Module 6: Deadbeat Response Design
Lecture Note 2

There are some practical issues in deadbeat response design.

Dead beat response design depends on the cancellation of poles and zeros of the plant transfer function.

If the poles are on or outside the unit circle, imperfect cancellation may lead to instability.

Thus, for practical constraints, one should not attempt to cancel poles which are on or outside the unit circle.

1 Deadbeat response design when some of the poles and zeros are on or outside the unit circle

Let the plant transfer function be

$$G_p(z) = \frac{\prod_{i=1}^{K}(1 - z_i z^{-1})}{\prod_{j=1}^{L}(1 - p_j z^{-1})} B(z)$$

where, K and L are the number of zeros and poles on or outside the unit circle and $B(z)$ is a rational transfer function in z^{-1} with poles and zeros inside the unit circle. This implies

$$D_c(z) = \frac{\prod_{j=1}^{L}(1 - p_j z^{-1})}{\prod_{i=1}^{K}(1 - z_i z^{-1})} \frac{M(z)}{B(z)(1 - M(z))}$$

Since we should not cancel poles or zeros which are on or outside unit circle by the controller $D_c(z)$, we have to choose $M(z)$ such that these get canceled out.

Thus $M(z)$ must contain the factors

$$\prod_{i=1}^{K}(1-z_i z^{-1})$$

and $(1-M(z))$ must contain the factors

$$\prod_{j=1}^{L}(1-p_j z^{-1})$$

So,

$$M(z)=\prod_{i=1}^{K}(1-z_i z^{-1})(m_k z^{-k}+m_{k+1}z^{-k-1}+........),\qquad k\geq n$$

and

$$1-M(z)=\prod_{j=1}^{L}(1-p_j z^{-1})(1-z^{-1})^P(1+a_1 z^{-1}+a_2 z^{-2}+..................)$$

P equals either the order of the poles of $R(z)$ or the order of poles of $G_p(z)$ at $z=1$ which ever is greater. Truncation depends on the following.

1. The order of poles of $M(z)$ and $(1-M(z))$ must be equal.

2. Total number of unknowns must be equal to the order of $M(z)$ so that they can be solved independently.

Example 1:

Let us consider the plant transfer function as

$$\begin{aligned}G_p(z)&=\frac{0.01(z+0.2)(z+2.8)}{z(z-1)(z-0.4)(z-0.8)}\\&=\frac{0.01z^{-2}(1+0.2z^{-1})(1+2.8z^{-1})}{(1-z^{-1})(1-0.4z^{-1})(1-0.8z^{-1})}\end{aligned}$$

For Unit Step Input:

$G_p(z)$ has a zero at -2.8 and pole at $z=1$. Therefore $M(z)$ must contain the term $1+2.8z^{-1}$ and $(1-M(z))$ should contain $1-z^{-1}$.

$G_p(z)$ has two more poles than zeros. This implies

$$M(z)=(1+2.8z^{-1})m_2 z^{-2}$$

$$1-M(z)=(1-z^{-1})(1+a_1 z^{-1}+a_2 z^{-2})$$

Since minimum order of $M(z)$ is 3, we have 3 unknowns in total. Combining the 2 equations

$$a_1 = 1$$

$$a_1 - a_2 = m_2$$

$$a_2 = 2.8m_2$$

$$\Rightarrow 1 - 2.8m_2 = m_2$$

$$m_2 = \frac{1}{3.8} = 0.26$$

$$a_2 = 2.8 \times 0.26 = 0.73$$

Thus

$$M(z) = 0.26z^{-2}(1 + 2.8z^{-1})$$

and

$$1 - M(z) = (1 - z^{-1})(1 + z^{-1} + 0.73z^{-2})$$

Putting the expressions of $M(z)$ and $1 - M(z)$ in the controller equation

$$\begin{aligned} D_c(z) &= \frac{(1 - z^{-1})}{(1 + 2.8z^{-1})} \frac{(1 + 2.8z^{-1})0.26z^{-2}}{(1 - z^{-1})(1 + z^{-1} + 0.73z^{-2})} \frac{(1 - 0.4z^{-1})(1 - 0.8z^{-1})}{0.01z^{-2}(1 + 0.2z^{-1})} \\ &= \frac{0.26z^{-2}(1 - 0.4z^{-1})(1 - 0.8z^{-1})}{0.01z^{-2}(1 + 0.2z^{-1})(1 + z^{-1} + 0.73z^{-2})} \\ &= \frac{26z(z - 0.4)(z - 0.8)}{(z + 0.2)(z^2 + z + 0.73)} \end{aligned}$$

Thus

$$\begin{aligned} C(z) &= \frac{0.26(z + 2.8)}{z^2(z - 1)} \\ &= 0.26z^{-2} + z^{-3} + z^{-4} + \ldots\ldots\ldots \end{aligned}$$

One should note that poles on or outside the unit circle are still present in the output expression.

$c(kT)$ tracks the unit step perfectly after 3 sampling periods (first term has a non unity coefficient). The output response is shown in Figure 1.

If $G_p(z)$ did not have any poles or zeros on or outside the unit circle it would take two sampling

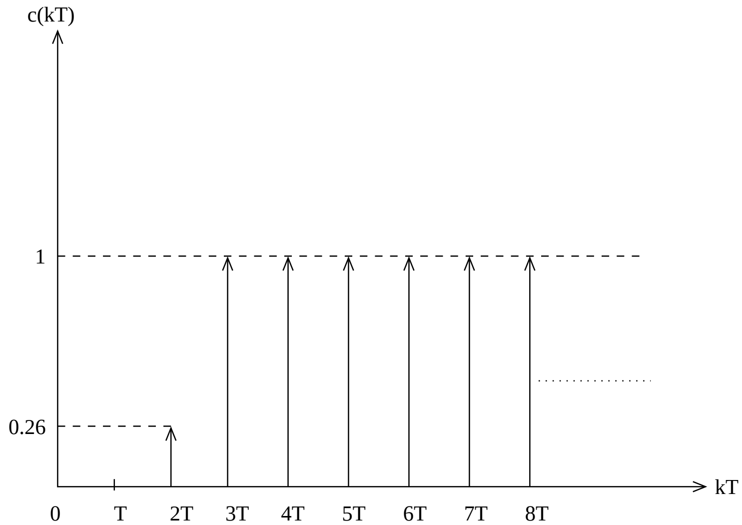

Figure 1: Deadbeat Response of The System in Example 1

periods to track the step input when $G_p(z)$ has two more poles than zeros.

Example 2:

Let us consider the plant transfer function as

$$\begin{aligned} G_p(z) &= \frac{0.0004(z+0.2)(z+2.8)}{(z-1)^2(z-0.28)} \\ &= \frac{0.0004z^{-1}(1+0.2z^{-1})(1+2.8z^{-1})}{(1-z^{-1})^2(1-0.28z^{-1})} \end{aligned}$$

For Unit Step Input:

$G_p(z)$ has a zero at -2.8 and two poles at $z = 1$. The number of poles exceeds the number of zeros by one.

$M(z)$ must contain the term $1 + 2.8z^{-1}$ and $1 - M(z)$ should contain $(1 - z^{-1})^2$.

This implies

$$M(z) = (1 + 2.8z^{-1})(m_1 z^{-1} + m_2 z^{-2})$$

$$1 - M(z) = (1 - z^{-1})^2(1 + a_1 z^{-1})$$

Combining the 2 equations and equating the like powers of z^{-1},

$$m_1 = 2 - a_1$$

$$2.8m_1 + m_2 = 2a_1 - 1$$

$$2.8m_2 = -a_1$$

The solutions of the above equations are $m_1 = 0.72$, $m_2 = -0.457$ and $a_1 = 1.28$. Thus

$$M(z) = (0.72z^{-1} - 0.457z^{-2})(1 + 2.8z^{-1})$$

and

$$1 - M(z) = (1 - z^{-1})^2(1 + 1.28z^{-1})$$

Putting the expressions of $M(z)$ and $1 - M(z)$ in the controller equation

$$\begin{aligned}
D_c(z) &= \frac{(1-z^{-1})^2}{(1+2.8z^{-1})}\frac{(0.72z^{-1} - 0.457z^{-2})(1+2.8z^{-1})}{(1-z^{-1})^2(1+1.28z^{-1})}\frac{(1-0.28z^{-1})}{0.0004z^{-1}(1+0.2z^{-1})} \\
&= \frac{(0.72z^{-1} - 0.457z^{-2})(1-0.28z^{-1})}{0.0004z^{-1}(1+0.2z^{-1})(1+1.28z^{-1})} \\
&= \frac{1800(z-0.635)(z-0.28)}{(z+0.2)(z+1.28)}
\end{aligned}$$

Thus

$$\begin{aligned}
C(z) = M(z)R(z) &= \frac{z(0.72z^2 + 1.56z - 1.28)}{z^3(z-1)} \\
&= 0.72z^{-1} + 2.28z^{-2} + z^{-3} + z^{-4} + \ldots\ldots\ldots
\end{aligned}$$

The output response is plotted in Figure 2.
Note that although $c(kT)$ tracks the unit step perfectly after 3 sampling periods, the maximum overshoot is 128 percent.

This is because of the fact that the digital plant is a type 2 system, hence a deadbeat response without overshoot cannot be obtained for a unit step input.

Thus one can conclude that it is not always possible to design dead beat response without any over shoot.

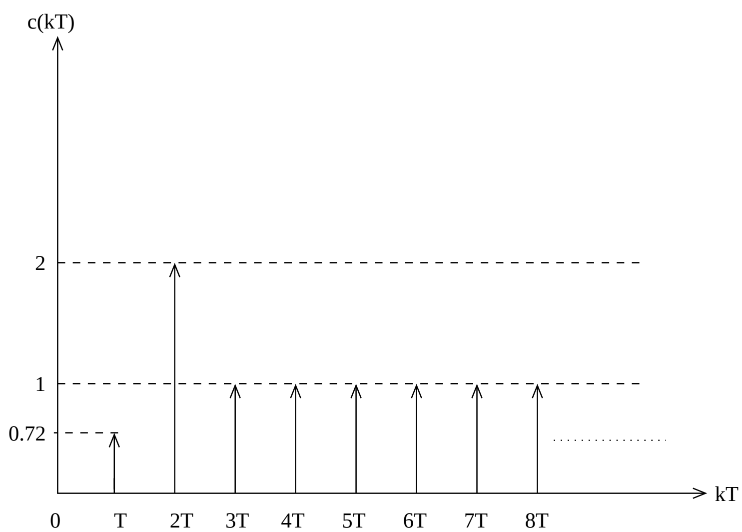

Figure 2: Deadbeat Response of The System in Example 2