Module 2 : Equipment and Stability Constraints in System Operation

Lecture 7 : Insert : Numerical Solution of Differential Equations

Objectives

- What does one mean by stability?
- What is angular stability?
- Dynamical equations for a single machine -infinite bus system. Equilibrium points.

What is Stability?

Stability is essentially the ability of a system to recover from disturbances, both large and small, and settle to an acceptable equilibrium. Randomly occurring load changes, faults resulting in line or generator tripping and changes in reference values of regulating controllers, are examples of disturbances.

One should note that equipment constraints are distinct from stability constraints in the sense that even though an equilibrium condition may exist (which is within equipment constraints), a system may not be able to "settle down" to it if it is perturbed or initially away from it.

If a system is not stable for even small disturbances, *it cannot be operated at all* since there are always small and random perturbations in the system due to load variations

If a system is stable for small disturbances but unstable if the disturbances are "large", then the system can be operated. However, the system may not be *secure*, i.e., it may be unstable if a large enough disturbance does actually occur .

The major stability problems which are inherent in AC interconnected grids are discussed next.

Angle Stability

An interesting physical characteristic of interconnected synchronous generators is their ability to generate restoring torques when disturbed from an equilibrium. These torques ensure that all machines stay in synchronism --- generator electrical speeds become equal in steady state. Equivalently, the phase angular differences between ac voltages at various points become constant if machines stay in equilibrium.

However, the restoring torques can become zero or negative for very large disturbances. This can result in machines falling out of step (i.e., they lose synchronism - the machines do not settle to the same electrical speed; this makes operation unviable - see **Module 1**). Sometimes, due to presence of automatic controllers, damping of the rotor oscillations is inadequate or negative, causing growing or sustained oscillations ('hunting') and may also lead to loss of synchronism.

The problem of loss of synchronism between synchronous machines is also known as the "angular stability problem". Note that if machines lose synchronism, then the phase angular difference between ac buses in the system will not settle to constant values.

To study the angular stability problem, consider the governing equations of the system.

The equations of motion of a synchronous machine rotor are given by :

$$\frac{d\delta}{dt} = (\omega - \omega_o)$$

$$\frac{d(\omega - \omega_o)}{dt} = \frac{\omega_B}{2H}(T_m - T_e)$$

where δ, ω are the rotor angle (with respect to a frame which rotates at ω_o) and rotor speed respectively

T_m, T_e are the mechanical and electrical torques respectively expressed in per-unit.

ω_B is the base frequency and H is the inertia constant in MJ/MVA.

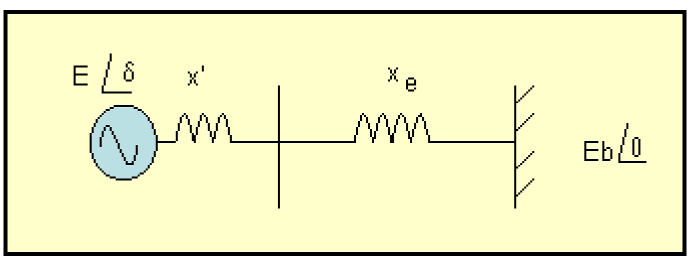
T_e is a **non-linear** function of δ. It is also a function of several other variables.

However, for simplicity we concentrate on the dependence on δ alone by considering a simple system.

SMIB System

A Single - Machine Infinite Bus system (SMIB), is an oversimplified model of a power system but it helps us to understand the essence of the large disturbance angular stability problem. The SMIB system represents a small generator connected to a large power system. The large power system is represented by an infinite bus (fixed voltage source with a constant frequency). The generator itself is represented as a constant magnitude voltage source behind its transient reactance. The infinite bus voltage is assumed to have a frequency wo. The phase angle of the internal voltage E with respect to the infinite bus is given by d. Note that this angle will change if the relative frequency between the generator and infinite bus changes.

Thus, if d does not reach a steady state value after a disturbance, then it implies that the generator has lost synchronism, or equivalently, is angular unstable.



For this simplified model, the electrical torque in per-unit is approximately given by,

$$T_e = \frac{EE_b \sin\delta}{X}, \quad X = x' + x_e$$

Thus,

$$\frac{d\delta}{dt} = (\omega - \omega_o)$$

$$\frac{d(\omega - \omega_o)}{dt} = \frac{\omega_B}{2H}\left(T_m - \frac{EE_b \sin\delta}{X}\right)$$

The set of equations are non-linear because of the $\sin\delta$ term.

xe is the transmission line reactance in W

The values of δ, ω at which $\frac{d\delta}{dt}$ and $\frac{d(\omega - \omega_o)}{dt} = 0$, represent the equlibrium values.

There are 2 equilibria $\delta = \delta_1, \omega = \omega_o$ and $\delta = \delta_2, \omega = \omega_o$, between 0-180 degrees as seen in the figures below.

However, $\delta = \delta_1, \omega = \omega_o$ is a stable equilibrium.

This is because, i.e., $(T_m - \frac{EE_b sin\delta}{X})$ is negative if the rotor angle is slightly perturbed from δ_1 in the positive direction, and is positive if the perturbation is negative. Thus the torque is restorative in nature.

On the other hand, $(T_m - \frac{EE_b sin\delta}{X})$ is positive if the rotor angle is slightly perturbed from δ_2 in the positive direction, and is negative if the perturbation is negative. Thus the torque enhances the disturbances!

Thus δ_2 is an unstable equilibrium.

Another interesting aspect is that $(T_m - \frac{EE_b sin\delta}{X})$ is a nonlinear function of δ, and may change sign with increasing value of δ.

Recap

In this lecture you have learnt the following

- A system is said to be stable if it can withstand disturbances and come to an equilibrium
- Angular stability refers to the study of whether machines remain in synchronism after a disturbance
- The equations of a single machine infinite bus system reveal that torque is a nonlinear function of rotor angle
- Congratulations, you have finished Lecture 7. To view the next lecture select it from the left hand side menu of the page.