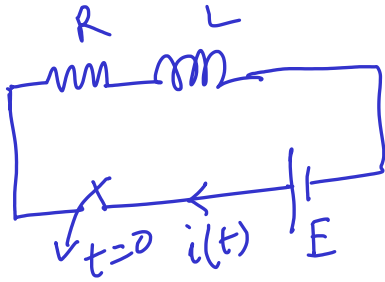


Dec - 18



$$E = L \frac{di}{dt} + Ri$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$$

→ 1st order diff. equⁿ.

Solving a linear diff. equⁿ

$$i(t) =$$

$$\frac{dy}{dt} + ay = x(t)$$

constant forcing fⁿ

$$x(t) \neq \underline{\underline{\delta(t)}}$$

$$y(t) = ?$$
$$y(0^-) \rightarrow y(0)$$

$$e^{at} \frac{dy}{dt} + a e^{at} y = e^{at} x(t)$$

$$\text{or } \frac{d}{dt} (e^{at} y) = e^{at} x(t)$$
$$d(e^{at} y) = e^{at} x(t) dt$$

$$\int d(e^{at} y) = \int e^{at} x(t) dt + A$$

$$\frac{dx}{dt} \frac{e^{at}}{a} = \dot{x} e^{at/a}$$

$$e^{at} y = \int x(t) e^{at} dt + A$$

$$\text{or } y(t) = e^{-at} \int \underbrace{x(t)}_{\text{1st f.?!}} e^{at} dt + A e^{-at} \quad \leftarrow \text{2nd f.?!}$$

$$= e^{-at} \left[x \frac{e^{at}}{a} - \int \dot{x} \frac{e^{at}}{a} dt \right] + A e^{-at}$$

$$= \frac{1}{a} x - \frac{e^{-at}}{a} \int \underbrace{\dot{x}}_{\text{1st f.?!}} e^{at} dt + A e^{-at} \quad \leftarrow \text{2nd f.?!}$$

$$= \frac{1}{a} x - \frac{e^{-at}}{a} \left[\dot{x} \frac{e^{at}}{a} - \int \ddot{x} \frac{e^{at}}{a} dt \right] + A e^{-at}$$

$$y(t) = \frac{1}{a} x - \frac{1}{a^2} \dot{x} + \frac{e^{-at}}{a^2} \int \underbrace{\ddot{x}}_{\text{1st f.?!}} e^{at} dt + A e^{-at}$$

$$y(t) = \frac{1}{a} x - \frac{1}{a^2} \dot{x} + \frac{e^{-at}}{a^2} \left[\ddot{x} \frac{e^{at}}{a} - \int \ddot{x} \frac{e^{at}}{a} dt \right] + A e^{-at}$$

$$y(t) = \frac{1}{a}x - \frac{1}{a^2}\dot{x} + \frac{e^{-at}}{a^2} \left[\ddot{x} \frac{e^{at}}{a} - \int \ddot{x} \frac{e^{at}}{a} dt \right] + A e^{-at}$$

$$= \frac{1}{a}x - \frac{1}{a^2}\dot{x} + \frac{1}{a^3}\ddot{x} - \frac{e^{-at}}{a^3} \int \ddot{x} e^{at} dt + A e^{-at}$$

$\frac{dy}{dt} + ay = x(t)$
forcing f.
system constant

$$y = \left[k_0 x + k_1 \dot{x} + k_2 \ddot{x} + k_3 \dddot{x} + \dots \text{upto } \infty \right] + A e^{-at}$$

$$y(t) = y_f(t) + y_n(t)$$

response of the system when $x(t) = 0$

$$\frac{dy_f}{dt} + ay_f = x(t)$$

Original

$$\text{diff } \frac{dy}{dt} + ay = x(t) \therefore$$

solution due to forcing f.
i.e., $x(t)$

natural response

$$\frac{dy_n}{dt} + ay_n = 0$$

i.e.,

$$\frac{dy}{dt} + ay = 0$$

$$\frac{d}{dt} (y_f + y_n) + a(y_f + y_n) = x(t)$$

$$\text{or } \frac{d}{dt} y_f + a y_f + \left(\frac{dy_n}{dt} + a y_n \right) = x(t)$$

$$y_n(t) = A e^{-at}$$

$$-A a e^{-at} + a A e^{-at} = 0$$

$$\frac{dy}{dt} + \textcircled{2}y = t^2 \quad \begin{array}{l} \swarrow a=2 \\ \swarrow x(t) = t^2 \\ \dot{x} = 2t \\ \ddot{x} = 2 \end{array} \quad \textcircled{y(0) = 2}$$

$$y(t) = \underset{\substack{\uparrow \\ \text{natural} \\ \text{response}}}{A e^{-2t}} + \left[k_1 t^2 + k_2 t + k_3 \right]$$

$$\frac{dy}{dt} + a y = x(t)$$

$$y(t) = \left(k_1 x + k_2 \dot{x} + k_3 \ddot{x} + \dots \infty \right) + A e^{-at}$$

Dec-19

output $\frac{dy}{dt} + ay = x(t)$ or forcing fⁿ. $\rightarrow$ 1st^{order} diff equⁿ.
input $y(0^-) = y(0^+) \rightarrow$ given

$$y(t) = y_n(t) + y_f(t)$$

$$y_f(t) = (k_1 x + k_2 \dot{x} + k_3 \ddot{x} + \dots \infty)$$

$$\frac{dy}{dt} + ay = 0 \Rightarrow y_n(t)$$

$$\frac{dy_n}{dt} + ay_n = 0$$

$$y_n(t) = A e^{-at} = A e^{mt}$$

$$(D+a)y_n = 0$$

$$D \equiv \frac{d}{dt}$$

$$m+a=0 \text{ is characteristic eqⁿ.}$$

$$\text{root } m = -a$$

Example :-

$$\frac{dy}{dt} + 2y = t^2$$

$$\boxed{y(0)=0} \text{ B.C}$$

$$\left(\frac{d}{dt} + 2\right)y = t^2$$

$$\text{ch. eqⁿ } m+2=0$$

$$\boxed{m=-2}$$

$$y(t) = A e^{-2t} + (k_1 t^2 + k_2 t + k_3)$$

$y_f(t)$ alone will satisfy $\frac{dy_f}{dt} + ay_f = x(t)$

$$\frac{dy_f}{dt} + 2y_f = t^2$$

$$y_f(t) = \underline{k_1 t^2 + k_2 t + k_3}$$

$$\underline{2k_1 t} + k_2 + \underline{2k_1 t^2} + \underline{2k_2 t} + 2k_3 = t^2$$

or $\underline{2k_1 t^2} + \underline{2(k_1 + k_2)t} + \underline{2k_3 + k_2} = t^2 \rightarrow \underline{\text{Identity}}$
 Equate the co-efficient of t^2 , t and t^0 on both sides:-

$$2k_1 = 1 \rightarrow k_1 = \frac{1}{2}$$

$$2(k_1 + k_2) = 0 \quad \text{or } k_1 + k_2 = 0 \therefore k_2 = -\frac{1}{2}$$

$$2k_3 + k_2 = 0 \quad \therefore 2k_3 = -k_2 \quad 2k_3 = \frac{1}{2} \quad \text{or } k_3 = \frac{1}{4}$$

$$\underline{y_f(t) = \frac{1}{2}t^2 - \frac{1}{2}t + \frac{1}{4}} \quad \text{soln due to forcing } f=1$$

$$y(t) = A e^{-2t} + \frac{1}{2}t^2 - \frac{1}{2}t + \frac{1}{4} \rightarrow \text{Apply B.C } \underline{y(0)=0}$$

$$\therefore A + \frac{1}{4} = 0 \quad \therefore A = -\frac{1}{4}$$

Total solution of $\frac{dy}{dt} + 2y = t^2$ with B.C $y(0) = 0$

is

$$y(t) = -\frac{1}{4}e^{-2t} + \frac{1}{2}t^2 - \frac{1}{2}t + \frac{1}{4}$$

Ex. 2

$$1 \times \frac{di}{dt} + 3i = 5$$

$$\frac{di}{dt} + 3i = \textcircled{5} \rightarrow \text{forcing } f^n$$

$$\text{ch. eqn. } (m+3) = 0$$

$$i_f(t) = \frac{5}{3}$$

$$\text{root } m = -3$$

$$i(t) = A e^{-3t} + \textcircled{5K_1}$$

soln due to forcing f^n

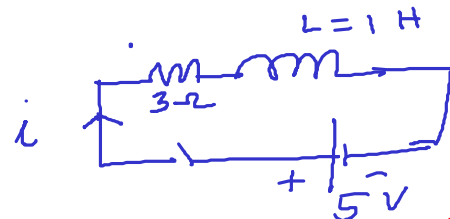
$$0 + 15K_1 = 5 \quad \text{or} \quad K_1 = \frac{1}{3}$$

$$i(t) = A e^{-3t} + \frac{5}{3}$$

$$i(0) = 0 \Rightarrow$$

$$0 = A + \frac{5}{3} \quad \therefore A = -\frac{5}{3}$$

$$i(t) = -\frac{5}{3}e^{-3t} + \frac{5}{3} = \frac{5}{3}(1 - e^{-3t})$$



$$i(0) = 0$$

$$\tau = \frac{1}{3}$$

$$m+a=0$$

$$m = -a$$

$$y(0)=0$$

$$\frac{dy}{dt} + ay = x(t)$$

$$y(t) = y_m(t) + y_f(t)$$

$$= A e^{mt} + (k_1 x + k_2 \dot{x} + k_3 \ddot{x} + \dots \infty)$$

An interesting exception

$$\frac{dy}{dt} + ay = e^{-at}$$

ch. root $-a$

$$e^{at} \frac{dy}{dt} + a e^{at} y = 1$$

$$d(e^{at} y) = dt$$

$$e^{at} y = t + A$$

$$y = t e^{-at} + A e^{-at} = (A+t) e^{-at}$$

$$\frac{d^2 y}{dt^2} + a \frac{dy}{dt} + by = x(t)$$

$$y(t) =$$

Dec-20

$$m^2 + am + b = 0$$

$$D^2 + aD + b = 0$$

$$(D - m_1)(D - m_2) = 0$$

$m_1 \neq m_2$
roots are distinct
def

$$\frac{d^2 y}{dt^2} + a \frac{dy}{dt} + by = x(t)$$

$y(0) = \cancel{u}$
 $\frac{dy}{dt}(0) = \cancel{u}$ where
 $D \equiv \frac{d}{dt}$

$$(D^2 + aD + b)y(t) = x(t)$$

$$(D - m_1)(D - m_2) \underbrace{y(t)}_{z(t)} = x(t)$$

$$(D - m_1)z(t) = x(t)$$

$$(D - m_2)y(t) = z(t)$$

$$(D - m_1)z = x(t) \Rightarrow z(t) = \bar{A} e^{m_1 t} + (\bar{k}_1 x + \bar{k}_2 \dot{x} + \bar{k}_3 \ddot{x} + \dots \infty)$$

$$(D - m_2)y(t) = \bar{A} e^{m_1 t} + (\bar{k}_1 x + \bar{k}_2 \dot{x} + \bar{k}_3 \ddot{x} + \dots \infty) = z(t)$$

ch. root m_2

$$y(t) = B e^{m_2 t} + (\bar{k}_1 z + \bar{k}_2 \dot{z} + \bar{k}_3 \ddot{z} + \dots \infty)$$

$$y(t) = B e^{m_2 t} + A e^{m_1 t} + k_1 x + k_2 \dot{x} + k_3 \ddot{x} + \dots$$

$$\text{let } \frac{d^2 y}{dt^2} + a \frac{dy}{dt} + by = x(t)$$

$$\text{let } (D-m)y = z(t)$$

$$\text{ch. equ.}^n (D-m)(D-m)y = x(t)$$

when roots are repeating $m_1 = m_2 = m$

$$(D-m)z = x(t)$$

$$\therefore z(t) = \bar{A} e^{mt} + (\bar{k}_1 x + \bar{k}_2 \dot{x} + \bar{k}_3 \ddot{x} + \dots \infty)$$

$$(D-m)y(t) = \bar{A} e^{mt} + (\bar{k}_1 x + \bar{k}_2 \dot{x} + \bar{k}_3 \ddot{x} + \dots \infty)$$

$$y(t) = (B+ct)e^{mt} + \bar{k}_1 z + \bar{k}_2 \dot{z} + \bar{k}_3 \ddot{z} + \dots \infty$$

$$= \underline{(B+ct)e^{mt}} + k_1 x + k_2 \dot{x} + k_3 \ddot{x} + \dots \infty$$

(if roots are repeating)

$$\frac{d^3 y}{dt^3} + a \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + cy = x(t)$$

$y(0)$ ✓
 $\frac{dy(0)}{dt}$ ✓ & $\frac{d^2 y}{dt^2}(0)$ ✓

$$(D-m_1)(D-m_2)(D-m_3)y = x(t)$$

$$y(t) = A e^{m_1 t} + B e^{m_2 t} + C e^{m_3 t} + (k_1 x + k_2 \dot{x} + k_3 \ddot{x} + \dots \infty)$$

if roots are distance $m_1 \neq m_2 \neq m_3$

If m_1, m_2, m_3
 $m_2 = m_3 = m$ 2 roots repeating

$$y(t) = A e^{m_1 t} + (Bt + C) e^{mt} + (k_1 x + k_2 \dot{x} + k_3 \ddot{x} + \dots \infty)$$

If $m_1 = m_2 = m_3 = m$

$$y(t) = (A + Bt + Ct^2) e^{mt} + (k_1 x + k_2 \dot{x} + \dots \infty)$$

classical way of solving differential eqn.

$$\frac{d^2 y}{dt^2} + 6 \frac{dy}{dt} + 5y = t \quad y(0)=0, \frac{dy}{dt}(0)=0$$

check

$$m^2 + 6m + 5 = 0$$

$$(m+5)(m+1) = 0$$

$$m_1 = -5 \quad \& \quad m_2 = -1$$

$$y(t) = A e^{-5t} + B e^{-t} + (k_1 t + k_2)$$

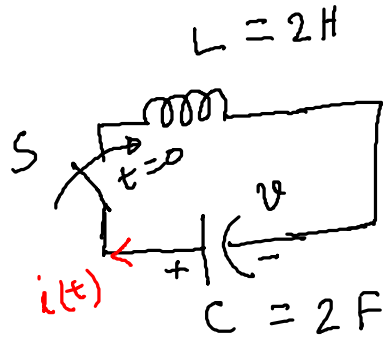
$k_1 t + k_2 \rightarrow$ solution due to forcing fn.

$$\frac{d^2}{dt^2} (k_1 t + k_2) + 6 \frac{d}{dt} (k_1 t + k_2) + 5(k_1 t + k_2) = t$$

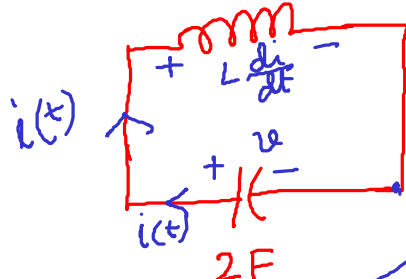
complete

$$v(t) = ?$$

$$i(t) = ?$$



for $t > 0$



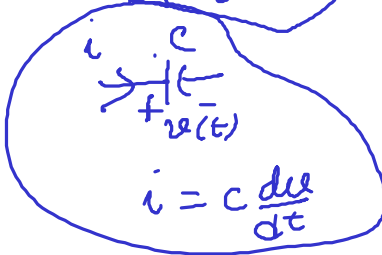
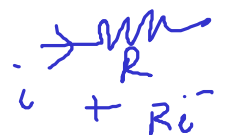
initial condition

$$v(0^-) = \underline{\underline{10V}}$$

$$i(0^-) = 0$$



Dec-21

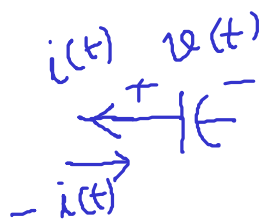


$$v - L \frac{di}{dt} = 0$$

$$C \frac{dv}{dt} = -i \Rightarrow i = -C \frac{dv}{dt}$$

Trying to solve for capacitor voltage $v(t)$

$$LC = 2 \times 2 = 4$$



$$v - L \frac{d}{dt} \left(-C \frac{dv}{dt} \right) = 0$$

$$\text{or } LC \frac{d^2 v}{dt^2} + v = 0$$

$$\Rightarrow \frac{d^2 v}{dt^2} + \frac{1}{LC} v = 0$$

$$\text{or } \frac{d^2 v}{dt^2} + \frac{1}{4} v = 0$$

$$\frac{d^2 v}{dt^2} + \frac{1}{4} v = 0$$

Ch. equⁿ :-

$$m^2 + \frac{1}{4} = 0$$

$$m^2 = -\frac{1}{4}$$

$$m_1 = j\frac{1}{2}, m_2 = -j\frac{1}{2}$$

$$\therefore v(t) = A e^{j\frac{1}{2}t} + B e^{-j\frac{1}{2}t}$$

B. conditions are $\left. \begin{array}{l} v(0) = 10 \text{ V} \\ i(0) = 0 \end{array} \right\}$

$$v(0) = 10 \Rightarrow \boxed{A + B = 10}$$

$$\frac{dv}{dt} = \frac{j}{2} A e^{j\frac{1}{2}t} - \frac{j}{2} B e^{-j\frac{1}{2}t}$$

$$\frac{dv}{dt}(t=0) = \frac{j}{2} A - \frac{j}{2} B = 0$$

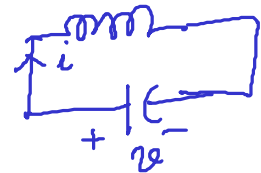
$$v(0^-)$$

$$\frac{dv}{dt}(0^-) \cdot v(0)$$

$$m_1, m_2$$

$$m_1, m_2 = \pm j\frac{1}{2}$$

$$j = \sqrt{-1}$$



$$v(0^-) = 10 \text{ V}$$

$$i(0^-) = 0$$

$$i(t) = -C \frac{dv}{dt}$$

$$= -2 \frac{dv}{dt}$$

$$i(0) = 0$$

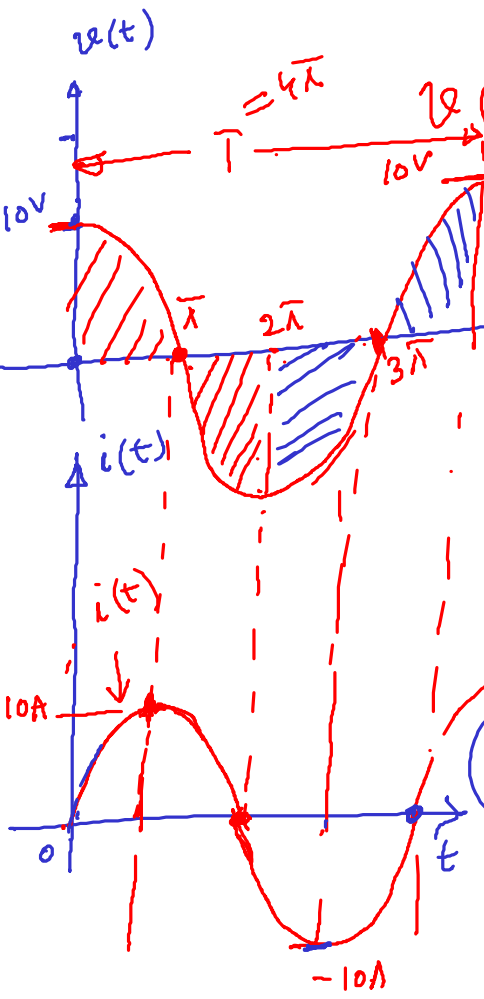
$$\Rightarrow \frac{dv}{dt} = 0 \text{ at } t=0$$

$$\therefore \boxed{A - B = 0}$$

$$\boxed{A = 5}$$

$$\boxed{B = 5}$$

$0 < t < \pi \text{ sec}$



$$v(t) = 5e^{j\frac{t}{2}} + 5e^{-j\frac{t}{2}} = 5(e^{j\frac{t}{2}} + e^{-j\frac{t}{2}})$$

$$= 5 \times 2 \cos \frac{t}{2} = 10 \cos \frac{t}{2}$$

$v(t) = 10 \cos \frac{t}{2}$

volt.

$\frac{t}{2} = \frac{\pi}{2} \quad j = \sqrt{-1}$

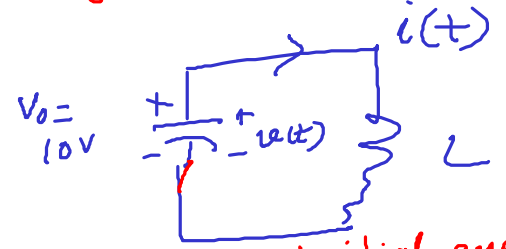
$\omega = \frac{1}{2} \text{ rad/sec} \quad e^{j\theta} + e^{-j\theta} = 2 \cos \theta$

$T = \frac{2\pi}{\frac{1}{2}} = 4\pi$

$e^{j\theta} = \cos \theta + j \sin \theta$

$e^{-j\theta} = \cos \theta - j \sin \theta$

$\omega = \frac{1}{\sqrt{LC}}$



initial energy stored in C

$\frac{1}{2} C V_0^2$

initial energy stored in L

$= 0 \Rightarrow \text{initial } t=0$

$$i(t) = -C \frac{dv}{dt} = -2 \frac{d}{dt} (10 \cos \frac{t}{2}) = 20 \times \frac{1}{2} \sin \frac{t}{2} = 10 \sin \frac{t}{2}$$

$i(t) = 10 \sin \frac{t}{2}$

at any time 't'

$\frac{1}{2} L i^2 + \frac{1}{2} C v^2$

$= i^2 + v^2 = 10^2 \sin^2 \frac{t}{2} + 10^2 \cos^2 \frac{t}{2} = \underline{100 \text{ J}}$

at $t = \pi \text{ sec}$

cap. energy $= \frac{1}{2} \times 2 \times 0^2 = 0$

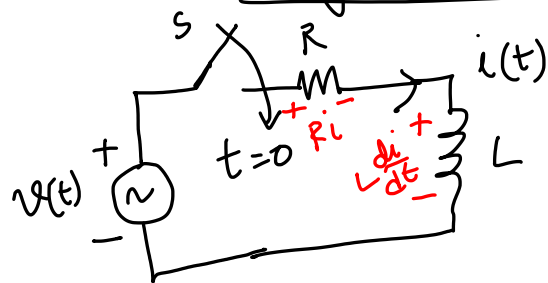
energy of inductor $= \frac{1}{2} \times 2 \times 100 = 100 \text{ J}$

$\frac{1}{2} \times 2 \times 100 = 100 \text{ J}$

Dec-22

R-L series ckt excited from a sinusoidal voltage source

$i(0^-) = 0$ let $v(t) = V_m \sin \omega t$

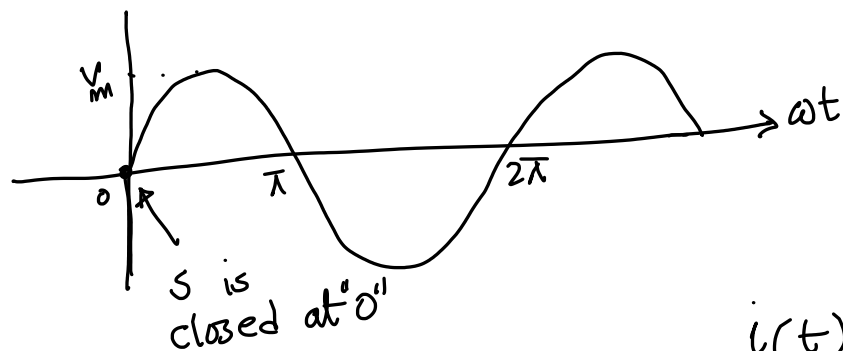


$L \frac{di}{dt} + Ri = v(t) = V_m \sin \omega t$

or $\boxed{\frac{di}{dt} + \frac{R}{L} i = \frac{V_m}{L} \sin \omega t}$

$(D + \frac{R}{L}) i = 0 \rightarrow \text{ch. equ.}$
 $m + \frac{R}{L} = 0$
 $m = -\frac{R}{L}$

will satisfy alone



$i(t) = A e^{-\frac{R}{L} t} + (\bar{K}_1 v + \bar{K}_2 \dot{v} + \bar{K}_3 \ddot{v} + \dots)$

$i(t) = A e^{-\frac{R}{L} t} + (K_1 \sin \omega t + K_2 \cos \omega t)$

↑ natural solⁿ

solⁿ due to forcing fⁿ.

$K_1 \omega \cos \omega t - K_2 \omega \sin \omega t + \frac{R}{L} K_1 \sin \omega t + \frac{R}{L} K_2 \cos \omega t = \frac{V_m}{L} \sin \omega t$

Identity

$\frac{R}{L} K_1 - \omega K_2 = \frac{V_m}{L}$
 $\omega K_1 + \frac{R}{L} K_2 = 0$

$$0 = A + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(-\theta)$$

$$\therefore A = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin \theta$$

$$\frac{R}{L} K_1 - \omega K_2 = \frac{V_m}{L}$$

$$\omega K_1 + \frac{R}{L} K_2 = 0$$

$$K_2 = -\frac{L\omega}{R} K_1$$

$$\frac{R}{L} K_1 + \frac{L\omega^2}{R} K_1 = \frac{V_m}{L}$$

$$\therefore K_1 = \frac{V_m/L}{\frac{R}{L} + \frac{\omega^2 L}{R}} = \frac{V_m}{R + \frac{\omega^2 L^2}{R}} = \frac{R V_m}{R^2 + \omega^2 L^2}$$

let

$$\frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \cos \theta \checkmark$$

Then

$$K_2 = -\frac{\omega L}{R} \cdot \frac{R V_m}{R^2 + \omega^2 L^2} = -\frac{\omega L V_m}{R^2 + \omega^2 L^2}$$

$$\frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \sin \theta$$

$$\frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \sin \theta$$

$$\tan \theta = \frac{\omega L}{R}$$

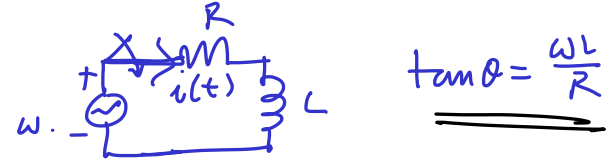
$$i(t) = A e^{-\frac{R}{L}t} + \frac{R V_m}{R^2 + \omega^2 L^2} \sin \omega t - \frac{\omega L V_m}{R^2 + \omega^2 L^2} \cos \omega t$$

$$= A e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \left[\frac{R}{\sqrt{R^2 + \omega^2 L^2}} \sin \omega t - \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} \cos \omega t \right]$$

$$i(t) = A e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \theta)$$

A is to be determined from boundary condition
 $i(0^-) = 0 = i(0)$

So total solution for $i(t)$ [$t \geq 0$]



$$i(t) = \frac{V_m \sin \theta}{\sqrt{R^2 + \omega^2 L^2}} e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \theta)$$

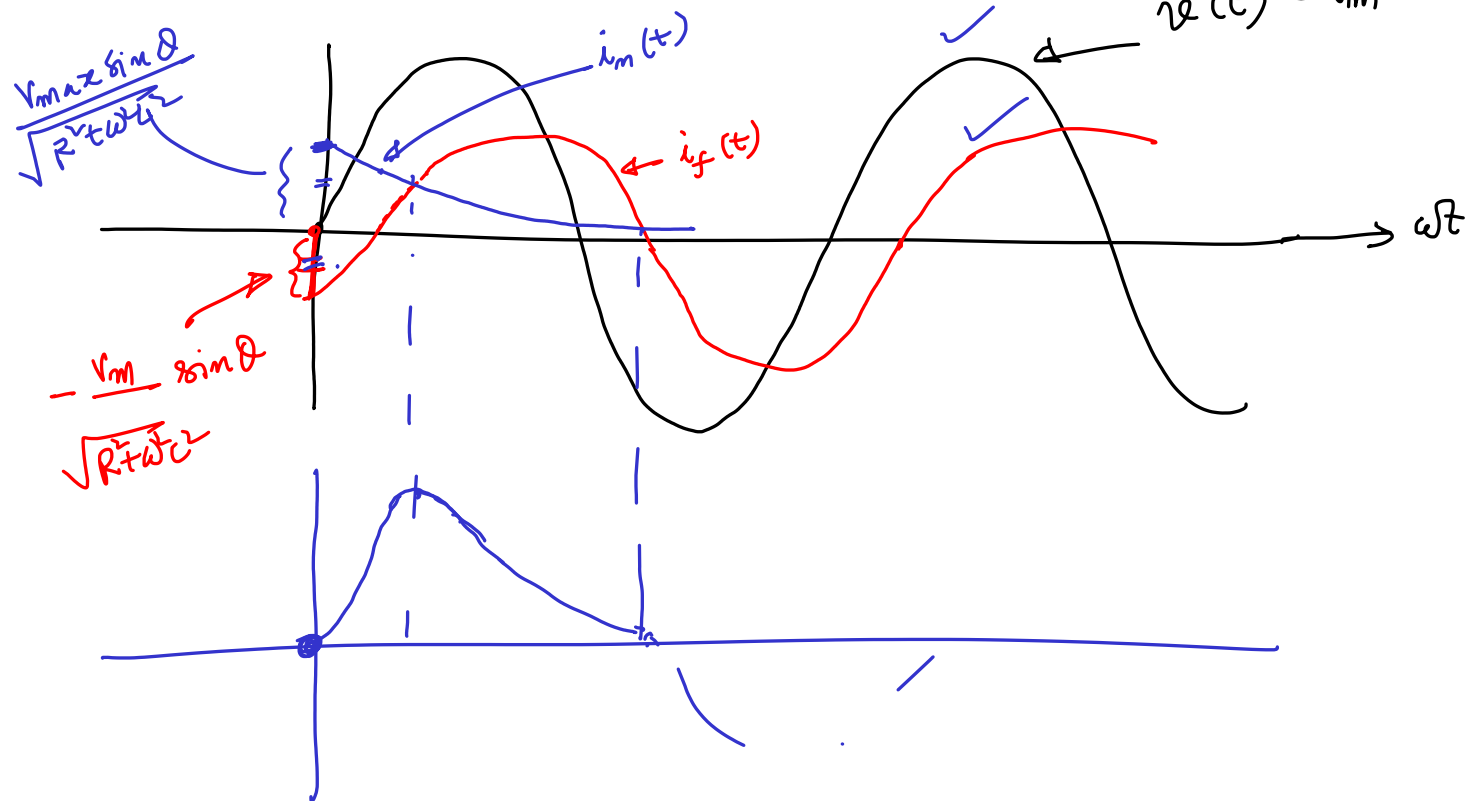
transient part

steady state solution

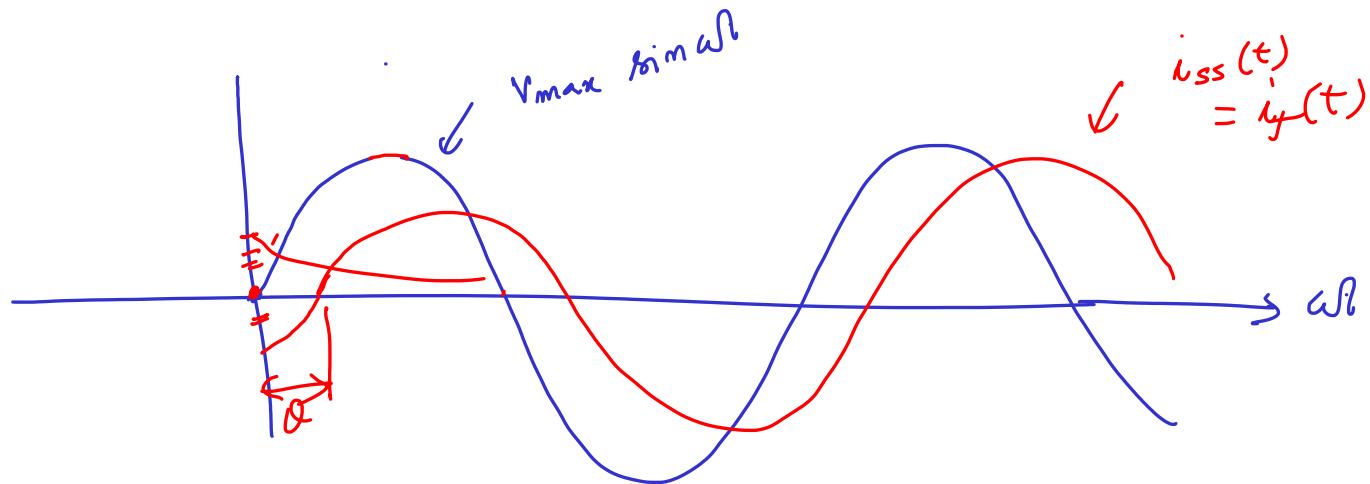
After sufficient time only this term

alone will be present

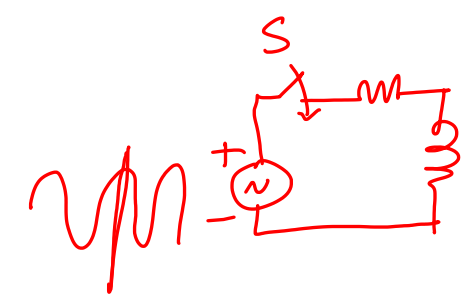
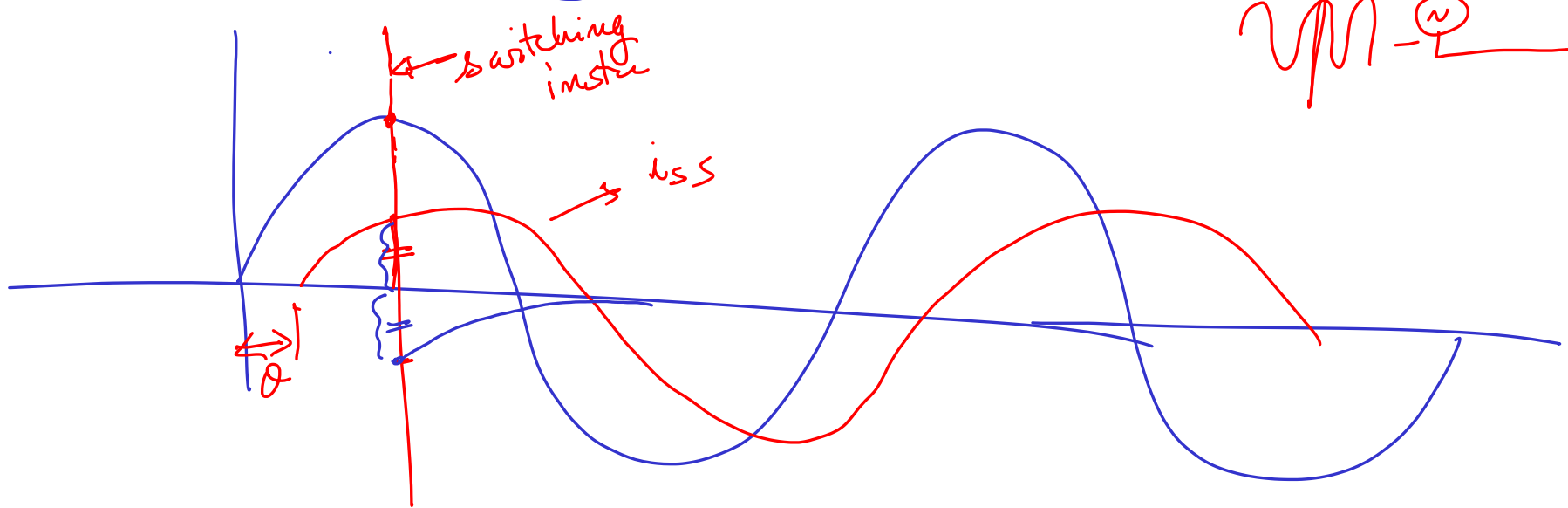
$$v(t) = V_m \sin \omega t$$

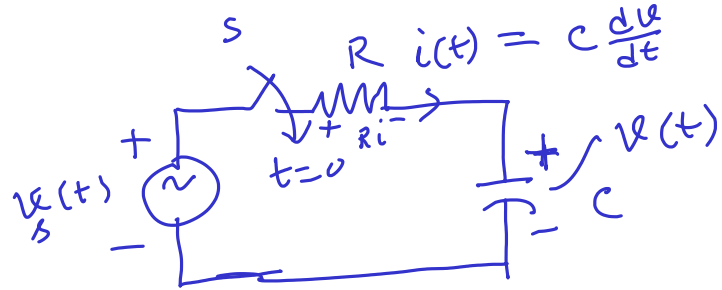
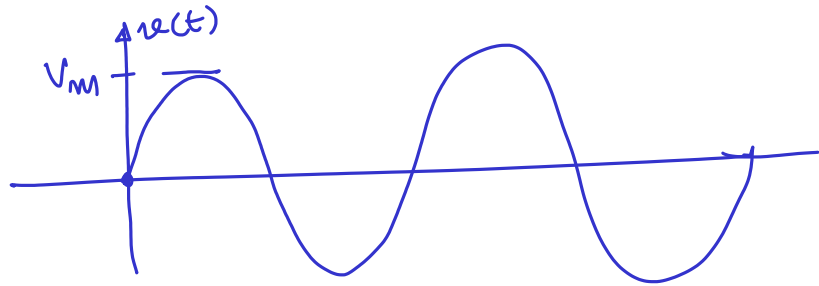


$$\therefore \frac{V_m \sin \theta}{\sqrt{R^2 + \omega^2 L^2}} e^{-\frac{R}{L}t} \rightarrow 0$$



$$\frac{V_{\max}}{Z} \sin(\omega t - \alpha)$$





Let $v_s(t) = V_m \sin \omega t$

what is $v(t) = ?$

$Ri + \frac{1}{C} \int i dt = v_s(t)$

$v(0^-) = 0 \rightarrow \text{B.C.}$
 $= v(0^+) = 0$

KVL :-

$v_s(t) - Rc \frac{dv}{dt} - v = 0$

or $Rc \frac{dv}{dt} + v = V_m \sin \omega t$

$\frac{dv}{dt} + \frac{1}{Rc} v = \frac{V_m}{Rc} \sin \omega t$

$v(t) = A e^{-\frac{t}{Rc}} + \left(\frac{K_1 \sin \omega t + K_2 \cos \omega t}{1} \right)$

ch. equⁿ $m + \frac{1}{Rc} = 0 \therefore m = -\frac{1}{Rc}$

$K_1 \omega \cos \omega t - K_2 \omega \sin \omega t + \frac{K_1}{Rc} \sin \omega t + \frac{K_2}{Rc} \cos \omega t = \frac{V_m}{Rc} \sin \omega t$

$\frac{K_1}{Rc} - \omega K_2 = \frac{V_m}{Rc} \quad (1)$

$\omega K_1 + \frac{K_2}{Rc} = 0$

Identifying
complete

find A

$$\frac{dy}{dt} + ay = x(t)$$

$$\frac{d^2y}{dt^2} + a \frac{dy}{dt} + by = x(t)$$

or d.c (constant)

ch. equⁿ.

$$(D^2 + aD + b)y = Qe^{st}$$

$$y(t) = \frac{Qe^{st}}{(D^2 + aD + b)} \quad D=b$$

If $x(t)$ happens to be an exponential / fⁿ.
Then the solution due to forcing function can be obtained rather easily.

we are looking for the response due to forcing fⁿ only

$$\frac{d^2y}{dt^2} + a \frac{dy}{dt} + by = x(t) = P e^{st}$$

forcing fⁿ.

Guessing the solⁿ to be

$$y = Q e^{st} \quad \left| \begin{array}{l} \frac{dy}{dt} = s Q e^{st} \\ \frac{d^2y}{dt^2} = s^2 Q e^{st} \end{array} \right.$$

$$s^2 Q e^{st} + a s Q e^{st} + b Q e^{st}$$

$$\text{or } Q e^{st} [s^2 + as + b] = P e^{st} = P e^{st}$$

input forcing fⁿ.

$$\therefore Q e^{st} = \frac{P e^{st}}{(s^2 + as + b)} = \frac{\text{input exponential}}{(s^2 + as + b)}$$

$$\frac{d^2 y}{dt^2} + a \frac{dy}{dt} + by = \underbrace{p e^{st}}_{= x(t)}$$

$$y(t) = \underline{A e^{m_1 t} + B e^{m_2 t}} + \left(\underline{k_1} x + \underline{k_2} \dot{x} + \dots \right)$$

$$m^2 + am + b = 0 \Rightarrow \text{roots are } m_1, m_2$$

$$y(t) = A e^{m_1 t} + B e^{m_2 t} + \left. \frac{p e^{st}}{(m^2 + am + b)} \right|_{m=s} = A e^{m_1 t} + B e^{m_2 t} + \frac{p e^{st}}{s^2 + as + b}$$

If excitation is constant

$$p = \text{const} = p e^{0 \cdot t}$$

$$A e^{m_1 t} + B e^{m_2 t} + \left. \frac{p}{(s^2 + as + b)} \right|_{s=0}$$

$$= A e^{m_1 t} + B e^{m_2 t} + \frac{p}{b}$$

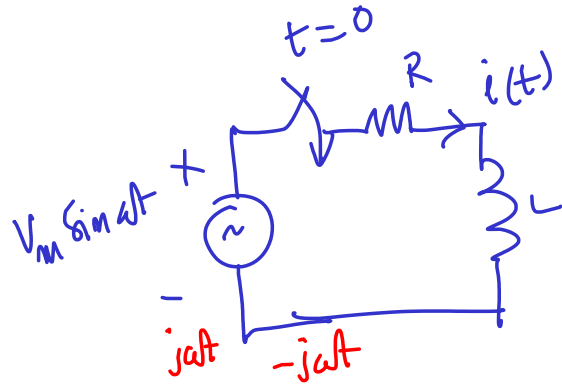
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$$\frac{d^2 y}{dt^2} + a \frac{dy}{dt} + by = \textcircled{P e^{st}}$$

$$(D^2 + aD + b)y = P e^{st}$$

$$D \equiv \frac{d}{dt}$$

$$y_f(t) = \frac{P e^{st}}{(D^2 + aD + b)} \Big|_{D=s} = \frac{P e^{st}}{(s^2 + as + b)}$$



$$\cos \omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$L \frac{di}{dt} + Ri = V_m \sin \omega t \quad \rightarrow \quad i(t) = A e^{-\frac{R}{L}t} + K_1 \sin \omega t + K_2 \cos \omega t$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V_m}{L} \sin \omega t$$

$$\frac{\cos \omega t + j \sin \omega t - (\cos \omega t - j \sin \omega t)}{2j}$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V_m}{L} \sin \omega t = \frac{V_m}{2jL} [e^{j\omega t} - e^{-j\omega t}]$$

$i(0^-) = 0$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V_m}{2jL} e^{j\omega t} - \frac{V_m}{2jL} e^{-j\omega t}$$

$s = j\omega$ $s = -j\omega$

$$i(t) = A e^{-\frac{R}{L}t} + \frac{\frac{V_m}{2jL} e^{j\omega t}}{(D + \frac{R}{L})} - \frac{\frac{V_m}{2jL} e^{-j\omega t}}{(D + \frac{R}{L})}$$

$D = j\omega$ $D = -j\omega$

$$i(t) = A e^{-\frac{R}{L}t} + \frac{V_m}{2jL} \frac{e^{j\omega t}}{(\frac{R}{L} + j\omega)} - \frac{V_m}{2jL} \frac{e^{-j\omega t}}{(\frac{R}{L} - j\omega)}$$

$$= A e^{-\frac{R}{L}t} + \frac{V_m}{2jL} \left[\frac{e^{j\omega t} (\frac{R}{L} - j\omega) - e^{-j\omega t} (\frac{R}{L} + j\omega)}{(\frac{R}{L} + j\omega)(\frac{R}{L} - j\omega)} \right]$$

$$\hat{i}(t)$$

$$= A e^{-\frac{R}{L}t} + \frac{V_m}{2jL} \left[\frac{e^{j\omega t} \left(\frac{R}{L} - j\omega \right) - e^{-j\omega t} \left(\frac{R}{L} + j\omega \right)}{\left(\frac{R}{L} + j\omega \right) \left(\frac{R}{L} - j\omega \right)} \right]$$

$$\begin{aligned} e^{j\omega t} + e^{-j\omega t} &= 2 \cos \omega t \\ e^{j\omega t} - e^{-j\omega t} &= 2j \sin \omega t \end{aligned}$$

$$= A e^{-\frac{R}{L}t} + \frac{V_m}{2jL} \frac{\frac{R}{L} (e^{j\omega t} - e^{-j\omega t}) - j\omega (e^{j\omega t} + e^{-j\omega t})}{\left(\frac{R}{L} \right)^2 + \omega^2}$$

$$= A e^{-\frac{R}{L}t} + \frac{V_m}{\cancel{2j}L} \frac{\frac{R}{L} \cancel{2j} \sin \omega t - \omega \cancel{2j} \cos \omega t}{\left(\frac{R}{L} \right)^2 + \omega^2}$$

$$= A e^{-\frac{R}{L}t} + \frac{V_m}{(R^2 + \omega^2 L^2)} \frac{L}{L} (R \sin \omega t - \omega L \cos \omega t)$$

$$= A e^{-\frac{R}{L}t} + \frac{V_m}{R^2 + \omega^2 L^2} (R \sin \omega t - \omega L \cos \omega t)$$

$$i(t) = A e^{-\frac{R}{L}t} + \frac{V_m}{(R^2 + \omega^2 L^2)} (R \sin \omega t - \omega L \cos \omega t) \quad \checkmark$$

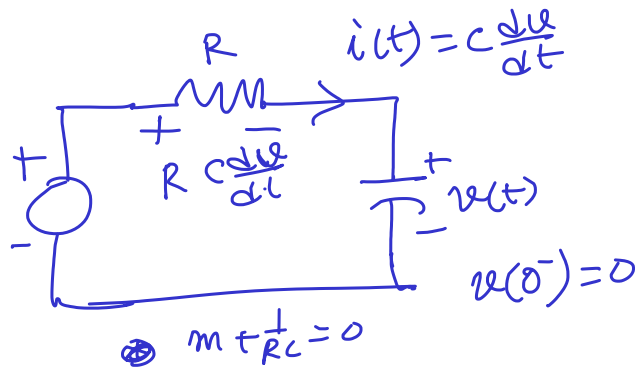


$$= A e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \left[\frac{R \sin \omega t}{\sqrt{R^2 + \omega^2 L^2}} - \frac{\omega L \cos \omega t}{\sqrt{R^2 + \omega^2 L^2}} \right]$$

$\rightarrow \cos \phi$
 $\downarrow \phi - \phi$

$$= A e^{-\frac{R}{L}t} + \frac{V_m}{Z} \sin(\omega t - \phi) \quad \tan \phi = \frac{\omega L}{R}$$

$$v_s(t) = V_m \sin \omega t$$



$$v(t) = A e^{-\frac{1}{Rc}t} + \underbrace{v_f(t)}$$

$$v_s(t) = V_m \sin \omega t = \frac{V_m}{2j} (e^{j\omega t} - e^{-j\omega t})$$

$$RC \frac{du}{dt} + v = v_s(t) = \frac{V_m}{2j} e^{j\omega t} - \frac{V_m}{2j} e^{-j\omega t}$$

$$\frac{du}{dt} + \frac{1}{RC} v = \frac{V_m}{2jRC} e^{j\omega t} - \frac{V_m}{2jRC} e^{-j\omega t}$$

$$v_f(t) = \frac{V_m}{2jRC} e^{j\omega t} - \frac{V_m}{2jRC} e^{-j\omega t}$$

$$\begin{aligned}
 v_f(t) &= \frac{V_m}{2jRC} \frac{e^{j\omega t}}{(\frac{1}{RC} + j\omega)} - \frac{V_m}{2jRC} \frac{e^{-j\omega t}}{(\frac{1}{RC} - j\omega)} \\
 &= \frac{V_m}{2jRC} \left[\frac{(\frac{1}{RC} - j\omega) e^{j\omega t} - (\frac{1}{RC} + j\omega) e^{-j\omega t}}{\frac{1}{R^2C^2} + \omega^2} \right]
 \end{aligned}$$

