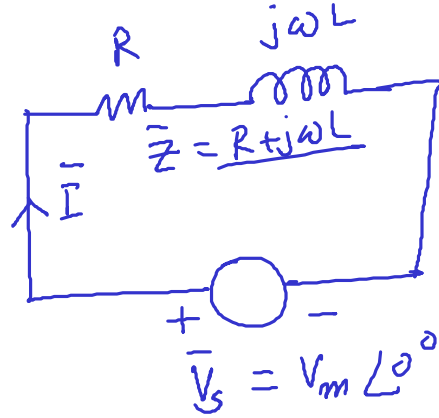
$$\bar{Z} = Z \angle 0^\circ$$

$$= Z \cos \theta + j Z \sin \theta$$

$$= R + j\omega L$$

$$= R + jX_L$$

write $\bar{Z}$
 then get $\bar{I}$
 then $i(t)$



$$\bar{V}_s = \bar{I} \bar{Z}$$

$$\bar{Z} = R + j\omega L$$

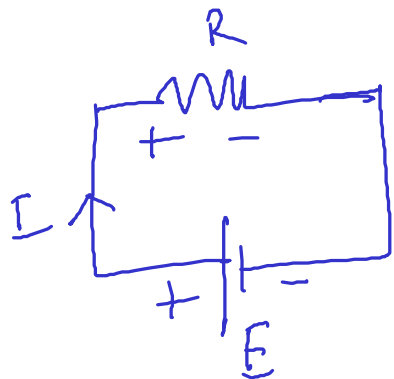
$$V = \bar{I} R$$

$$\bar{I} = \frac{\bar{V}_s}{\bar{Z}}$$

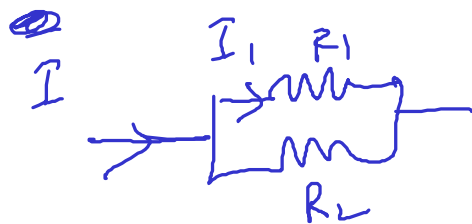
$$= \frac{V_m \angle 0^\circ}{Z \angle \theta}$$

Phasor current $\bar{I} = \frac{V_m}{Z} \angle -\theta$

$$i(t) = \frac{V_m}{Z} \sin(\omega t - \theta)$$

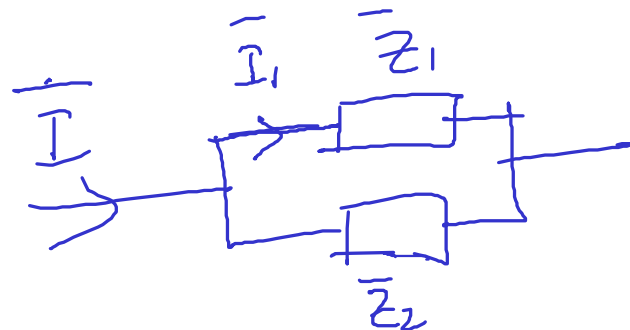
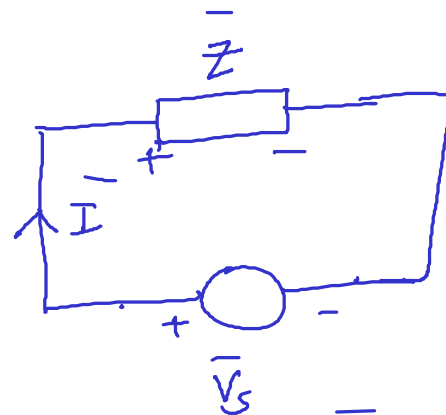


$$E = IR$$



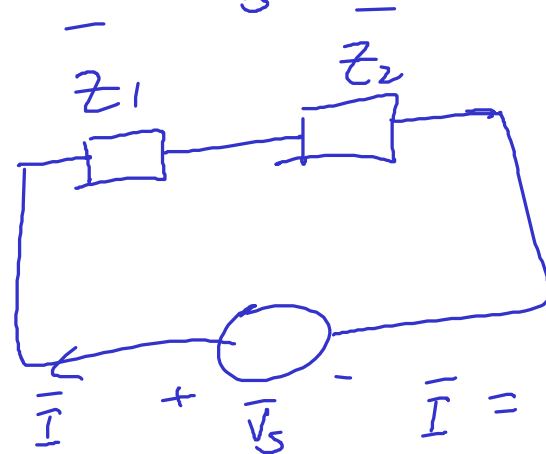
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = I \frac{R_2}{R_1 + R_2}$$



$$\bar{V}_s = \bar{Z} \bar{I}$$

$$\bar{V}_s - \bar{Z} \bar{I} = 0$$



$$\bar{I} = \frac{\bar{V}_s}{\bar{Z}_1 + \bar{Z}_2}$$

$$\bar{Z}_{eq} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$\bar{I}_1 = \bar{I} \frac{\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

Phasor Analysis of electrical circuit:

Dec - 28

(Applicable only to sinusoidal excitation of single frequency ω).

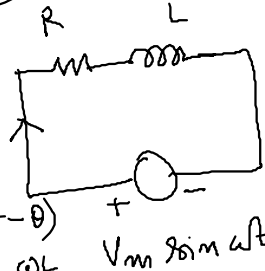
time domain

$i(t) = i_s(t) = \frac{V_m \sin(\omega t - \theta)}{Z}$
 $\tan \theta = \frac{\omega L}{R}$

$\frac{v(t)}{i(t)} = ?$

$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$
 $v(t) = V_m \sin \omega t$

$i(t) = \frac{V_m}{Z} \sin(\omega t + \theta)$
 $\tan \theta = \frac{1}{\omega C R} = \frac{1}{\omega C R}$



$\bar{Z} = Z \angle \theta$
 $\bar{V} = V_m \angle 0^\circ = V_m e^{j0}$

$\bar{Z} = R - jX_c = -j/\omega C$
 $\bar{V} = \bar{I} \bar{Z}$

$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V_m \angle 0^\circ}{Z \angle \theta} = \frac{V_m}{Z} \angle -\theta$

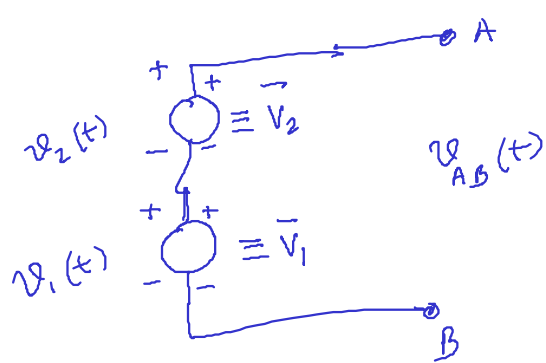
$\omega L = \text{inductive reactance}$

$\frac{1}{\omega C} = \text{capacitive reactance}$

$\bar{Z} = R - \frac{j}{\omega C} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \angle -\theta$
 $= Z \angle -\theta$

$\bar{I} = \frac{V_m \angle 0^\circ}{Z \angle -\theta} = I_m \angle \theta$

$V = IR$



let $v_1(t) = V_{m1} \sin(\omega t + \frac{\pi}{6})$

$\sin(-\alpha) = -\sin \alpha$

and $v_2(t) = V_{m2} \cos \omega t$

$= V_{m2} \sin(\frac{\pi}{2} - \omega t)$

$v_2(t) = -V_{m2} \sin(\omega t - \frac{\pi}{2})$

$v_1(t) = V_{m1} \sin(\omega t + \frac{\pi}{6}) \equiv V_{m1} \angle +\frac{\pi}{6} = \vec{V}_1$

$v_{AB}(t) = V_{m1} \sin(\omega t + \frac{\pi}{6}) + V_{m2} \cos \omega t$

$v_2(t) = -V_{m2} \angle -\frac{\pi}{2} = -V_{m2} \angle -\frac{\pi}{2} = \vec{V}_2$

$\vec{V}_{AB} = \vec{V}_1 + \vec{V}_2 = (V_{m1} \angle +\frac{\pi}{6}) - V_{m2} \angle -\frac{\pi}{2} = V_m \angle \theta'$

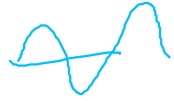
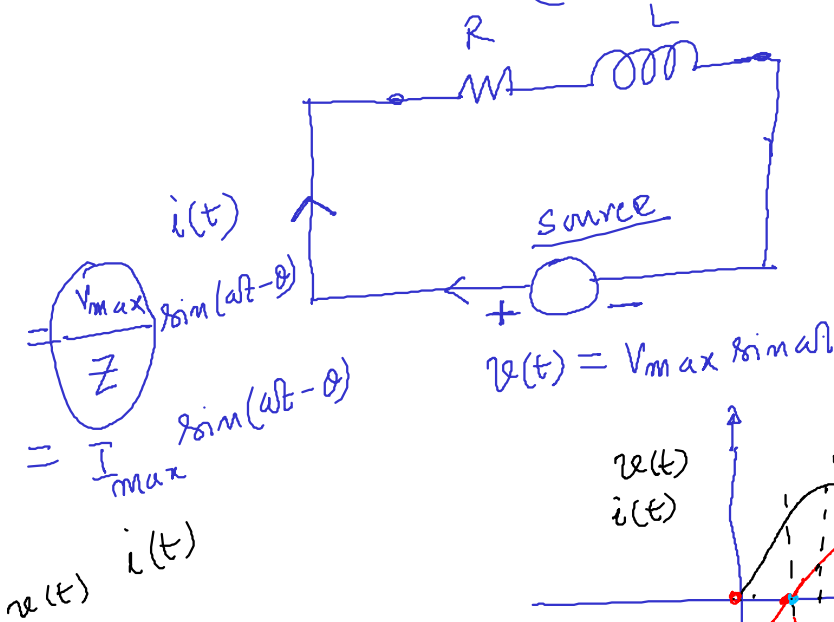
$v_{AB}(t) = V_m \sin(\omega t + \theta')$

Power in A.C circuit
(sinusoidal)

$$\bar{Z} = R + j\omega L$$

$$|\bar{Z}| = Z = \sqrt{R^2 + \omega^2 L^2}$$

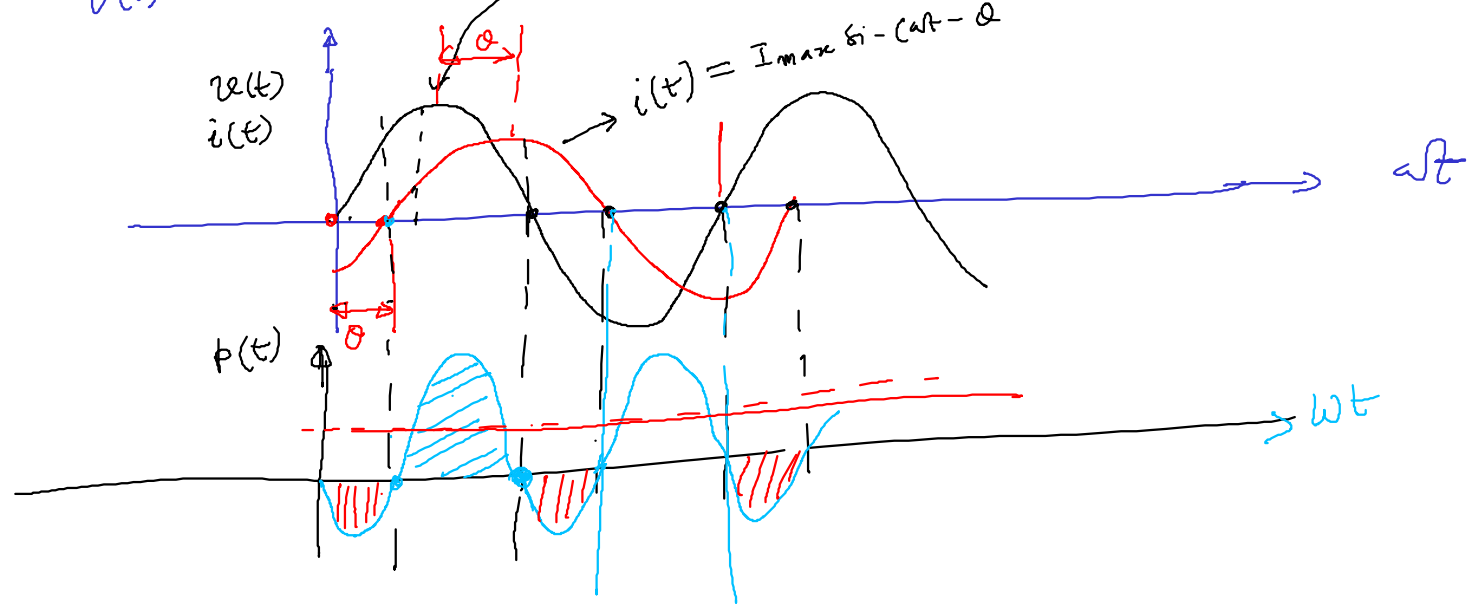
Instantaneous
Power delivered by the
source to the load



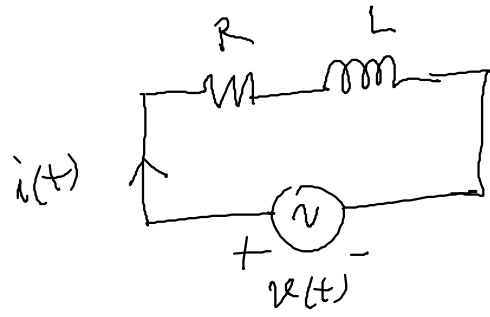
$$p(t) = v(t) i(t)$$

$$= V_{\max} \sin \omega t \cdot I_{\max} \sin(\omega t - \phi)$$

$$= V_{\max} I_{\max} \sin \omega t \sin(\omega t - \phi)$$



Instantaneous Power in A-C circuit



$$\text{Let } v(t) = V_{\max} \sin \omega t$$

$$i(t) = I_{\max} \sin(\omega t - \phi)$$

$$I_{\max} = \frac{V_{\max}}{Z}$$

$$\tan \phi = \frac{\omega L}{R}$$

$$p(t) = V_{\max} I_{\max} \sin \omega t \sin(\omega t - \phi)$$

$$p(t) = V_{\max} I_{\max} \sin \omega t (\sin \omega t \cos \phi - \cos \omega t \sin \phi)$$

$$= V_{\max} I_{\max} \cos \phi \sin^2 \omega t - V_{\max} I_{\max} \sin \phi \sin \omega t \cos \omega t$$

$$p(t) = \frac{V_{\max} I_{\max} \cos \phi (1 - \cos 2\omega t)}{2} - \frac{V_{\max} I_{\max} \sin \phi \sin 2\omega t}{2}$$

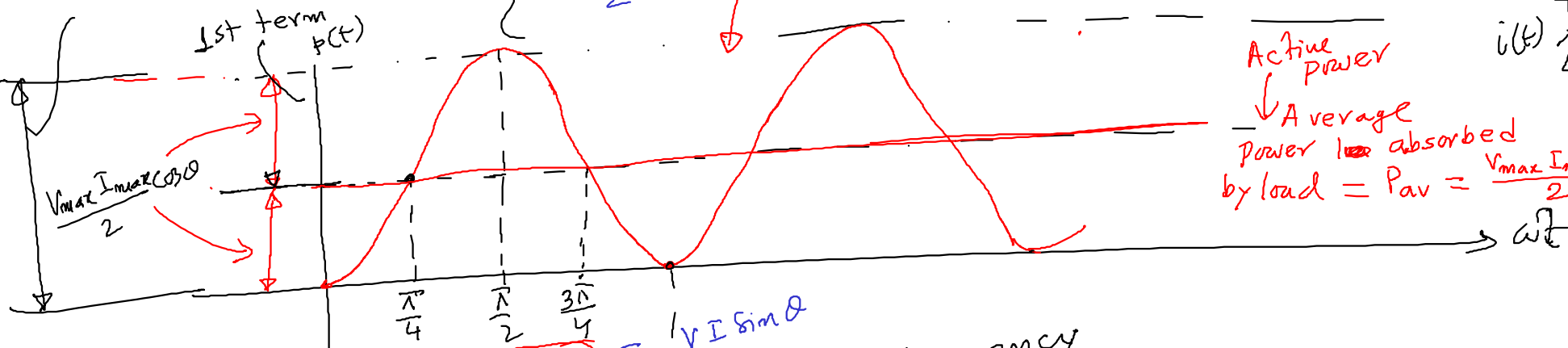
* This term will never become -ve

→ It will be equal having both +ve & -ve excursion about ωt axis

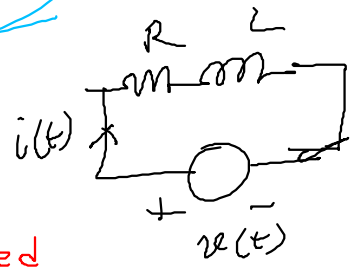
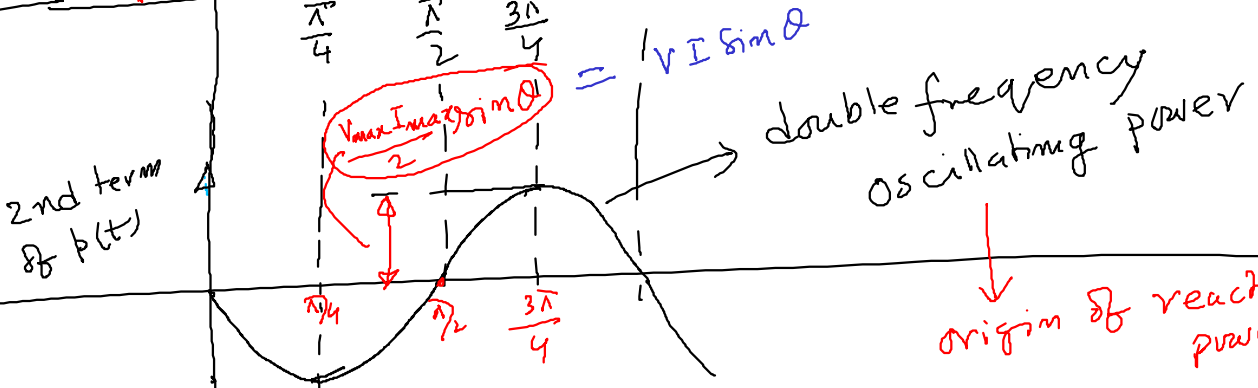
$$\frac{V_{max} I_{max}}{2} \cos \phi - \frac{V_{max} I_{max}}{2} \cos \phi \cos 2\omega t$$

$$V_{max} I_{max} \cos \phi$$

$$p(t) = \left\{ \frac{V_{max} I_{max} \cos \phi}{2} (1 - \cos 2\omega t) \right\} - \left(\frac{V_{max} I_{max} \sin \phi}{2} \sin 2\omega t \right)$$



Active power
↓
Average power absorbed by load = $P_{av} = \frac{V_{max} I_{max}}{2} \cos \phi$



$$\left(\frac{V_{max} I_{max} \sin \phi}{2} \right) \sin 2\omega t$$

In A.C circuit:-

$$\bar{I} = \frac{\bar{V}}{Z}$$

average Power

$$P_{av} = \frac{V_{max} I_{max}}{2} \cos \theta \quad (\text{Real power})$$

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{V_{max} \angle 0^\circ}{I_{max} \angle -\theta}$$

for R-L
Circuit

$$= \frac{\frac{V_{max}}{\sqrt{2}} \angle 0^\circ}{\frac{I_{max}}{\sqrt{2}} \angle -\theta} = \frac{V_{RMS} \angle 0^\circ}{I_{RMS} \angle -\theta}$$

$$= \frac{V_{max}}{\sqrt{2}} \frac{I_{max}}{\sqrt{2}} \cos \theta$$

$$P_{av} = V_{RMS} I_{RMS} \cos \theta$$

Power
factor
of the circuit

In d.c

$$I = \frac{V}{R}$$

$$P = VI$$

$$\bar{Z} = R + j\omega L$$

Suppose

$$v(t) = 245 \sin \omega t$$

$$\bar{V} = \frac{245}{\sqrt{2}} \angle 0^\circ$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = I_{RMS} \angle -\theta$$

Active & Reactive Power in A.C Circuit :-

2nd Lec-30

average
power
absorbed
by load

Reactive
power

$$P = V I \cos \theta$$

RMS

=

$$\frac{1}{2\pi} \int_0^{2\pi} v(t) i(t) d(\omega t) = \frac{1}{2\pi} \int_0^{2\pi} V_{\max} I_{\max} \sin(\omega t) \sin(\omega t - \theta) d(\omega t)$$

$$Q = \frac{V_{\max} I_{\max}}{2} \sin \theta$$

inductive

$$= V I \sin \theta$$

= It is peak value of the

oscillating power between
the source and the load.

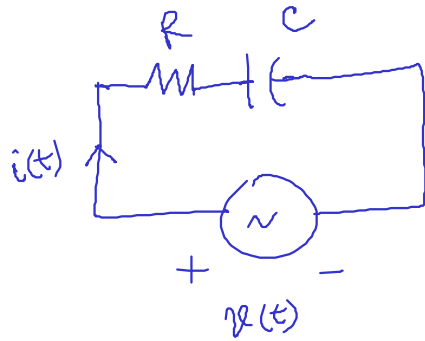
$$= \frac{V_{\max} I_{\max}}{2\pi \times 2} \int_0^{2\pi} 2 \sin \omega t \sin(\omega t - \theta) d(\omega t)$$

$$= \frac{V_{\max} I_{\max}}{4\pi} \int_0^{2\pi} [\cos \theta - \cos(2\omega t - \theta)] d(\omega t)$$

$$= \frac{V_{\max} I_{\max}}{4\pi} \int_0^{2\pi} \cos \theta d(\omega t) - 0$$

$$= \frac{V_{\max} I_{\max} \cos \theta \times 2\pi}{4\pi}$$

$$= \frac{V_{\max} I_{\max}}{2} \cos \theta = V I \cos \theta$$



$$v(t) = V_{max} \sin \omega t$$

$$i(t) = I_{max} \sin(\omega t + \theta)$$

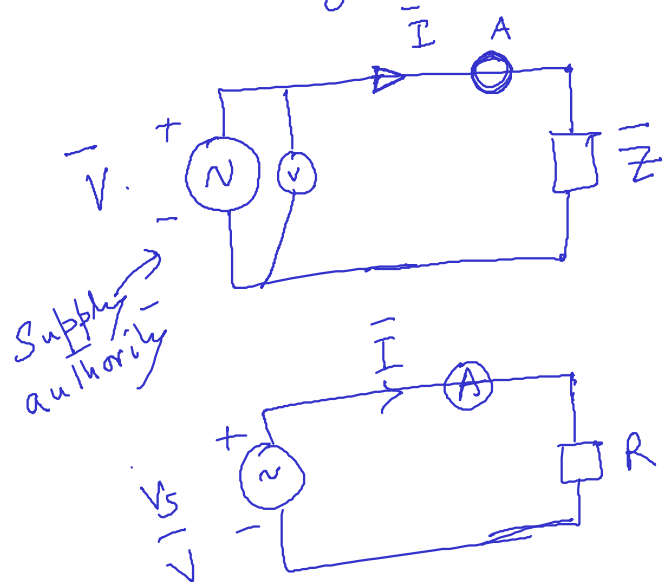
$$Z = \frac{V_{max}}{I_{max}} = \frac{V}{I}$$

$$\tan \theta = \frac{1/\omega C}{R} = \frac{1}{\omega CR}$$

$$P = VI \cos \theta$$

$$Q = VI \sin \theta \rightarrow \text{Reactive Power capacitive.}$$

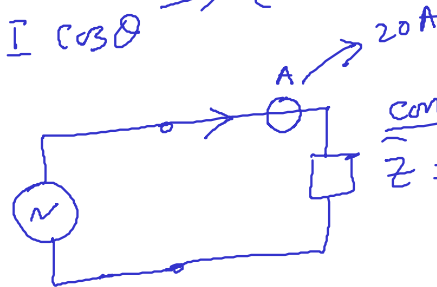
Significance of power factor $\cos \theta$



Supply
authority

$$P = VI \cos \theta \rightarrow (0 < \cos \theta < 1)$$

RMS
200V
50Hz



$$\bar{V} = 200 \angle 0^\circ$$

$$P = 200 \times 20 \cos 53^\circ$$

$$= \underline{2400 \text{ W}}$$

$$\bar{Z} = 6 + j8 \, \Omega = 10 \angle 53^\circ \, \Omega$$

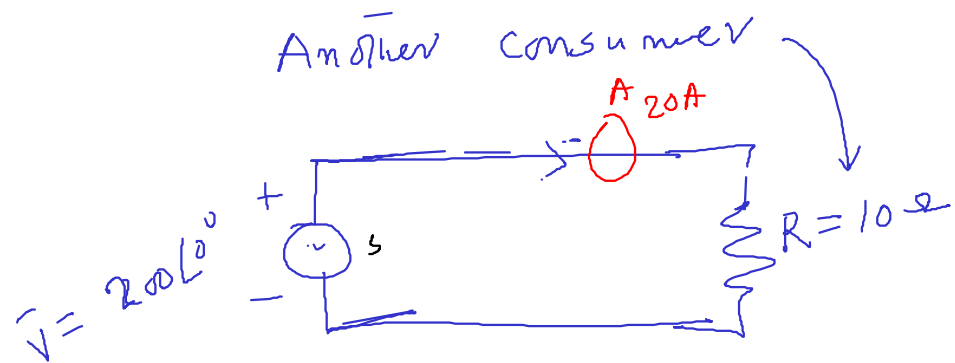
$$\bar{I} = \frac{200 \angle 0^\circ}{10 \angle 53^\circ}$$

$$= 20 \angle -53^\circ$$

$$Q = 200 \times 20 \sin 53^\circ$$

$$= \underline{3200 \text{ VAR}}$$

volt ampere
reactive



$$\bar{V} = 200\angle 0^\circ \text{ V}$$

$$\bar{I} = \frac{200\angle 0^\circ}{10\angle 0^\circ} = 20\angle 0^\circ \text{ A}$$

$$\bar{Z} = 10 + j0$$

$$= 10\angle 0^\circ \Omega \quad \theta = 0$$

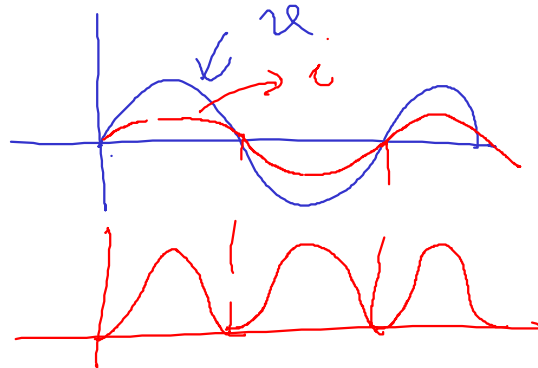
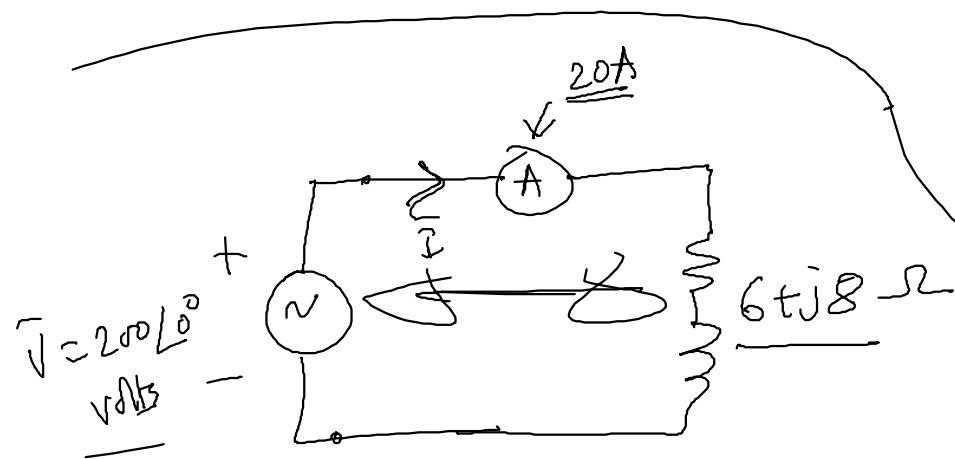
$$P_{av} = 200 \times 20 \cos 0^\circ$$

$$= 4000 \text{ W}$$

$$Q_{\text{reactive}} = 200 \times 20 \sin 0^\circ$$

$$= 0$$

P.f. $\rightarrow 0.8 \text{ p.f. (lagg)}$



$$200 \times 20$$

$$4000 \text{ VA}$$

$$\bar{I} = \frac{200\angle 0^\circ}{6 + j8 \Omega} = \frac{200\angle 0^\circ}{10\angle 53^\circ} = 20\angle -53^\circ$$

$$P = 2400 \text{ W} \rightarrow \text{active}$$

$$Q = 3200 \text{ VAR} \rightarrow \text{Reactive}$$

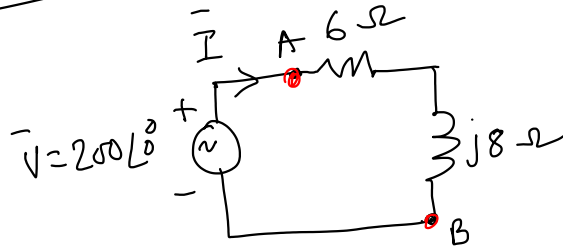
Apparent Power, (S)
 VI

Active Power or Real (P)
 $VI \cos \theta$ $\nearrow$ p.f.

Reactive Power (Q)
 $VI \sin \theta$

total complex power $\rightarrow$

$$\bar{S} = P + jQ$$



$$\bar{Z} = 6 + j8 \Omega$$

Complex Power supplied by the source :
" " absorbed by the load

$$\bar{V}_{AB} \bar{I}^* = 200 \angle 0^\circ \times 20 \angle 53^\circ$$

$$P = 2400 \text{ W} \quad Q = 3200 \text{ VAR}$$

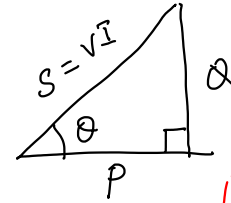
$$= 2400 \text{ W} + j3200 \text{ VAR}$$

$$\bar{V} = 200 \angle 0^\circ \text{ volts.}$$

$$\bar{I} = \frac{200 \angle 0^\circ}{6 + j8} = 20 \angle -53^\circ \text{ A}$$

$$P = 200 \times 20 \cos(-53^\circ)$$

$$Q =$$



$$\bar{V} = \bar{I} \bar{Z}$$

Lec 31

$$\bar{I} = 20 \angle -53^\circ$$

Power absorbed by load

$$200 \angle 0^\circ \times 20 \angle -53^\circ = 4000 - j3200 \text{ VAR}$$

$$\therefore \bar{I}^* = 20 \angle +53^\circ$$

(10 - j50 VAR)

$$\bar{S} = \bar{V} \bar{I}^* = 200 \angle 0^\circ \times 20 \angle 53^\circ$$

$$= 4000 \angle 53^\circ = \left(\frac{2400}{P} + j \frac{3200}{Q} \right) \text{ VA}$$

In general

$\theta > 0$

Let $\bar{V} = V \angle \alpha$

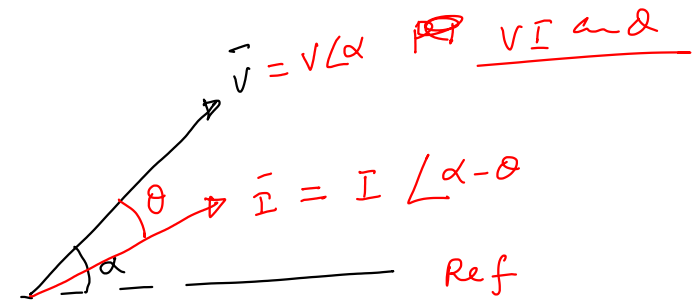
Then $\text{If } \bar{Z} = Z \angle \theta$

$$\therefore \bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{V}{Z} \angle \alpha - \theta$$

$$\bar{S} = \bar{V} \bar{I}^*$$

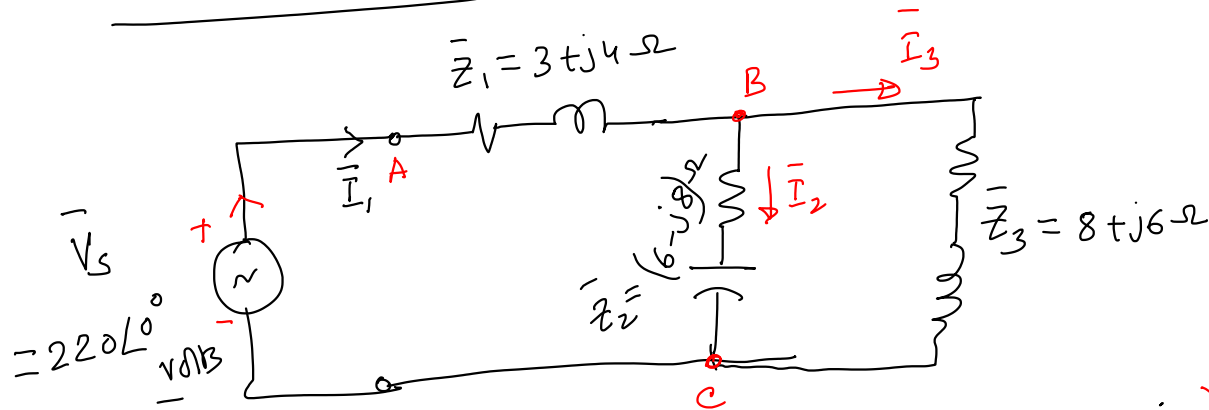
$$= V \angle \alpha \left(I \angle \alpha - \theta \right)^*$$

$$= \left(V \angle \alpha \times I \angle \theta - \alpha \right) = VI \angle \theta = VI \cos \theta + j VI \sin \theta$$



$$\begin{aligned} \bar{S} \neq \bar{V} \bar{I} &= V \angle \alpha \times I \angle \alpha - \theta \\ &= VI \angle (2\alpha - \theta) \neq P \\ &= VI \cos(2\alpha - \theta) + j VI \sin(2\alpha - \theta) \neq Q \end{aligned}$$

Numerical Problem :



$$\bar{I}_1 = \frac{220 \angle 0^\circ}{\left(\bar{Z}_1 + \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3} \right)} = \frac{220 \angle 0^\circ}{\bar{Z}_{eq}}$$

$$\bar{Z}_{eq} = (3 + j4) + \frac{(6 - j8)(8 + j6)}{14 - j2}$$

$$= 10 + j3 \Omega$$

$$\bar{I}_1 = \frac{220 \angle 0^\circ}{10 + j3} = 21.07 \angle -16.69^\circ \text{ A} = \bar{I}_1$$

$$\bar{I}_2 = \bar{I}_1 \times \frac{\bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3} = \frac{21.07 \angle -16.69^\circ \times (8 + j6)}{14 - j2}$$

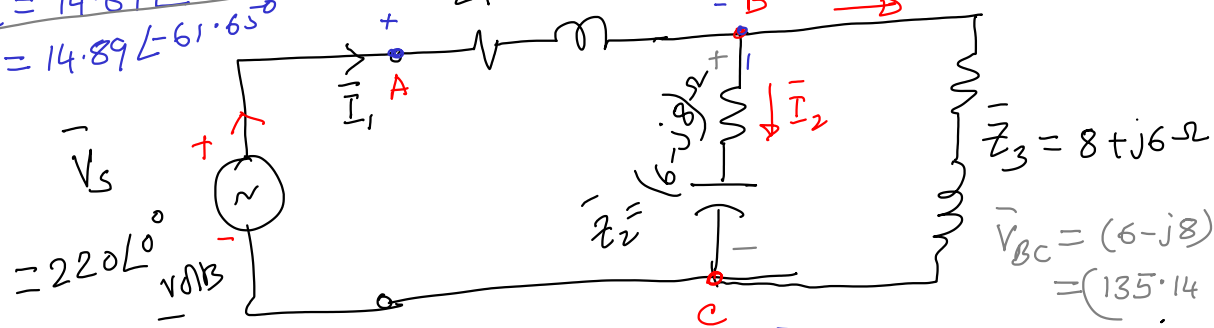
$$= 13.116 + j7.06 = 14.89 \angle 28.31^\circ \text{ A} = \bar{I}_2$$

$$\bar{I}_3 = \bar{I}_1 - \bar{I}_2 = 21.07 \angle -16.69^\circ - 14.89 \angle 28.31^\circ = 14.89 \angle -61.65^\circ = \bar{I}_3$$

Complex Power supplied by source = $\bar{V}_s \times \bar{I}_1^* = 220 \angle 0^\circ \times 21.07 \angle 16.69^\circ$

$$= 4393.76 \text{ W} + j1331.25 \text{ VAR}$$

$$\begin{aligned}\bar{I}_1 &= 21.07 \angle -16.69^\circ \\ \bar{I}_2 &= 14.89 \angle 28.31^\circ \\ \bar{I}_3 &= 14.89 \angle -61.65^\circ\end{aligned}$$



(complex)
Total Power supplied by the source :-
 $= 4393.76 \text{ W} + j1331.25 \text{ VAR}$

$$\begin{aligned}\bar{V}_{BC} &= (6-j8) \times 14.89 \angle 28.31^\circ \\ &= (135.14 - j62.5) \text{ V}\end{aligned}$$

Total power absorbed by all loads (i.e., $\bar{Z}_1, \bar{Z}_2$ & $\bar{Z}_3$)
=

Total Power absorbed by $\bar{Z}_1$
 $\bar{V}_{AB} \bar{I}_1^*$

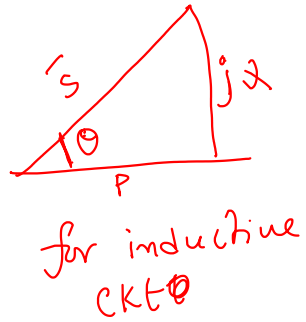
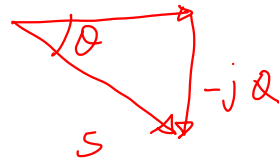
$$\begin{aligned}\text{now } \bar{V}_{AB} &= \bar{I}_1 \bar{Z}_1 = 21.07 \angle -16.69^\circ \times (3+j4) \text{ vdB.} \\ &= 84.8 + j62.6 \text{ vdB.}\end{aligned}$$

$$\bar{S}_1 = (84.8 + j62.6) \times 21.07 \angle 16.69^\circ = 1332 + j1777$$

Power absorbed by $\bar{Z}_2$: $\bar{S}_2 = \bar{V}_{BC} \bar{I}_2^* = (135.14 - j62.5) \times 14.89 \angle -28.31^\circ$

Power " by $\bar{Z}_3$: $\bar{S}_3 = \bar{V}_{BC} \bar{I}_3^* = (135.14 - j62.5) \times 14.89 \angle 61.65^\circ$

$$\Sigma \quad 4438 + j1332$$



$\bar{V}$ known
Get $\bar{I} (= \bar{V}/\bar{Z})$

Get Power supplied by source
Power absorbed by load

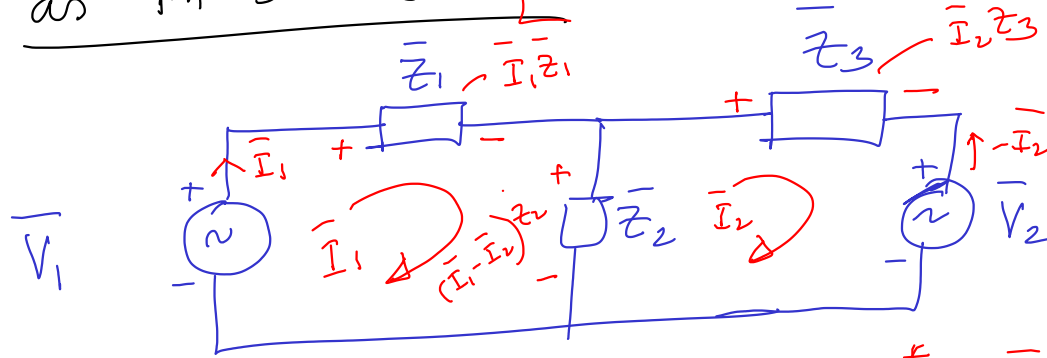
$$P = V I \cos \theta$$

$$= I Z I \cos \theta$$

$$= I^2 (Z \cos \theta)$$

$$\therefore R I^2$$

Nodal & Mesh analysis's can be adopted to solve network problems in the same way as in D.C ckt. $V = IR \equiv \bar{V} = \bar{I} \bar{Z}$



KVL in mesh 1 & 2

$$(\bar{Z}_1 + \bar{Z}_2) \bar{I}_1 - \bar{Z}_2 \bar{I}_2 = \bar{V}_1$$

$$-\bar{Z}_2 \bar{I}_1 + (\bar{Z}_2 + \bar{Z}_3) \bar{I}_2 = -\bar{V}_2$$

Solve for $\bar{I}_1$ & $\bar{I}_2$

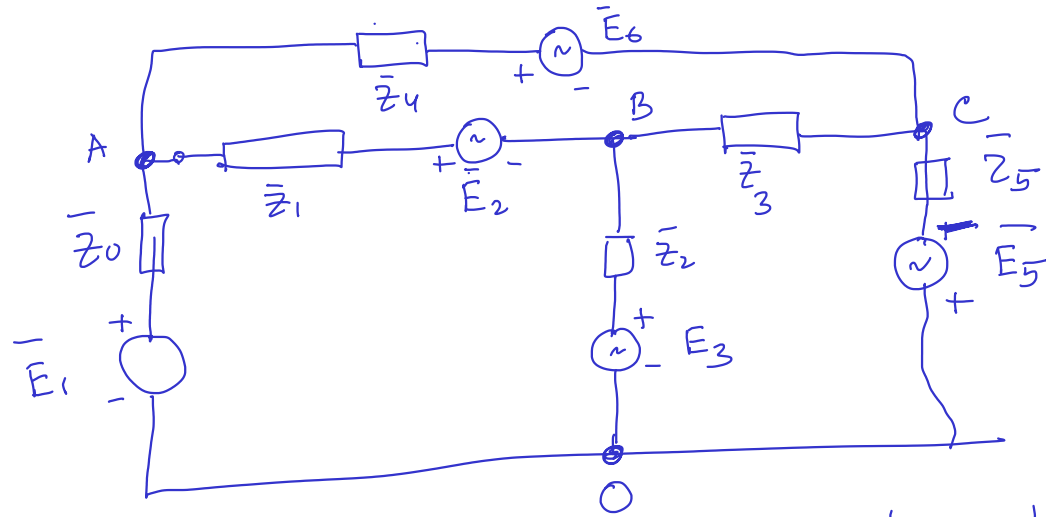
Power delivered by sources:

$$\bar{V}_1 \bar{I}_1 + \bar{V}_2 (-\bar{I}_2)$$

$$= \bar{V}_1 \bar{I}_1^* - \bar{V}_2 \bar{I}_2^*$$

Power Absorbed by loads

$$\bar{V}_{Z1} \bar{I}_1^* + \bar{V}_{Z2} (\bar{I}_1 - \bar{I}_2)^* + \bar{V}_{Z3} \bar{I}_2^* = \bar{V}_1 \bar{I}_1^* - \bar{V}_2 \bar{I}_2^*$$



KCL at node A:-

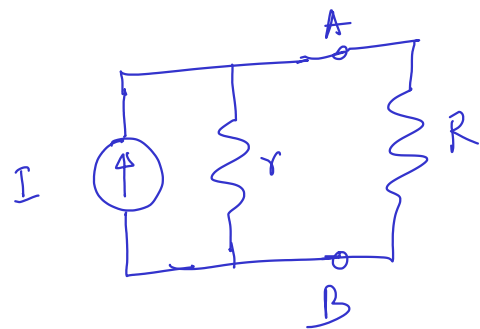
$$\left(\frac{1}{\bar{Z}_0} + \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_4} \right) \bar{V}_{A0} - \frac{1}{\bar{Z}_1} \bar{V}_{B0} - \frac{1}{\bar{Z}_4} \bar{V}_{C0} = \frac{\bar{E}_1}{\bar{Z}_0} + \frac{\bar{E}_6}{\bar{Z}_4} + \frac{\bar{E}_2}{\bar{Z}_1} \dots (1)$$

at node B:-

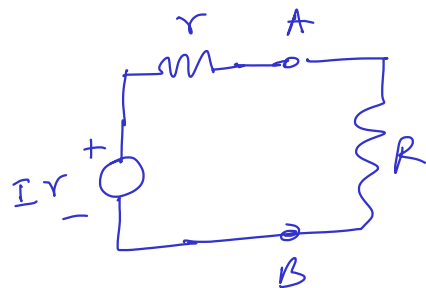
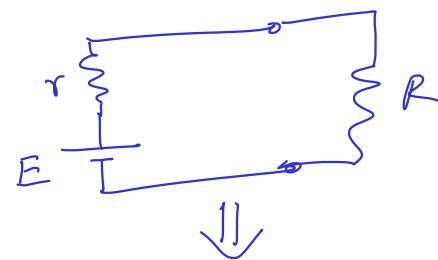
$$-\frac{1}{\bar{Z}_1} \bar{V}_{A0} + \left(\frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3} \right) \bar{V}_{B0} - \frac{1}{\bar{Z}_3} \bar{V}_{C0} = \frac{\bar{E}_3}{\bar{Z}_2} - \frac{\bar{E}_2}{\bar{Z}_1} \dots (2)$$

at node C:-

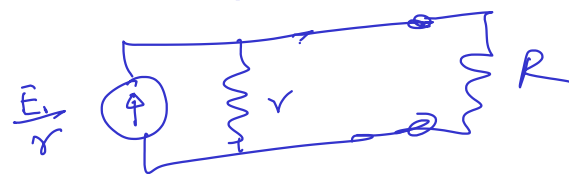
$$-\frac{1}{\bar{Z}_4} \bar{V}_{A0} - \frac{1}{\bar{Z}_3} \bar{V}_{B0} + \left(\frac{1}{\bar{Z}_3} + \frac{1}{\bar{Z}_5} + \frac{1}{\bar{Z}_4} \right) \bar{V}_{C0} = -\frac{\bar{E}_5}{\bar{Z}_5} - \frac{\bar{E}_6}{\bar{Z}_4} \dots (3)$$



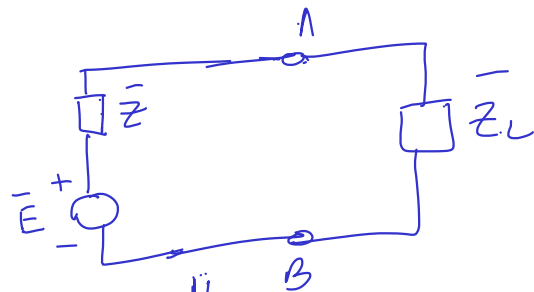
$\Rightarrow$



$\Downarrow$

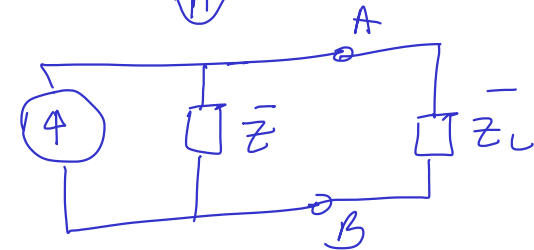
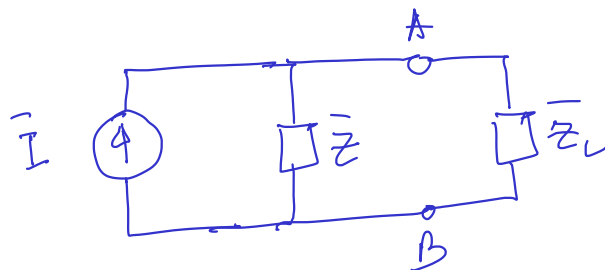


$\Downarrow$

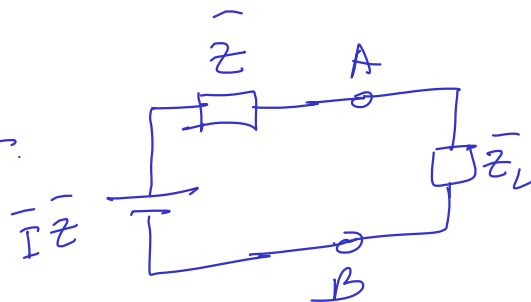
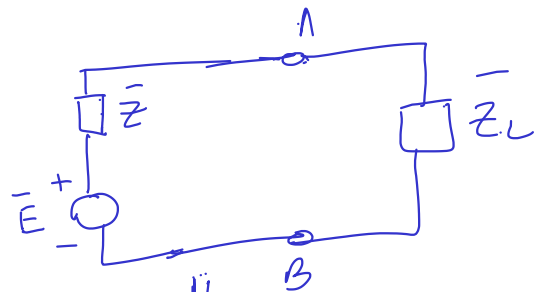


A

$\frac{E}{r}$

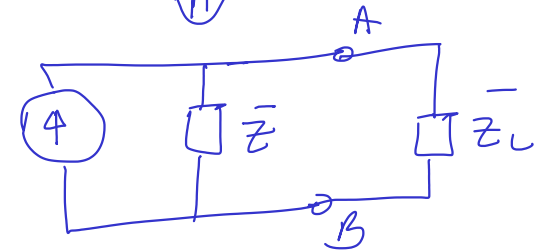


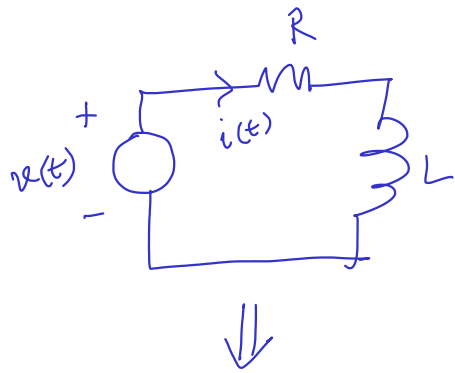
$\Rightarrow$



A

$\frac{\bar{E}}{\bar{Z}}$

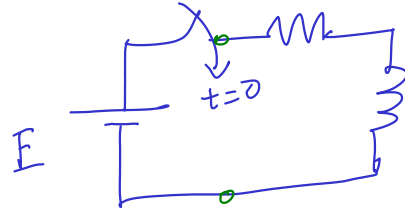




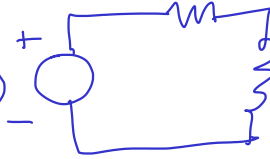
1

$$i(t) = \frac{E}{R} \left(1 - e^{-\frac{R}{L}t}\right)$$

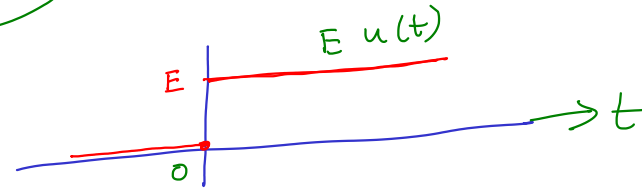
if $v(t) = E$
with $i(0) = 0$



$$v(t) = E u(t)$$



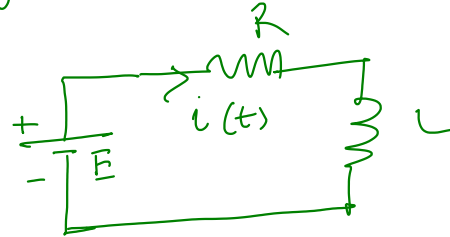
write step
 $u(t)$ for



for $t < 0$



for $t > 0$

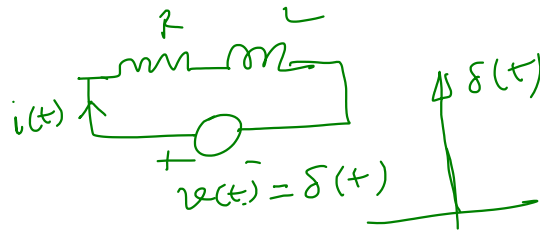
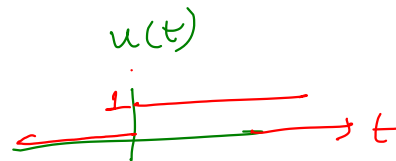


$$R i + L \frac{di}{dt} = E u(t) = E \text{ for } t > 0$$

$$Ri + L \frac{di}{dt} = v(t)$$

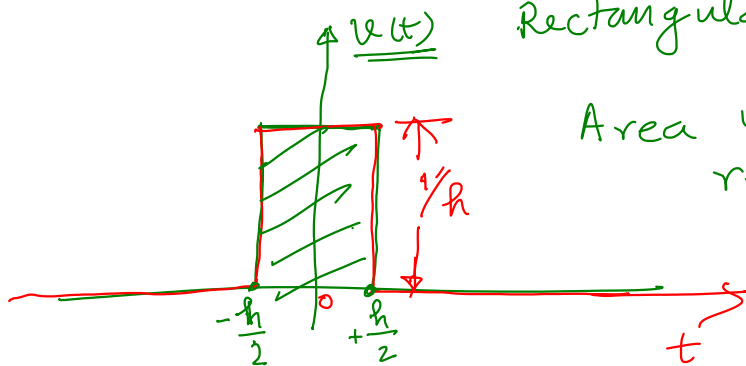
$$i(0^-) = i(0^+)$$

excitation
or input voltage
 $v(t)$ is not an
impulse voltage



What happens if an impulse is applied to
R-L circuit

Rectangular Pulse



$\rightarrow h \leftarrow$

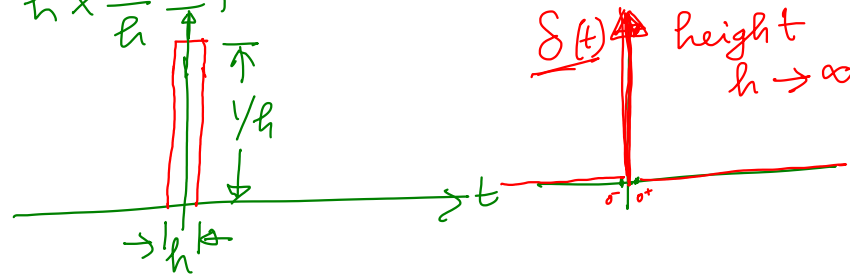
Area under this
rectangular pulse

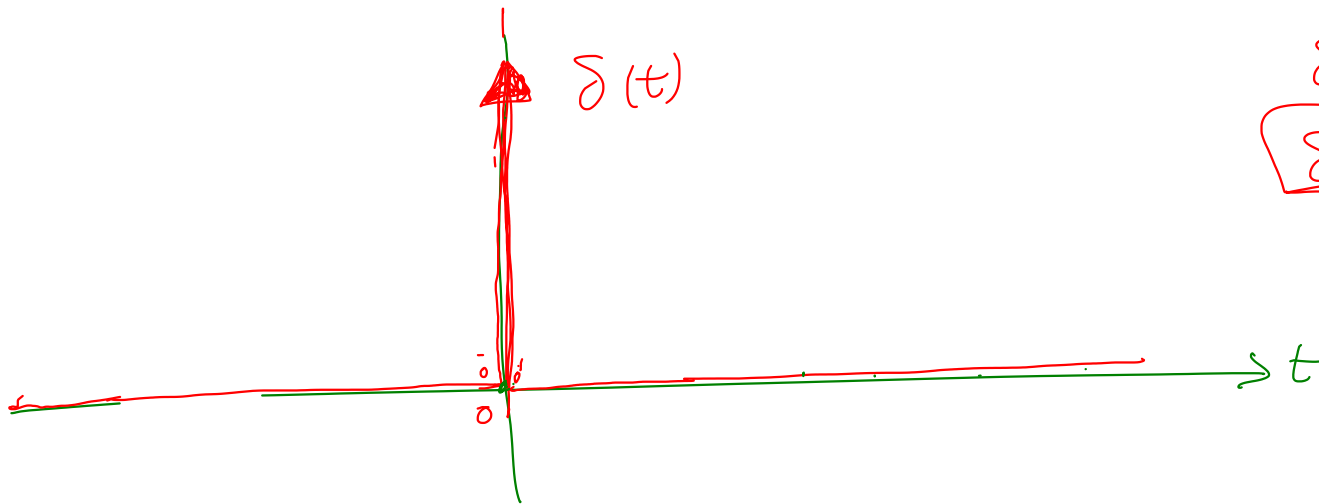
$$= h \times \frac{1}{h} = 1$$



Area
under
the pulse
 $= \frac{1}{2} \times 2$
 $= 1$

Let $h \rightarrow 0$





$\delta(t) = \text{going to } \infty$ (can not be defined)

$$\delta(t) = 0 \text{ if } t \neq 0$$

$$\int_{t=0^-}^{0^+} \delta(t) dt = 1$$

