

# An introduction to coding theory

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## Lecture #8B: Introduction to convolutional codes-II: state diagram, trellis diagram



# Convolutional codes

Outline of the lecture:

- State Diagram



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- State Diagram
- Trellis Diagram



# State Diagram

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$$S_l = (x_{l-1}, x_{l-2}, \dots, x_{l-m})$$

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- There are  $2^m$  number of possible states for a  $(n, 1, m)$  convolutional code.
- The output of a convolutional code at each time instant  $l$ , depends on the input and the current state.

$$v_l = f(u_l, S_l)$$



# State Diagram

- The convolutional encoder undergoes a state transition whenever a new information bit is input to the encoder.

Time unit	Message bit	State
$l$	$u_l$	$S_l = (x_{l-1}, x_{l-2}, \dots, x_{l-m})$
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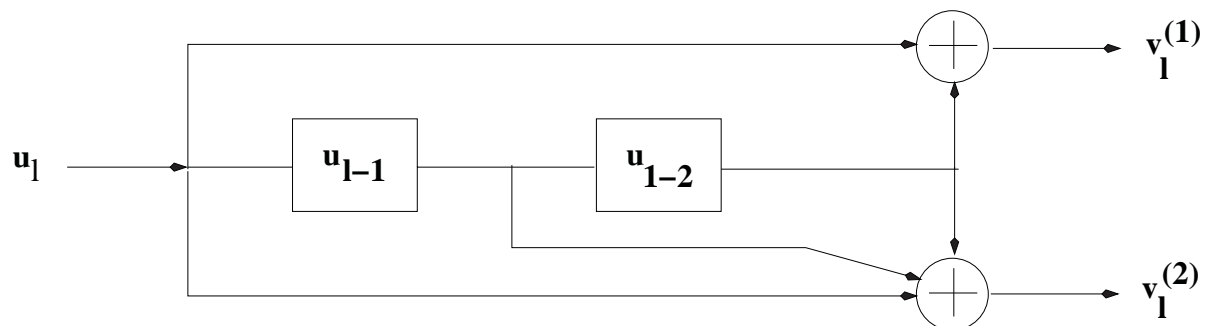
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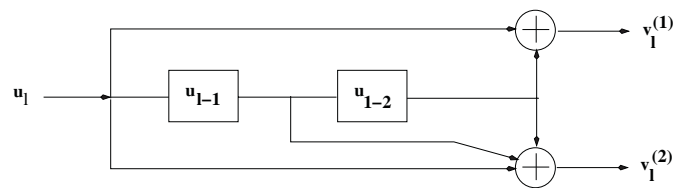
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- The state transitions are labeled with the information and coded bits corresponding to that transition.

# State Diagram



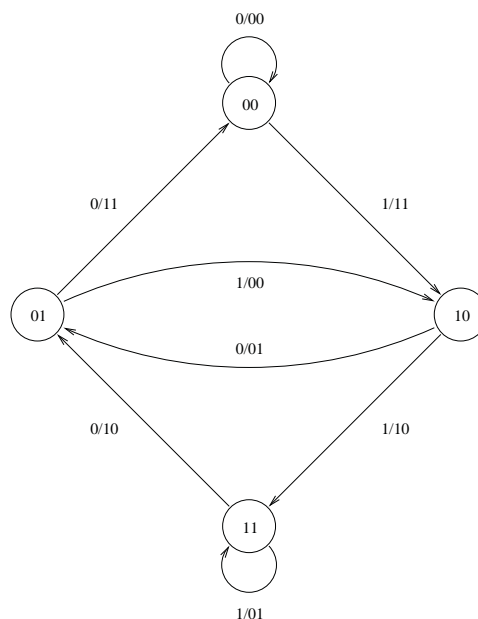
# State Diagram



## State Table

Input	Initial State	Next State	Output

# State Diagram



- There are four states: (0,0), (0,1), (1,1), and (1,0) for the (2,1,2) convolutional code.

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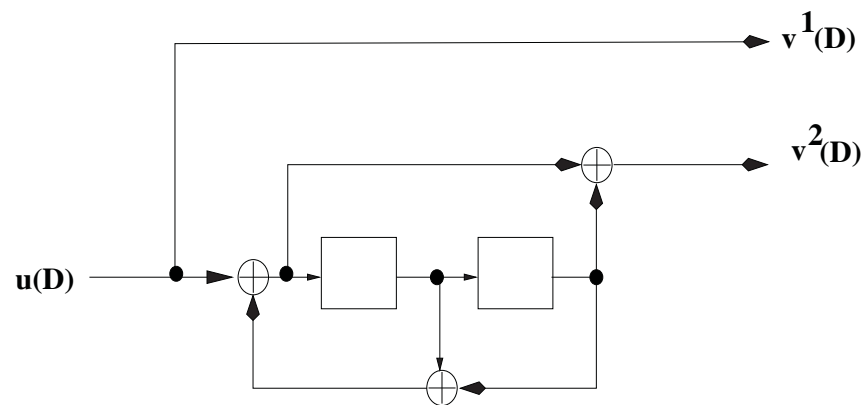
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- A trellis diagram for a convolutional code is obtained by joining the trellis sections at different time units.
- Each code word is made up of the labels on the trellis transitions that correspond to a specific path through the trellis.

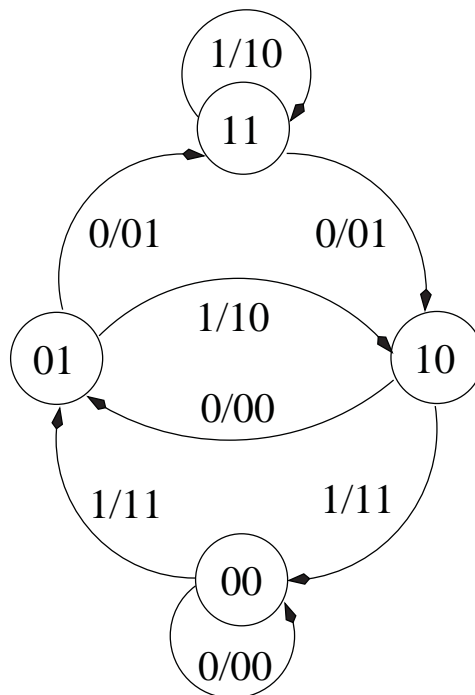
## Example: Convolutional Encoder



Rate  $R=1/2$  Systematic Feedback Convolutional Encoder

Navigation icons: back, forward, search, etc.

## Example: State Diagram



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## Example: Trellis Diagram

