Storage Systems

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(Lecture 28)

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Performance Modelling

- Storage devices often the slowest component in a system
- Often determine the speed and responsiveness of the system
- Modelling useful
  - Simple models sometimes very useful!
  - Effectiveness of, say, caching may/may not be useful
    - Two zones of operation

What/How to model?

CPU utilization for processing storage requests

Compression, Encryption, Deduplication, QoS for mm

I/O Completion Processing and Interrupt handling

Network packet processing

QoS for packets

Scheduling groups of processes across nodes: distributed search

Two main types

- Operational models: limited assumptions
- Stochastic models: mathematically tractable (eg. Queuing theory)

Operational Laws

Little's Laws: true if during observed period T,

- arrivals (a) ~ completions (c)
- ie: (a-c) small compared to c

Let J = time in system

Mean time spent in system = J/N

Mean # in system = J/T = J/N * N/T = response time * throughput

Or: Q_i (mean # in device i) = $X_i R_i$

Usually written as $L=\lambda W$:

Mean number in system= arrival rate * mean time spent in system

Poisson Arrivals

Assumes that in a small interval δ

# of arrivals: $\lambda*\delta$

Prob of more than 1 arrival in δ: negligible

Arrivals in nonoverlapping intervals statistically indep

Expected arrival time = $1/\lambda$

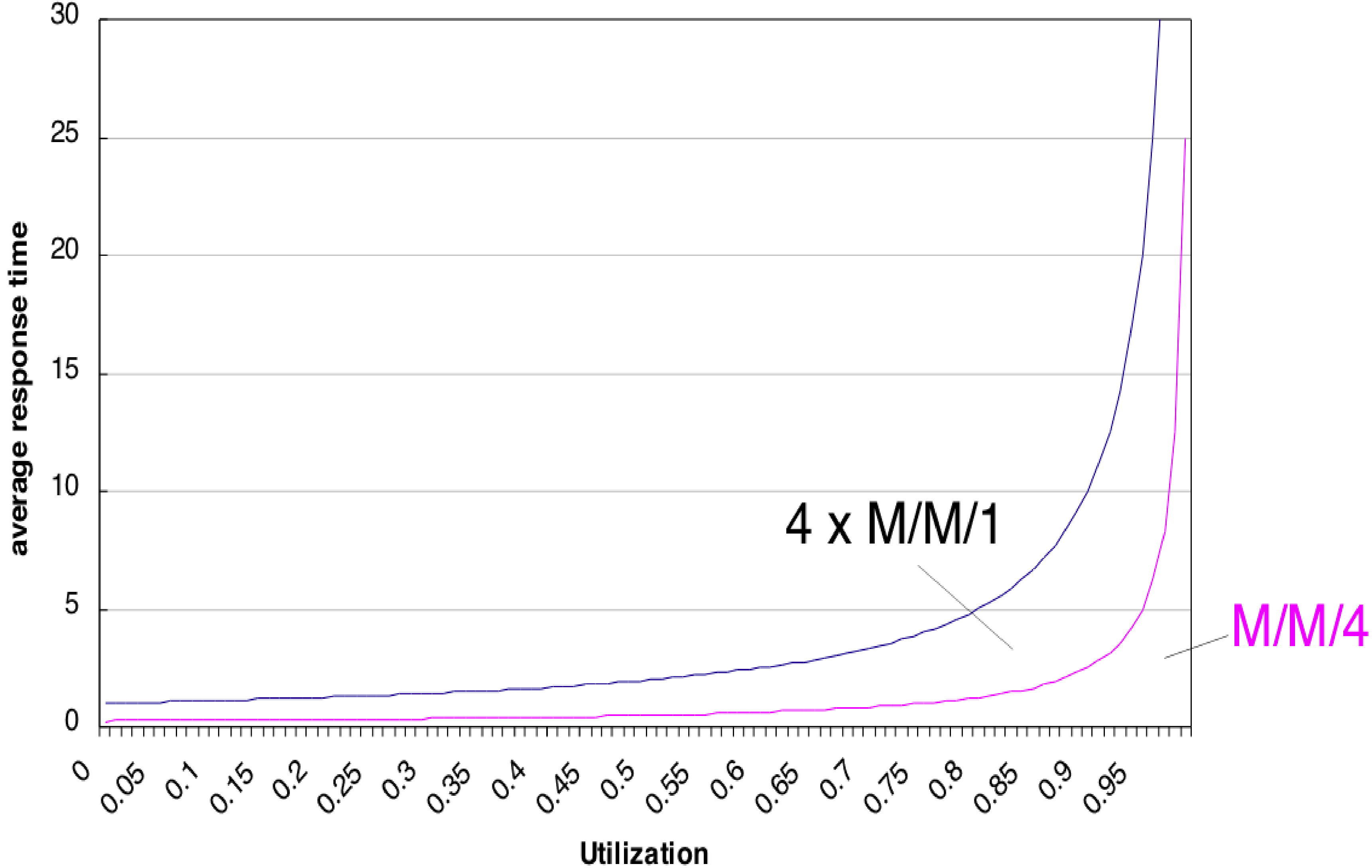
Probability of an arrival in time t = $1 - \exp(-\lambda t)$

Probability of no arrivals in time $t = \exp(-\lambda t)$

Probability of k arrivals in time t = $\exp(-\lambda t)(\lambda t)^k/k!$

Similarly, Poisson departures

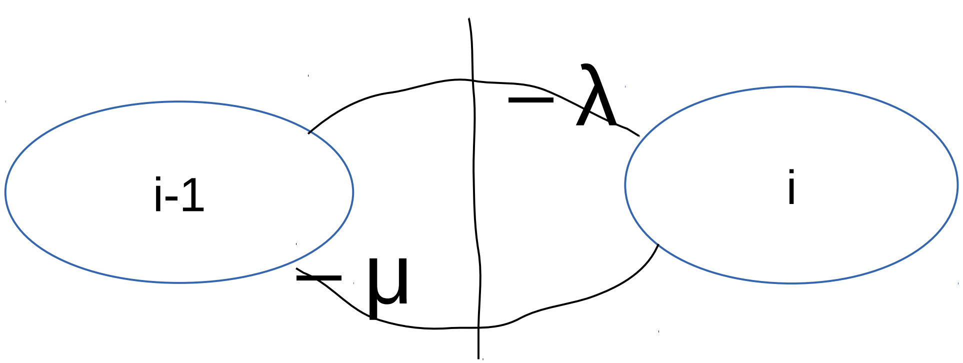
M/M/4!



30
25
20
15
10
5
0
average response time
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
Utilization
4 x M/M/1
M/M/4

M/M/1 Analysis

- Arrival rate Poisson λ
- Service rate exponential μ
- Model as birth-death process
- Steady state ($\lambda < \mu$): Probability stable that i customers in system ($\pi(i)$). Given by
  - $\pi(i)*\mu = \pi(i-1)*\lambda, i = 1, \ldots$
  - ie. $\pi(i) = \rho\pi(i-1), i = 1, \ldots$



M/M/1

- $\pi(i) = \rho^i \pi(0)$, $i = 1, \ldots$
- Since sum of all $\pi(i)$, $i = 1, \ldots$ should be 1
  - $\pi(i) = (1-\rho)\,\rho^i$, $i = 1, \ldots$
- Mean number of customers $E[N] = \rho / (1-\rho)$
- Expected Waiting time $E[W] = E[N]/\mu$
- Expected service time $E[T] = E[W] + 1/\mu = 1/(\mu-\lambda)$

M/M/1 vs M/M/2

Response time for separate Qs (M/M/1) = $1/\mu(1-\rho)$

- with $\rho=(\lambda/2)/\mu$,
  - E[W]= $2/(2\mu-\lambda) = (4\mu+2\lambda)/(4\mu^2-\lambda^2)$

Response time for combined Qs (M/M/2):

E[N] $= 2\rho/(1-\rho^2)$ where $\rho = \lambda/2\mu$

E[W] = E[N]/λ (Little's Law) $= 4\mu/(4\mu^2-\lambda^2)$

Less than that for separate Qs !

Utilization Law

Utilization = Busy Time/T = (completions/T) * (Busy Time/completions) = throughput * service time

Or, at device i, $U_i = X_i S_i$

Forced Flow Law: Device throughput of device i = completions(i)/T = completions(i)/completions of jobs * completions of jobs/T = visit ratio * system throughput

Or, $X_i = X v_i$

Utilization of device U_i = throughput_i * service time_i = visit ratio_i * system throughput * service time_i = system throughput * total service demand on device_i

Or, $U_i = X D_i$

Example (from Jain'91)

Each prog requires 5 secs of CPU time & 80 I/O reqs to disk A and 100 I/O reqs to disk B. Disk A takes 50ms; disk B takes 30ms. Total of 17 terminals with disk A's thruput 15.7 I/O reqs/s. Ave think time: 18s. What is the system thruput? Device utilization?

$D_{diskA} = V_A S_A = 80*1/20=4s$

$D_{diskB} = V_B S_B = 100*3/100=3s$

$X_A=15.7 = XV_A \Rightarrow X=15.7/80$

$D_{cpu}=5s$; $V_{cpu} = V_A + V_B + 1 = 181$;

$U_{cpu}= XD_{cpu} = (15.7/80) * 5 = 98\%$; $U_A = X D_A$; $U_B = X D_B$

If $Q_{cpu}=8.88$, $Q_A=3.2$; $Q_B=1.4$, what is response time?

Use Little's law: $R_i = Q_i/X_i$

General Response Time Law: $Q=Q_1+...+Q_n$; $Q_i=X_iR_i$ (Little's Law); $Q=XR= \Sigma Q_i = \Sigma X_iR_i$ $R=\Sigma V_iR_i$

Interactive Response Time Law: Z (think time) + response (R). In time T, T/(Z+R) requests.

With N users, system thruput X = (N*T/(Z+R)) /T = N/(Z+R)

=> R = N/X - Z

Bottleneck Analysis: $X(N) <= \min(1/D_{max}, N/(D+Z))$ where $D= \text{sum}(D_i)$ (the sum of total service demands on all devices)

$U_{\text{bottleneck device}} = XD_{max} <=1 => X <=1/D_{max}$

With 1 job: $R(1) = D_1+...+D_M = D$

R(N)>=R(1) =D; X(N) = N/(R(N)+Z) <=N/D+Z

Conclusion

- Simple Models often give quick insight
  - Esp if devices are of widely varying speeds!