NPTEL ONLINE CERTIFICATION COURSES

Course Name: Deep Learning

Faculty Name: Prof. P. K. Biswas
Department : E & ECE, IIT Kharagpur

Topic

Lecture 06: Discriminant Function and Decision Surface

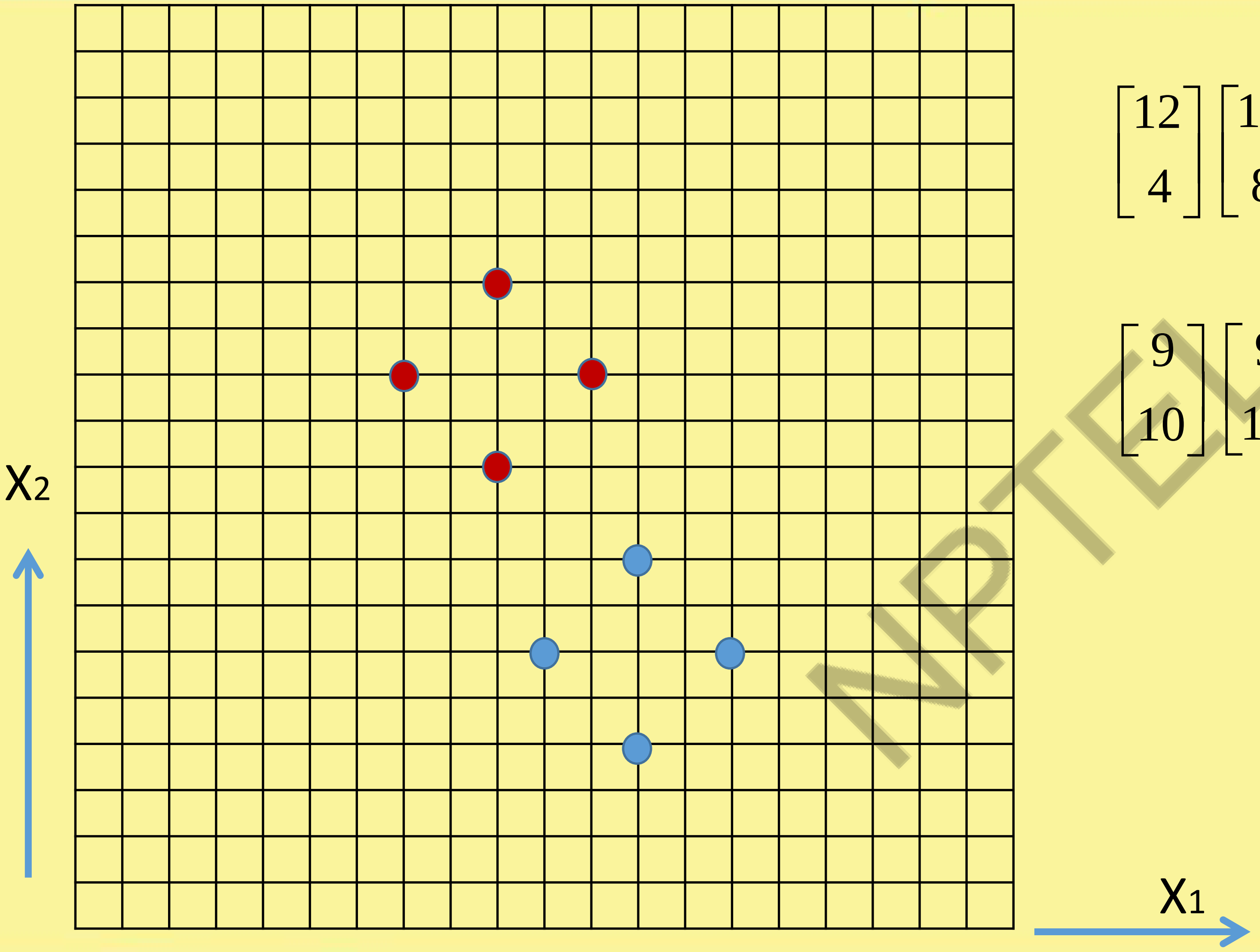
CONCEPTS COVERED

Concepts Covered:

- Bayes Minimum Error Classifier
- Bayes Minimum Risk Classifier
- Discriminant Function
- Decision Boundary

Discriminant Function

Discriminant Function under Multivariate Normal Distribution



$$\begin{bmatrix}12\\4\end{bmatrix}\begin{bmatrix}12\\8\end{bmatrix}\begin{bmatrix}10\\6\end{bmatrix}\begin{bmatrix}14\\6\end{bmatrix} \Rightarrow \omega_1$$

$$\begin{bmatrix}9\\10\end{bmatrix}\begin{bmatrix}9\\14\end{bmatrix}\begin{bmatrix}7\\12\end{bmatrix}\begin{bmatrix}11\\12\end{bmatrix} \Rightarrow \omega_2$$

$$\begin{bmatrix}12\\4\end{bmatrix}\begin{bmatrix}12\\8\end{bmatrix}\begin{bmatrix}10\\6\end{bmatrix}\begin{bmatrix}14\\6\end{bmatrix} \Rightarrow \omega_1 \qquad \mu_1 = \frac{1}{4}\left[\begin{bmatrix}12\\4\end{bmatrix}+\begin{bmatrix}12\\8\end{bmatrix}+\begin{bmatrix}10\\6\end{bmatrix}+\begin{bmatrix}14\\6\end{bmatrix}\right]=\begin{bmatrix}12\\6\end{bmatrix}$$

$$[X_1-\mu_1][X_1-\mu_1]^t = \begin{bmatrix}0\\-2\end{bmatrix}\begin{bmatrix}0 & -2\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 4\end{bmatrix} = M_1$$

$$[X_2-\mu_1][X_2-\mu_1]^t = \begin{bmatrix}0\\2\end{bmatrix}\begin{bmatrix}0 & 2\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 4\end{bmatrix} = M_2$$

$$[X_3-\mu_1][X_3-\mu_1]^t = \begin{bmatrix}-2\\0\end{bmatrix}\begin{bmatrix}-2 & 0\end{bmatrix} = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} = M_3$$

$$[X_4-\mu_1][X_4-\mu_1]^t = \begin{bmatrix}2\\0\end{bmatrix}\begin{bmatrix}2 & 0\end{bmatrix} = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} = M_4$$

$$\Sigma_1 = \frac{1}{4}[M_1+M_2+M_3+M_4]$$

$$= \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix} = 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = 2I$$

$$\begin{bmatrix} 9 \\ 10 \end{bmatrix} \begin{bmatrix} 9 \\ 14 \end{bmatrix} \begin{bmatrix} 7 \\ 12 \end{bmatrix} \begin{bmatrix} 11 \\ 12 \end{bmatrix} \Rightarrow \omega_2 \qquad \mu_2 = \frac{1}{4}\left[\begin{bmatrix} 9 \\ 10 \end{bmatrix} + \begin{bmatrix} 9 \\ 14 \end{bmatrix} + \begin{bmatrix} 7 \\ 12 \end{bmatrix} + \begin{bmatrix} 11 \\ 12 \end{bmatrix}\right] = \begin{bmatrix} 9 \\ 12 \end{bmatrix}$$

$$\Sigma_2 = 2I$$

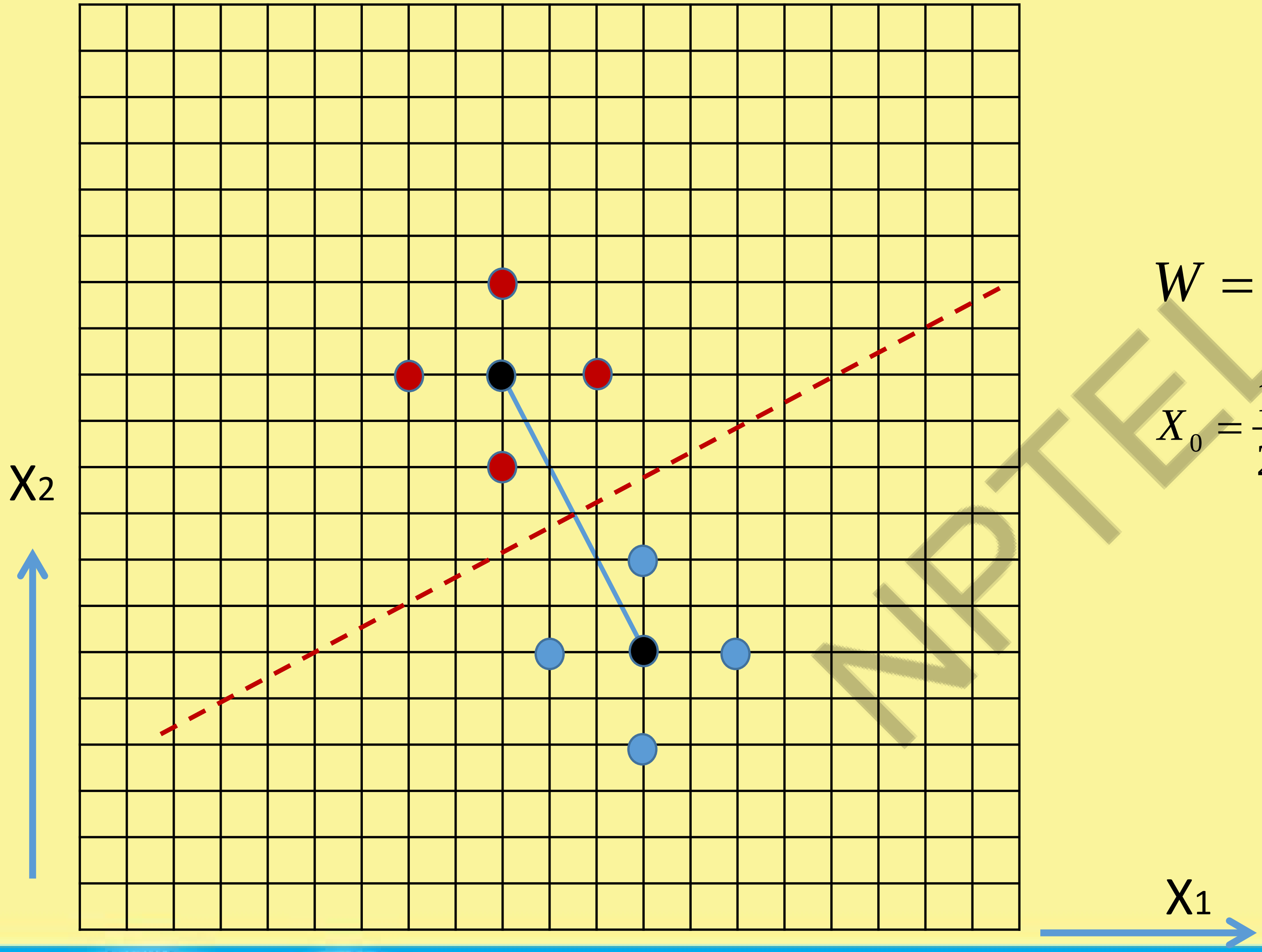
$$\Downarrow$$

$$\Sigma_1 = \Sigma_2 = 2I \approx \sigma^2 I$$

Where

$$\sigma = \sqrt{2}$$

Decision Surface



$$W^t(X - X_0) = 0$$

$$W = \mu_2 - \mu_1$$

$$X_0 = \frac{1}{2}(\mu_1 + \mu_2) - \frac{\sigma^2}{\|\mu_1 - \mu_2\|^2} \ln \frac{P(\omega_1)}{P(\omega_2)} (\mu_1 - \mu_2)$$

Discriminant Function under Multivariate Normal Distribution

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Thank you

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Topic

Lecture 07: Discriminant Function and Decision Surface - II

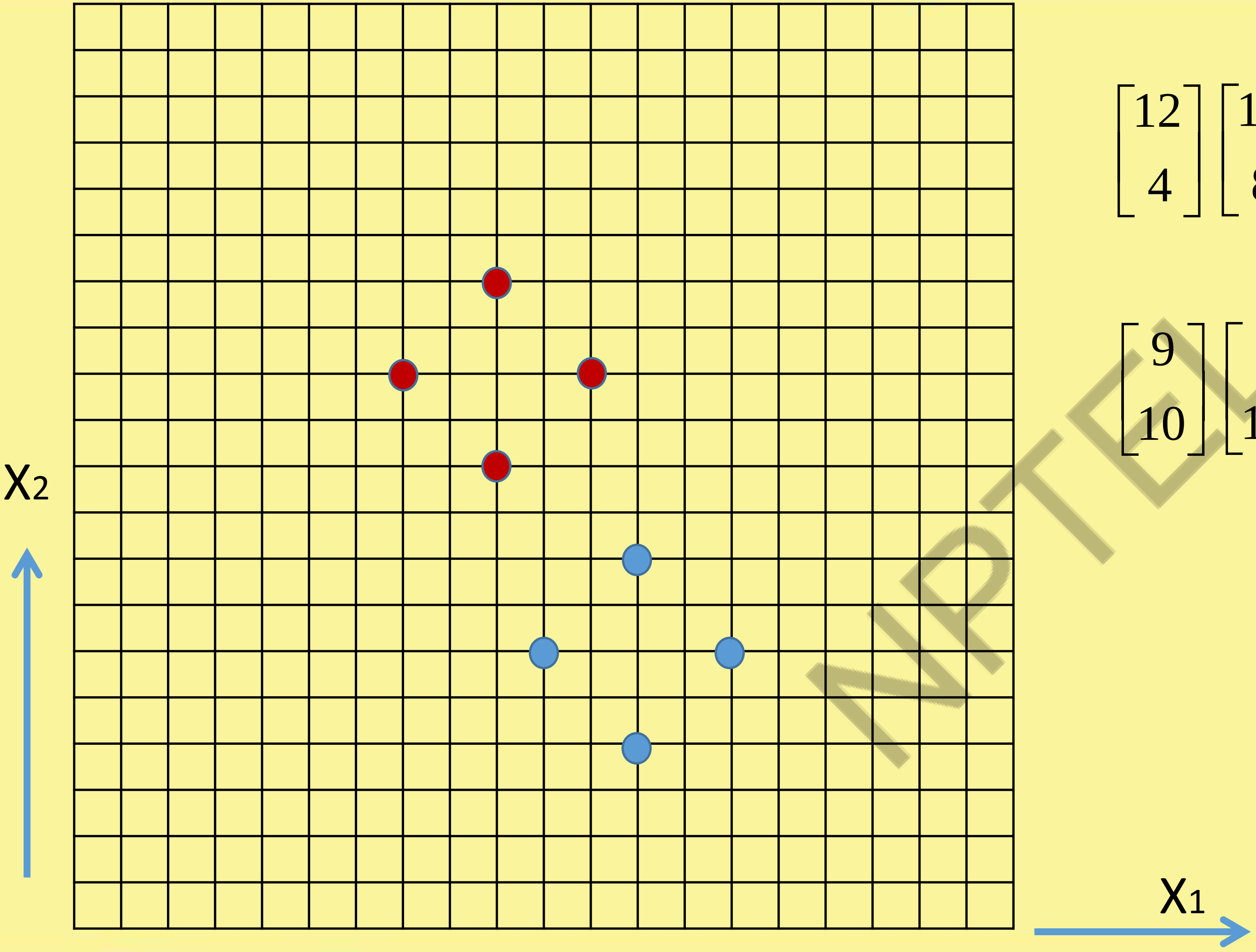
CONCEPTS COVERED

Concepts Covered:

- Discriminant Function under Multivariate Normal Distribution
- Decision Boundary under Various Cases of Covariance Matrices
- Examples

Discriminant Function under Multivariate Normal Distribution

Decision Surface



$$\begin{bmatrix}12\\4\end{bmatrix}\begin{bmatrix}12\\8\end{bmatrix}\begin{bmatrix}10\\6\end{bmatrix}\begin{bmatrix}14\\6\end{bmatrix} \Rightarrow \omega_1$$

$$\begin{bmatrix}9\\10\end{bmatrix}\begin{bmatrix}9\\14\end{bmatrix}\begin{bmatrix}7\\12\end{bmatrix}\begin{bmatrix}11\\12\end{bmatrix} \Rightarrow \omega_2$$

Decision Surface

$$\begin{bmatrix}12\\4\end{bmatrix}\begin{bmatrix}12\\8\end{bmatrix}\begin{bmatrix}10\\6\end{bmatrix}\begin{bmatrix}14\\6\end{bmatrix} \Rightarrow \omega_1$$

$$\mu_1 = \frac{1}{4}\left[\begin{bmatrix}12\\4\end{bmatrix}+\begin{bmatrix}12\\8\end{bmatrix}+\begin{bmatrix}10\\6\end{bmatrix}+\begin{bmatrix}14\\6\end{bmatrix}\right] = \begin{bmatrix}12\\6\end{bmatrix}$$

$$[X_1-\mu_1][X_1-\mu_1]^t = \begin{bmatrix}0\\-2\end{bmatrix}[0 \quad -2] = \begin{bmatrix}0 & 0\\0 & 4\end{bmatrix} = M_1$$

$$[X_2-\mu_1][X_2-\mu_1]^t = \begin{bmatrix}0\\2\end{bmatrix}[0 \quad 2] = \begin{bmatrix}0 & 0\\0 & 4\end{bmatrix} = M_2$$

$$[X_3-\mu_1][X_3-\mu_1]^t = \begin{bmatrix}-2\\0\end{bmatrix}[-2 \quad 0] = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} = M_3$$

$$[X_4-\mu_1][X_4-\mu_1]^t = \begin{bmatrix}2\\0\end{bmatrix}[2 \quad 0] = \begin{bmatrix}4 & 0\\0 & 0\end{bmatrix} = M_4$$

$$\Sigma_1 = \frac{1}{4}[M_1+M_2+M_3+M_4]$$

$$= \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix} = 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = 2I$$

Decision Surface

$$\begin{bmatrix} 9 \\ 10 \end{bmatrix} \begin{bmatrix} 9 \\ 14 \end{bmatrix} \begin{bmatrix} 7 \\ 12 \end{bmatrix} \begin{bmatrix} 11 \\ 12 \end{bmatrix} \Rightarrow \omega_2 \qquad \mu_2 = \frac{1}{4}\left[\begin{bmatrix} 9 \\ 10 \end{bmatrix} + \begin{bmatrix} 9 \\ 14 \end{bmatrix} + \begin{bmatrix} 7 \\ 12 \end{bmatrix} + \begin{bmatrix} 11 \\ 12 \end{bmatrix}\right] = \begin{bmatrix} 9 \\ 12 \end{bmatrix}$$

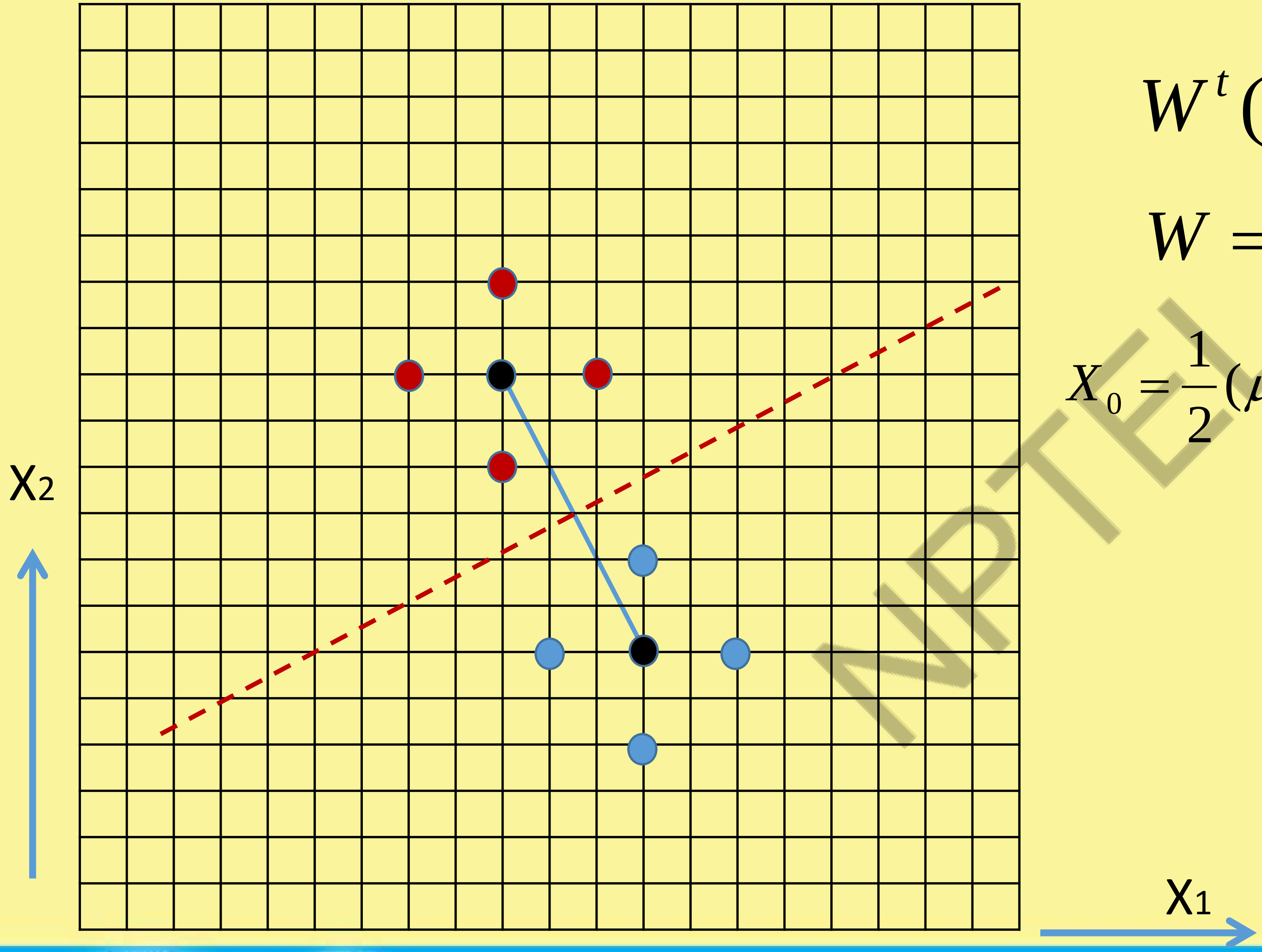
$$\Sigma_2 = 2I$$

$$\Sigma_1 = \Sigma_2 = 2I \approx \sigma^2 I$$

Where

$$\sigma = \sqrt{2}$$

Decision Surface



$$W^t(X - X_0) = 0$$

$$W = \mu_2 - \mu_1$$

$$X_0 = \frac{1}{2}(\mu_1 - \mu_2) - \frac{\sigma^2}{\|\mu_1 - \mu_2\|^2} \ln \frac{P(\omega_1)}{P(\omega_2)} (\mu_1 - \mu_2)$$

Discriminant Function under Multivariate Normal Distribution

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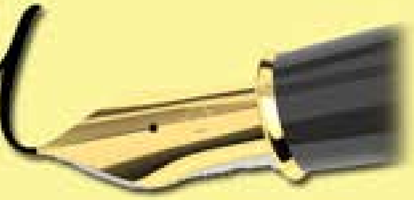
Thank you



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Course Name: Deep Learning

Faculty Name: Prof. P. K. Biswas

Department : E & ECE, IIT Kharagpur

Topic

Lecture 08: Discriminant Function and Decision Surface - III

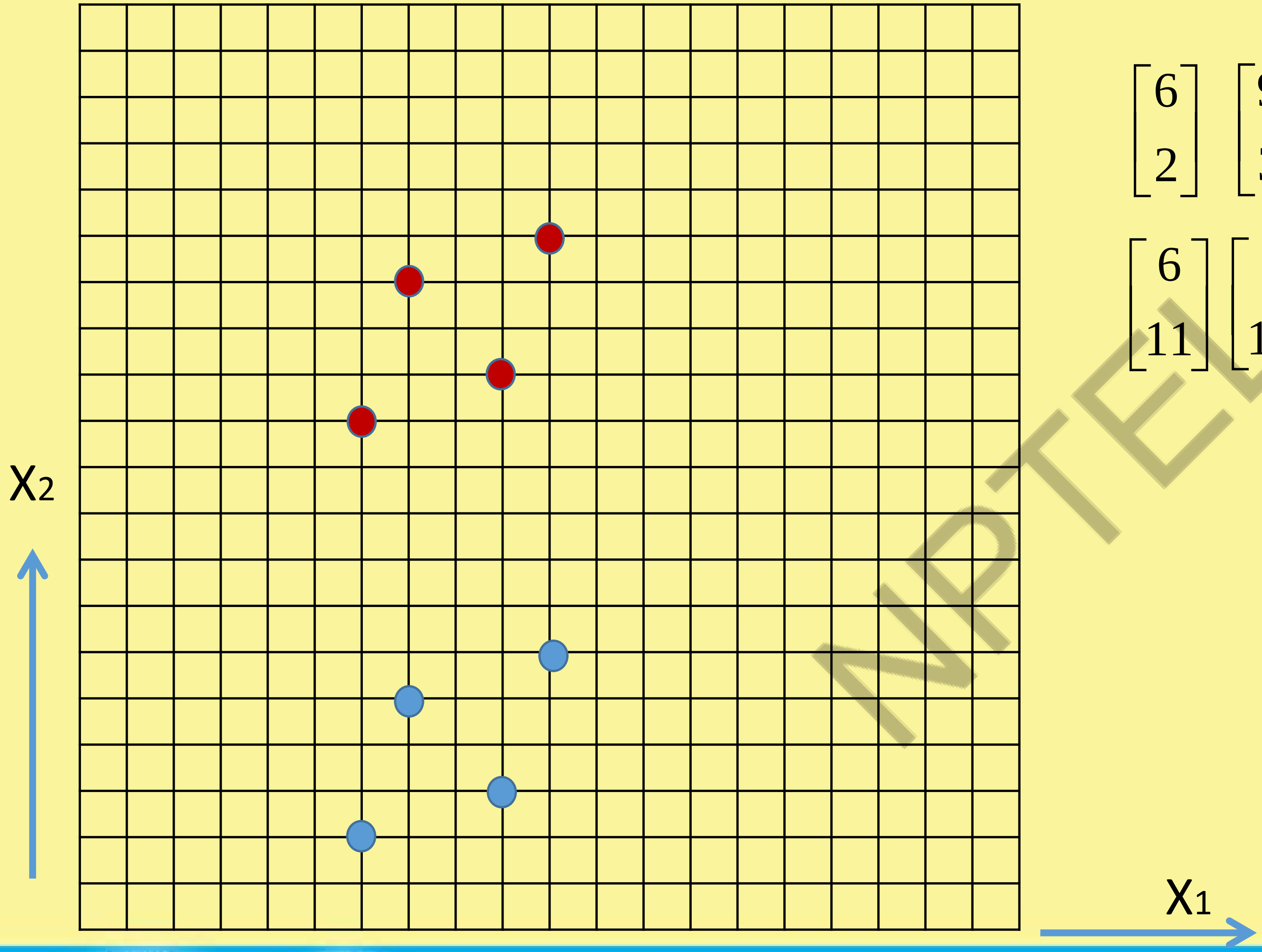
CONCEPTS COVERED

Concepts Covered:

- Decision Boundary under Various Cases of Covariance Matrices
- Examples

Discriminant Function under Multivariate Normal Distribution

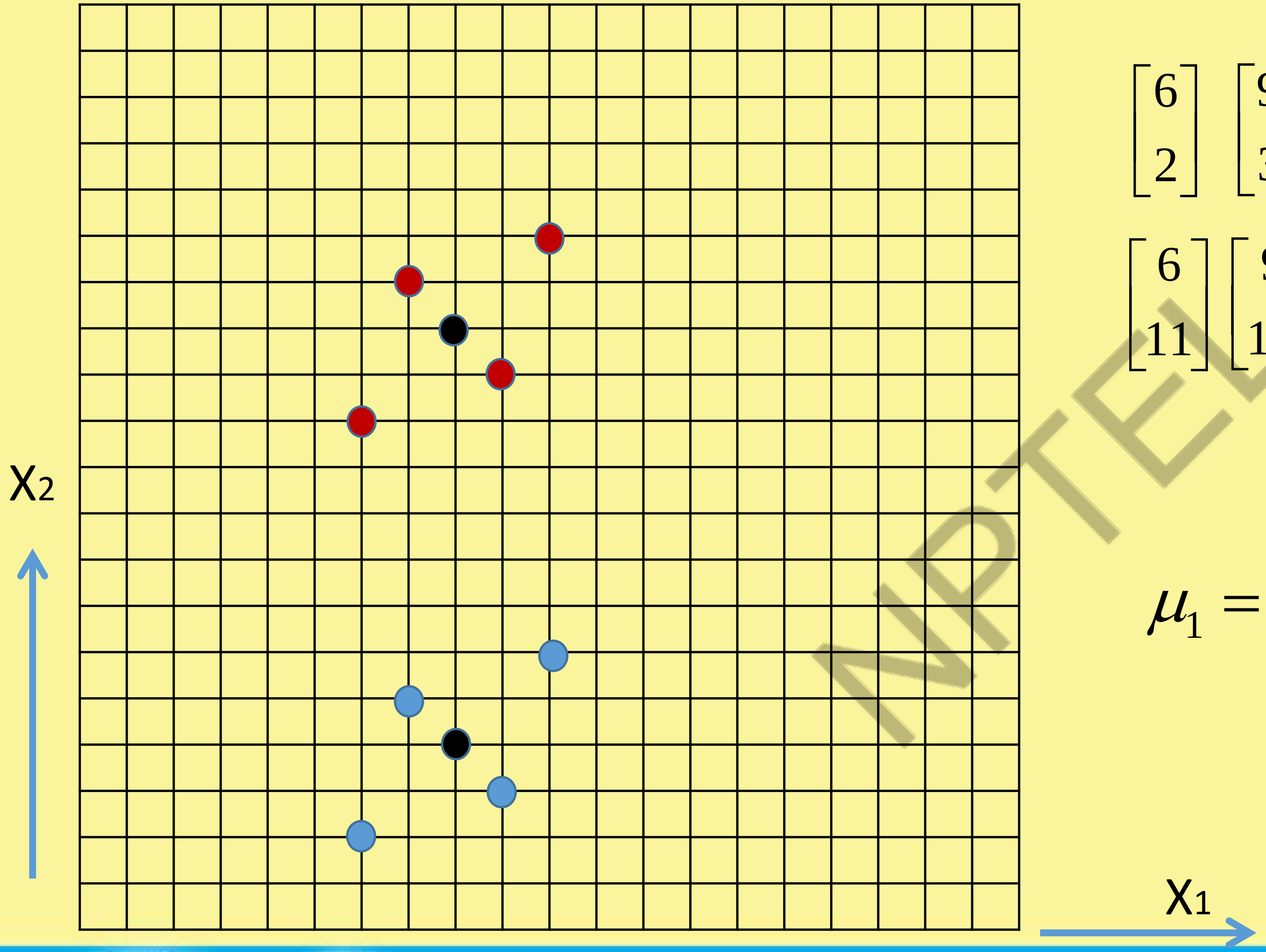
Decision Surface



$$\begin{bmatrix}6\\2\end{bmatrix} \begin{bmatrix}9\\3\end{bmatrix} \begin{bmatrix}7\\5\end{bmatrix} \begin{bmatrix}10\\6\end{bmatrix} \Rightarrow \omega_1$$

$$\begin{bmatrix}6\\11\end{bmatrix} \begin{bmatrix}9\\12\end{bmatrix} \begin{bmatrix}7\\14\end{bmatrix} \begin{bmatrix}10\\15\end{bmatrix} \Rightarrow \omega_2$$

Decision Surface



$$\begin{bmatrix}6\\2\end{bmatrix} \begin{bmatrix}9\\3\end{bmatrix} \begin{bmatrix}7\\5\end{bmatrix} \begin{bmatrix}10\\6\end{bmatrix} \Rightarrow \omega_1$$

$$\begin{bmatrix}6\\11\end{bmatrix} \begin{bmatrix}9\\12\end{bmatrix} \begin{bmatrix}7\\14\end{bmatrix} \begin{bmatrix}10\\15\end{bmatrix} \Rightarrow \omega_2$$

$$\mu_1 = \begin{bmatrix}8\\4\end{bmatrix} \qquad \mu_1 = \begin{bmatrix}8\\13\end{bmatrix}$$

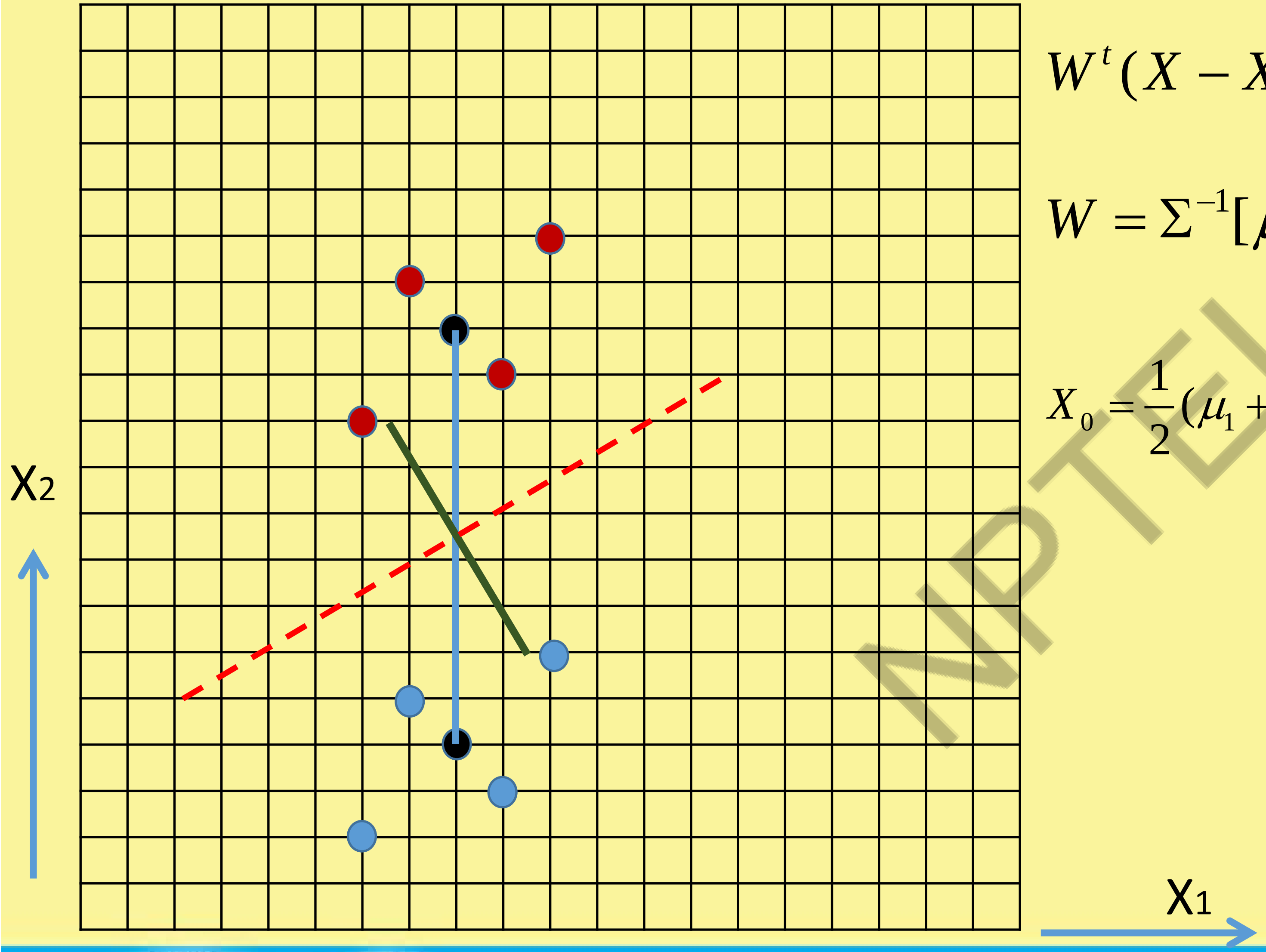
Decision Surface

$$\begin{bmatrix}6\\2\end{bmatrix}\begin{bmatrix}9\\3\end{bmatrix}\begin{bmatrix}7\\5\end{bmatrix}\begin{bmatrix}10\\6\end{bmatrix} \Rightarrow \omega_1 \qquad \begin{bmatrix}6\\11\end{bmatrix}\begin{bmatrix}9\\12\end{bmatrix}\begin{bmatrix}7\\14\end{bmatrix}\begin{bmatrix}10\\15\end{bmatrix} \Rightarrow \omega_2$$

$$\mu_1 = \begin{bmatrix}8\\4\end{bmatrix} \qquad \mu_2 = \begin{bmatrix}8\\13\end{bmatrix}$$

$$\Sigma = \frac{1}{2}\begin{bmatrix}5 & 3\\3 & 5\end{bmatrix} \qquad \Sigma^{-1} = \frac{1}{8}\begin{bmatrix}5 & -3\\-3 & 5\end{bmatrix}$$

Decision Surface



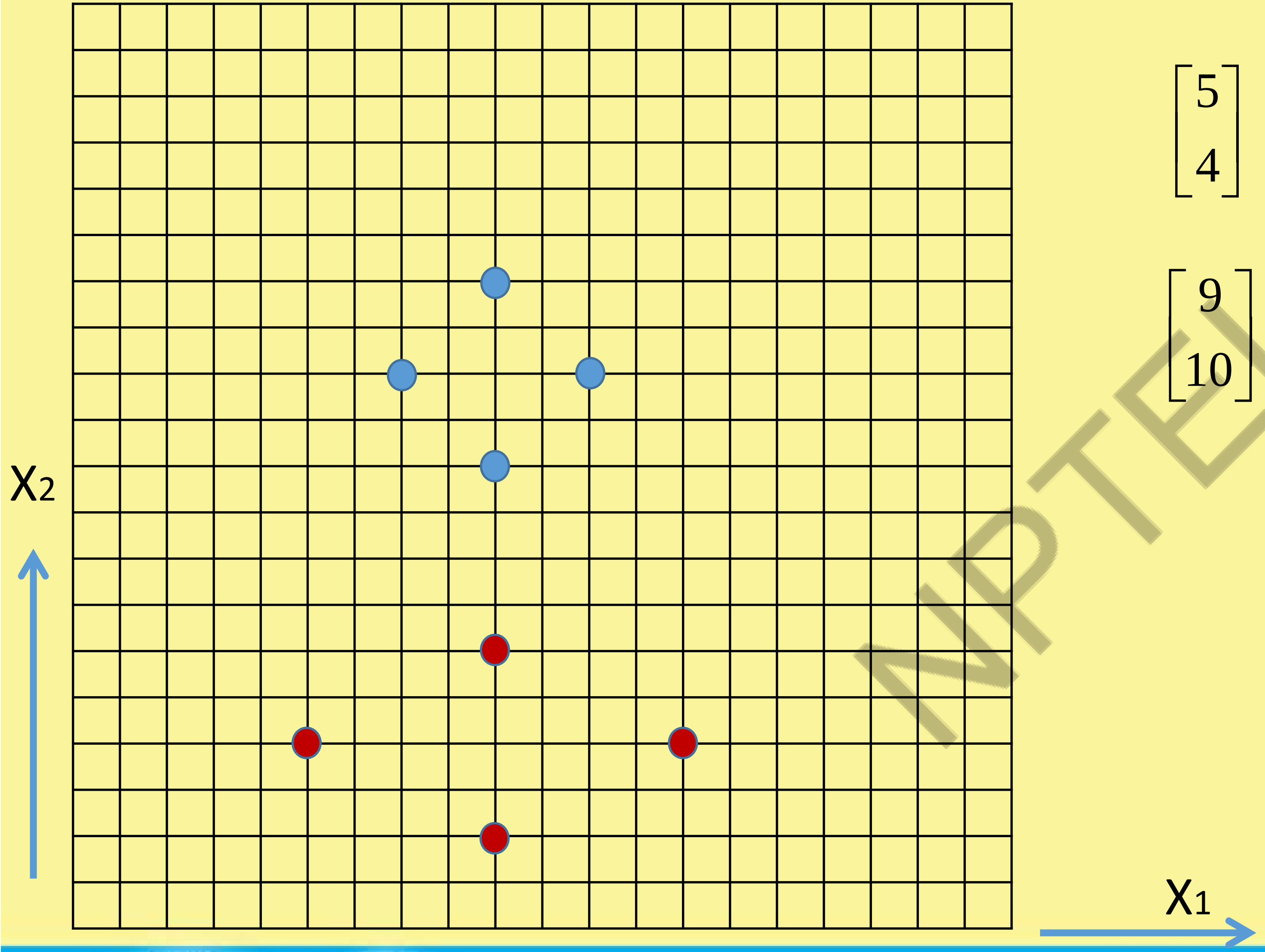
$$W^t(X - X_0) = 0$$

$$W = \Sigma^{-1}[\mu_2 - \mu_1] = \frac{1}{8}\begin{bmatrix} 5 & -3 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$$

$$X_0 = \frac{1}{2}(\mu_1 + \mu_2) - \frac{1}{(\mu_1 - \mu_2)^t \Sigma^{-1}(\mu_1 - \mu_2)} \ln\frac{P(\omega_1)}{P(\omega_2)}(\mu_1 - \mu_2)$$

Discriminant Function under Multivariate Normal Distribution

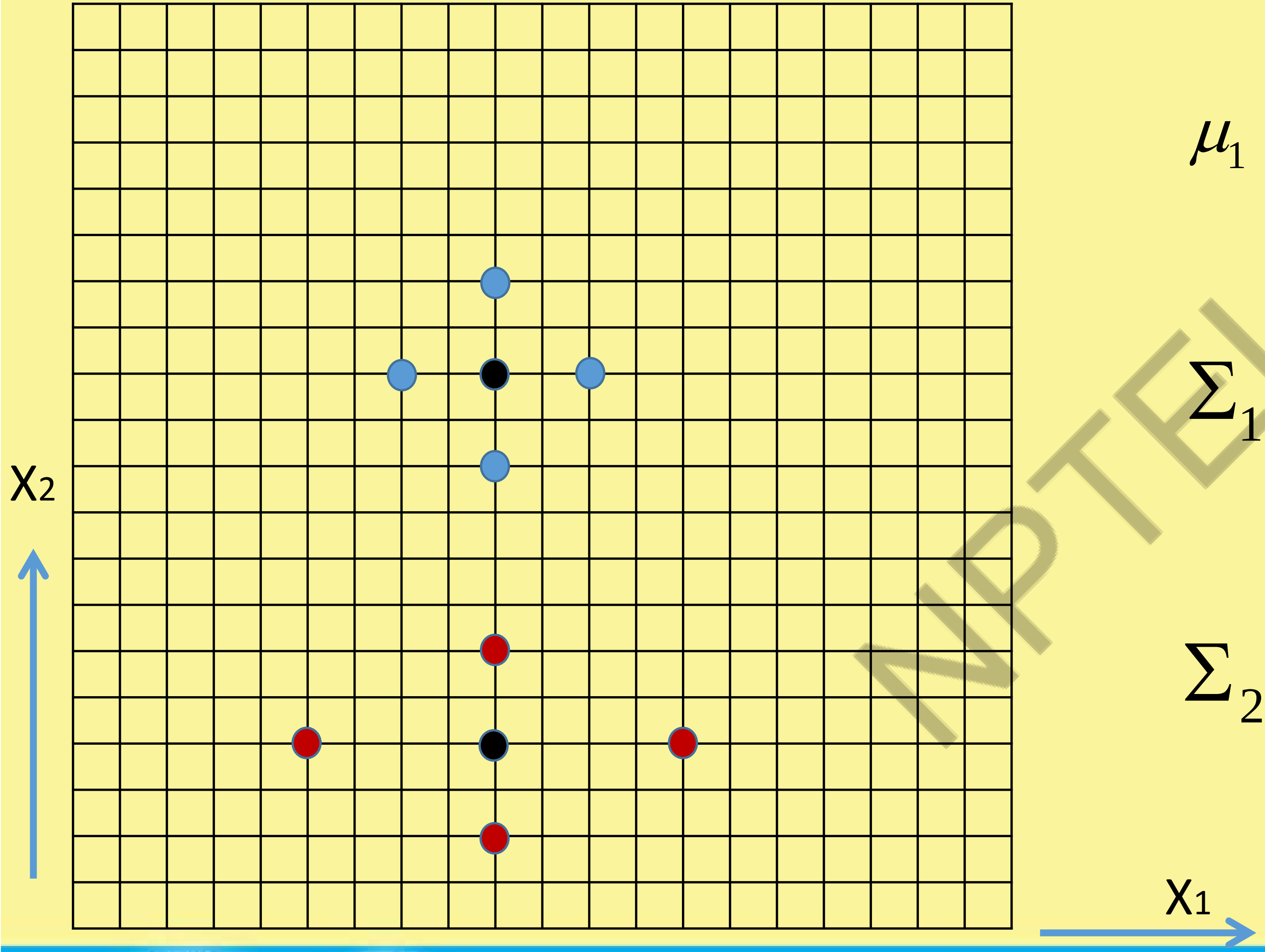
Decision Surface



$$\begin{bmatrix}5\\4\end{bmatrix}\ \begin{bmatrix}9\\2\end{bmatrix}\ \begin{bmatrix}9\\6\end{bmatrix}\ \begin{bmatrix}13\\6\end{bmatrix} \Rightarrow \omega_1$$

$$\begin{bmatrix}9\\10\end{bmatrix}\ \begin{bmatrix}9\\14\end{bmatrix}\ \begin{bmatrix}7\\12\end{bmatrix}\ \begin{bmatrix}11\\12\end{bmatrix} \Rightarrow \omega_2$$

Decision Surface



$$\mu_1 = \begin{bmatrix} 12 \\ 6 \end{bmatrix} \quad \mu_2 = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} 8 & 0 \\ 0 & 2 \end{bmatrix}$$

Discriminant Function

$$g_i(X) = -\frac{d}{2}\ln 2\pi - \frac{1}{2}\ln|\Sigma_i| - \frac{1}{2}\left[(X-\mu_i)^t \Sigma_i^{-1}(X-\mu_i)\right] + \ln P(\omega_i)$$

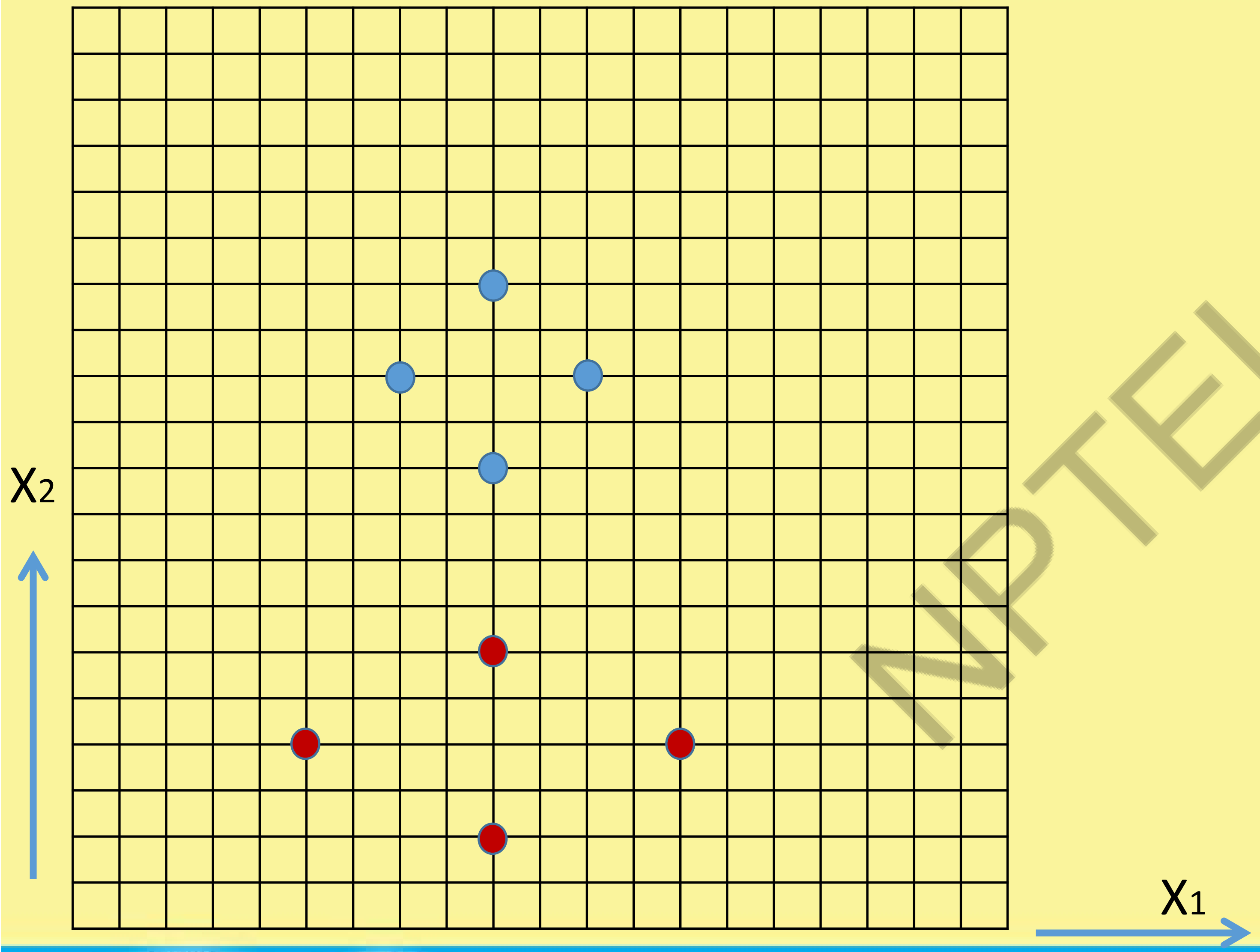
$$= X^t A_i X + B_i^t X + C_i$$

$$A_i = -\frac{1}{2}\Sigma_i^{-1}$$

$$B_i = \Sigma_i^{-1}\mu_i$$

$$C_i = -\frac{1}{2}\mu_i^t \Sigma_i^{-1}\mu_i - \frac{1}{2}\ln|\Sigma_i| + \ln P(\omega_i)$$

Decision Surface



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Course Name: Deep Learning

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Department : E & ECE, IIT Kharagpur

Topic

Lecture 09: Linear Classifier

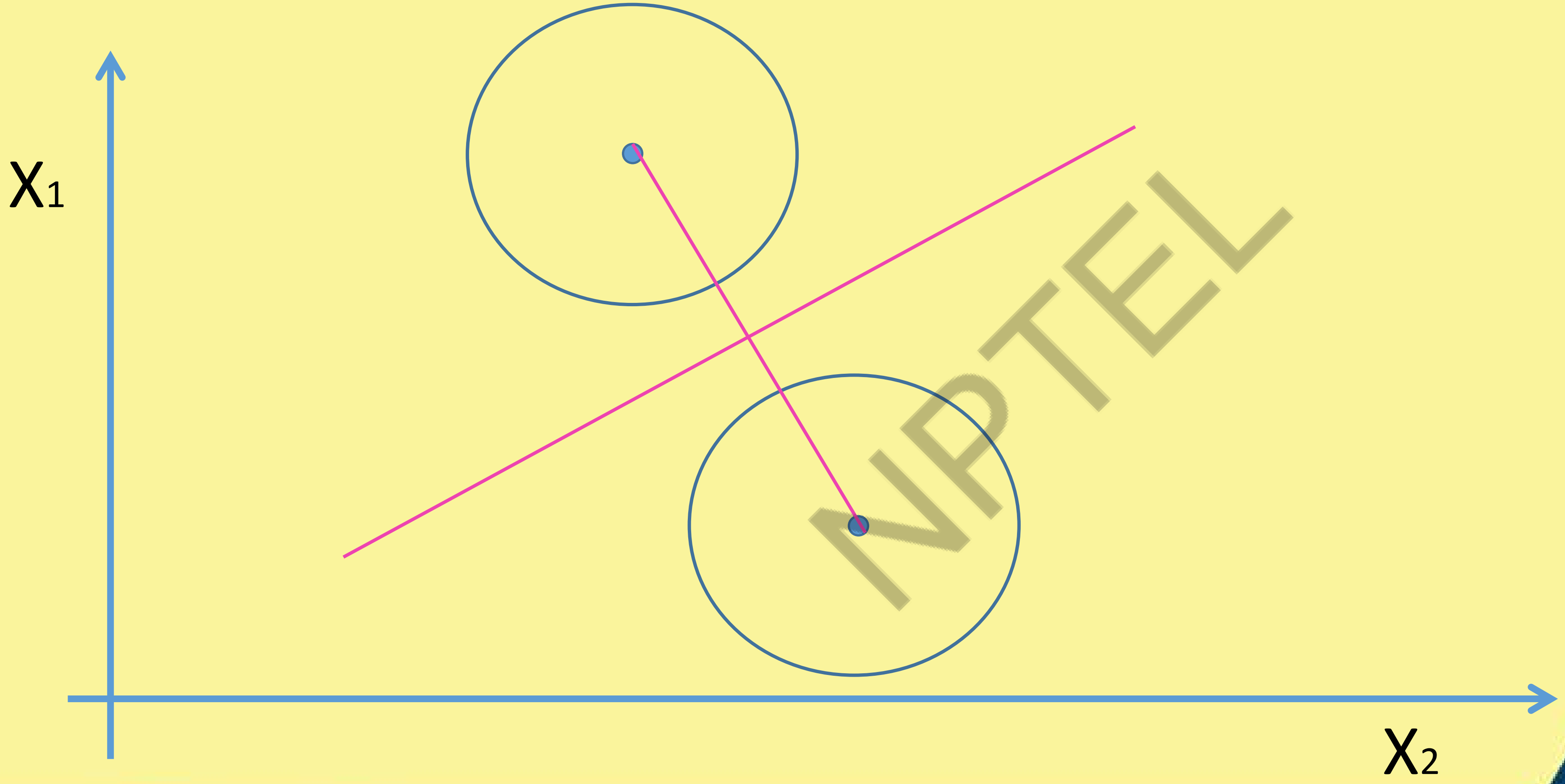
CONCEPTS COVERED

Concepts Covered:

- Discriminant Function and Decision Boundary
- Nearest Neighbour and k-NN Classifier
- Linear Classifier
- Support Vector Machine (SVM)

Nearest Neighbour Rule

Minimum Distance Classifier



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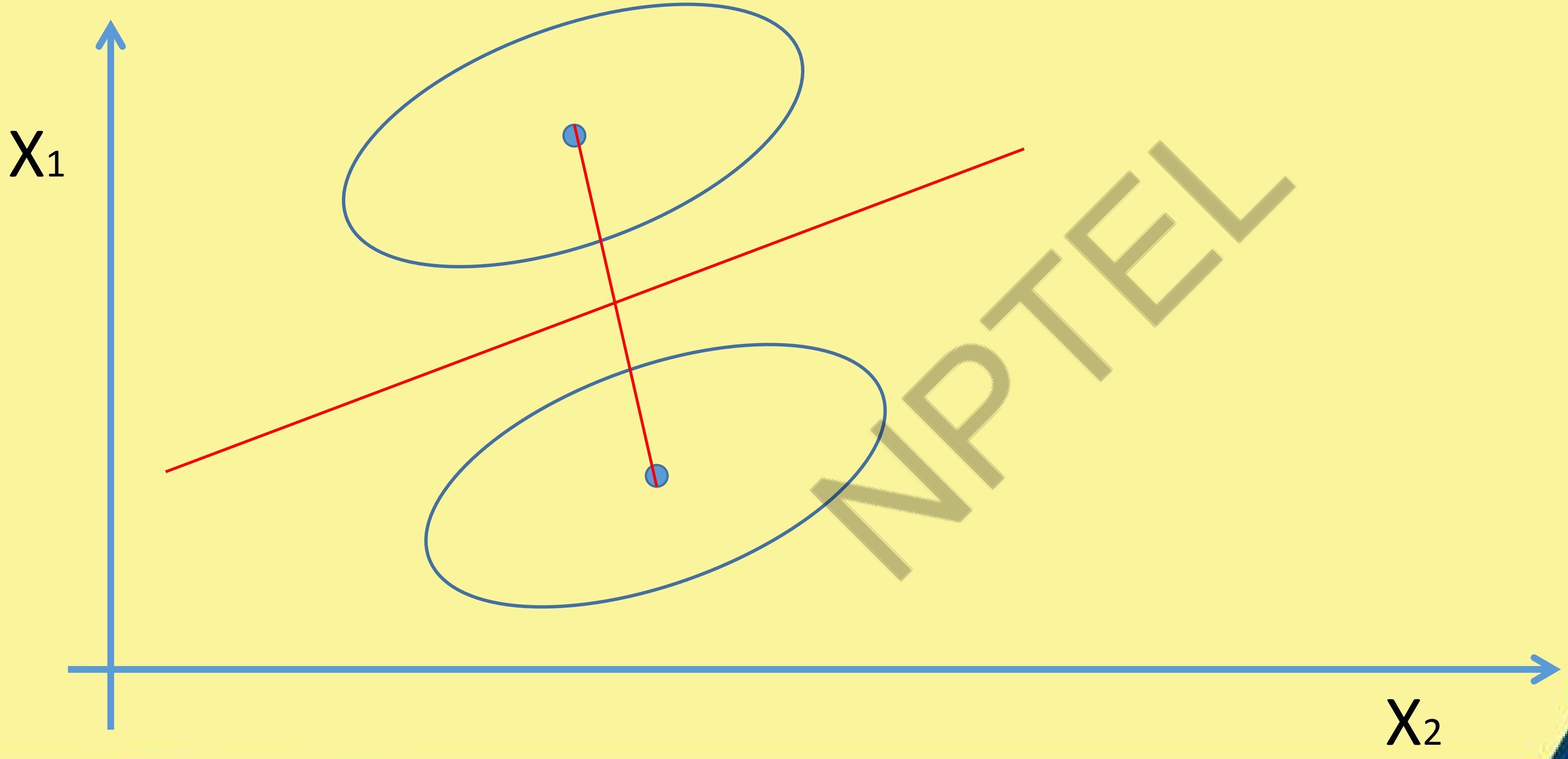
Minimum Distance Classifier



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Nearest Neighbour Rule

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Nearest Neighbour Rule

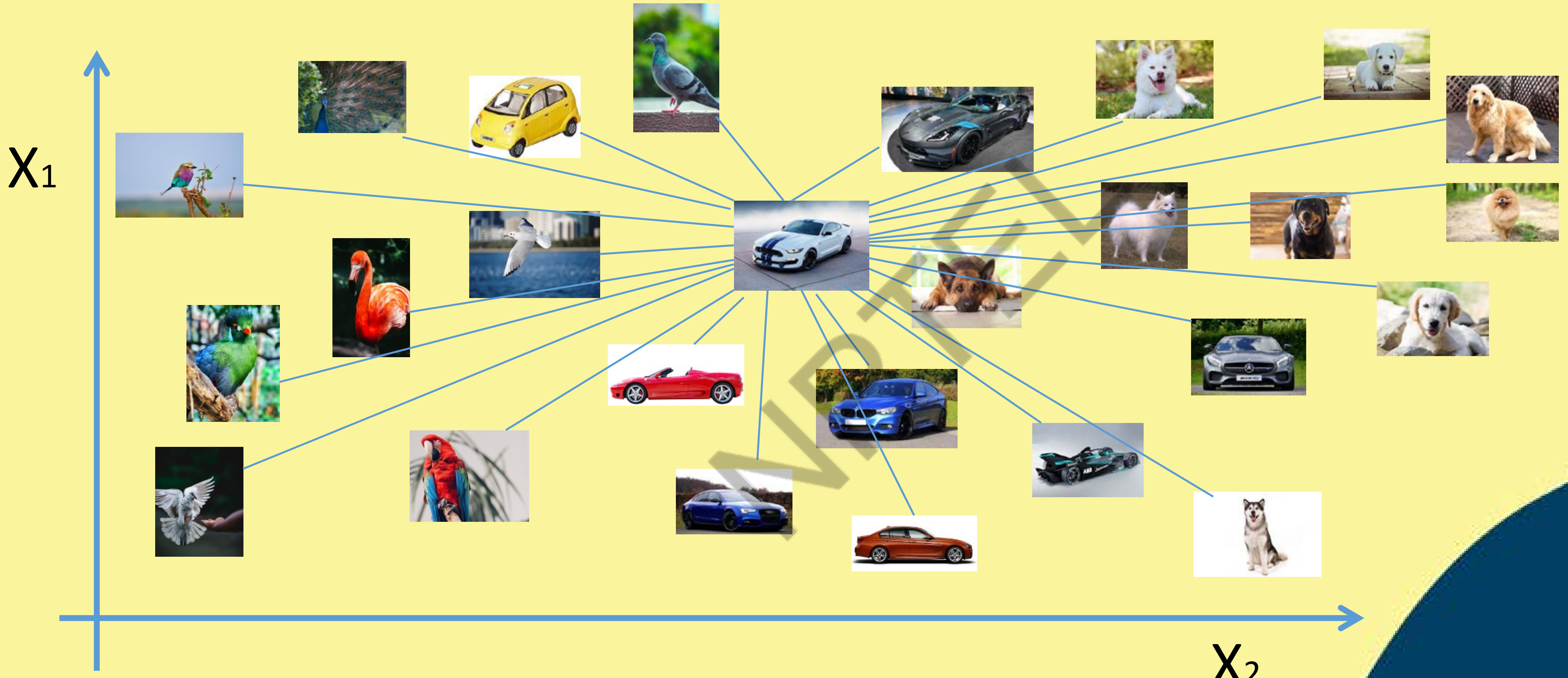


Image Source: Internet

Nearest Neighbour Rule

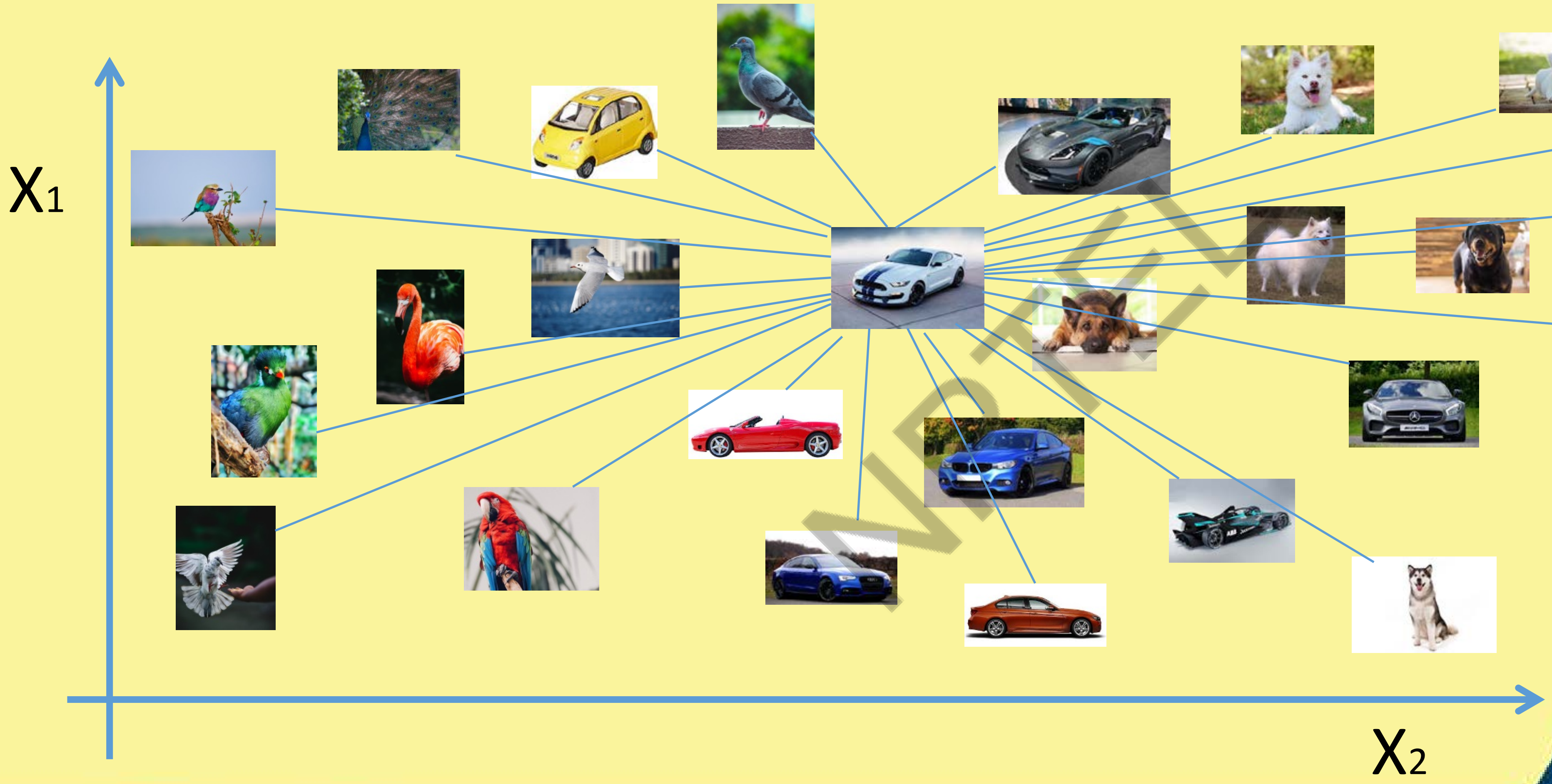
Image Source: Internet

k-Nearest Neighbour Rule

K-Nearest Neighbour Rule (k-NN Rule)

Image Source: Internet

K-Nearest Neighbour Rule (k-NN Rule)



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K-Nearest Neighbour Rule (k-NN Rule)

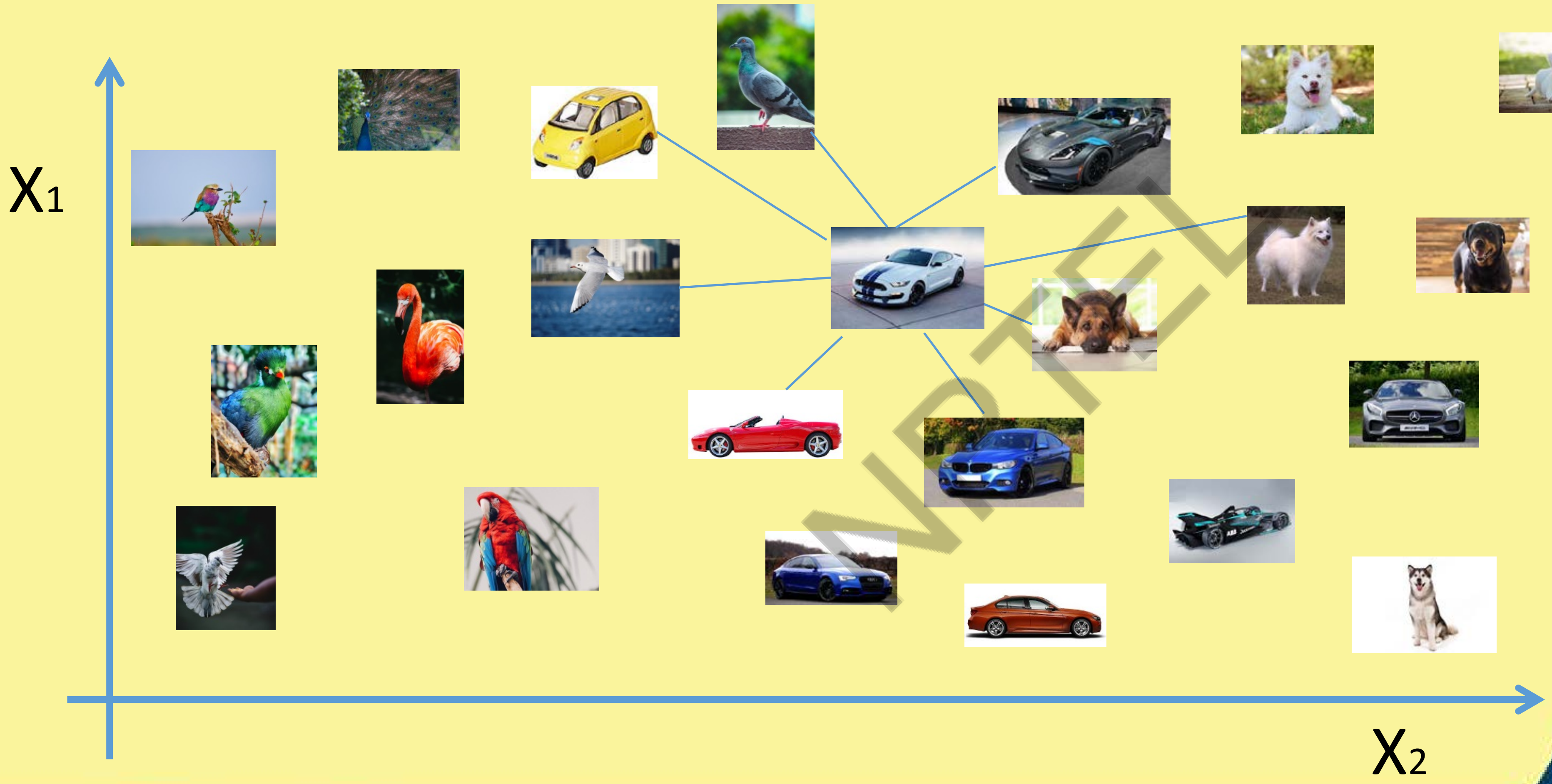


Image Source: Internet

K-Nearest Neighbour Rule (k-NN Rule)

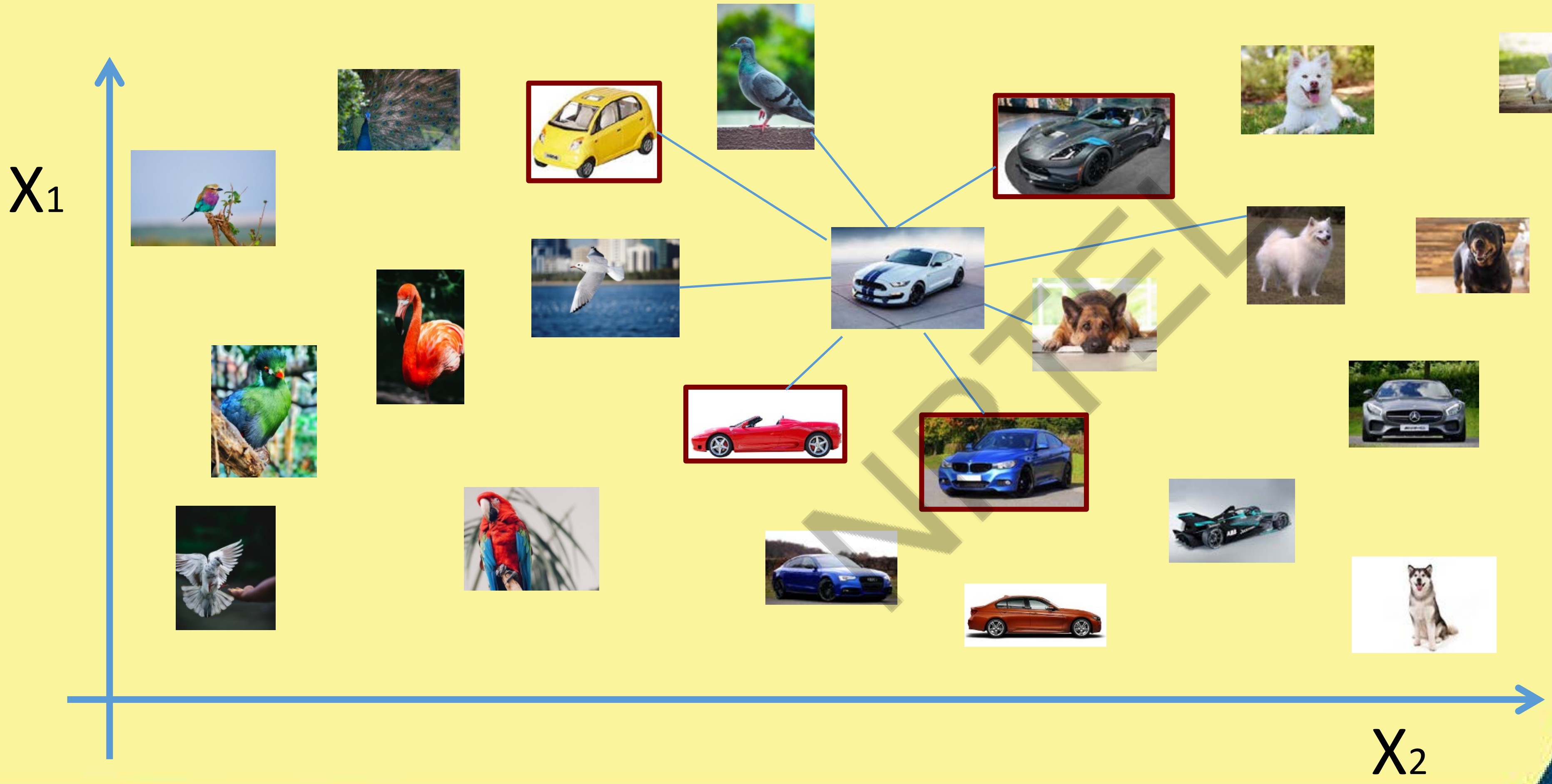
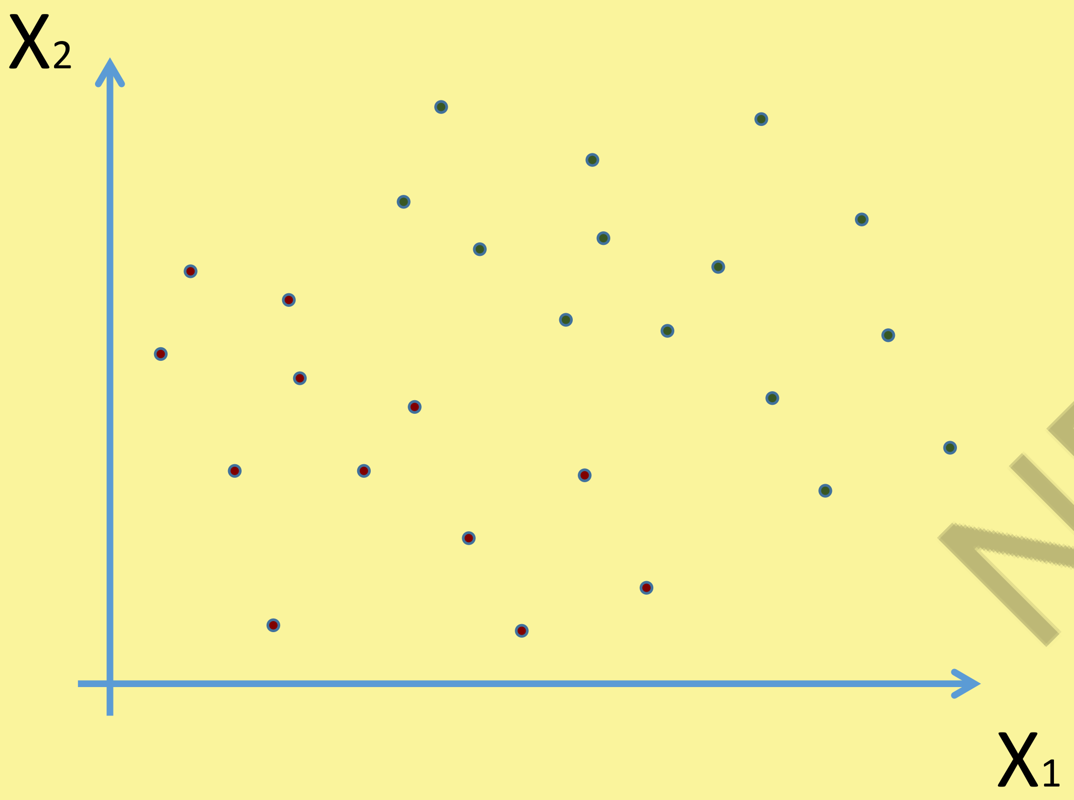
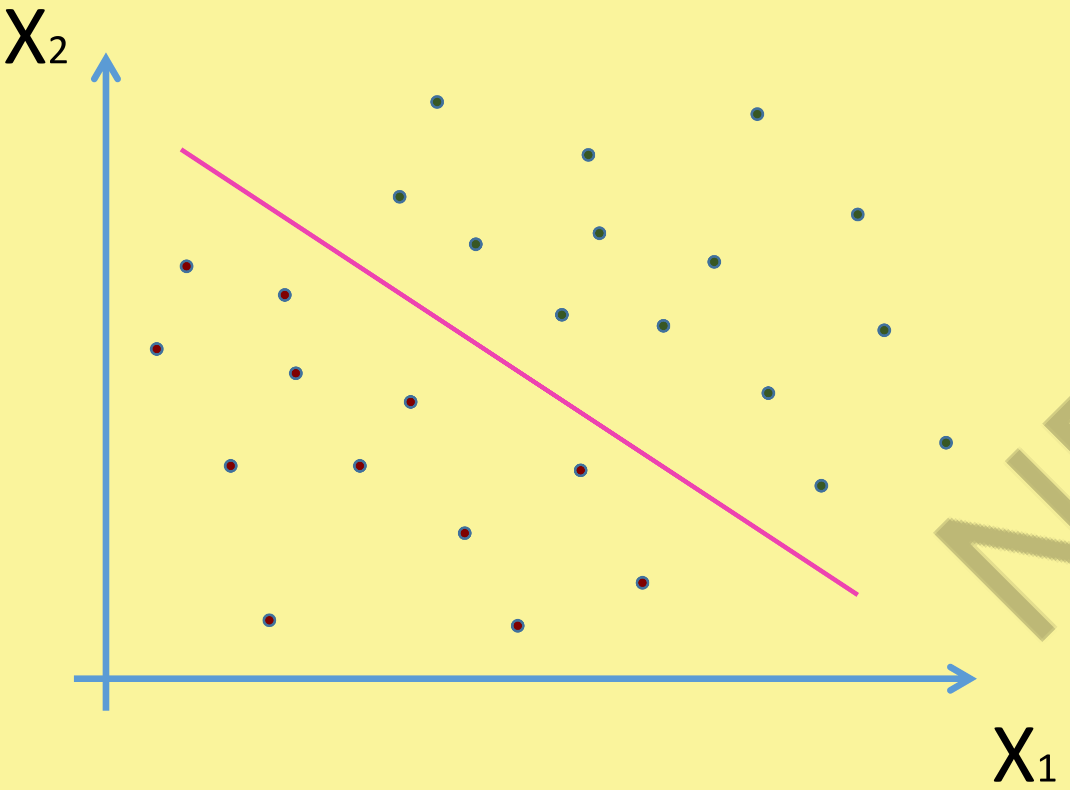


Image Source: Internet

Linear Classifier – 2 Class Problem

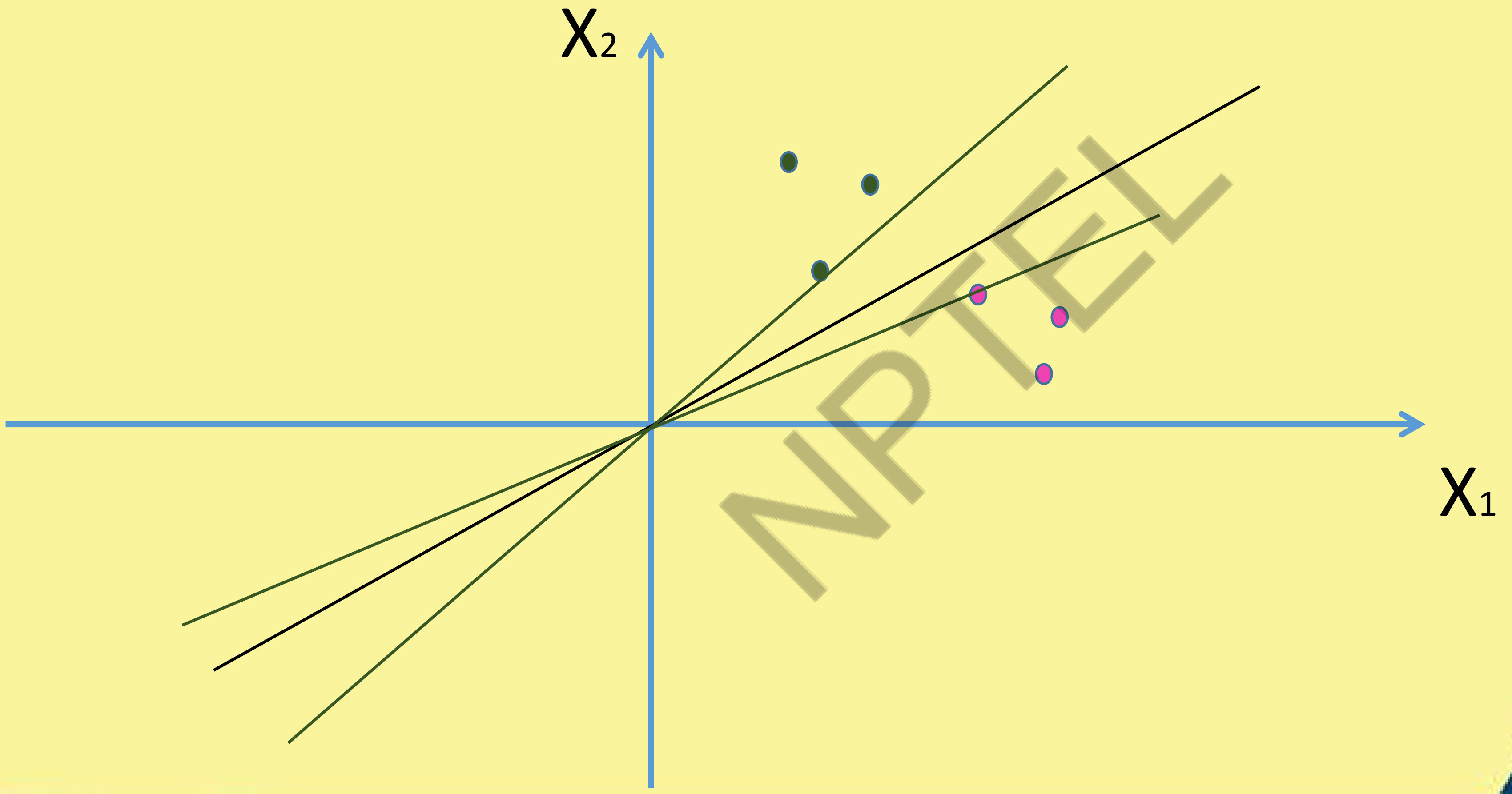


Linear Classifier – 2 Class Problem

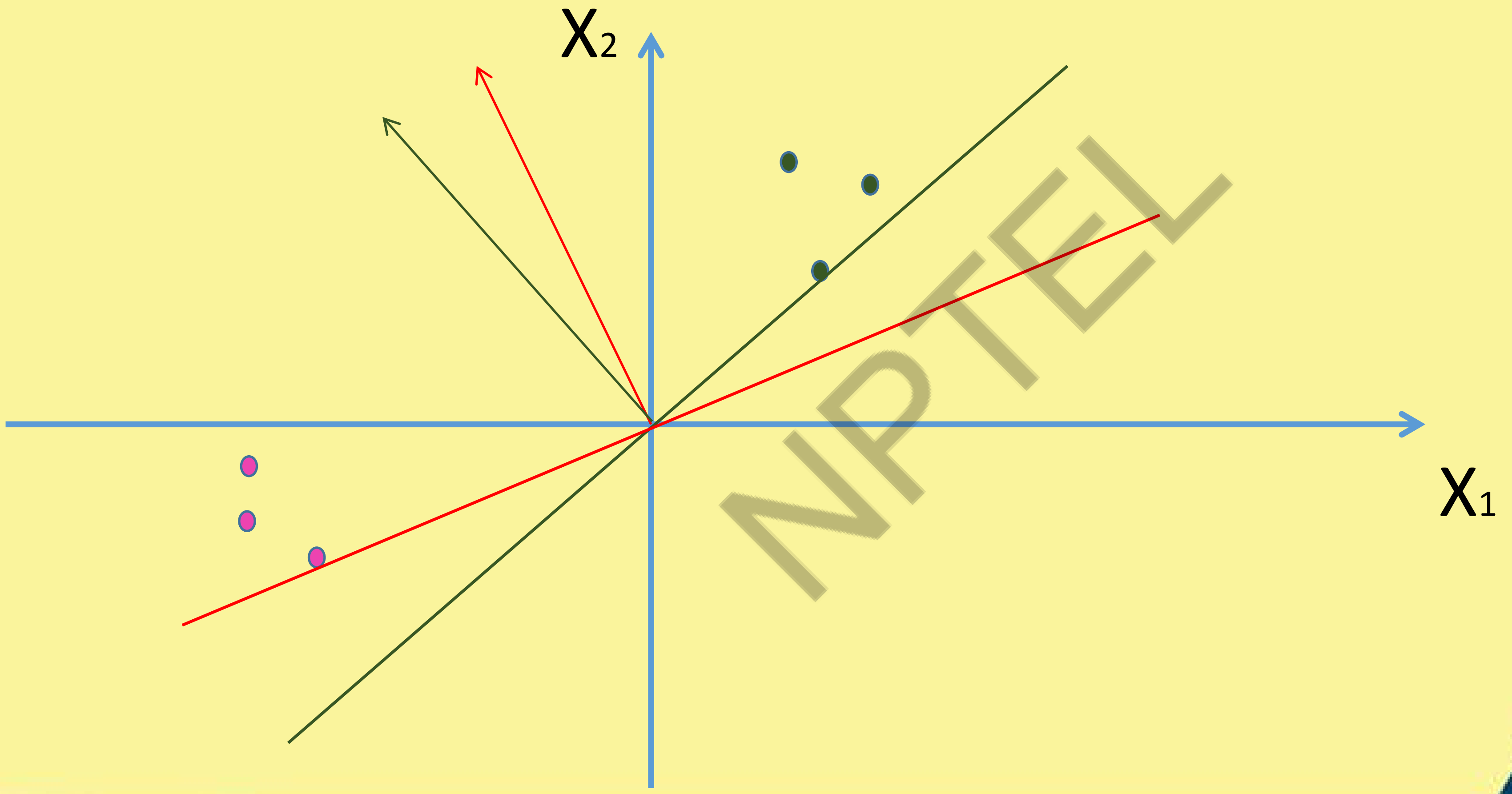


Linear Classifier – 2 Class Problem

Linear Classifier – 2 Class Problem



Linear Classifier – 2 Class Problem



Linear Classifier – 2 Class Problem

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Course Name: Deep Learning

Faculty Name: Prof. P. K. Biswas

Department : E & ECE, IIT Kharagpur

Topic

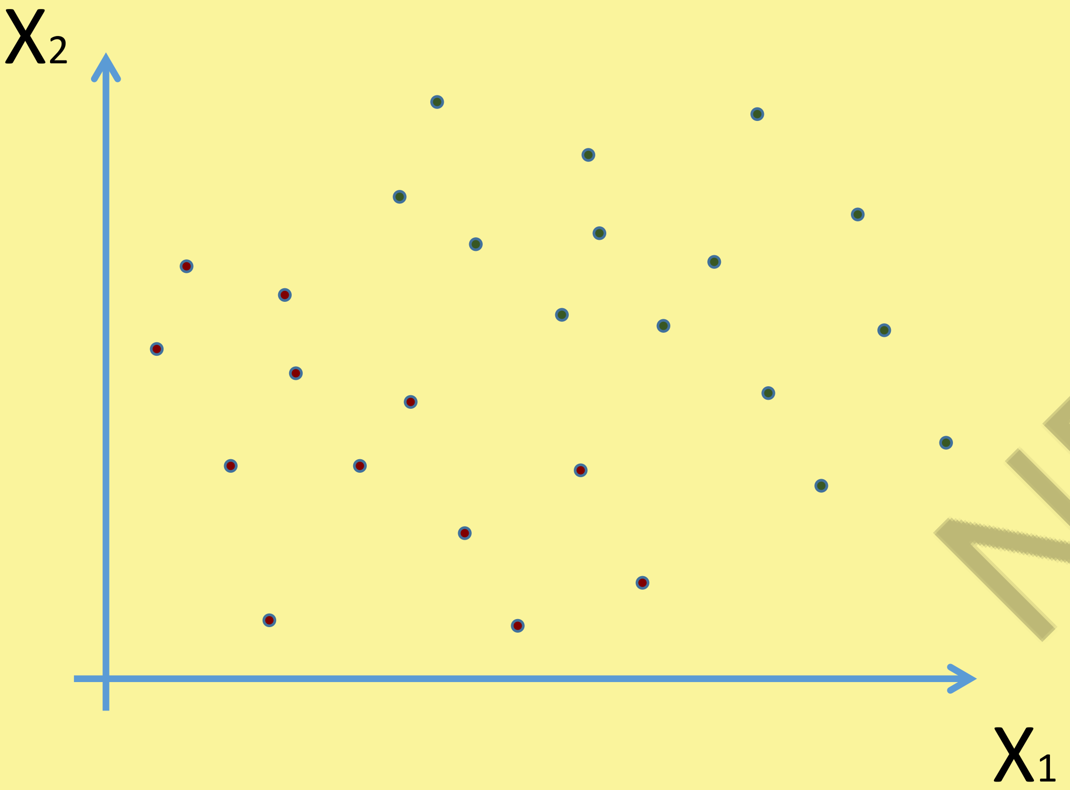
Lecture 10: Linear Classifier

CONCEPTS COVERED

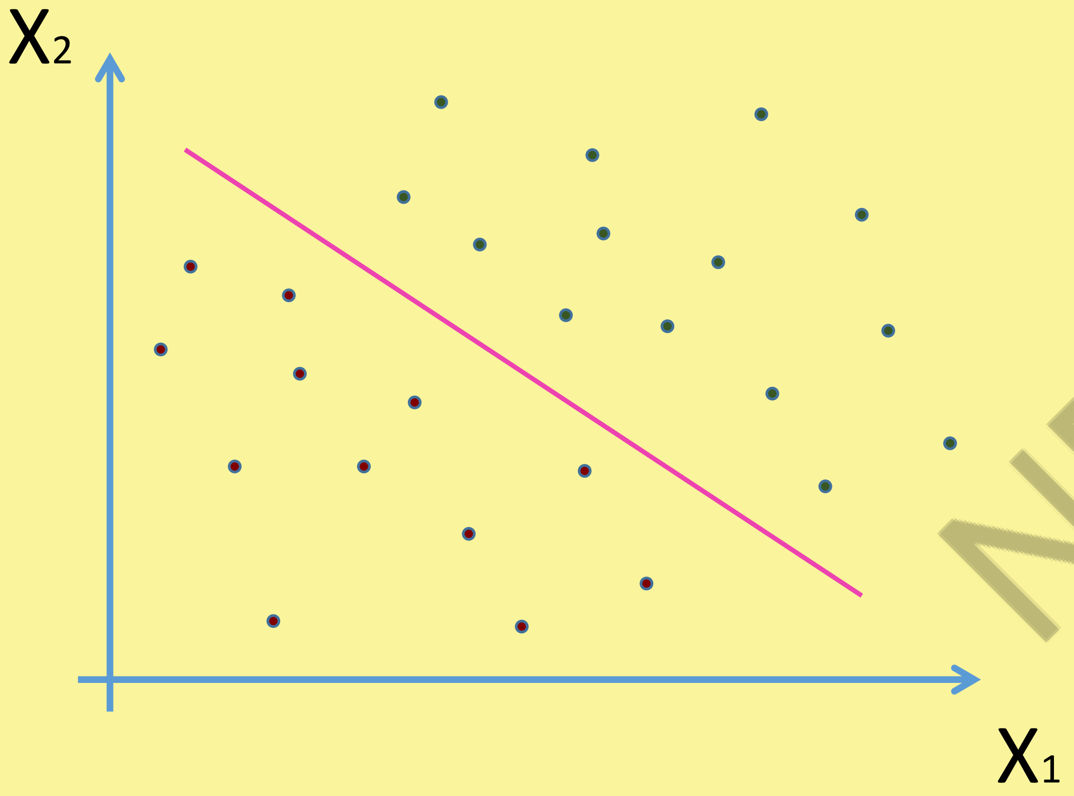
Concepts Covered:

- Discriminant Function and Decision Boundary
- Nearest Neighbour and k-NN Classifier
- Linear Classifier
- Support Vector Machine (SVM)

Linear Classifier – 2 Class Problem

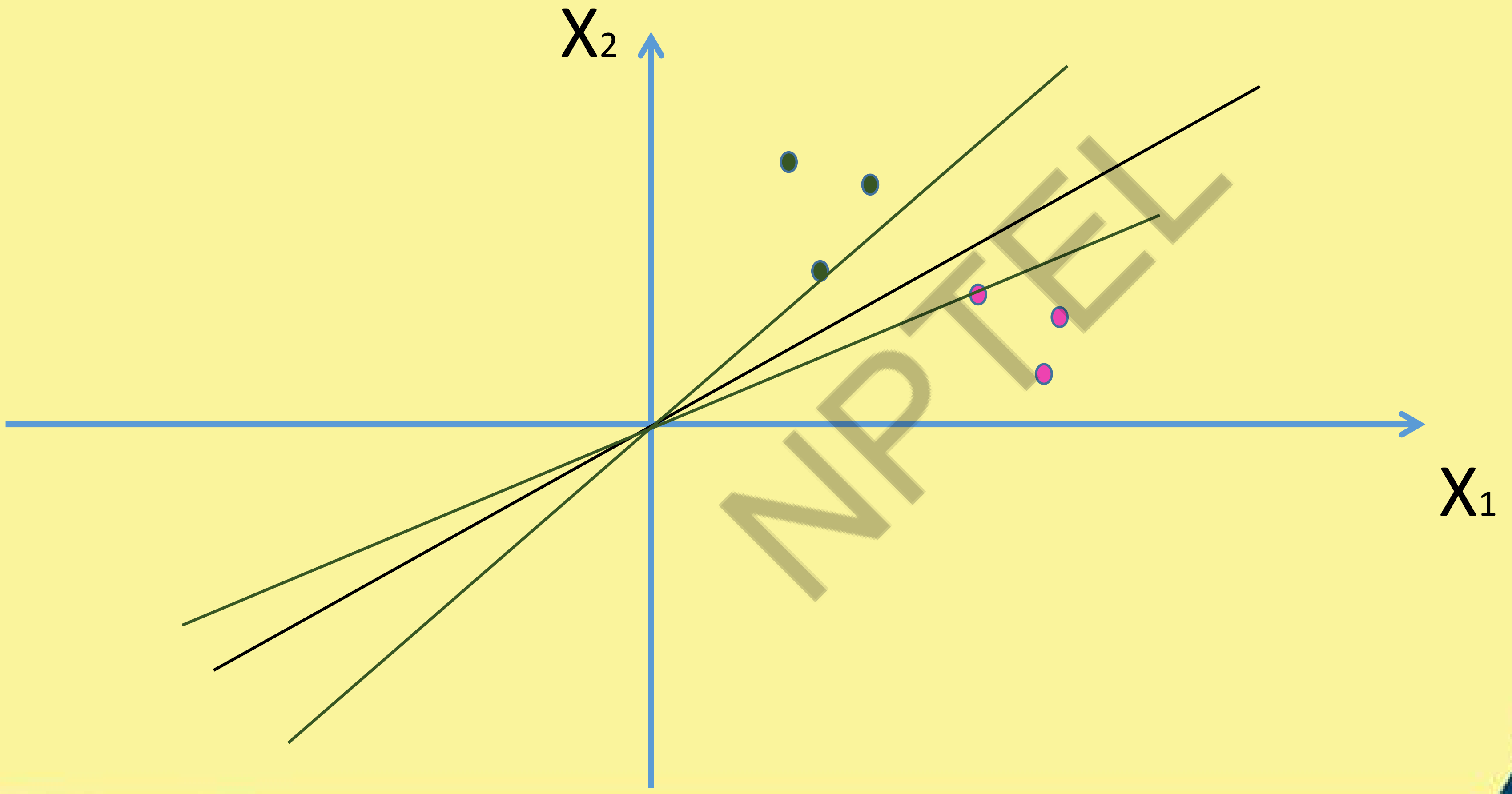


Linear Classifier – 2 Class Problem

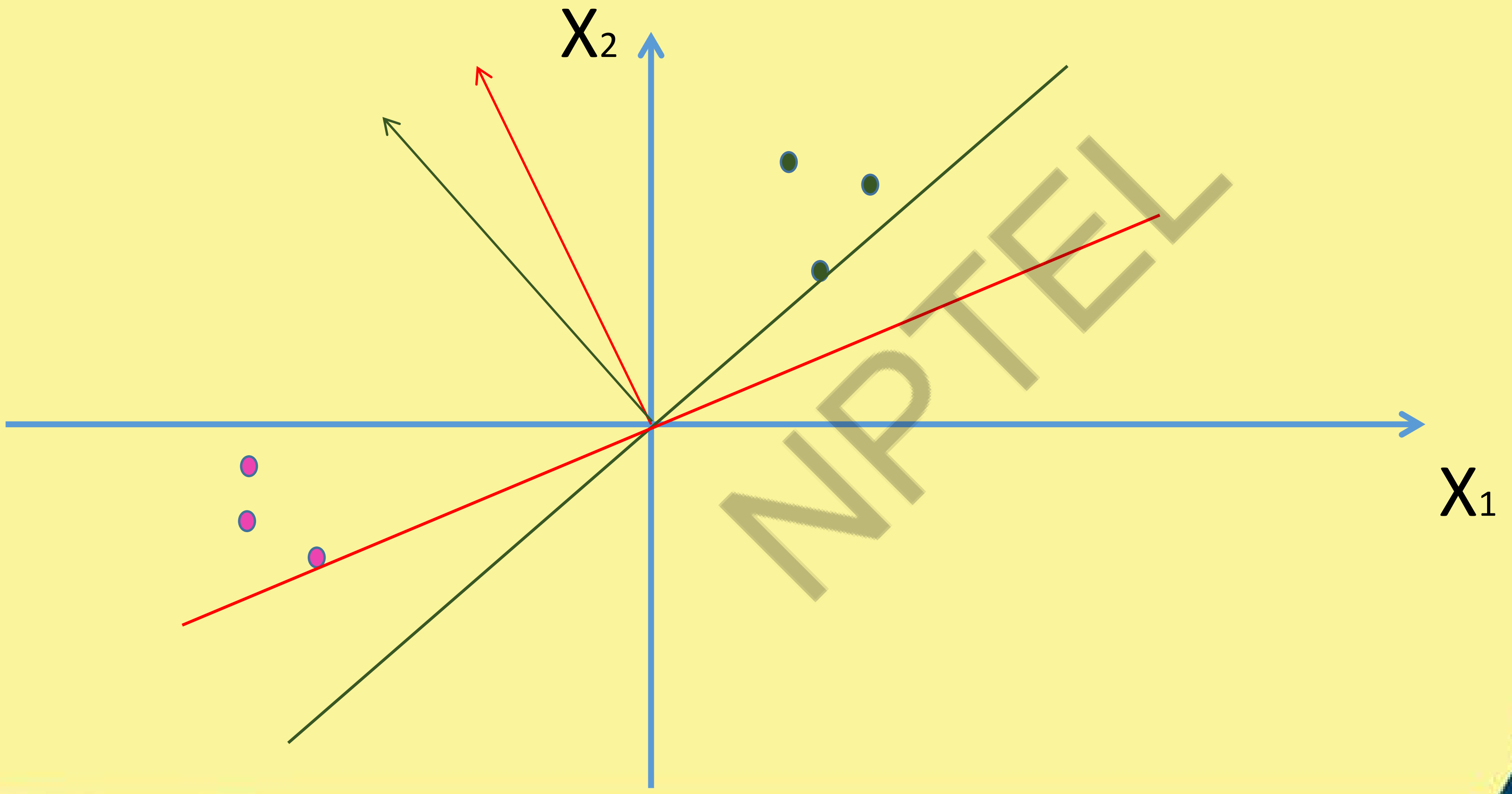


Linear Classifier – 2 Class Problem

Linear Classifier – 2 Class Problem

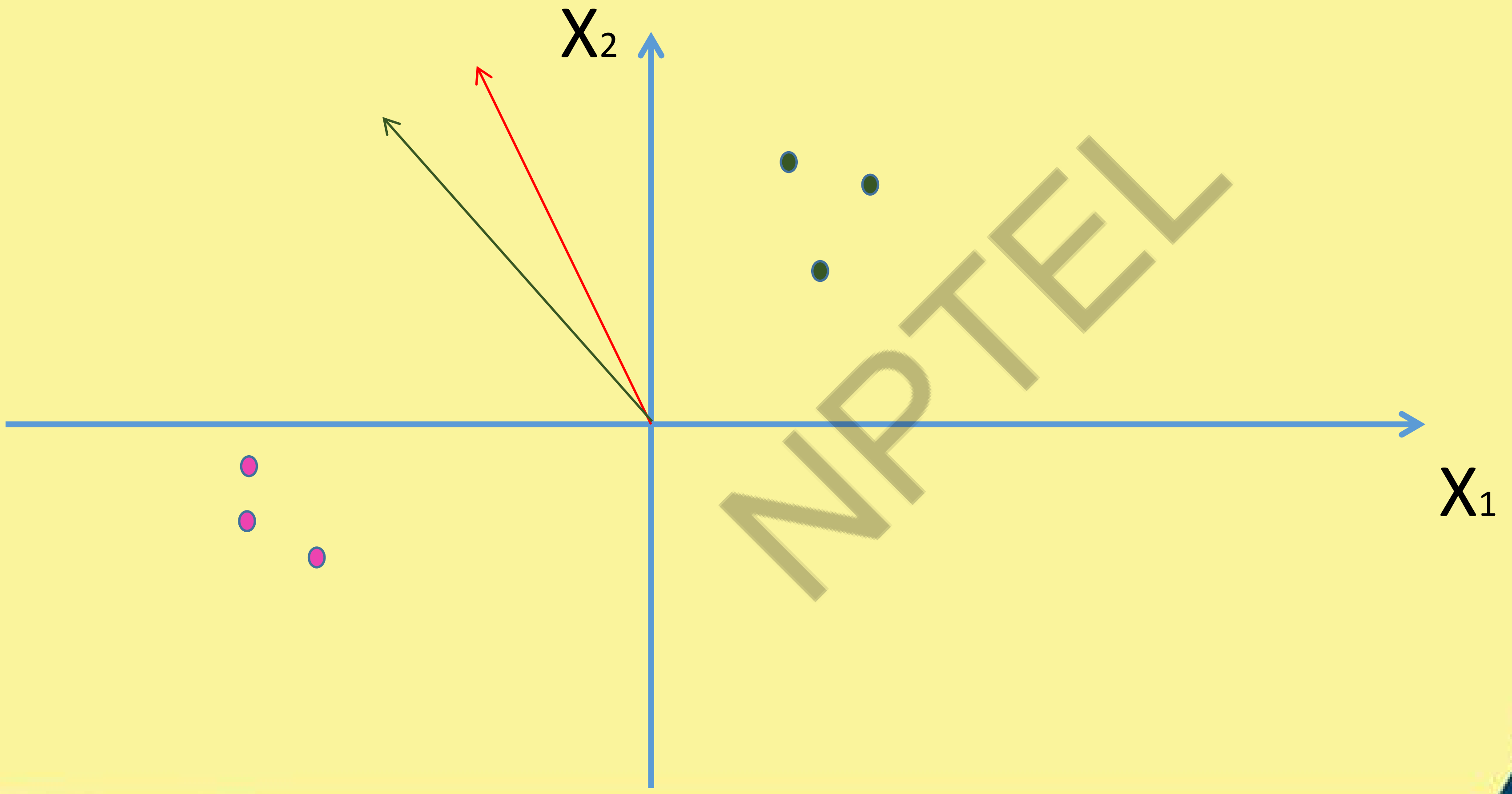


Linear Classifier – 2 Class Problem

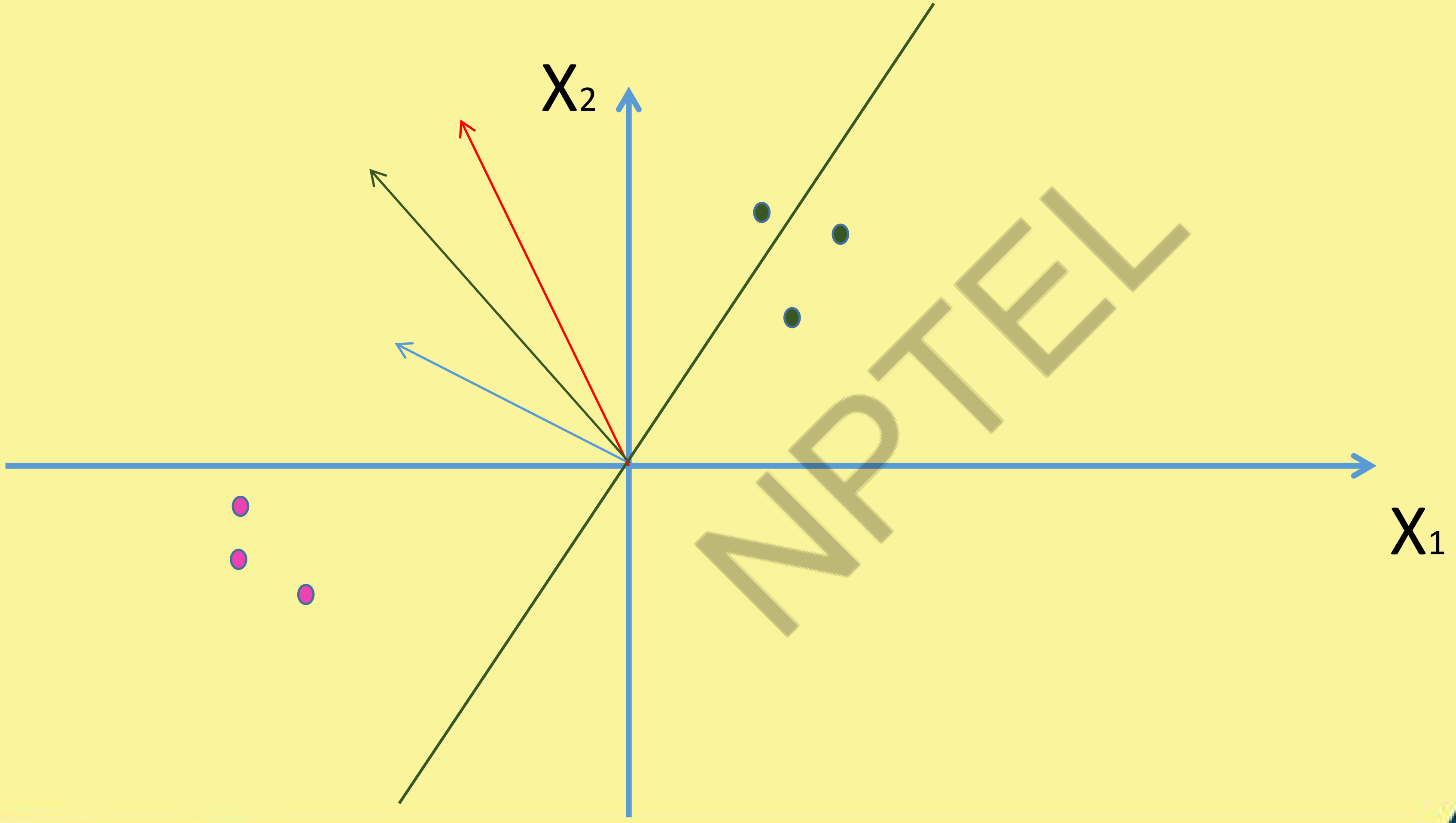


Linear Classifier – 2 Class Problem

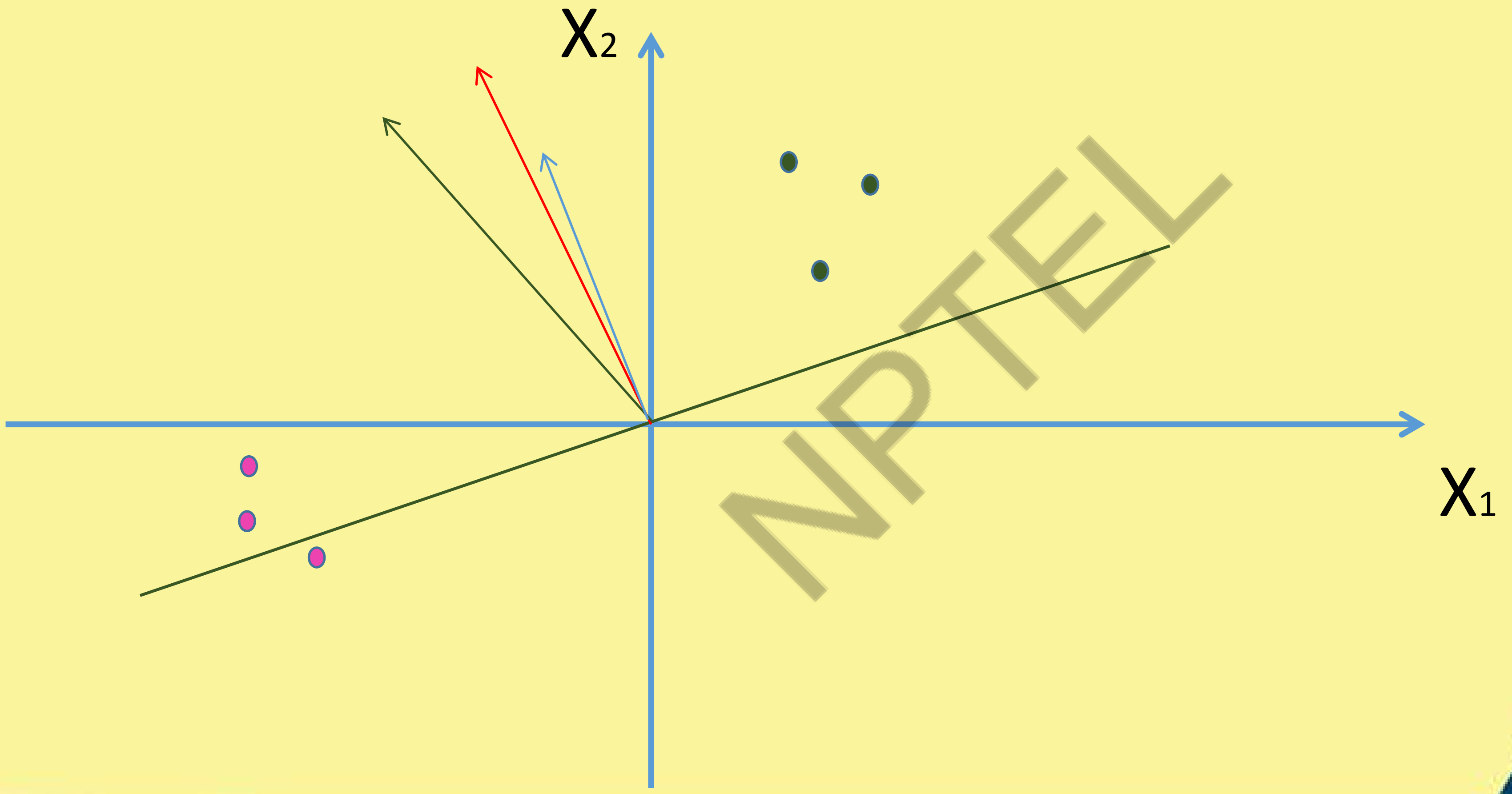
Linear Classifier – 2 Class Problem



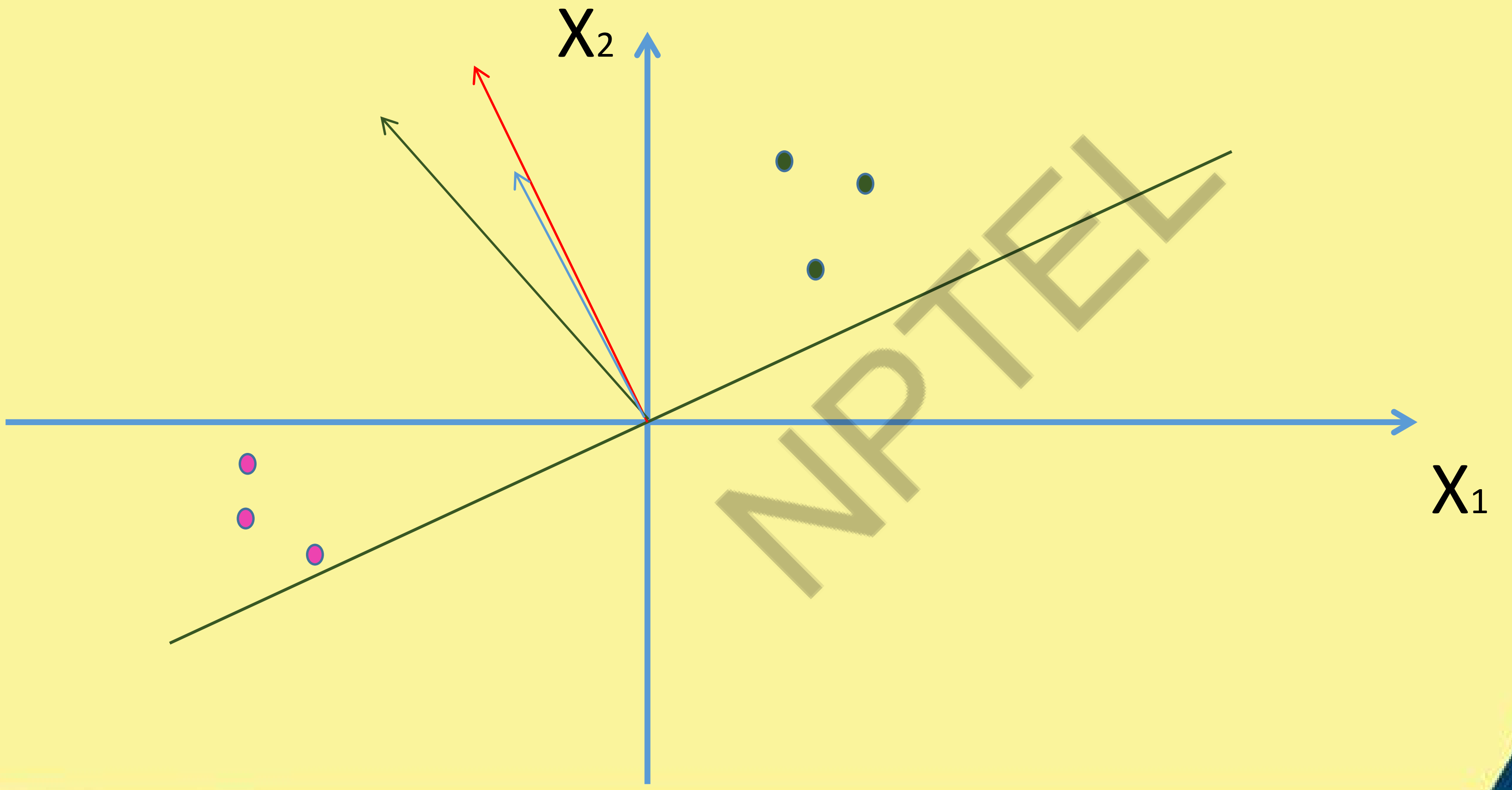
Linear Classifier – 2 Class Problem



Linear Classifier – 2 Class Problem



Linear Classifier – 2 Class Problem



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