Module 6: Deadbeat Response Design
Lecture Note 3

1 Sampled data control systems with Dead beat response

In case of a continuous time controlled process, the output $c(t)$ is a function of time t and the dead beat response design, based on cancellation of stable poles and zeros, may lead to inter sampling ripples in the output.

The reason behind this is since the process zeros are canceled by controller poles, the continuous dynamics are excited by the input and are not affected by feed back.

The strategy of designing dead beat response for a sampled data system with the process plant transfer function $G_{h0}G_p(z)$ having at least one zero is not to cancel the zeros, whether they are inside or outside the unit circle.

H.P Sirisena gave a mathematical formulation and analysis to dead beat response.

If $G_{h0}G_p(z^{-1}) = \dfrac{Q(z^{-1})}{P(z^{-1})}$, then according to Sirisena the digital controller for ripple free dead beat response to step input is

$$D_c(z) = \frac{P(z^{-1})}{Q(1) - Q(z^{-1})}$$

The design of ripple free dead beat response can still be done using similar approach as discussed in the previous chapters except for an added constraint which will increase the response time of the system.

Following example will illustrate the design procedure

Example 1:

Let us consider a sampled data system as shown in Figure 1, where,

$$G_p(s) = \frac{2}{s(s+2)}$$

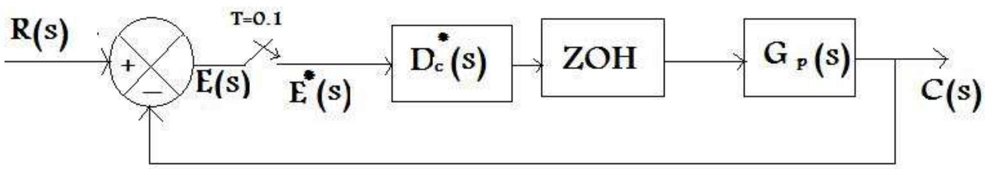

Figure 1: A sampled data control system

Thus

$$G_{h0}G_p(z) = \frac{0.01(z+0.9)}{(z-1)(z-0.8)}$$

If we design $D_c(z)$ without bothering about the inter sample ripples then

$$M(z) = z^{-1}, \qquad 1 - M(z) = 1 - z^{-1}$$

$$D_c(z) = \frac{100(z-0.8)}{(z+0.9)}$$

$$C(z) = \frac{1}{z-1} = z^{-1} + z^{-2} + ...$$

This implies that the output response is deadbeat only at sampling instants. However, the true output $c(t)$ has inter sampling ripples which makes the system response as shown in Figure 2.

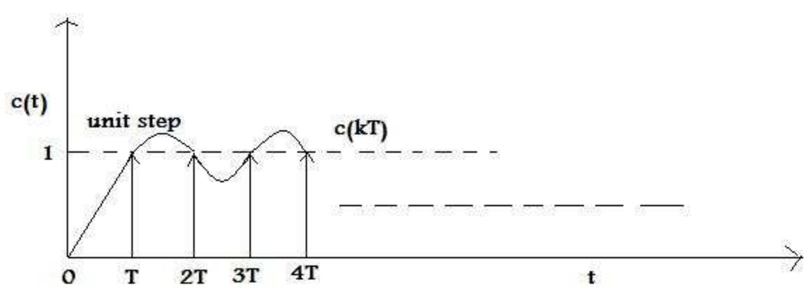

Figure 2: Rippled output response for Example 1

Thus the system takes forever to reach its steady state. The necessary and sufficient condition for $c(t)$ to track a unit step input in finite time is

$$c(NT) = 1 \qquad \left.\frac{dc(t)}{d(t)}\right|_{t=NT} = 0$$

for finite N and all the higher derivatives should equal to zero. Let

$$w(t) = \frac{dc(t)}{d(t)}$$

Taking Z-transform,

$$W(z) = \frac{D_c(z)(1-z^{-1})Z[G_p(s)]}{1 + D_c(z)(1-z^{-1})Z\left[\frac{G_p(s)}{s}\right]}R(z)$$

$$= \frac{A_1(z-1)}{z(z+0.9)}R(z)$$

where A_1 is a constant. Unit step response of $W(z)$ will not go to zero in finite time since poles of $\frac{W(z)}{R(z)}$ are not all at $z = 0$.

If we now apply the condition that zero of $G_{h0}G_p(z)$ at $z = -0.9$ should not be canceled by $D_c(z)$, then

$$M(z) = (1 + 0.9z^{-1})m_1 z^{-1}$$

$$1 - M(z) = (1 - z^{-1})(1 + a_1 z^{-1})$$

Solving

$$\Rightarrow m_1 = 0.53, \qquad a_1 = 0.47$$

Thus

$$M(z) = \frac{0.53(z+0.9)}{z^2}$$

$$D_c(z) = \frac{A_2(z-0.8)}{z+0.47}$$

$$C(z) = A_3 z^{-1} + z^{-2} + z^{-3} + ...$$

where A_2 and A_3 are constants. This implies that the dead beat response reaches the steady state after two sampling periods.

To show that the output response is indeed deadbeat, we derive the z-transform of $w(t)$ as

$$W(z) = 2z^{-1}$$

Thus $c(t)$ will actually reach its steady state in two sampling periods with no inter sample ripples which is shown in Figure 3.

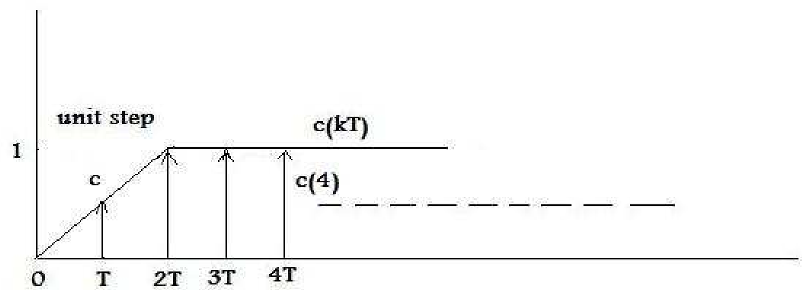

Figure 3: Ripple free deadbeat response for Example 1

Example 2: Consider the plant transfer function as

$$G_{h0}G_p(z) = \frac{0.01(z+0.2)(z+2.8)}{z(z-1)(z-0.4)(z-0.8)}$$

If we apply the condition that zeros of $G_{h0}G_p(z)$ at $z = -0.2$ and $z = -2.8$ should not be canceled by $D_c(z)$, then

$$M(z) = (1 + 0.2z^{-1})(1 + 2.8z^{-1})m_1 z^{-2}$$

$$1 - M(z) = (1 - z^{-1})(1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3})$$

While considering $M(z)$ and $1 - M(z)$, following points should be kept in mind

1. $M(z)$ should contain all the zeros of $G_{h0}G_p(z)$.
2. The number of poles over zeros of $M(z)$ should be at least equal to that of $G_{h0}G_p(z)$ which is 2 in this case.
3. $1 - M(z)$ must include the term $1 - z^{-1}$.
4. The orders of $M(z)$ and $1-M(z)$ should be same and should equal the number of unknown coefficients.

Solving for the coefficients of $M(z)$ and $1 - M(z)$, we get

$$1 - a_1 = 0$$

$$m_1 = a_1 - a_2$$

$$3m_1 = a_2 - a_3$$

$$0.56m_1 = a_3$$

The solutions of the above are $m_1 = 0.219$, $a_1 = 1$, $a_2 = 0.781$ and $a_3 = 0.123$. The closed loop transfer function is

$$M(z) = \frac{0.219z^2 + 0.657z + 0.123}{z^4}$$

The transfer function of the digital controller is obtained as

$$D_c(z) = \frac{21.9z(z - 0.4)(z - 0.8)}{z^3 + z^2 + 0.781z + 0.123}$$

The output for a unit step input is written as

$$\begin{aligned} C(z) &= \frac{0.219z^2 + 0.657z + 0.123}{z^3(z-1)} \\ &= 0.219z^{-2} + 0.876z^{-3} + z^{-4} + z^{-5}....... \end{aligned}$$

Thus the output response $c(kT)$ reaches the steady state in 4 sampling instants. This is one more sampling instant than the previous example where we considered the plant to be all digital.

This implies that for sampled data control system, the dead beat response $c(t)$ reaches the steady state after three sampling periods but inter sample ripples occur. After four sampling instants the inter sample ripples disappear.

To show that the output response is indeed deadbeat, we derive the z-transform of $w(t)$ which will come out to be

$$W(z) = A_1z^{-2} + A_2z^{-3}$$

where A_1, A_2 are constants.

Thus the derivative of $c(t)$ is zero for $kT \geq 4T$, which implies that the step response reaches the steady state in 4 sampling instants with no inter sample ripples, as shown in Figure 4.

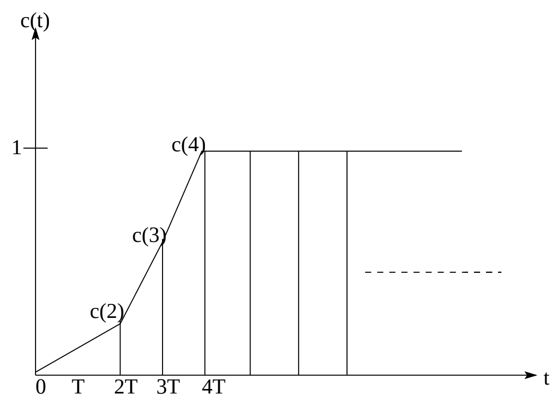

Figure 4: Ripple free deadbeat response for Example 2