

Module 3 : Frequency Control in a Power System

Lecture 13 : Calculation of System Frequency

Objectives

In this lecture you will learn the following

- How is equilibrium system frequency calculated if load parameters are frequency dependent ?

Calculation of System Frequency

The equilibrium system frequency $\left(f = \frac{\omega_e}{2\pi}\right)$ is a solution of the following equation :

$$\sum (P_{mi}(\omega_e) - \sum P_{Lk}(\omega_e) - losses(\omega_e)) = 0$$

If it is assumed that:

- a) Loads are not voltage dependent but are frequency dependent. Load real power variation is characterised by:

$$P_L = P_{L0} \left(1 + k_{pf} \frac{f - f_0}{f_0} \right)$$

- b) P_{mi} is not a function of frequency (generator mechanical power input is constant).
- c) Losses are a small fixed proportion of the total generation and are frequency independent. Strictly speaking, losses *are* frequency dependent because transmission line current flows are dependent on the loads, which are themselves frequency dependent. However this effect is indirect and small.
- d) Network parameters (reactance/ susceptances) are not frequency dependent.

Calculation of System Frequency (Con td..)

then equilibrium frequency is given by:

$$\sum P_{mi} - \sum P_{L0k} - \text{losses} = \sum P_{mi}(1-l) - \sum P_{L0k} - \sum P_{L0k} k_{pfk} \left(\frac{f-f_0}{f_0} \right) = 0$$

where *losses* are assumed to be *l* times the total generation.

therefore :

$$\frac{f-f_0}{f_0} = \frac{\sum P_{mi}(1-l) - \sum P_{L0k}}{\sum P_{L0k} k_{pfk}}$$

Let $\sum (P_{mi}(1-l) - P_{L0k}) = 0$, i.e., $\sum P_{L0k} = \sum P_{mi}(1-l)$.

then it is clear that $f = f_0$.

Now suppose 10% of the load is switched out, i.e. $\sum \hat{P}_{L0k} = 0.9 \sum P_{L0k}$, and

it is assumed that load frequency characteristics are the same for each load, i.e., $k_{pfk} = k_{pf}$,

then

$$\frac{\hat{f}-f_0}{f_0} = \frac{\sum P_{mi}(1-l) - \sum \hat{P}_{L0k}}{\sum \hat{P}_{L0k} k_{pf}} = \frac{\sum P_{L0k} - \sum \hat{P}_{L0k}}{\sum \hat{P}_{L0k} k_{pf}} = \frac{0.1}{0.9 k_{pf}}$$

if $k_{pf} = 1.5$, it is clear that $\left(\frac{\hat{f}-f_0}{f_0} \right) = 0.074$ implying that $\hat{f} = 53.7 \text{ Hz}$.

Consideration of Real & Reactive Power Losses

If we consider voltage dependence of real and reactive loads and the marginal effect of change in losses, then the calculation of equilibrium frequency is a bit more complicated. The solution for equilibrium frequency has to consider real and reactive power balance at all nodes to obtain estimates of voltages and losses. Therefore a complete load-flow solution is required.

To illustrate this, we shall now consider an example in which we tackle three cases:

a) A "usual" load flow for a 4 bus system.

b) A load flow for a 4 bus system considering voltage dependence of loads, load at nominal voltage is specified and load is assumed to vary as follows:

$$P_L = P_{L0} \left(\frac{V}{V_0} \right)^2 \approx P_{L0} \left(1 + 2 \frac{V-V_0}{V_0} \right)$$

$$Q_L = Q_{L0} \left(\frac{V}{V_0} \right)^2 \approx Q_{L0} \left(1 + 2 \frac{V-V_0}{V_0} \right)$$

It is assumed that voltage deviation from the nominal is small.

Consideration of Real & Reactive Power Losses

c) A load flow in which **all** generator (including slack bus) powers are specified and frequency dependence of loads is considered.

$$P_L = P_{L0} \left(\frac{V}{V_0} \right)^2 \left(1 + 2 \frac{f - f_0}{f_0} \right) \approx P_{L0} \left(1 + 2 \frac{V - V_0}{V_0} \right) \left(1 + 2 \frac{f - f_0}{f_0} \right)$$
$$Q_L = Q_{L0} \left(\frac{V}{V_0} \right)^2 \left(1 - 2 \frac{f - f_0}{f_0} \right) \approx Q_{L0} \left(1 + 2 \frac{V - V_0}{V_0} \right) \left(1 - 2 \frac{f - f_0}{f_0} \right)$$

Cases (b) and (c) require minor modification of a "conventional" load flow program.

For (b), a simple modification is to *vary* the P-Q specification at load buses in accordance with the voltage magnitude at that iteration step.

For (c), at every step the calculated slack generator power is compared with the specified slack generator power, and frequency is *incremented* by some proportion of the difference. The P-Q specification at load buses is varied in accordance with the voltage magnitude and frequency at that iteration step.

With these two hints, can you write a load flow program which includes voltage and frequency dependence of loads? You will need to use a method to solve non-linear algebraic set of equations, as discussed in the previous lecture.

We now give the [solved load flow cases \(along with the specifications\) for a 4 bus system](#).

Recap

In this lecture you have learnt the following

- We have calculated system frequency for a simple case of loads being frequency dependent but not voltage dependent.
- If both voltage and frequency dependence is to be considered, then we need to use a modified load flow program. This is demonstrated with an example.

Congratulations, you have finished Lecture 13. To view the next lecture select it from the left hand side menu of the page.