

## Module 5 : Real and Reactive Power Scheduling

### Lecture 24 : Real and Reactive Power Scheduling

#### Objectives

In this lecture you will learn the following

- Cost minimization considering network constraints and losses.
- An illustrative example

#### Optimal Power Flow

Until now, we have considered fairly simple examples pertaining to the economic dispatch problem. In an actual system, there are many variables (other than generated power) that need to be scheduled. These may be set under the direction of a system operator. If the quantity which is to be scheduled belongs to an independent entity, a system operator may "buy" the quantity based on the price at which it is sold.

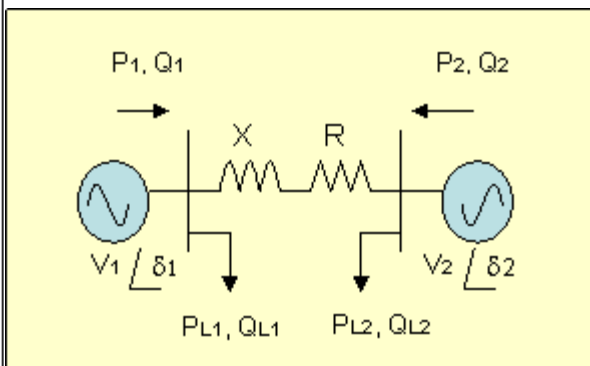
The formulation of the minimum cost problem, by including the network, reactive power demand, voltage and current constraints is known as an "optimal power flow". Note that optimal power flow takes into account of losses in the network.

Let us consider an optimal power flow problem in which :

- Control variables (which can be scheduled independently) are the generated real and reactive power
- Load demand is fixed
- Cost is a function of only the real power (this assumes that the service of providing reactive power for maintaining voltages is free!)
- Real powers (control variables) are constrained by maximum limits.
- Voltage magnitudes (auxiliary variable) is constrained by maximum and minimum limits.

In actual practice, several additional constraints and control variables are present, but for the sake of simplicity let us consider only the ones mentioned above.

#### Optimal Power Flow - An example



Consider a 2 generator system which supplies 2 loads at buses 1 and 2. Compute the minimum cost real and reactive power schedule of the generators if the cost of real power (power is in per-unit) is:

$$C1 = 50 * P1 + 300 * P1^2$$

$$C2 = 50 * P2 + 250 * P2^2$$

Reactive power is assumed to be free. The following limits are applicable: Voltages at both buses should be between 0.975 pu and 1.025 pu. Generated real power for each generator should be less than 3 pu. Reactive power capability is assumed to be unlimited.

$$R+j X = 0.02 + j 0.1, PL1 + j QL1 = 4.0 + j*2.0, PL2 + jQL2 = 1.5 + j*0.75 \text{ pu}$$

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The problem can be stated formally as follows:

Total cost is given by :  $C = C_1 + C_2$  which is to be minimized.

subject to

$$P_1 - PL_1 - V_1^2 \cos f / Z - V_1 V_2 \cos(f + d_1 - d_2) / Z = 0$$

$$P_2 - PL_2 - V_2^2 \cos f / Z - V_1 V_2 \cos(f + d_2 - d_1) / Z = 0$$

and

$$V_1 - 1.025 < 0, V_2 - 1.025 < 0,$$

$$0.975 - V_1 < 0, 0.975 - V_2 < 0,$$

$$P_1 < 3.0, P_2 < 3.0$$

where  $Z = |R + jX|$ , and  $f = \arctan(X/R)$

Note that there are implicit equality constraints which are present due to real and reactive power balance equations at the 2 nodes. The auxiliary variables are  $V_1$ ,  $V_2$  and  $d_1 - d_2$ . Note that  $d_1$  and  $d_2$  cannot be obtained as independently and uniquely since they appear as  $(d_1 - d_2)$  in all equations (i.e., if  $d_1$  and  $d_2$  satisfy the equations, then  $d_1 + g$  and  $d_2 + g$  also satisfy them,  $g$  being an arbitrary choice).

## Optimal Power Flow : An example (contd.)

To obtain the solution to the problem, we first form an augmented cost function which factors the equality and inequality constraints as follows:

$$C = 50 * P_1 + 300 * P_1^2 + 50 * P_2 + 250 * P_2^2 -$$

$$/1 * (P_1 - PL_1 - V_1^2 \cos f / Z + V_1 V_2 \cos(f + d_1 - d_2) / Z) -$$

$$/2 * (P_2 - PL_2 - V_2^2 \cos f / Z + V_1 V_2 \cos(f + d_2 - d_1) / Z) +$$

$$p(P_1 - 3.0) + p(P_2 - 3.0) + p(V_1 - 1.025) + p(V_2 - 1.025) + p(0.975 - V_1) + p(0.975 - V_2)$$

where the penalty function has the form :  $p(t) = e^{\beta t}$ ,  $\beta$  being set to 100.

We then set all the partial derivatives to zero and solve the resulting equations to obtain the values of the variables which result in the minimum value of  $C$ . The equations to be solved are *non-linear algebraic equations* and can be solved by a variant of Gauss Siedel method. (You may try the same using gradient descent method which was discussed in the previous lecture). A matlab program to do can be downloaded here: [opf.m](http://opf.m). This may be run by typing `opf` at the MATLAB prompt.

## The solution and comments

The final schedule is calculated to be:

$$P_1 = 2.5982, P_2 = 2.9413, Q_1 = 1.1795, Q_2 = 0.7677$$

The voltages are  $V_1 = 1.0062$ ,  $V_2 = 1.0263$ , and the phase angular difference  $d_1 - d_2 = -8$  degrees.

The power flow from bus 2 to bus 1 equals 1.4413 pu, and the loss is 0.0395 pu

Comments:

- 1) It is cheaper to import some power from generator 2 for the local load (4 pu) at the bus 1.
- 2) However, this implies that there will be some loss due to transmission which will increase the cost.
- 3) Therefore we have to tradeoff : obtain power from a cheaper remote generator and at the same time ensure losses due to transmission are low.
- 4) Voltages at the 2 ends can be adjusted so as to reduce losses. Obtaining power at voltages above the nominal results in lower losses (due to lesser current) but then voltages have to be restricted within the limits.
- 5) Voltage at bus 2 is slightly higher than the specified limits. This can be corrected if the penalty function parameter  $b$  is increased; however, this may cause divergence in the numerical method used for solving the optimization (try doing it with the program given to you).

## Optimal Power Flow : Security constrained Optimal Power Flow

In a realistic power system, the following **additional** points have to be considered.

- a) Reactive power capability is not free and unlimited.

Instead of specifying the reactive power schedule, an operator may prefer to schedule the voltage at the buses at which reactive power is controllable.

- b) There may be additional controllable variables (possibly having service costs), like tap settings of tap changing transformers, SVC output, TCSC reactance, HVDC power flow etc. Each controllable variable is likely to have a limit which is dictated by the associated equipment.
- c) Current in the transmission line has to be within values as dictated by the thermal limit.
- d) Security Constraints: Even if we have ensured that we have a least cost real and reactive power schedule for a given network and load condition, we would also like to ensure that this schedule is "secure". This means that *if* there is a disturbance when the system is operating as per this schedule, it should be :
  - 1) stable, i.e., it should "settle down" after a credible(realistic) disturbance to a post-fault equilibrium (see Module 2 for a discussion of stability).
  - 2) transmission line voltage and current constraints are not violated in the post-disturbance steady state (which may be different from the pre-disturbance steady state due to loss of a component like transmission line).

Incorporating security constraints rigorously is not an easy task since there can be a large number of realistic disturbances which can take place. Restricting pre-disturbance power flows or phase angular differences across lines is a convenient way to specify Angular Stability constraint.

An optimal power flow which includes security constraints is called a "*security constrained optimal power flow*".

If an optimal power flow study is carried out *without* considering security constraints, i.e., the constraints imposed by the steady state / dynamic behaviour following a potential contingency, then this may call for changes (or usually some minor adjustments) in the final "schedule" in order to satisfy security constraints. This is known as preventive re-scheduling (we shall see an example of this in the next module).

## Recap

In this lecture you have learnt the followinga

- Optimal Power Flow: Cost minimization, by considering network constraints and losses.
- Illustration of optimal power flow by an example.

Congratulations, you have finished Lecture 24. Please view the next slide for concluding remarks for this module.

We now summarize what we have learnt in this Module

- 1) A system operator can schedule several quantities (control variables) which ensure that the system operates with minimum cost of energy, while satisfying constraints of equipment and stability.
- 2) Apart from generated real power there are several other control variables, which may be available for scheduling - at a service cost.
- 3) In systems where generation is not a regulated monopoly, the price of electricity need not necessarily reflect actual costs, but may be sold at a price determined by load demand and competition.
- 4) Real and reactive power scheduling problems can be formulated as constrained optimization problems.

Congratulations, you have finished Module V.

To view the next lecture select it from the left hand side menu of the page.