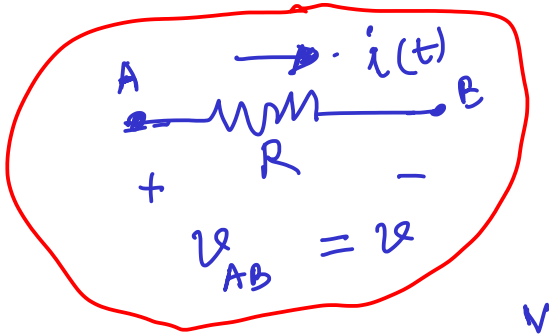
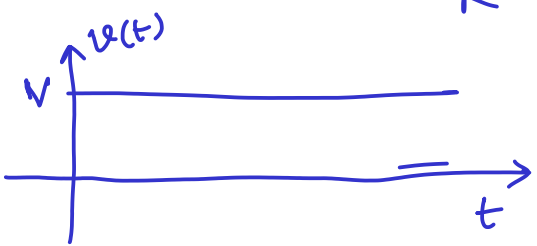
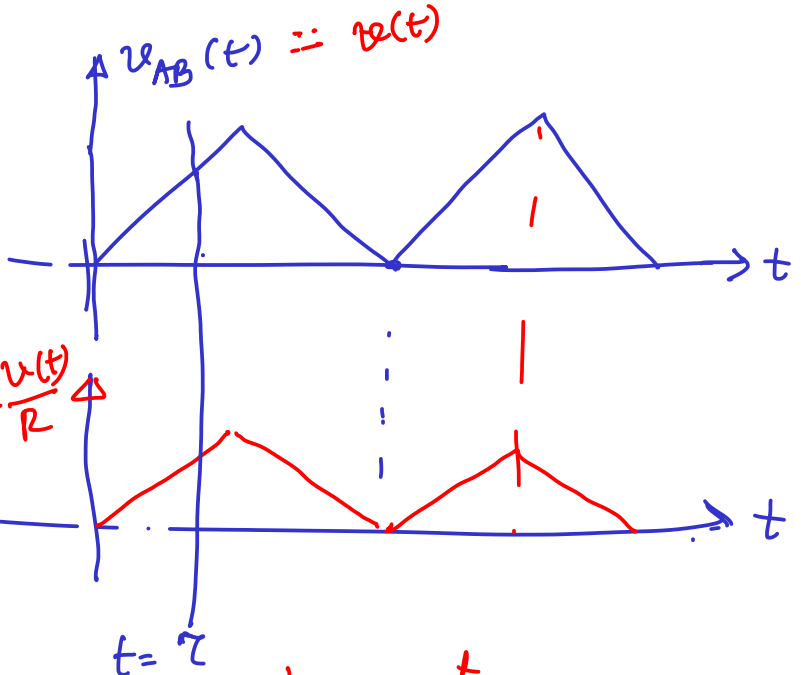


$$i(t) = \frac{v_{AB}(t)}{R} = \frac{v(t)}{R}$$

→ Terminal v-i relation

Lec - 9



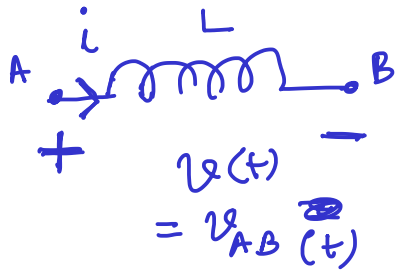


Power $= vi$

$= Ri^2$ → will be lost

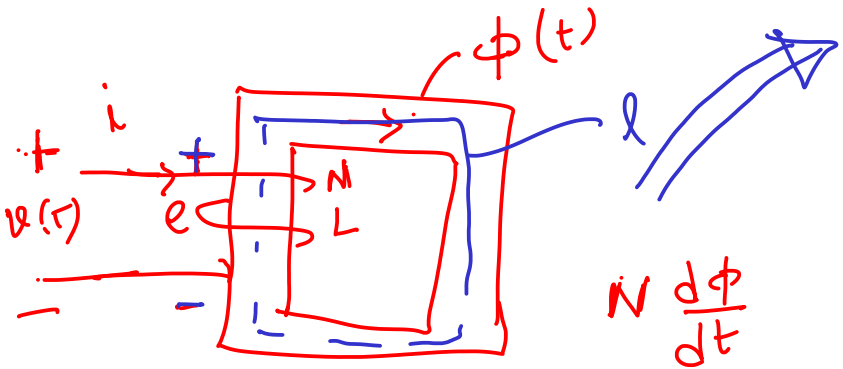
Resistance → is memory less element

Inductor (L)



$v = v_{AB}(t) = L\frac{di}{dt}$

$v = Ri$



$N\frac{d\phi}{dt}$

$e = \frac{N^2 \mu_0 \mu_r A}{l}\frac{di}{dt}$

$e = \frac{d}{dt}(N\phi)$

$= L\frac{di}{dt}$

$= N\frac{d\phi}{dt}$

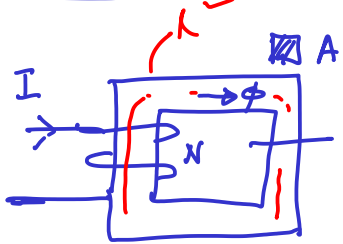
$= N\frac{d}{dt}(B(t)\,A)$

$= N\frac{d}{dt}\left[\mu_0\mu_r \frac{Ni}{l} A\right]$

$L = \mu_0\mu_r \frac{N^2 A}{l}$

$R = \rho\frac{l}{a}$

Li = instantaneous flux linkage with the coil



$H = \frac{NI}{l}$

$L = \frac{\text{flux linkage}}{I}$

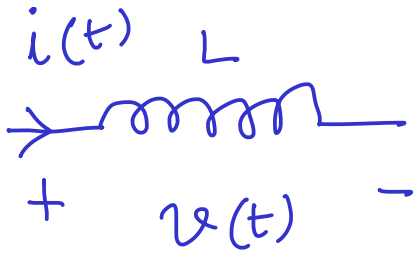
$= \frac{N\phi}{I} \rightarrow \frac{\text{Wb-turns}}{A}$ (H)

$L = \frac{N}{I}\cdot BA$

& $B = \mu_0\mu_r H$

$= \frac{N}{I} A \mu_0\mu_r H$

$L = \frac{NA}{I}\mu_0\mu_r \frac{NI}{l}$



Inductor is a memory ~~device~~ element

$$L\frac{di}{dt} = v$$

$$\frac{di}{dt} = \frac{1}{L}v(t)$$

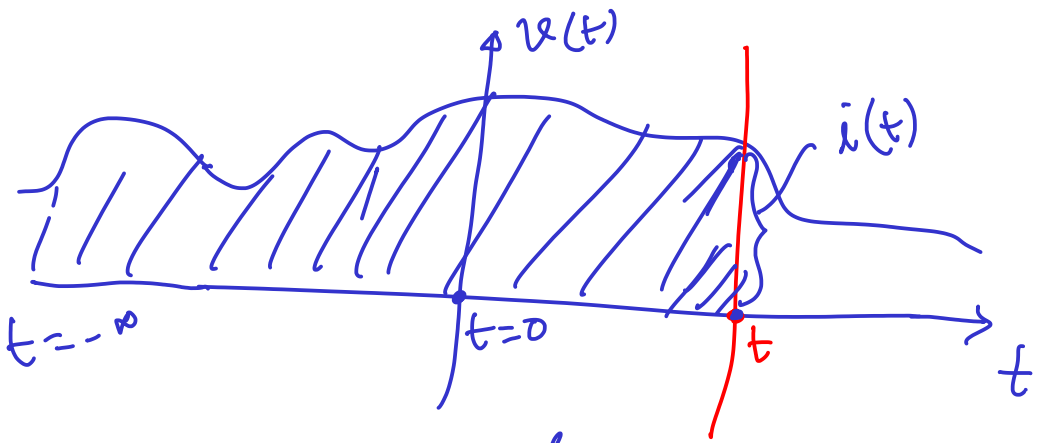
$$di = \frac{1}{L}v(\tau)\,d\tau$$

$$i(t) = \frac{1}{L}\int_{\tau=-\infty}^{t} v(\tau)\,d\tau$$

$$\frac{di}{dt} = \frac{1}{L}v(t)$$

$$\int di = \frac{1}{L}\int_{t=-\infty}^{t} v(t)\,dt$$

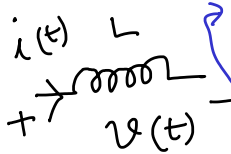
$$i(t) = \frac{1}{L}\int_{t=-\infty}^{t} v(t)\,dt$$

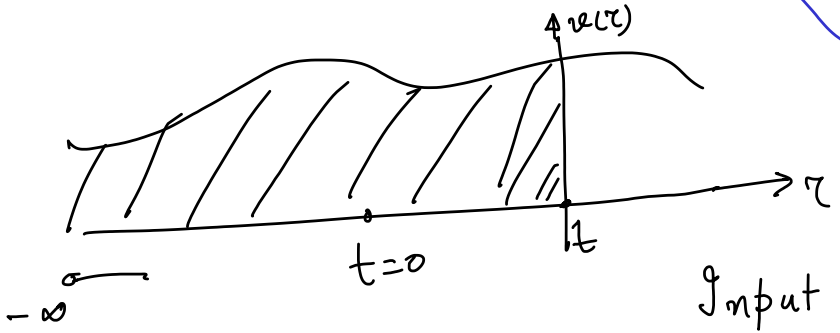


If an inductor carries a fixed current I (which is constant) the voltage across the inductor will be $L\frac{d}{dt}(I) = 0$

Inductor :

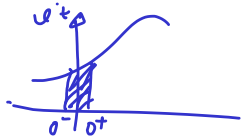
memory.





$$i(0^-) = \frac{1}{L}\int_{t=-\infty}^{0^-} v(t)\,dt$$

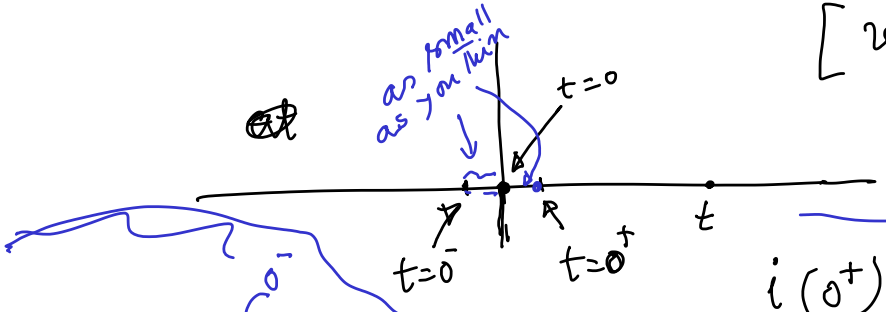
$$v(t) = L\frac{di}{dt}$$



$$i(t) = \frac{1}{L}\int_{t=-\infty}^{t} v\,dt = \frac{1}{L}\int_{t=-\infty}^{t} v(\tau)\,d\tau$$

Input voltage $v(t)$ is a reasonable signal.

[$v(t)$ excludes and impulse $\delta(t)$]



$$i(0^+) = \frac{1}{L}\int_{t=-\infty}^{0^-} v(t)\,dt + 0 = i(0^-) \Rightarrow \boxed{i(0^+) = i(0^-)}$$

$$i(0^+) = \frac{1}{L}\int_{t=-\infty}^{0^-} v(t)\,dt + \frac{1}{L}\int_{t=0^-}^{0^+} v(t)\,dt = 0$$

$$i(0^+) = \frac{1}{L}\int_{t=-\infty}^{t=0^+} v(t)\,dt$$

$$i(0^-) = \frac{1}{L}\int_{t=-\infty}^{0^-} v(t)\,dt$$

$t = 0^-$ or 0 or $0^+ \equiv 0$

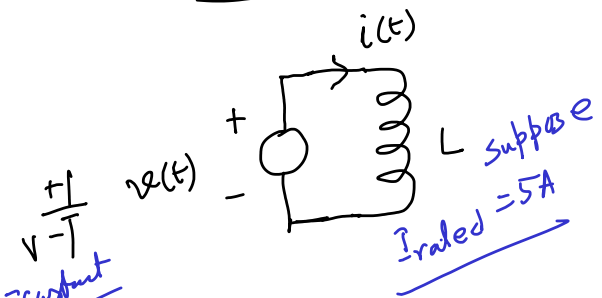
$t > 0$

$$i(t) = \frac{1}{L}\int_{t=-\infty}^{t} v(t)\,dt = \frac{1}{L}\int_{t=-\infty}^{0^-} v(t)\,dt + \underbrace{\frac{1}{L}\int_{t=0^-}^{0^+} v(t)\,dt}_{=0} + \frac{1}{L}\int_{t=0^+}^{t} v(t)\,dt$$

$$i(t) = i(0^-) + \frac{1}{L}\int_{t=0^+}^{t} v(t)\,dt$$

$$i(0^+) = i(0^-)$$

Inductor can store energy



for $t < 0$ $\quad v(t) = 0$

$i(0^-) = 0 \rightarrow$ initial current

$\Rightarrow i(0^+) = 0$

$v(t)$ for $t \geq 0$

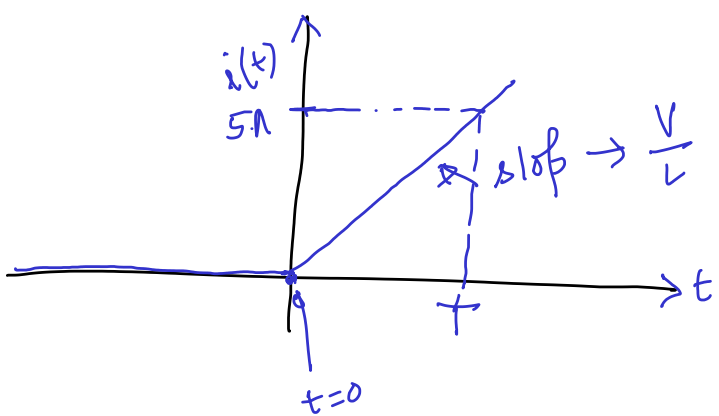
$v(t) = V$

$$i(t) = i(0^-) + \frac{1}{L}\int_{0^+}^{t} v(t)\,dt$$

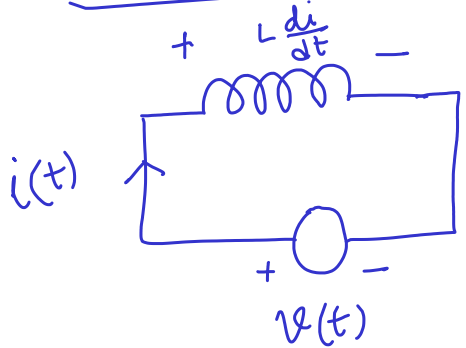
$$i(t) = \frac{1}{L}\int_{0^+}^{t} v(t)\,dt = \frac{1}{L}\int_{0}^{t} v(t)\,dt$$

$$= \frac{1}{L}\int_{0}^{t} V\,dt = \frac{V}{L}t$$

$$i(t) = \frac{V}{L}t$$



Inductor can store energy



$$p(t) = L\frac{di}{dt}\, i$$

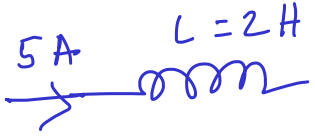
energy supplied to L over a time "dt" ($dt \to 0$)

$$dW = L\frac{di}{dt}\, i\, dt$$

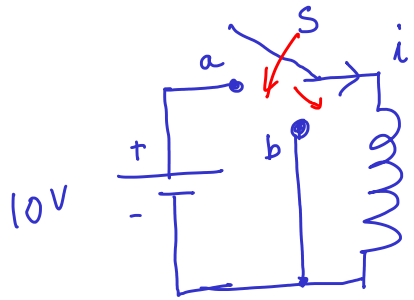
$$dW = L\, i\, di$$

total Energy supplied

$$W = L\int_0^i i(di) = \frac{1}{2} L i^2$$



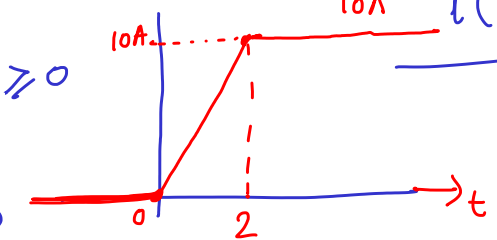
$$\rightarrow \quad \frac{1}{2} \times 2 \times 5^2 = 25\ J$$



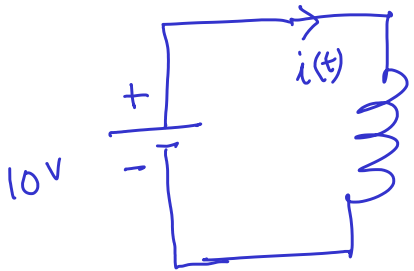
at $t=0$ move the switch to position "a"

at $t=2$ move the switch from "a" to position "b"

$i(0^-) = 0 = i(0^+) = i(0)$



for $t \geq 0$



$v = L\frac{di}{dt} = 10$

or $2\frac{di}{dt} = 10$

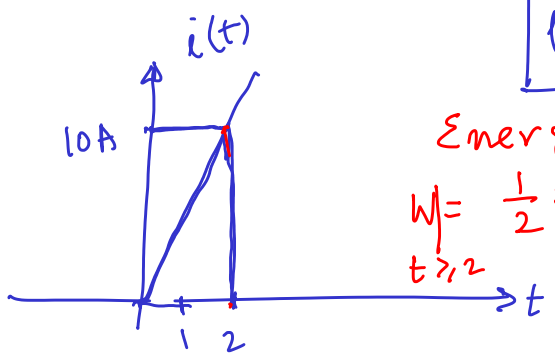
$\frac{di}{dt} = 5$

or $i = 5t + A$

$i(0) = 0 \quad \therefore A = 0$

$\boxed{i = 5t}$ *

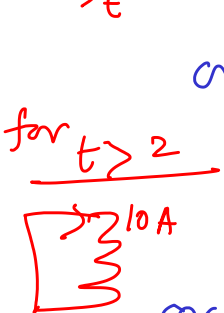
slope



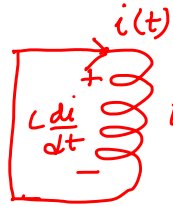
Energy stored

$W = \frac{1}{2} \times 2 \times 100 = 100\ J$ for $t \geq 2$

for $t > 2$



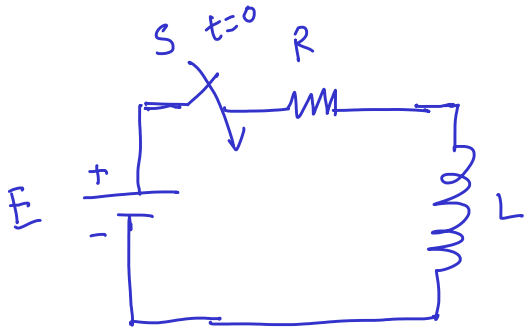
for $t \geqslant 2$



$i(t = 2^-) = 10A$

$i(t = 2^+)$

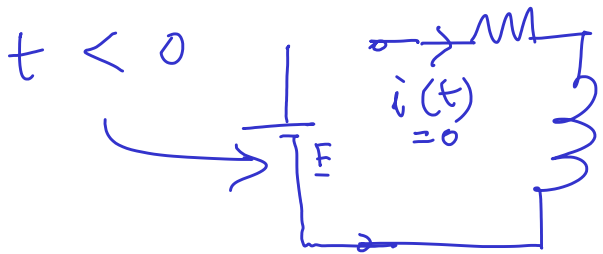
$2\frac{di}{dt} = 0 \Rightarrow$ i must be a constant for $t > 2$



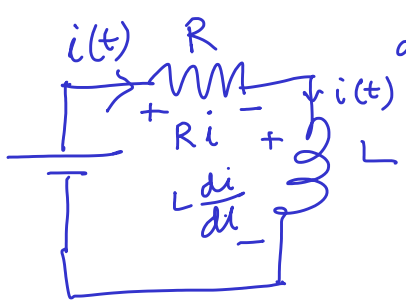
for $t<0$ S → opened.

$i(0^-) = 0$

at $t=0$ S is closed.



ckt for $t \geq 0$



at $t = 0^+$

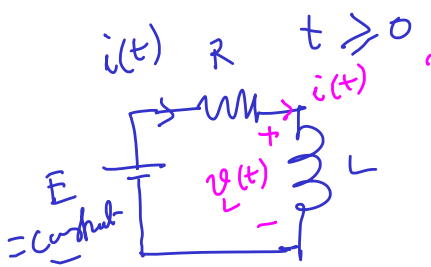
$i(0^+) = i(0^-) = 0$

$= i(0)$

$$E - Ri - L\frac{di}{dt} = 0$$

$$\boxed{L\frac{di}{dt} + Ri = E}$$

1st order linear differential equation.



$v_L(t) = L\frac{di}{dt}$

$L\frac{di}{dt} + Ri = E$

or $\frac{di}{dt} + \frac{R}{L}i = \frac{E}{L}$

$i(0^-) = 0 = i(0^+) = i(0)$

Integrating factor $e^{\frac{R}{L}t}$

$$\left\{ e^{\frac{R}{L}t}\frac{di}{dt} + \frac{R}{L}e^{\frac{R}{L}t} i \right\} = \frac{E}{L} e^{\frac{R}{L}t}$$

$$\int d\left(e^{\frac{R}{L}t} i \right) = \int \frac{E}{L} e^{\frac{R}{L}t} dt + A \quad \text{(A: constant)}$$

$$e^{\frac{R}{L}t} i = \frac{E}{L}\frac{e^{\frac{R}{L}t}}{\frac{R}{L}} + A = \frac{E}{R} e^{\frac{R}{L}t} + A$$

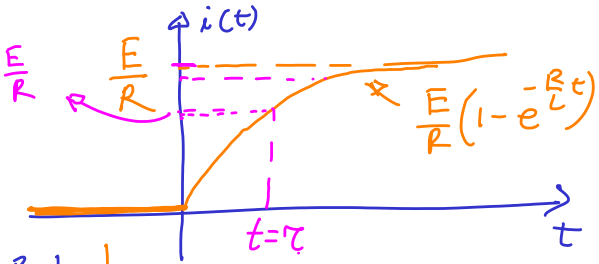
$$i(t) = \frac{E}{R} + A e^{-\frac{R}{L}t}$$

$i(0) = 0$

$0 = \frac{E}{R} + A$ or $A = -\frac{E}{R}$

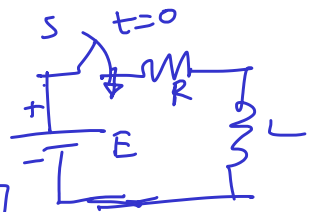
$$i(t) = \frac{E}{R} - \frac{E}{R} e^{-\frac{R}{L}t} = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$i\Big|_{t=\tau} = \frac{E}{R}\left(1 - e^{-1}\right) = 0.632\frac{E}{R}$$



$\frac{L}{R} = \tau$ = time constant

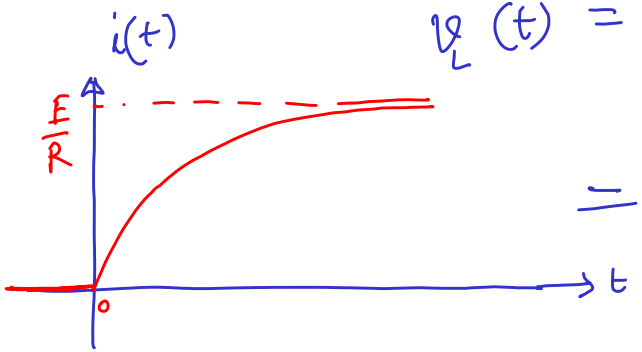
$i(t) = \frac{E}{R}\left(1 - e^{-t/\tau}\right)$



$$i(t) = \frac{E}{R}\left(1 - e^{-t/\tau}\right) \qquad \tau = \frac{L}{R}$$

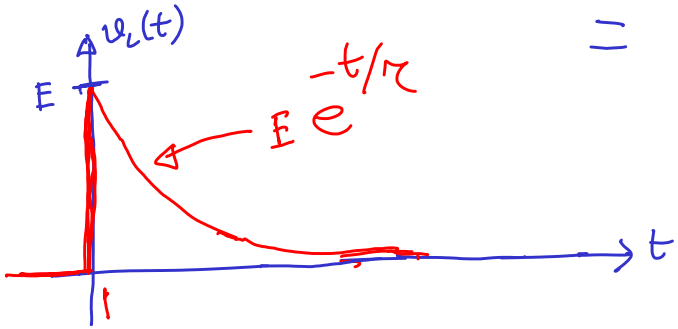
voltage across the inductor.

$$v_L(t) = L\frac{di}{dt} = L\frac{d}{dt}\left[\frac{E}{R}\left(1 - e^{-t/\tau}\right)\right]$$



$$= L\frac{d}{dt}\left(-e^{-t/\tau}\frac{E}{R}\right) = \frac{EL}{R}\frac{d}{dt}\left(-e^{-t/\tau}\right)$$

$$= \frac{EL}{R}\frac{1}{\frac{L}{R}}e^{-t/\tau} = E\,e^{-t/\tau} = v_L(t)$$

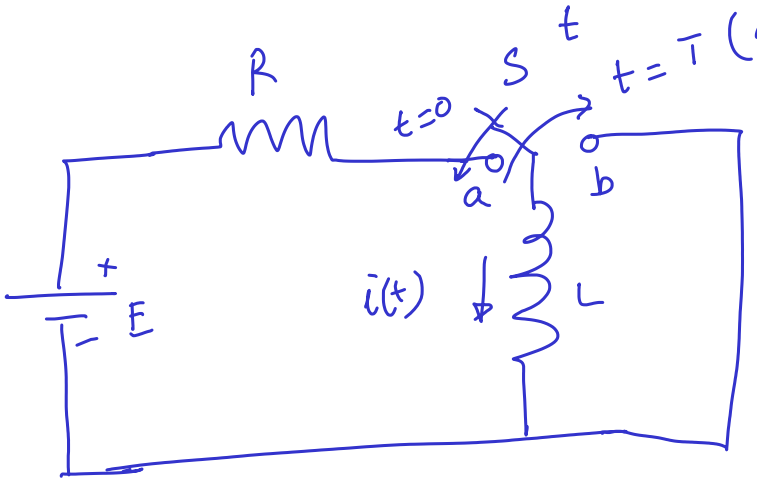


Current through an inductor can not have a step jump!

$v_L(0^-) = 0$

$v_L(0^+) = E\,e^{-0/\tau} = E$

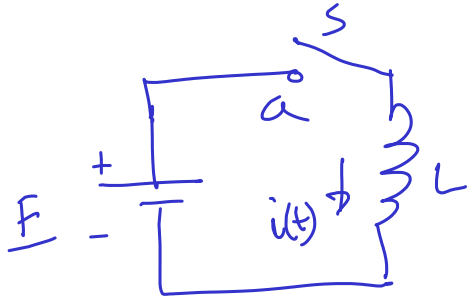
voltage of course can have a step jump.

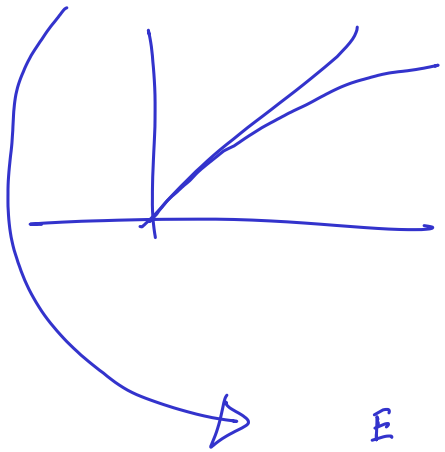


at $t=0$ S to pos. "a"

$$i(t) = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \qquad \tau = \frac{L}{R}$$

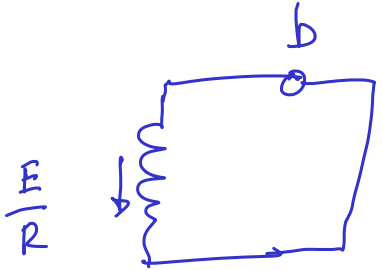
$$= \frac{E}{R}\left(1 - e^{-t/\tau}\right)$$





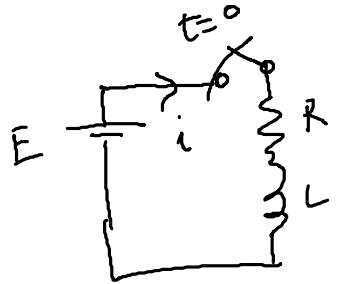
final energy stored in the inductor

$$= \frac{1}{2} L \left(\frac{E}{R}\right)^2 \text{ J}$$



S is moved to pos. 'b'.

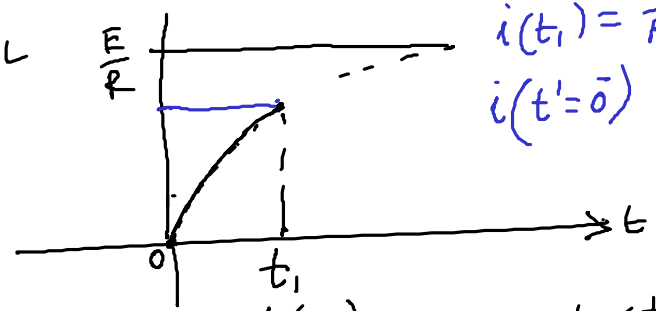
Lec - 13



$$i(t) = \frac{E}{R}\left(1 - e^{-t/\tau}\right) \qquad \tau = \frac{L}{R}$$

$$i(t_1) = \frac{E}{R}\left(1 - e^{-t_1/\tau}\right)$$

$$i(t'=0) = i(t_1)$$



$$\frac{di(t')}{dt'} + \frac{R_L}{L} i(t') = 0$$

$$e^{\frac{R_L}{L}t'} \frac{di}{dt'} + \frac{R_L}{L} e^{\frac{R_L}{L}t'} i = 0$$

$$\frac{d}{dt'}\left(e^{\frac{R_L}{L}t'} i\right) = 0$$

$$\Downarrow$$

$$e^{\frac{R_L}{L}t'} i = A$$

or

$$i(t') = A e^{-\frac{R_L}{L}t'}$$

$$i(t) = \frac{E}{R}\left(1 - e^{-t/\tau}\right) \qquad 0 \le t \le t_1$$

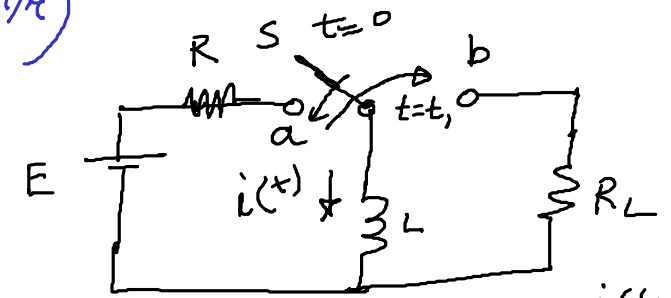
at $t = t_1$ S is moved from pos. a to pos b very quickly (practically in no time)

Let $t' = t - t_1$

$$L\frac{di(t')}{dt} + R_L i(t') = 0$$

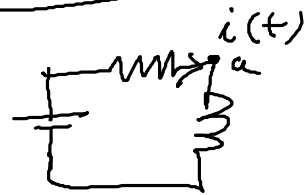
$$i(0) = i(t'=0)$$

$$\therefore \quad i(0) = A = i(t_1)$$

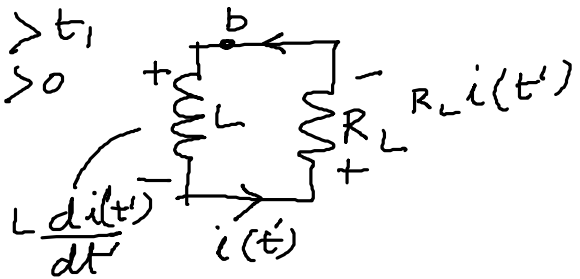


$0 < t < t_1$

$i(t_1) =$

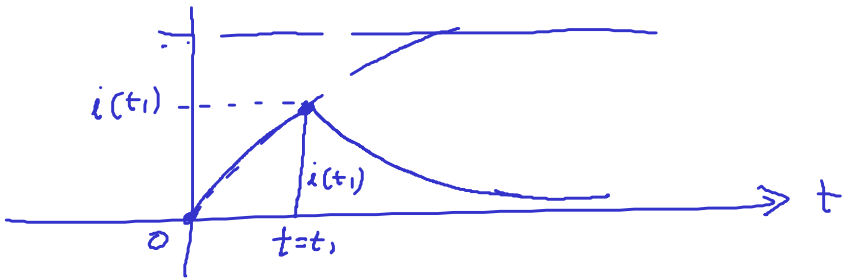


for $t > t_1$ or $t' > 0$



$$i(t') = i(t=t_1)\, e^{-\frac{R}{L}t'}$$

$$= i(t_1)\, e^{-\frac{R}{L}(t-t_1)} \quad \Rightarrow t \geq t_1$$



$\frac{1}{2} L\, i(0^-)^2$

$$L\frac{di}{dt} + R_L i = 0$$

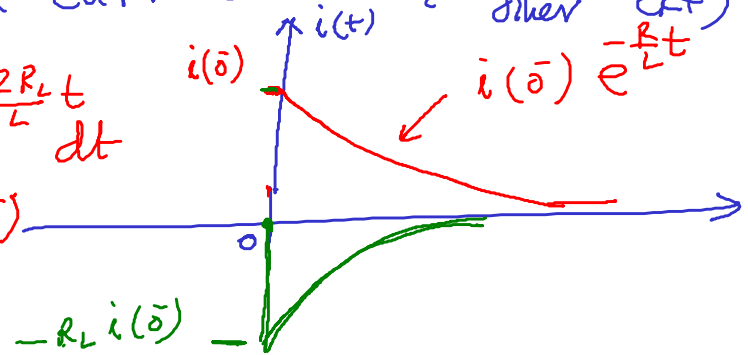
$$\frac{di_L}{dt} + \frac{R_L}{L} i = 0 \Rightarrow i_L(t) = i(0^-)\, e^{-\frac{R_L}{L}t}$$

Energy dissipated in R_L

at $t=0$, R_L is connected across the inductor with initial current $i(0^-)$ { inherited by L from some other ckt

$$W = \int_0^\infty i^2 R_L\, dt = \int_0^\infty i(0^-)^2\, e^{-\frac{2R_L}{L}t}\, dt$$

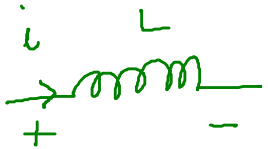
$$= \frac{1}{2} L\, i^2(0^-)$$



$$v_L(t) = L\frac{di}{dt} = L\left(-\frac{R_L}{L}\right) i(0^-)\, e^{-\frac{R_L}{L}t}\, e^{-\frac{R_L}{L}t}$$

$$= -R_L \left(i(0^-)\, e^{-\frac{R_L}{L}t}\right)$$

$$= -R_L\, i(t)$$

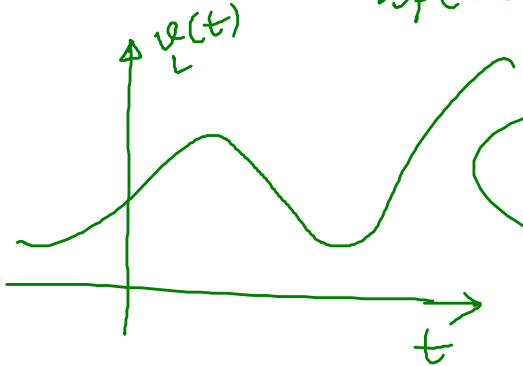


$$v_L = L\frac{di}{dt}$$

To store energy in an inductor we have to pump current into it

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v_L\,dt \rightarrow \text{Volt-sec}$$

To establish current in an inductor we have spend Volt-sec.



$$L\frac{di}{dt} = v_L$$

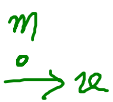
$$\frac{di}{dt} = \frac{v_L}{L}$$

if $v_L = +ve \therefore \frac{di}{dt} > 0$ & current has to rise

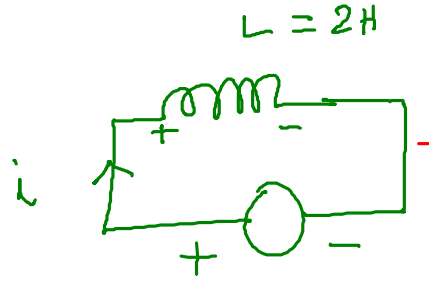
if $v_L = -ve \quad \frac{di}{dt} < 0$

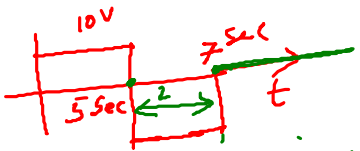
$$m\frac{dv}{dt} = F$$

$$v(t) = \frac{1}{m}\int_{-\infty}^{t} F\,dt$$

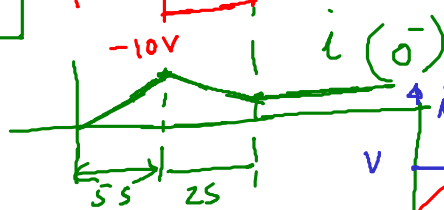


Lec - 14





$$L\frac{di}{dt} = v(t)$$

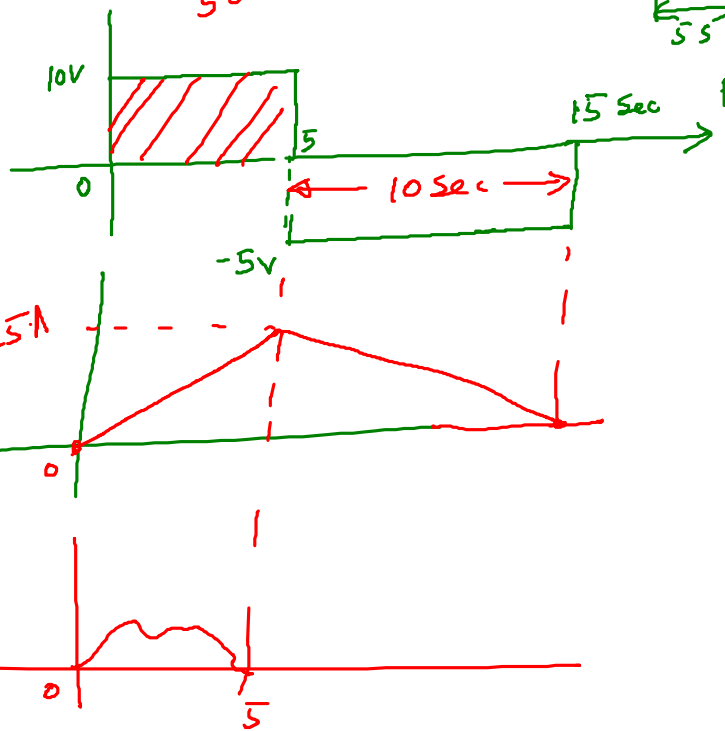


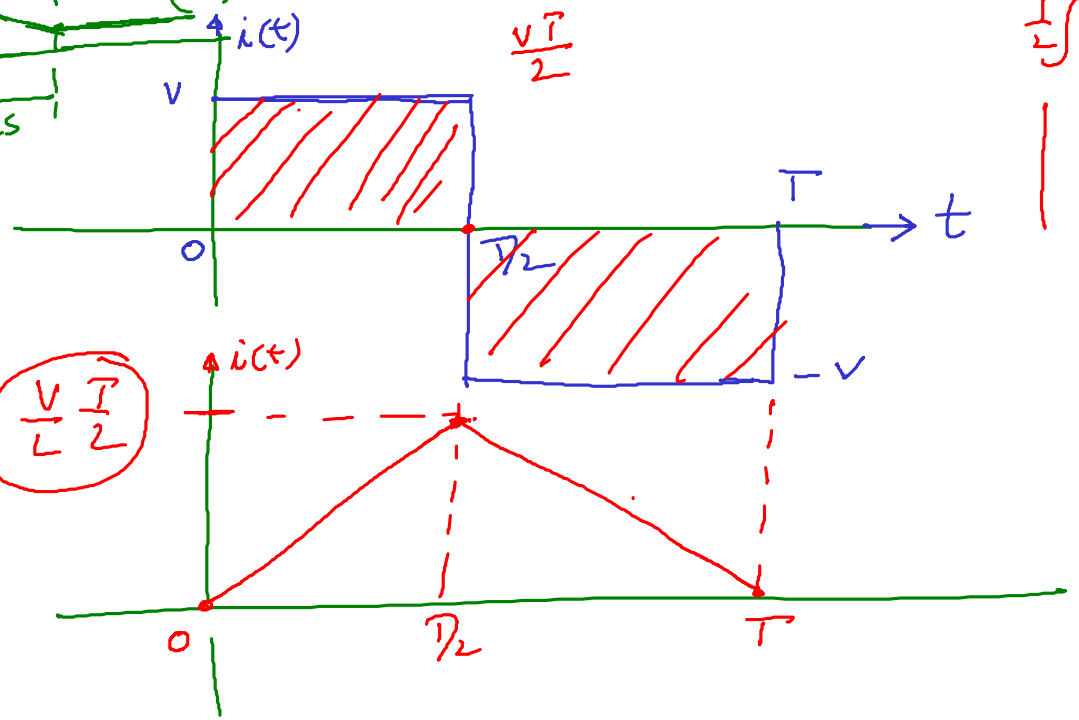
$i(0^-) = 0$

$$\frac{di}{dt} = \frac{v(t)}{L}$$

$$i = \frac{1}{L}\int v\,dt$$

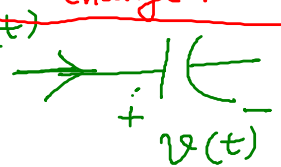
50 V-sec



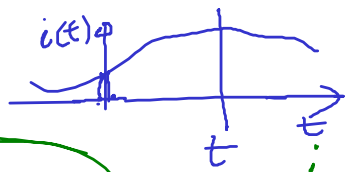


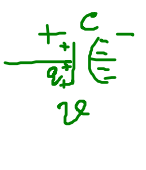
Capacitor C

voltage across the capacitor cannot change instantaneously



$$i(t) = C\frac{dv}{dt}$$





$$v = \frac{q}{C}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v\,dt$$

$$v = \frac{q}{C} = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

→ element with capacitor

$$v(t) = \frac{1}{C}\int_{-\infty}^{0^-} i\,dt + \frac{1}{C}\int_{0^-}^{0^+} i\,dt + \frac{1}{C}\int_{0^+}^{t} i\,dt$$

$\int_{0^-}^{0^+} i\,dt = 0$

current source $i(t)$ is not an impulse source

$$v(0^+) = \frac{1}{C}\int_{-\infty}^{0^-} i\,dt + \frac{1}{C}\int_{0^-}^{0^+} i\,dt = 0$$

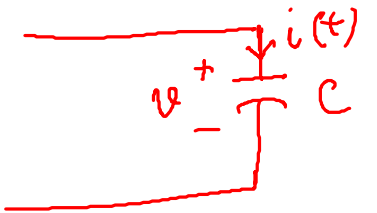
$$v(0^+) = v(0^-)$$

$i \equiv$ Amp $= \frac{\text{Coulomb}}{\text{S}}$

$$q = \int_{-\infty}^{t} i\,dt$$

= present charge on the plates of the capacitor

Capacitor stores energy :



Let at $t=0$ $\quad v(0^-) = 0 = v(0^+) = v(0)$

instantaneous power absorbed

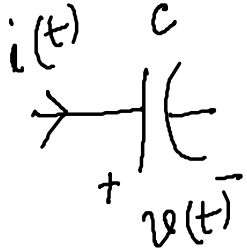
$$p(t) = v\, C\frac{dv}{dt} = C\, v \frac{dv}{dt}$$

$\therefore$ energy absorbed by the capacitor in time dt

$$dW = p(t)\,dt = C\, v \frac{dv}{dt} dt = C\, v\, dv$$

$$W = C \int_{v=0}^{v=V} v\, dv = \frac{1}{2} C V^2$$

Lec-15



$$i(t) = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt = \frac{1}{C}\int_{-\infty}^{0^-} i\,dt + \int_{0^-}^{0^+} i\,dt + \int_{0^+}^{t} i\,dt$$

$0^- \Downarrow 0$

$$v(t) = v(0^-) + \int_{0^+}^{t} i\,dt$$

$$v(0^+) = v(0^-) + \int_{0^+}^{0^+} i\,dt = v(0^-)$$

$\frac{1}{2}Cv^2$ Joules

$C \to F$

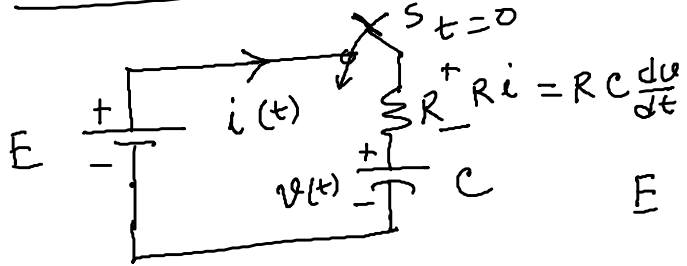
$v \to$ volts.

$\tau = \frac{L}{R}$

$i(0^+) = i(0) = \frac{E}{R}$

Charging a capacitor

$\frac{1}{2}CE^2$



$v(0^-) = 0 = v(0^+) = v(0)$

$i = C\frac{dv}{dt}$

$Ri = RC\frac{dv}{dt}$

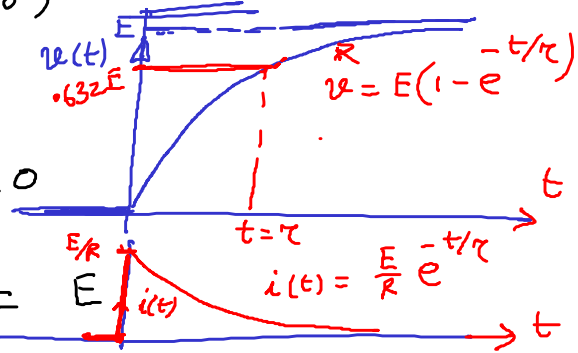
$E - RC\frac{dv}{dt} - v = 0$

or

$RC\frac{dv}{dt} + v = E$

or

$$\frac{dv}{dt} + \frac{1}{RC}v = \frac{E}{RC}$$



$e^{\frac{t}{RC}}\,v = \frac{E}{RC\frac{1}{RC}}\,e^{t/RC} + A$

$$v = E + Ae^{-t/RC}$$

$\tau = RC$ = time constant

$v(0) = 0 \Rightarrow 0 = E + A$

$A = -E$

$v(t) = E - Ee^{-t/RC} = E\left(1 - e^{-t/RC}\right)$

$v(t=\infty) = E = E\left(1 - e^{-t/\tau}\right)$

$e^{\frac{1}{RC}t}\frac{dv}{dt} + \frac{1}{RC}e^{\frac{1}{RC}t}v = \frac{E}{RC}e^{\frac{1}{RC}t}$

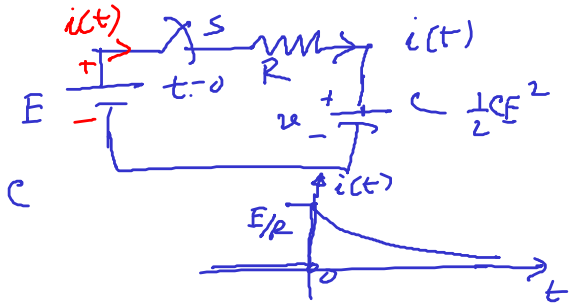
$\int d\left(e^{+\frac{t}{RC}}v\right) = \int \frac{E}{RC}e^{\frac{1}{RC}t}\,dt$

$i(t) = C\frac{dv}{dt}$

$= CE\frac{1}{RC}e^{-t/RC}$

$i(t) = \frac{E}{R}e^{-t/RC}$

Energy dissipated in R while charging the capacitor to the supply voltage (E)



$\tau = RC$

$v(t) = E\left(1 - e^{-t/\tau}\right)$ → is independent of the value of R

$i(t) = \frac{E}{R} e^{-t/RC}$

Energy dissipated in R = energy stored in cap = $\frac{1}{2}CE^2$

$p(t) = i^2 R$ = instantaneous power loss in the ckt

$dw = p\,dt = i^2 R\,dt = \frac{E^2}{R^2} e^{-\frac{2t}{RC}} R\,dt$

Energy dissipated in R in time dt

∴ total energy dissipated in R

$$W = \int_{t=0}^{\infty} \frac{E^2}{R^2} e^{-\frac{2t}{RC}} R\,dt = \frac{E^2}{R} \left. \frac{e^{-\frac{2t}{RC}}}{-\frac{2}{RC}} \right|_{t=0}^{\infty}$$

$$= \frac{E^2}{R} \frac{RC}{2} \left. e^{-2t/RC} \right|_{\infty}^{0} = \frac{1}{2} C E^2 \left[1 - 0\right] = \frac{1}{2} C E^2$$

Energy delivered @ $W = \int_0^{\infty} E\,i\,dt$ (E i(t))

$= E \int_0^{\infty} \frac{E}{R} e^{-t/RC}\,dt$

$= \frac{E^2}{R} \left. \frac{e^{-t/RC}}{-1/RC} \right|_0^{\infty} = CE^2 \left. e^{-t/RC} \right|_{\infty}^{0}$

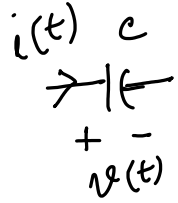
$= CE^2$

Total Amount of power delivered by the source

Energy diss. in R = energy stored in cap = $\frac{1}{2}CE^2$

1 | 100 mJ

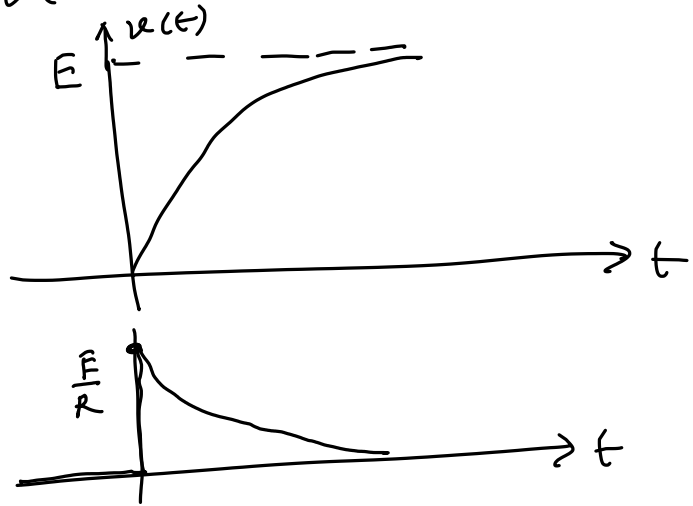
Lec 16

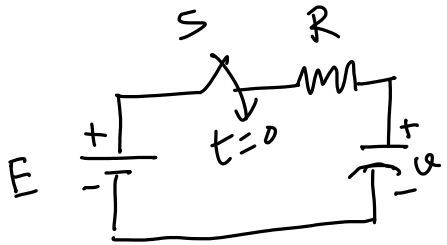


$$i(t) = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$v(0^+) = v(0^-) = v(0)$$





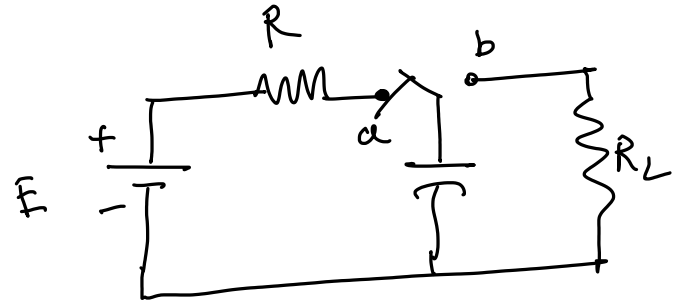
$$v(0^-) = 0$$

$$v(t) = E(1 - e^{-t/\tau})$$

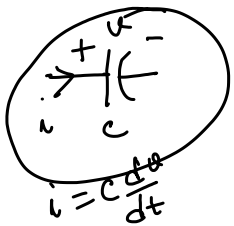
$$\tau = RC$$

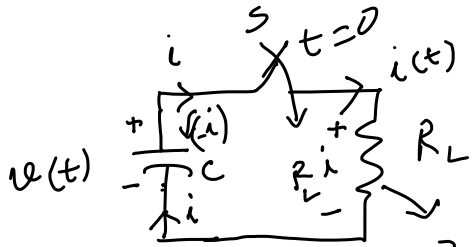
$$i(t) = \frac{E}{R}e^{-t/\tau}$$

$$W_C(t) = \frac{1}{2}Cv^2$$



$$v(0^-) = 0$$

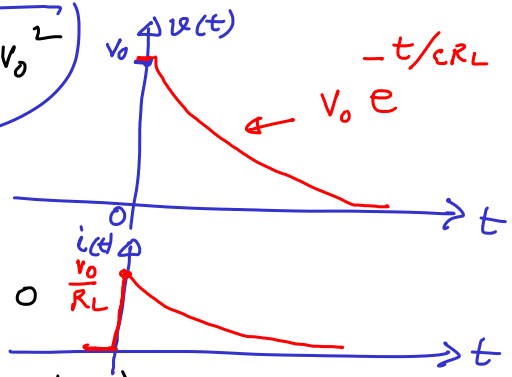




$i(t) = \frac{v(t)}{R_L}$

$v(0^-) = V_0$

$\frac{1}{2} C V_0^2$



KVL :-

$i(t) = \frac{R_L i}{-R_L}$

$= -C\frac{dv}{dt}$

$v(0) = V_0$

$-i = C\frac{dv}{dt}$

$i = -C\frac{dv}{dt}$

$\Rightarrow i(t) = -C\frac{d}{dt}\left(V_0 e^{-t/CR_L}\right)$ or

$= -C\left(-\frac{1}{CR_L}\right)V_0 e^{-t/CR_L}$ or

KVL :-

$v - R_L i = 0$

$v - R_L\left(-C\frac{dv}{dt}\right) = 0$

$CR_L\frac{dv}{dt} + v = 0$

$\frac{dv}{dt} + \frac{v}{CR_L} = 0$

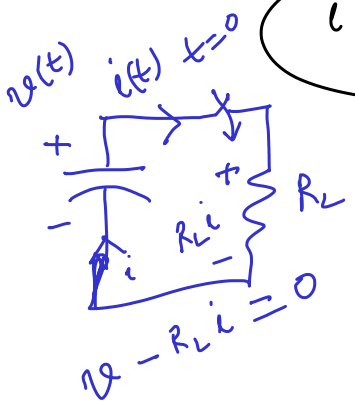
$e^{\frac{t}{CR_L}}\frac{dv}{dt} + \frac{1}{CR_L}e^{t/CR_L} v = 0$

or $\frac{d}{dt}\left(e^{t/CR_L} v\right) = 0$

$\int_{t=0}^{\infty} i^2 R_L\, dt$

= Total Energy dissipated in R_L

$= \frac{1}{2} C V_0^2$



$\therefore e^{t/CR_L} v = A$

or $v = A e^{-t/CR_L}$

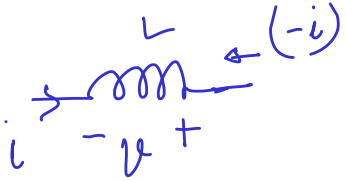
$v(0^-) = v(0^+) = v(0) = V_0$

Get $A = V_0$

$v(t) = V_0 e^{-t/CR_L}$

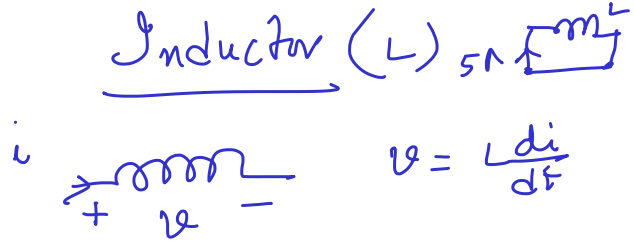
$-i = C\frac{dv}{dt}$

$i(t) = \frac{V_0}{R_L} e^{-t/CR_L}$



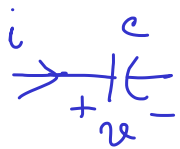
$$v = L \frac{d}{dt}(-i)$$

Inductor (L)



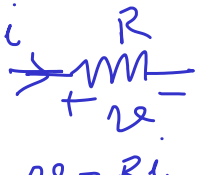
$$v = L\frac{di}{dt}$$

Capacitor



$$i = C\frac{dv}{dt}$$

Resistance



$$v = Ri$$

Are these element linear or not-?

v is input signal
i is the output response

$v = 0 \qquad i = 0$

Homogeneity cond.

Principle Superposition:-

$v_1 \longrightarrow i_1$
$v_2 \longrightarrow i_2$

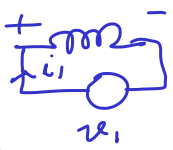
$v_1 + v_2 \longrightarrow i_1 + i_2$

$$i_3(t) = \frac{1}{L}\int_0^t (v_1 + v_2)\,dt$$

$$= i_1 + i_2$$

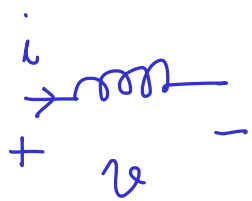
$$i_1(t) = \frac{1}{L}\int_0^t v_1(t)\,dt$$

$$i_2(t) = \frac{1}{L}\int_0^t v_2\,dt$$



Add

$$i_1 + i_2 = \frac{1}{L}\int_0^t (v_1 + v_2)\,dt$$

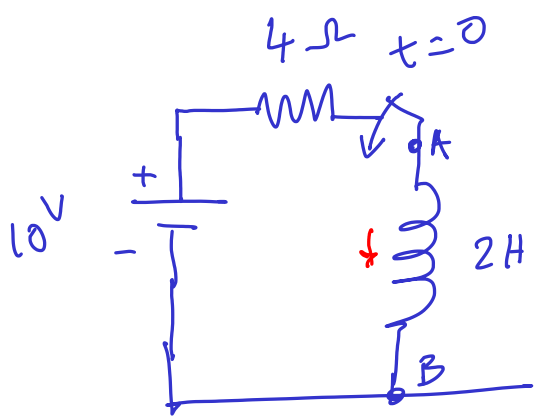


$$i = \frac{1}{L}\int_0^t v\,dt$$

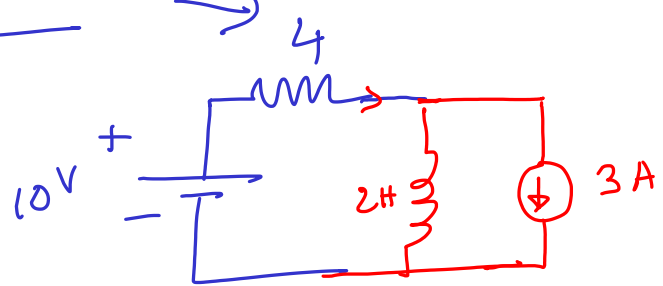
$$i(0^-) = 0$$

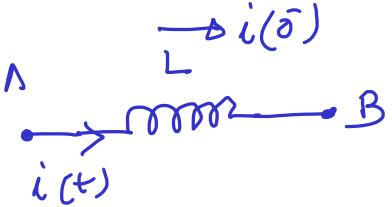
What happens when inductor has finite initial current $i(0^-) \neq 0$

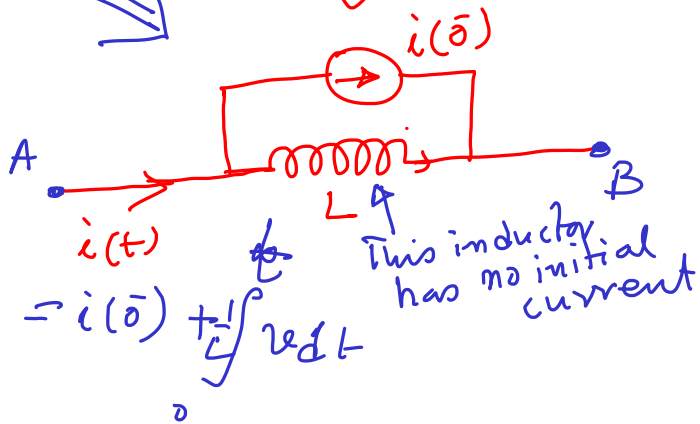
$$i(t) = i(0^-) + \frac{1}{L}\int_0^{\infty} v(t)\,dt$$



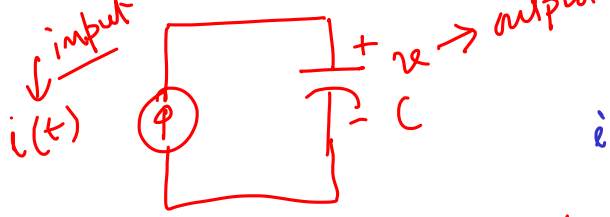
$t > 0$







Is capacitor a linear element



$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt = \left(\frac{1}{C}\int_{-\infty}^{0^-} i\,dt\right) + \frac{1}{C}\int_{0}^{t} i\,dt$$

V-S

A-S → C_n

i → not an impulse

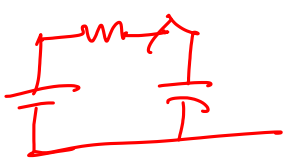
$i_1 + i_2$

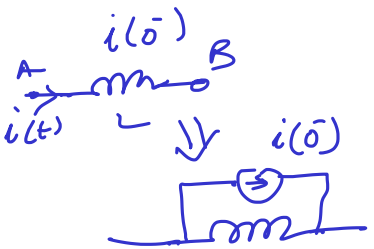
$$v_3 = 2 + C^{-1}\int_0^t (i_1+i_2)\,dt^{\infty}$$

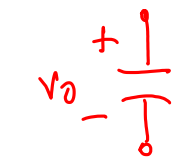
0 input → 0 output

$$v(t) = v(0^-) + \frac{1}{C}\int_0^t i\,dt$$

let $v(0^-) = 0$







let $v(0^-) = 2V$

input → output

$i_1 \longrightarrow v_1$

$$v_1 = 2 + \frac{1}{C}\int_0^t i_1\,dt$$

$$v_1(t) = \frac{1}{C}\int_0^t i_1\,dt$$

$i_2 \longrightarrow v_2$

$$v_2 = 2 + \frac{1}{C}\int_0^t i_2\,dt$$

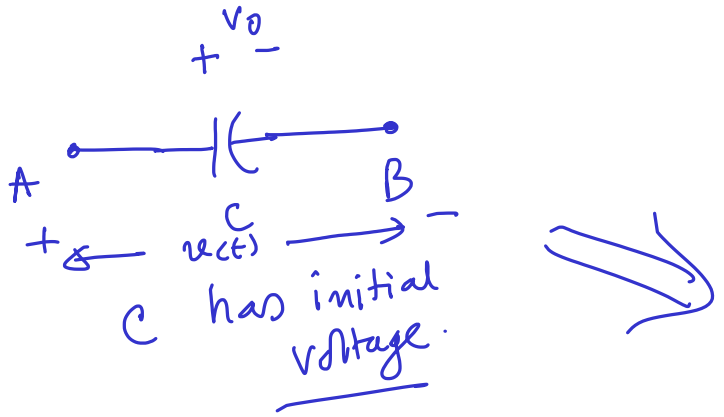
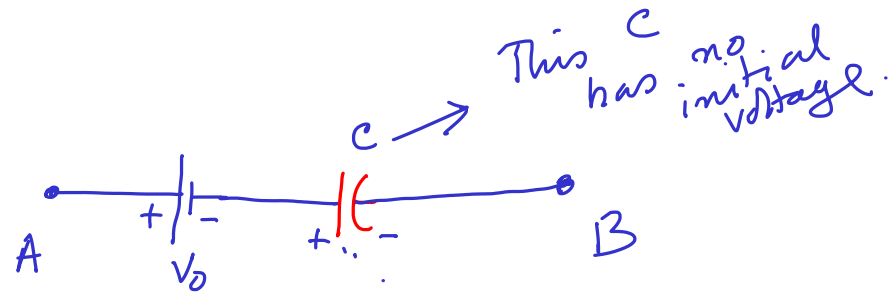
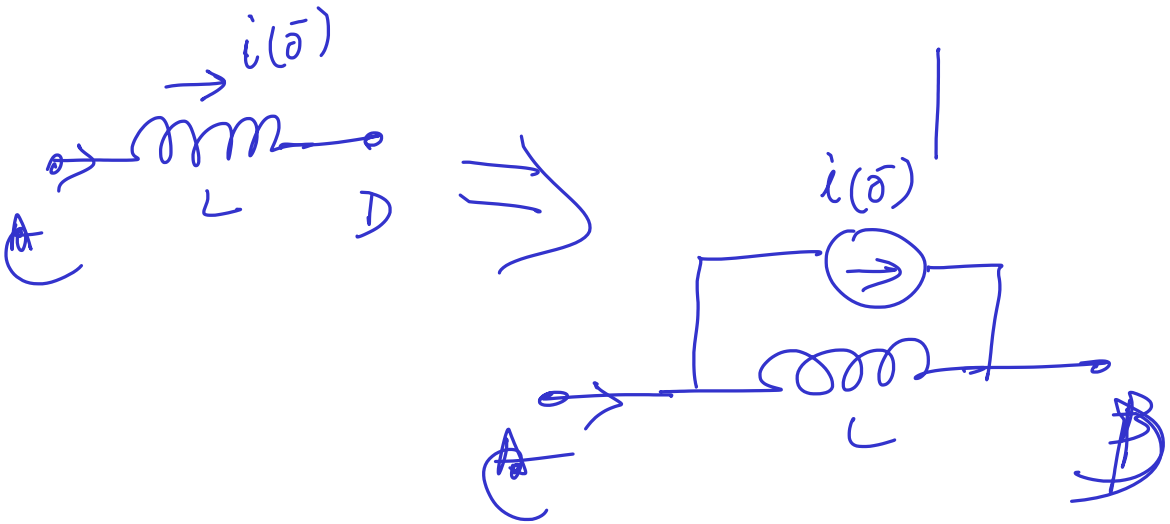
$$v_2(t) = \frac{1}{C}\int_0^t i_2\,dt$$

$i_1 + i_2 \longrightarrow v_3$

$$v_3(t) = \frac{1}{C}\int_0^t (i_1+i_2)\,dt = \frac{1}{C}\int_0^t i_1\,dt + \frac{1}{C}\int_0^t i_2\,dt$$

$$= v_1 + v_2$$

Add $v_1 + v_2 = 4 + \frac{1}{C}\int_0^t (i_1+i_2)\,dt$



$$v(t) = \underset{\text{Constant}}{V_0} + \frac{1}{C}\int_0^t i\,dt$$