NPTEL ONLINE CERTIFICATION COURSES

Course Name: Deep Learning

Faculty Name: Prof. P. K. Biswas
Department : E & ECE, IIT Kharagpur

Topic

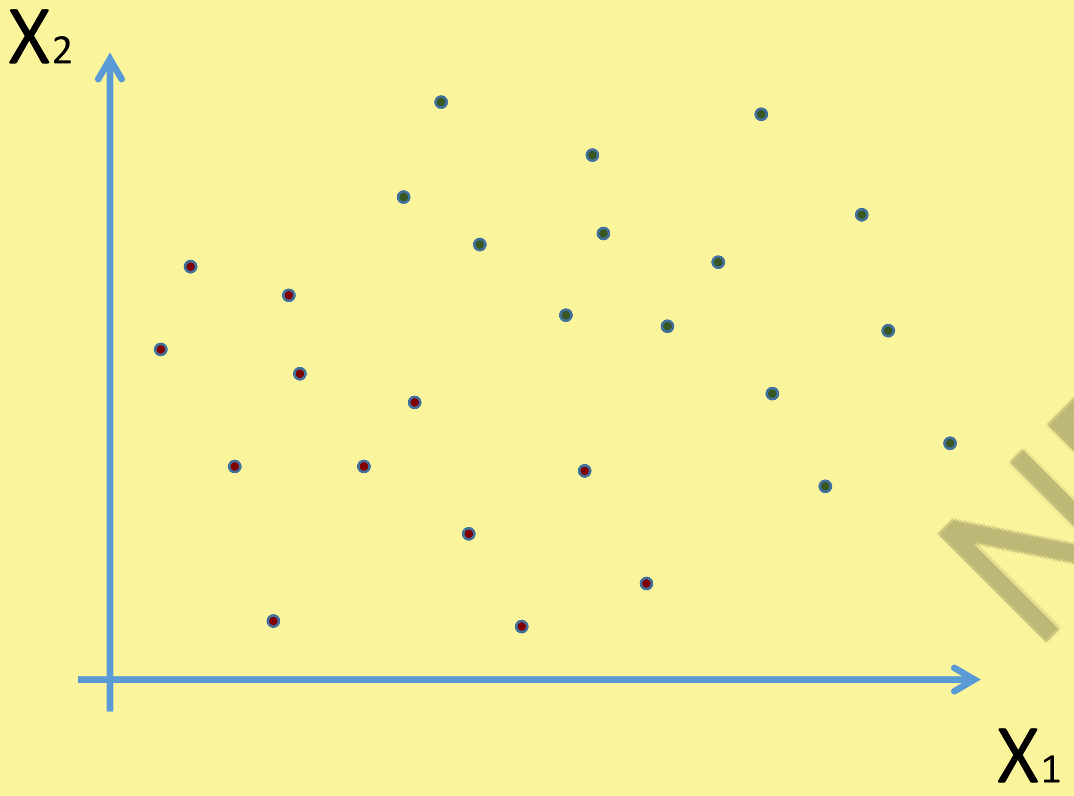
Lecture 11: Support Vector Machine

CONCEPTS COVERED

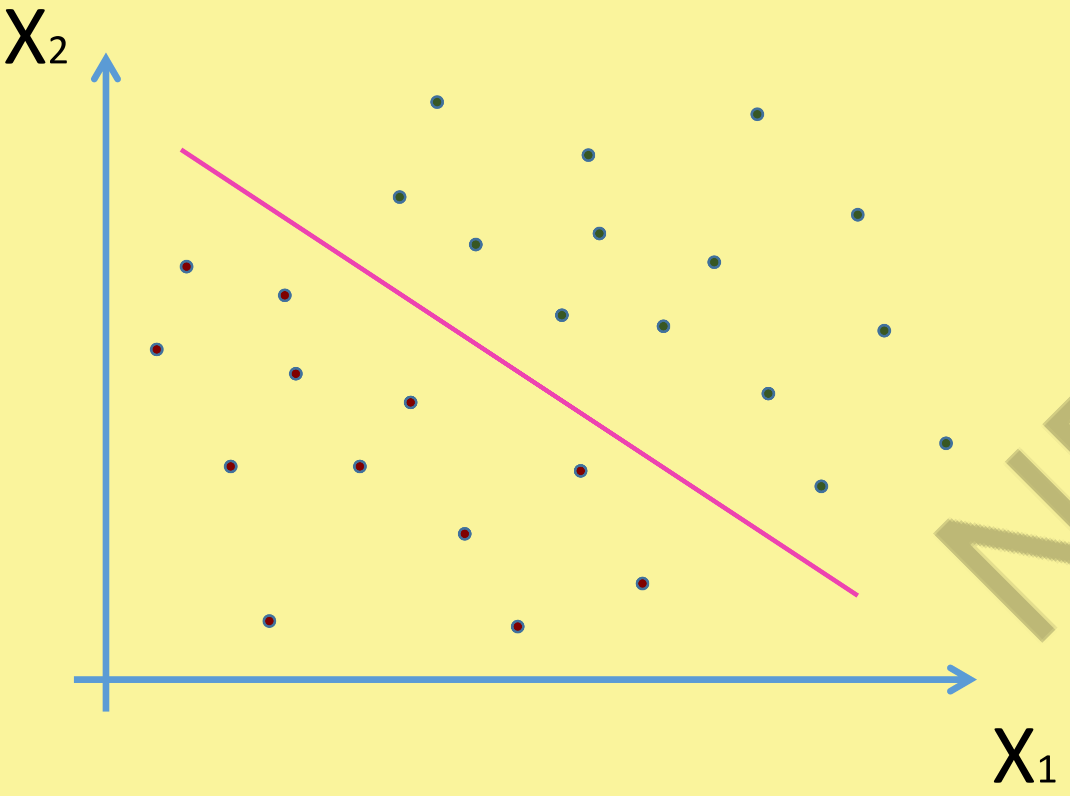
Concepts Covered:

- Linear Discriminator
- Perceptron Algorithm
- Support Vector Machine (SVM)

Linear Classifier – 2 Class Problem



Linear Classifier – 2 Class Problem



$$a^t X + b = 0$$

$$For\ X \in \omega_1 : \quad a^t X + b > 0$$

$$For\ X \in \omega_2 : \quad a^t X + b < 0$$

Linear Classifier – 2 Class Problem

$$a^t X + b = 0 \;\Rightarrow\; a^t X = 0$$

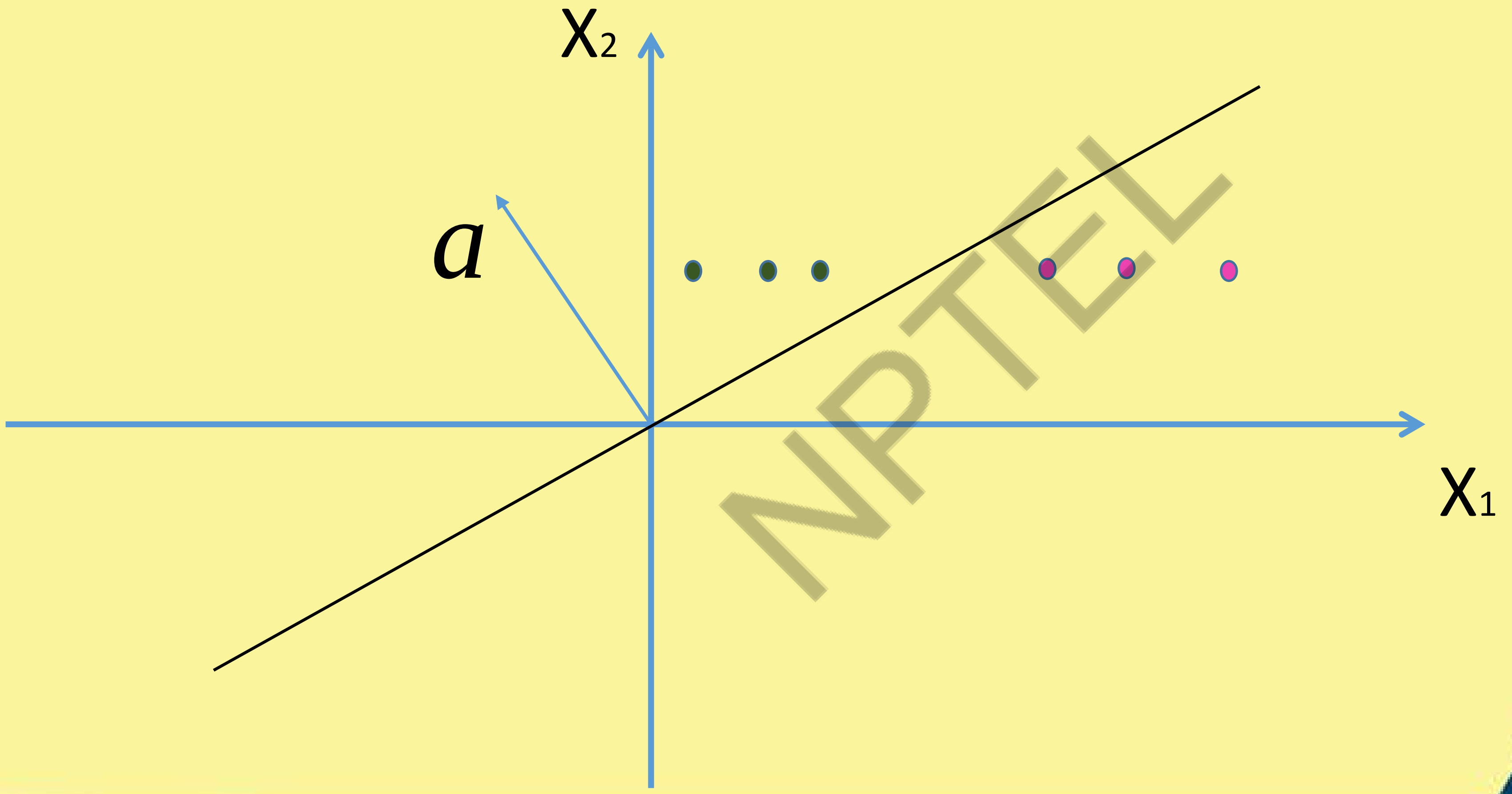
$$a = \begin{bmatrix} a_1 \\ a_2 \\ : \\ a_d \\ b \end{bmatrix} \qquad X = \begin{bmatrix} X_1 \\ X_2 \\ : \\ X_d \\ 1 \end{bmatrix}$$

Classification Rule

$$For Y \in \omega_1 :\; a^t X > 0$$

$$For Y \in \omega_2 :\; a^t X < 0$$

Linear Classifier – 2 Class Problem



Linear Classifier – 2 Class Problem

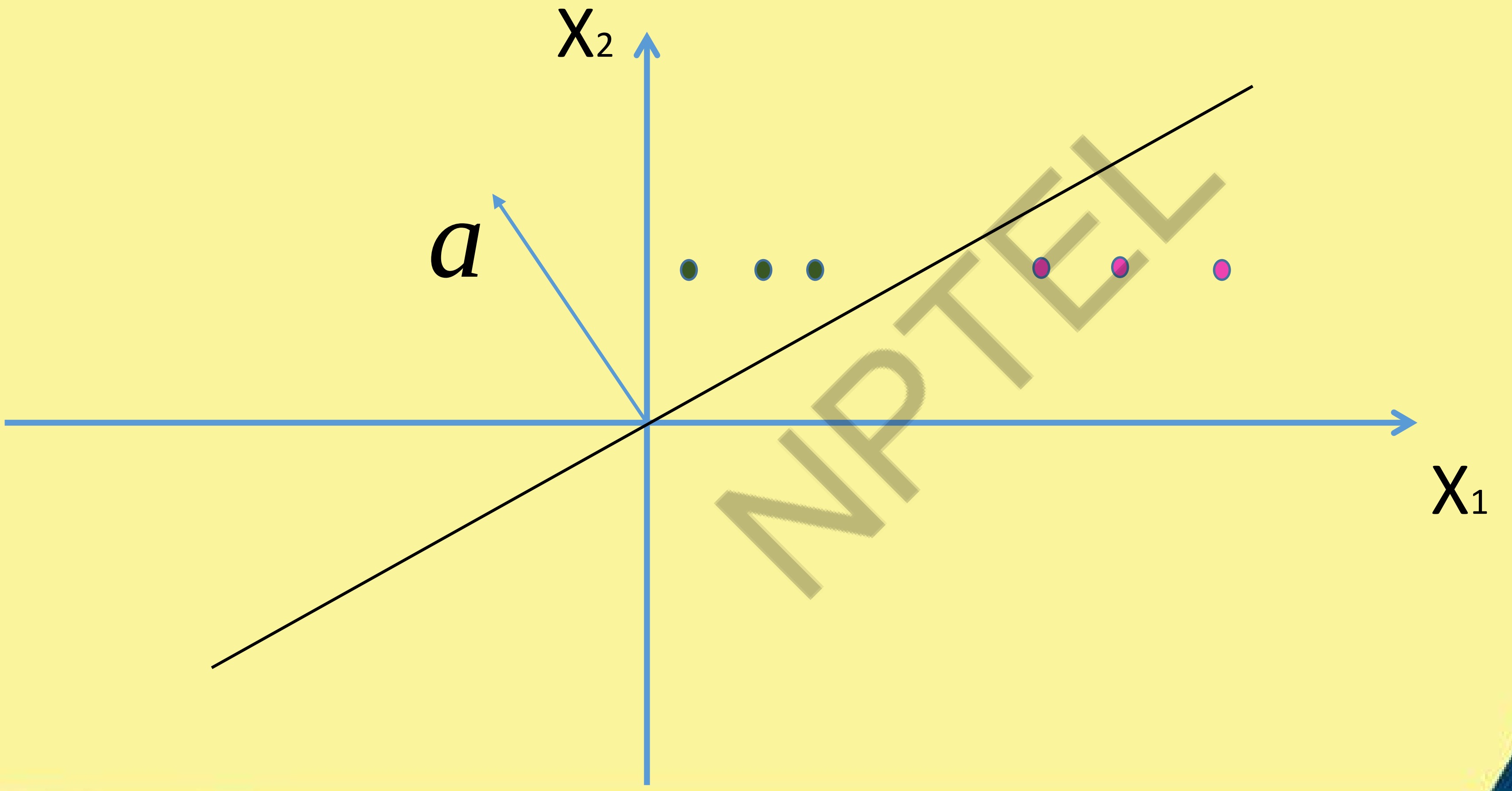
Negating all X from ω_2 : $X \leftarrow -X$

Classification Rule: $a^t X > 0$

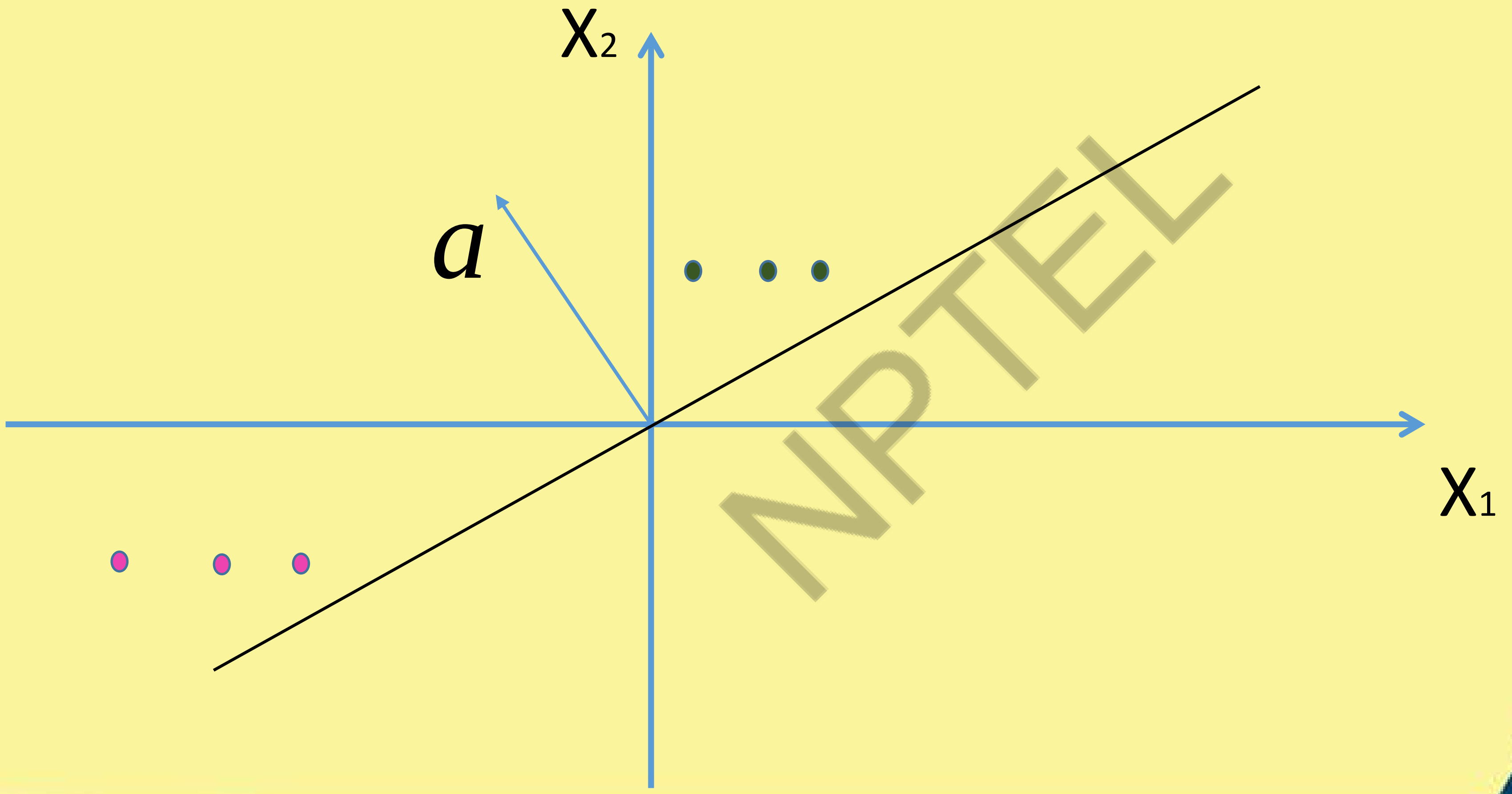
$If\ a^t X < 0;\ for\ any\ X\ irrespective\ of\ class$

$\Rightarrow "a"\ misclassifies\ that\ particular\ X$

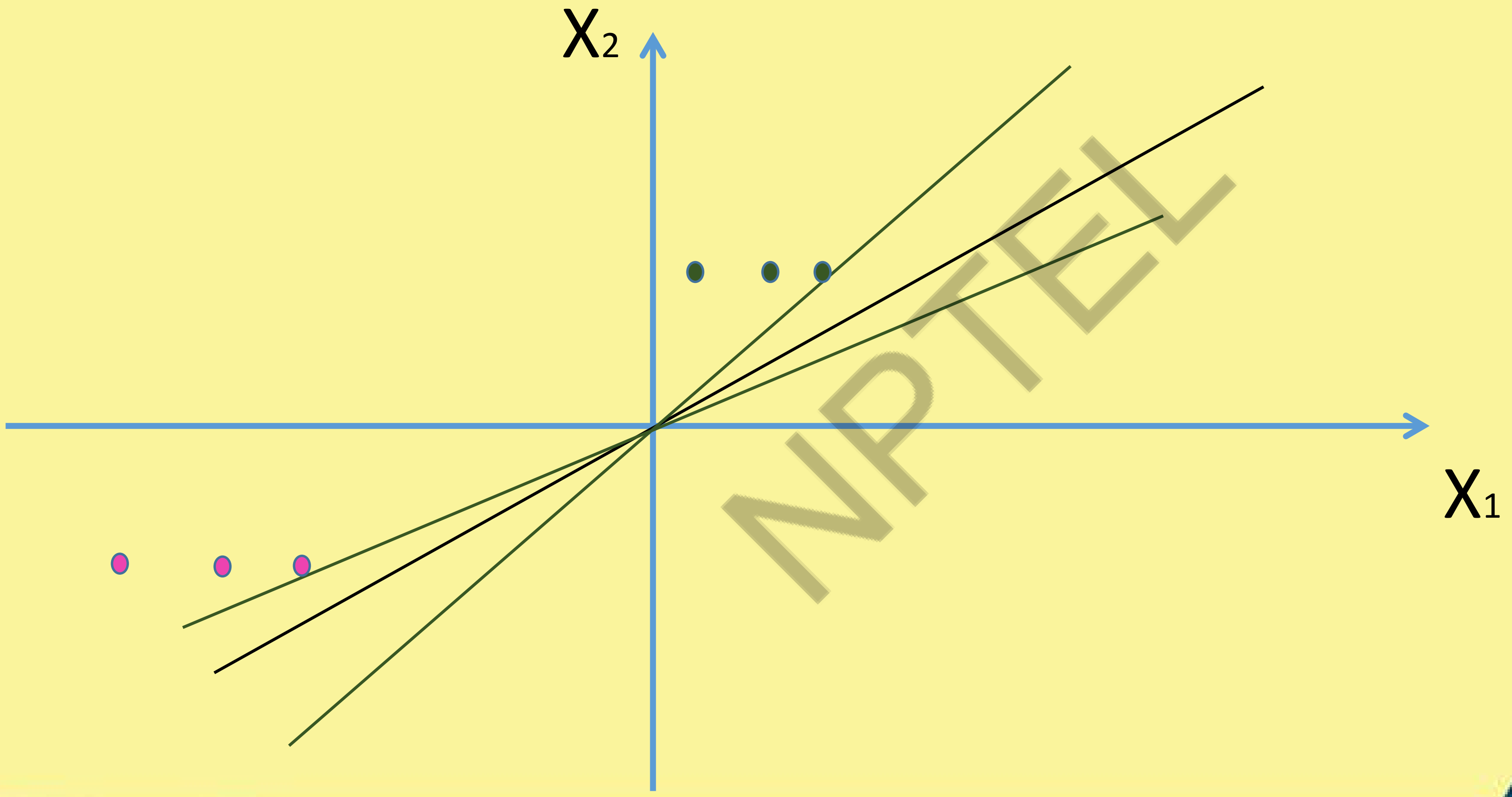
Linear Classifier – 2 Class Problem



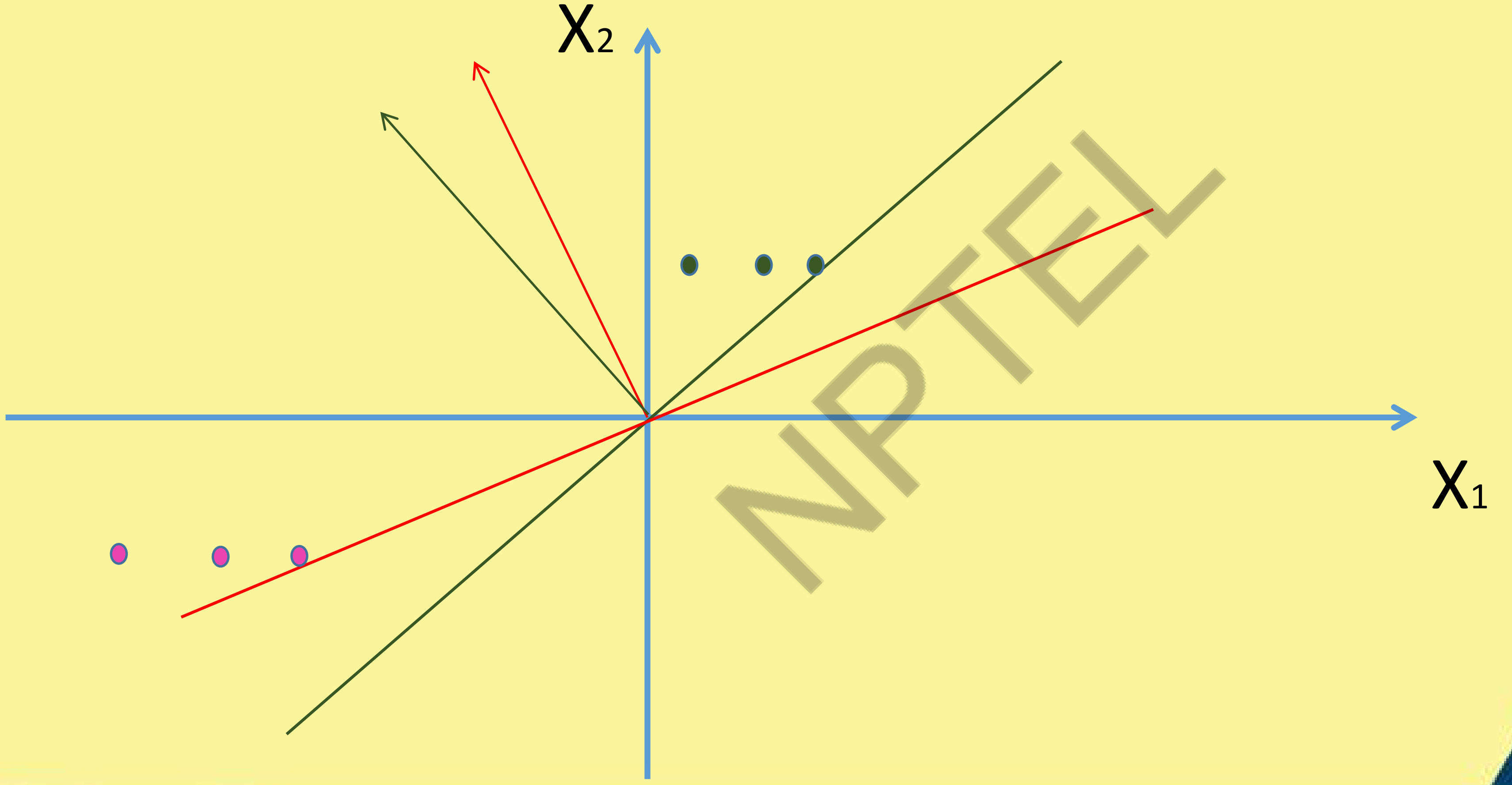
Linear Classifier – 2 Class Problem



Linear Classifier – 2 Class Problem



Linear Classifier – 2 Class Problem



Linear Classifier – Learning

$$Any\ "a"\ misclassifies\ X \Rightarrow a^t X < 0$$

$$This\ leads\ to\ an\ error:\quad J_P(a) = \sum_{\forall Y:\,misclassified} -a^t X$$

$$Follow\ Gradient\ Descent\ Algorithm$$

$$a \leftarrow a - \eta\ \nabla_a J_P(a)$$

Linear Classifier – Learning

Perceptron Criteria

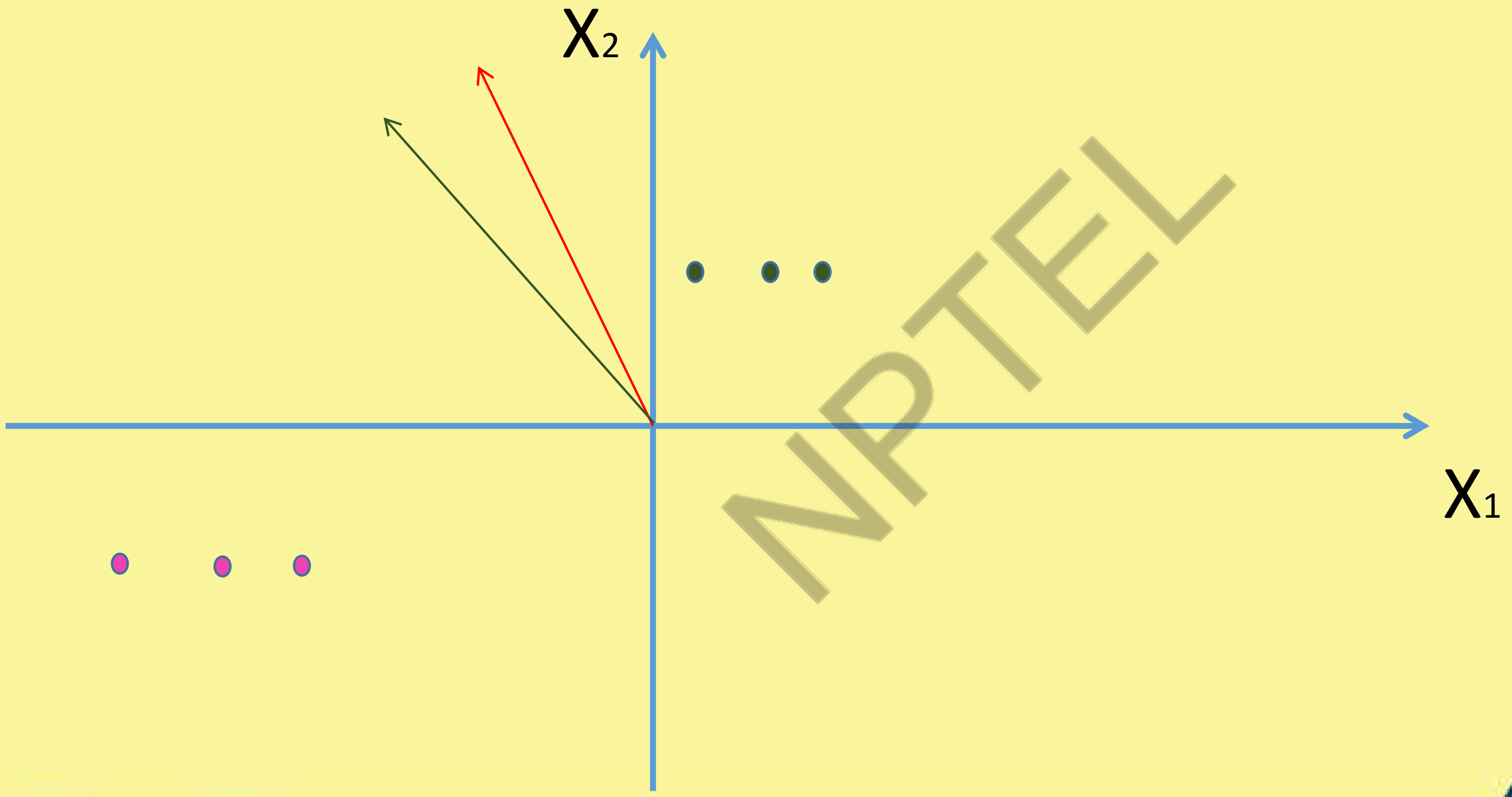
$$J_P(a) = \sum_{\forall X : misclassified} -a^t X \quad \Rightarrow \quad \nabla_a J_p(a) = -\sum_{\forall X : misclassified} X$$

Weight Updation Rule

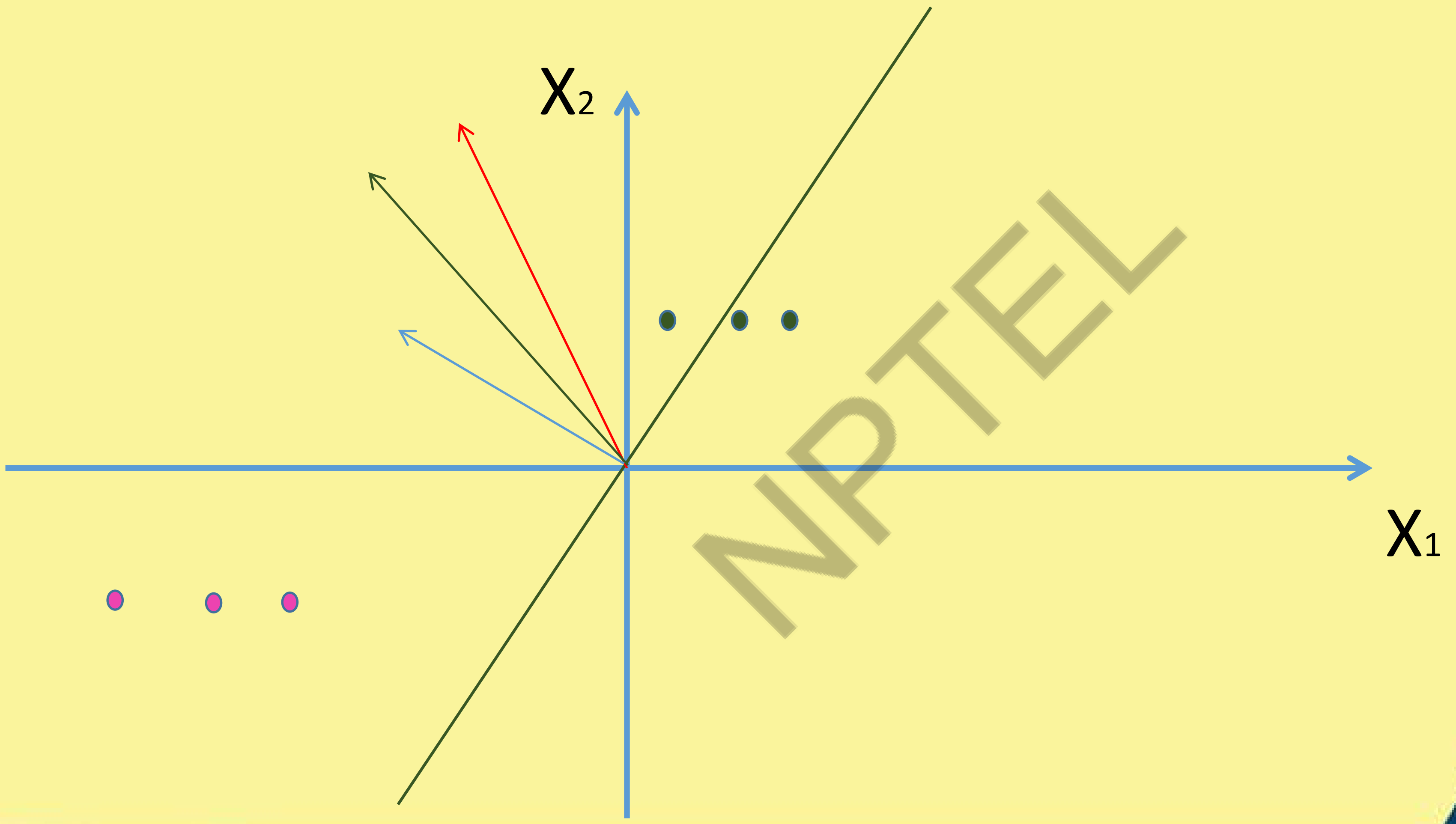
$$a(0) \leftarrow Random$$

$$a(k+1) \leftarrow a(k) + \eta \sum_{\forall X : Misclassified} X$$

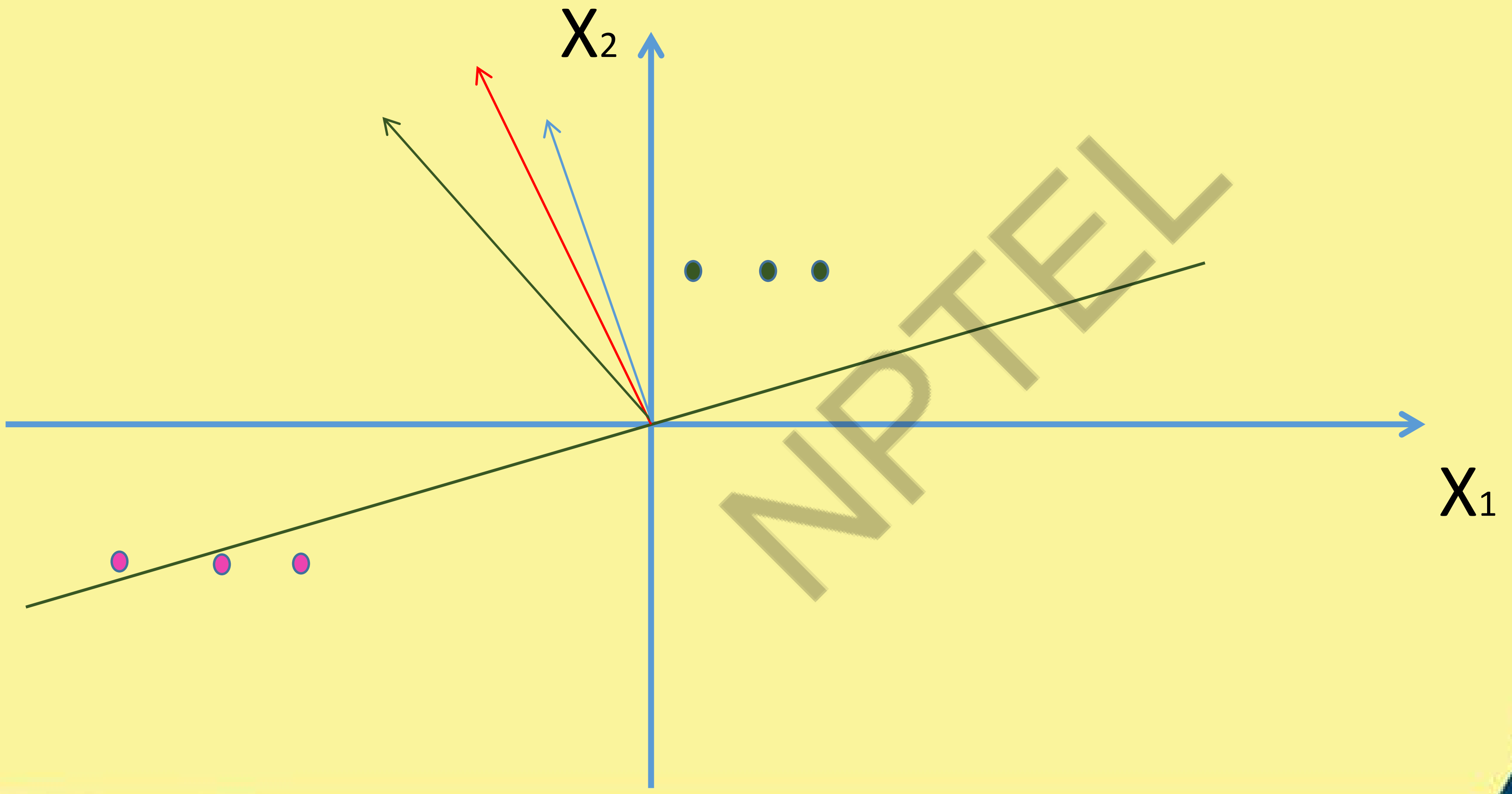
Linear Classifier – 2 Class Problem



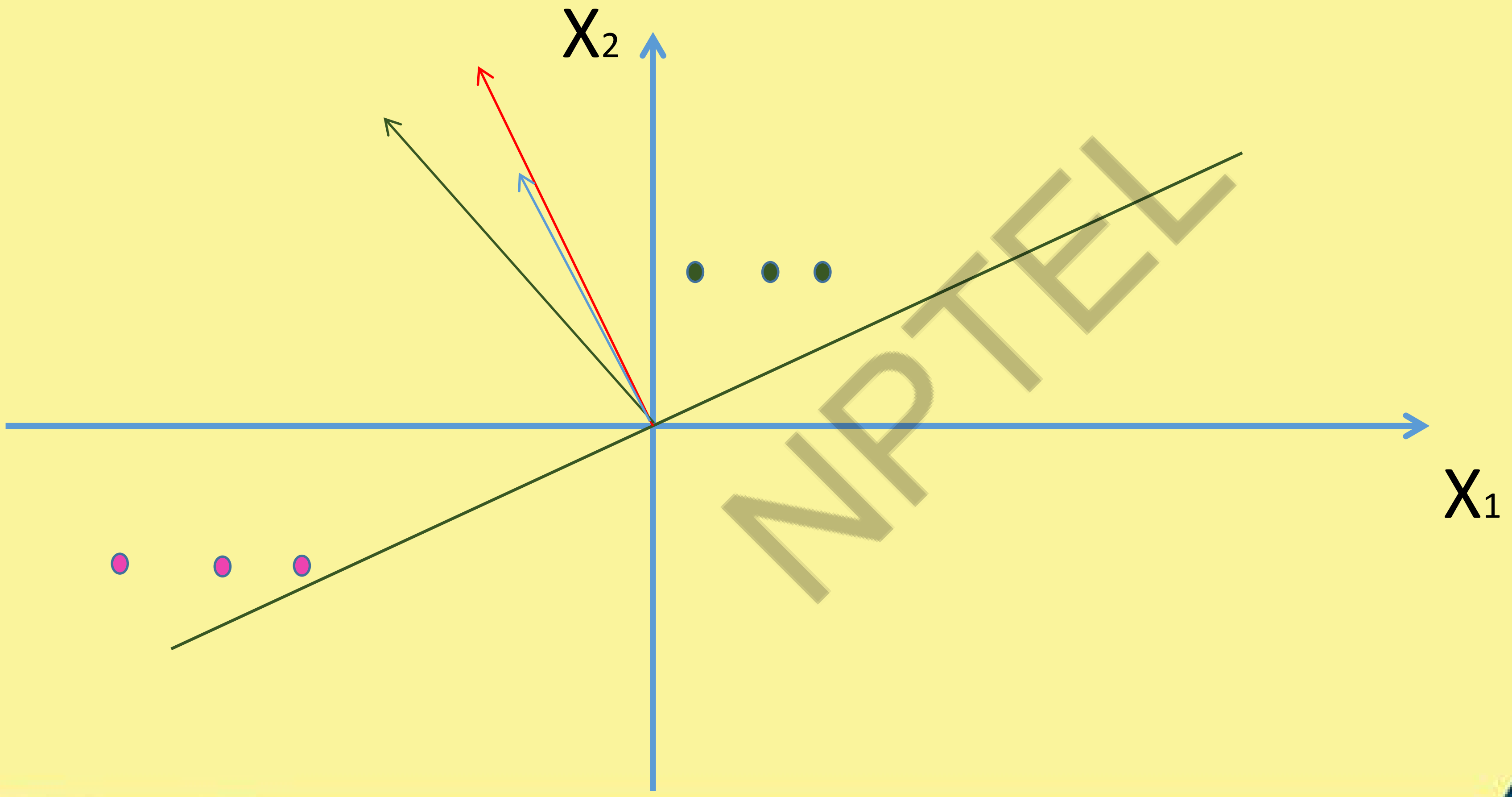
Linear Classifier – 2 Class Problem



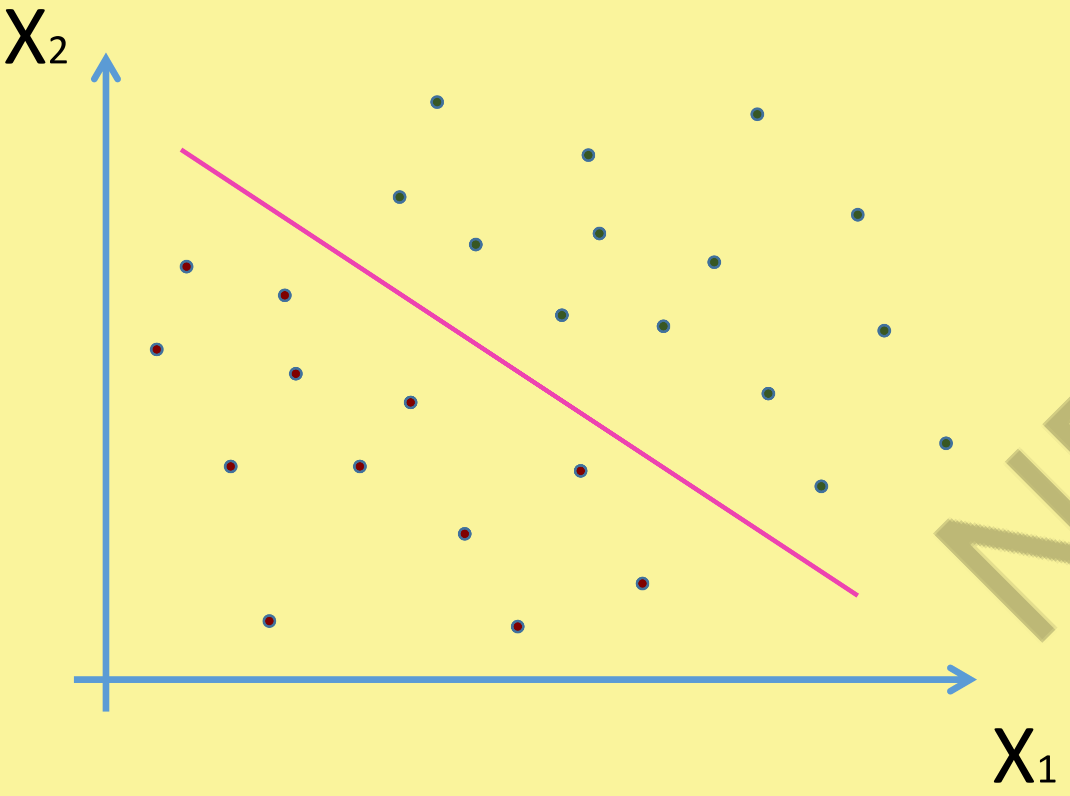
Linear Classifier – 2 Class Problem



Linear Classifier – 2 Class Problem



Support Vector Machine



$$a^t X + b = 0$$

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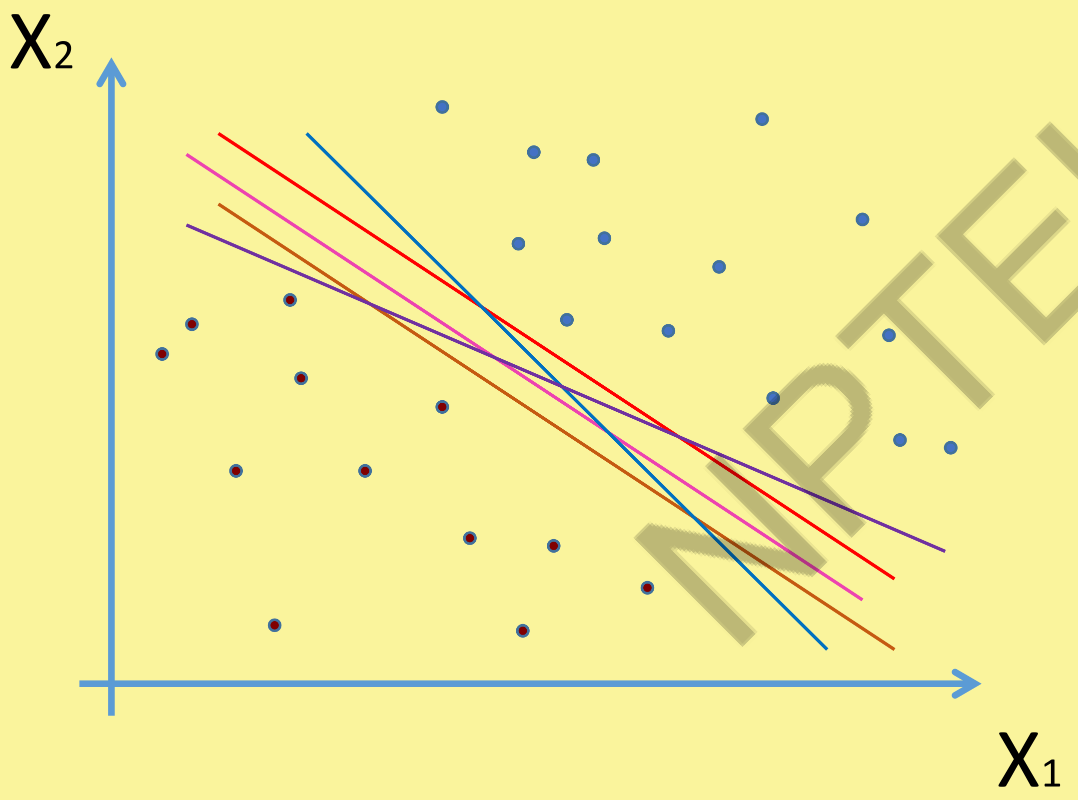
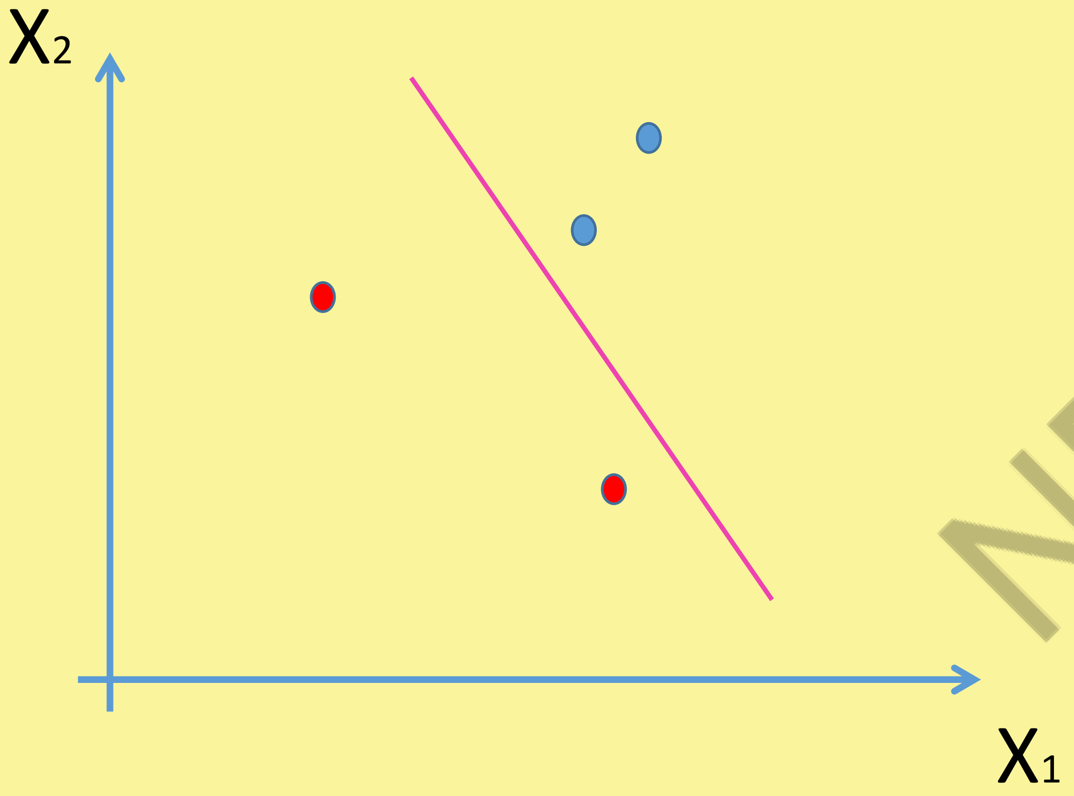
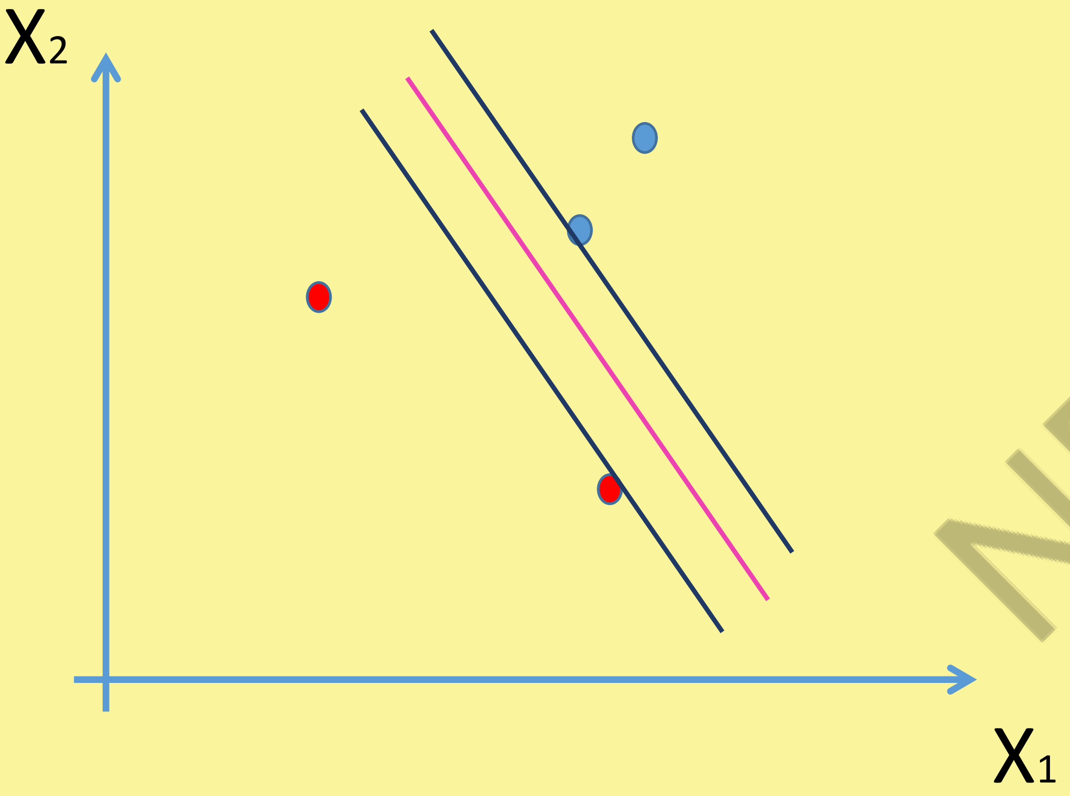
Support Vector Machine



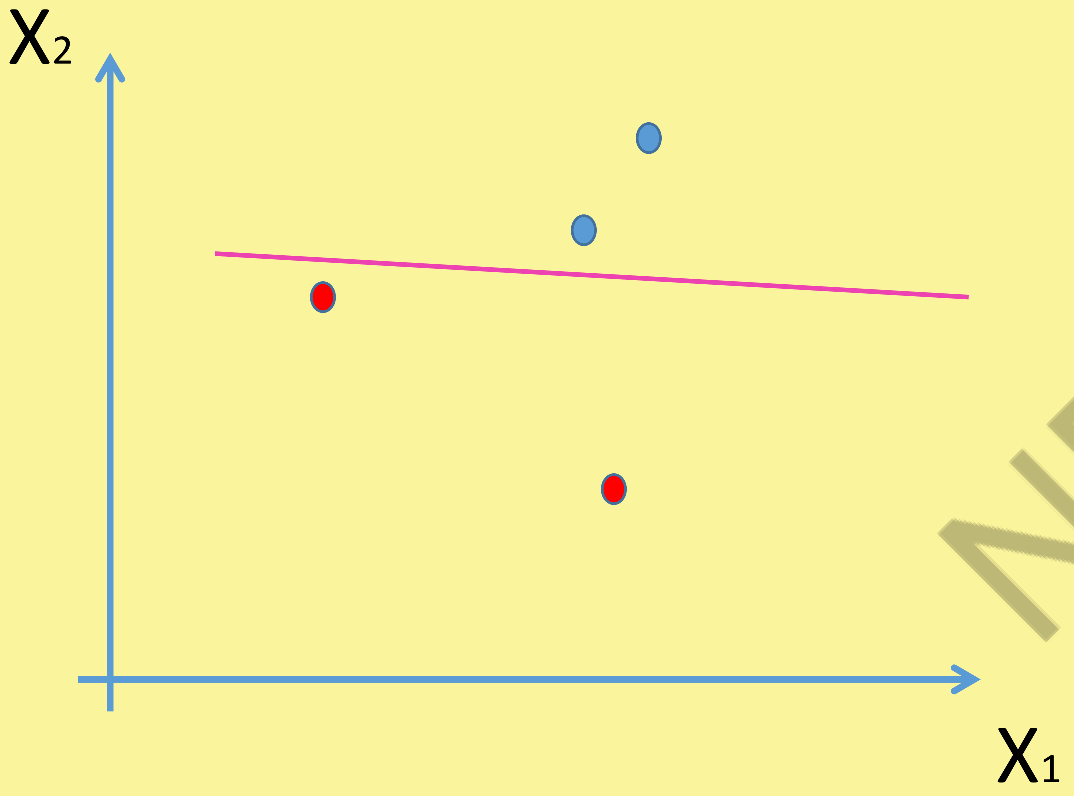
Support Vector Machine



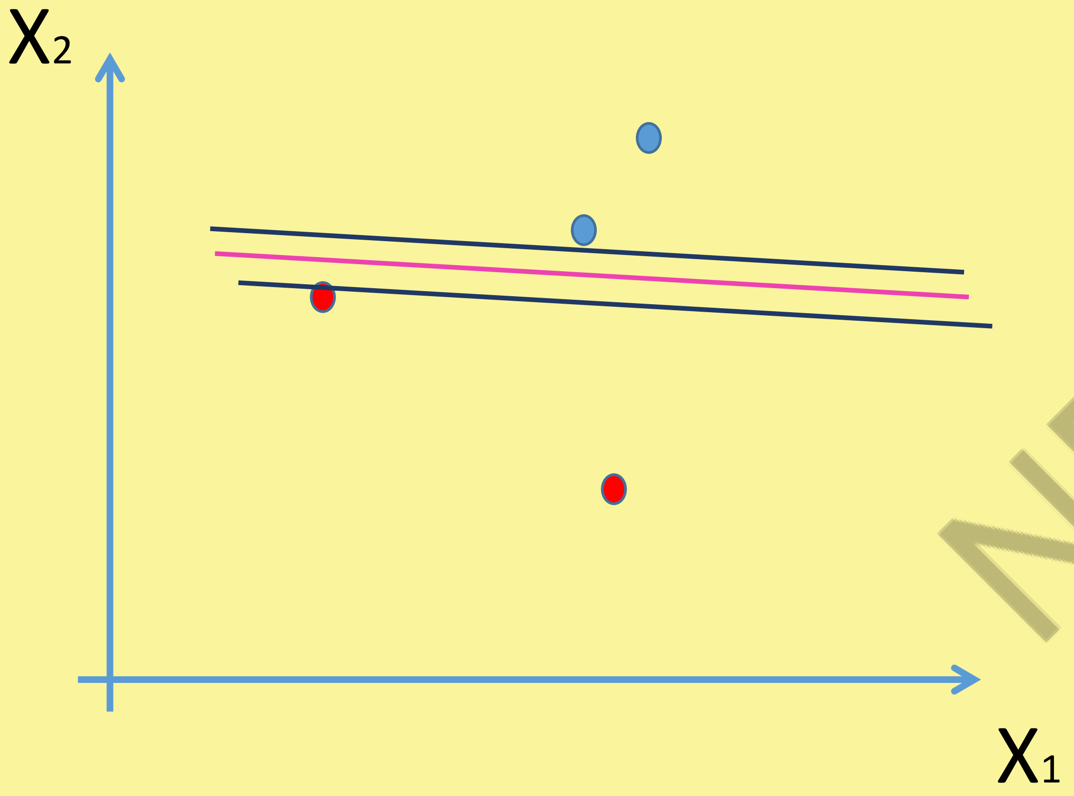
Support Vector Machine



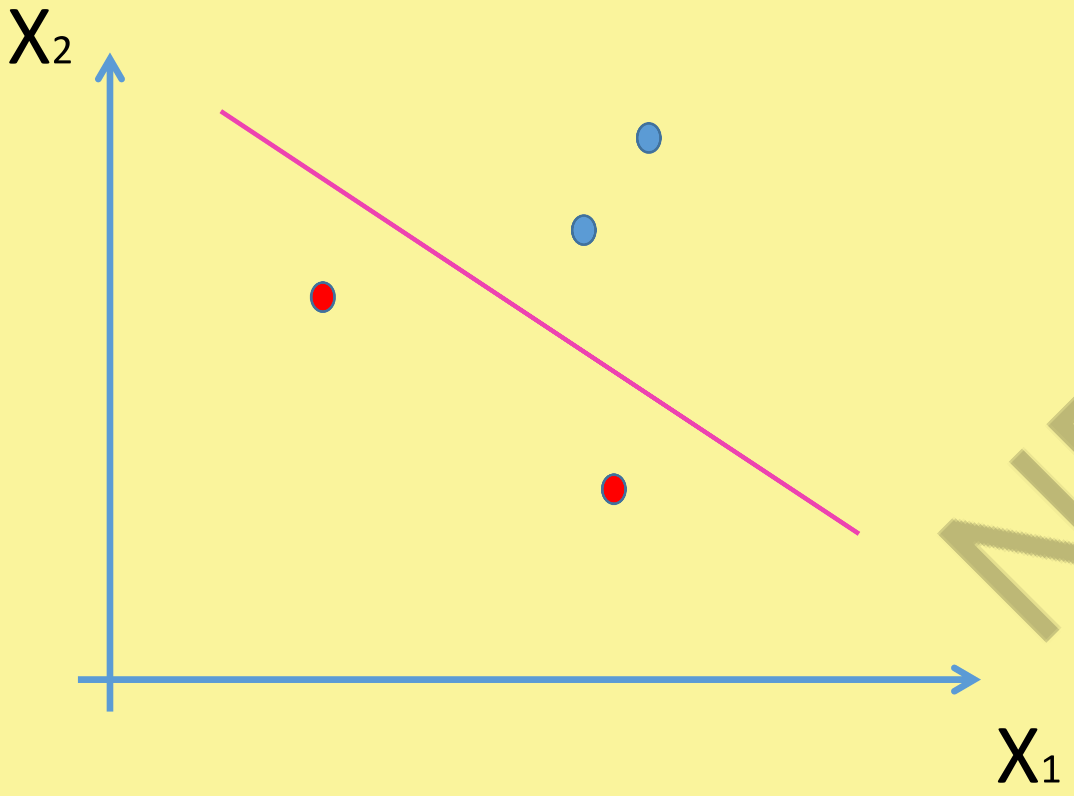
Support Vector Machine



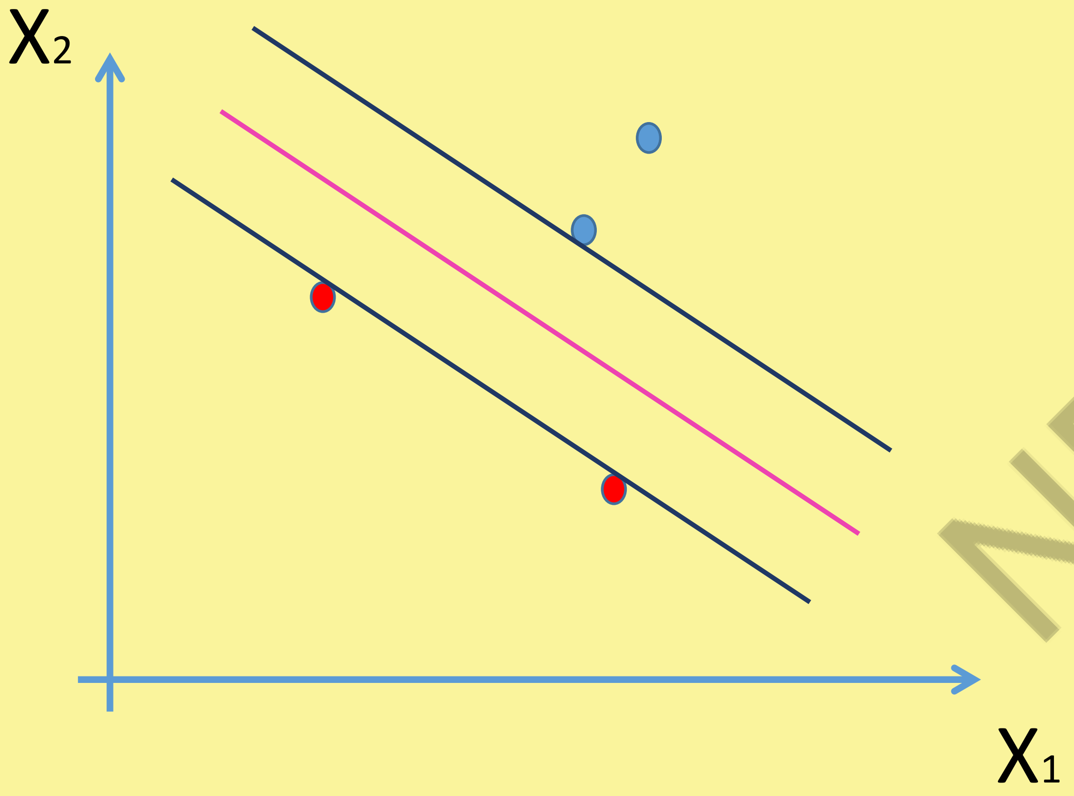
Support Vector Machine



Support Vector Machine



Support Vector Machine



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Course Name: Deep Learning

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Topic

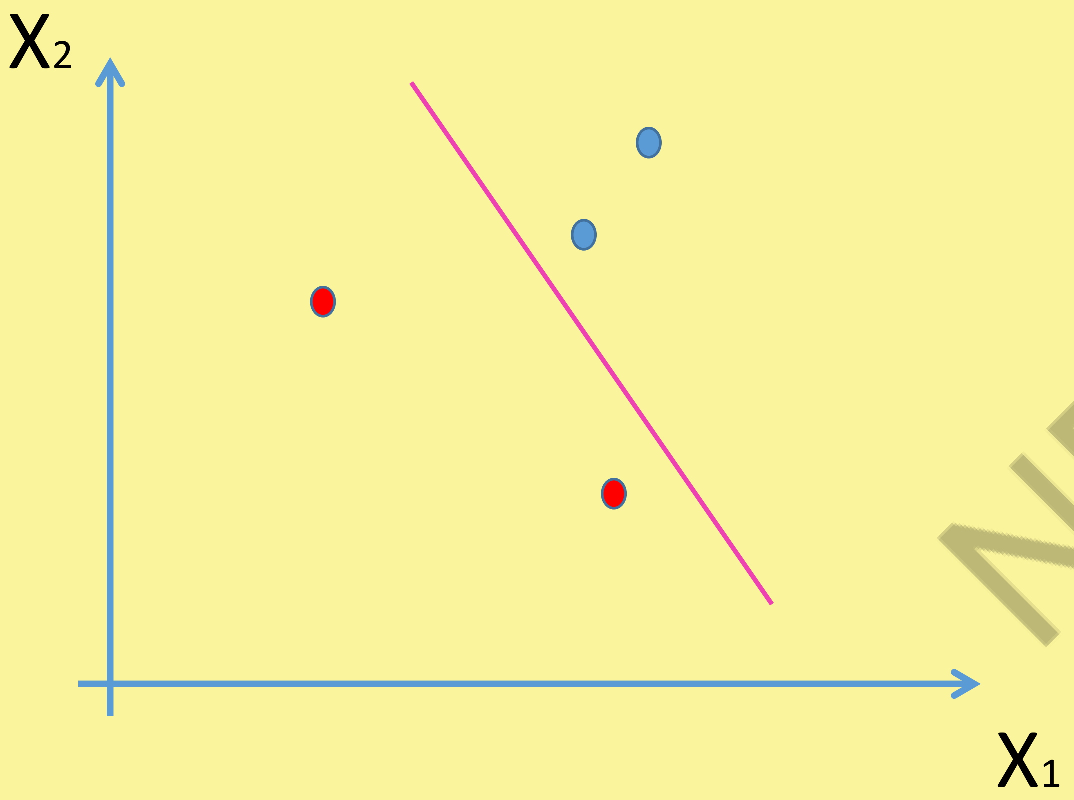
Lecture 12: Support Vector Machine II

CONCEPTS COVERED

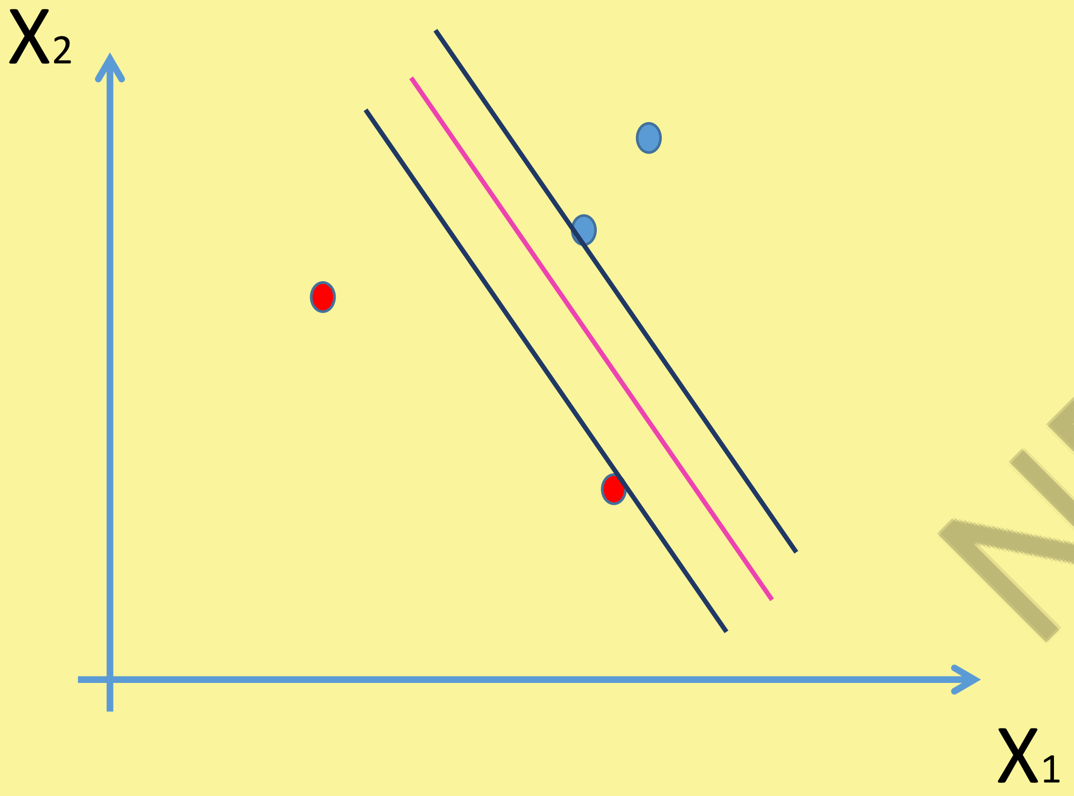
Concepts Covered:

- Support Vector Machine (SVM)
- Support Vector Machine Design

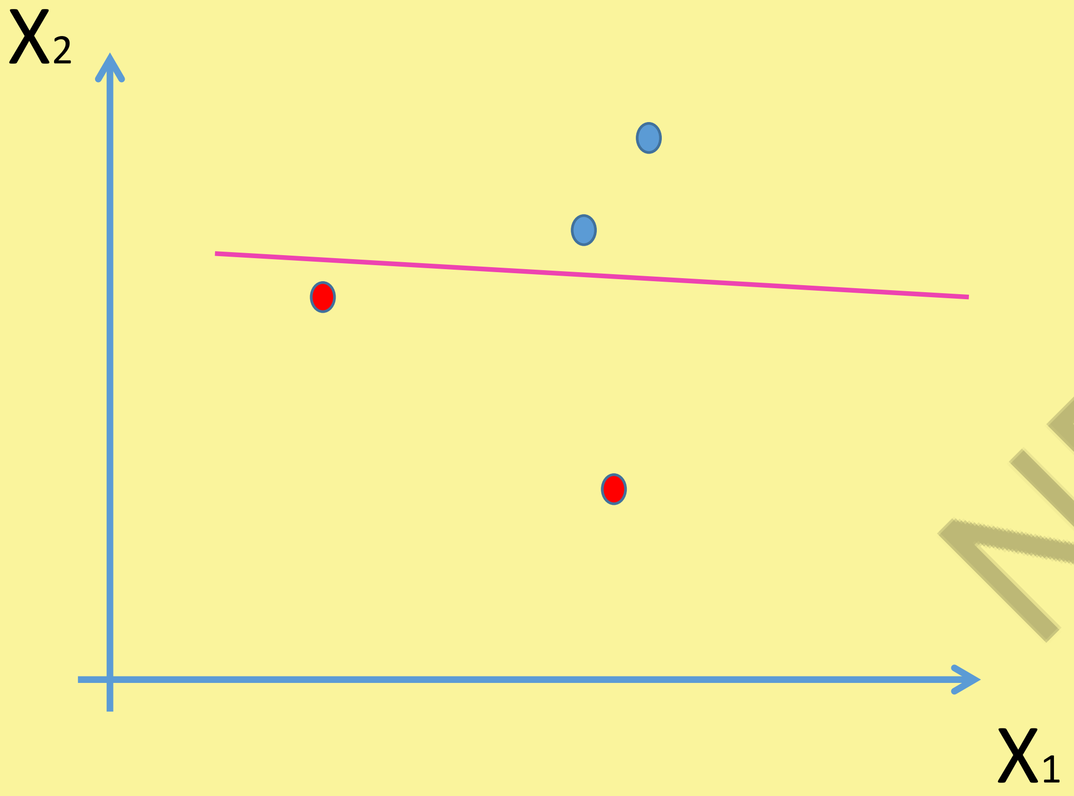
Support Vector Machine



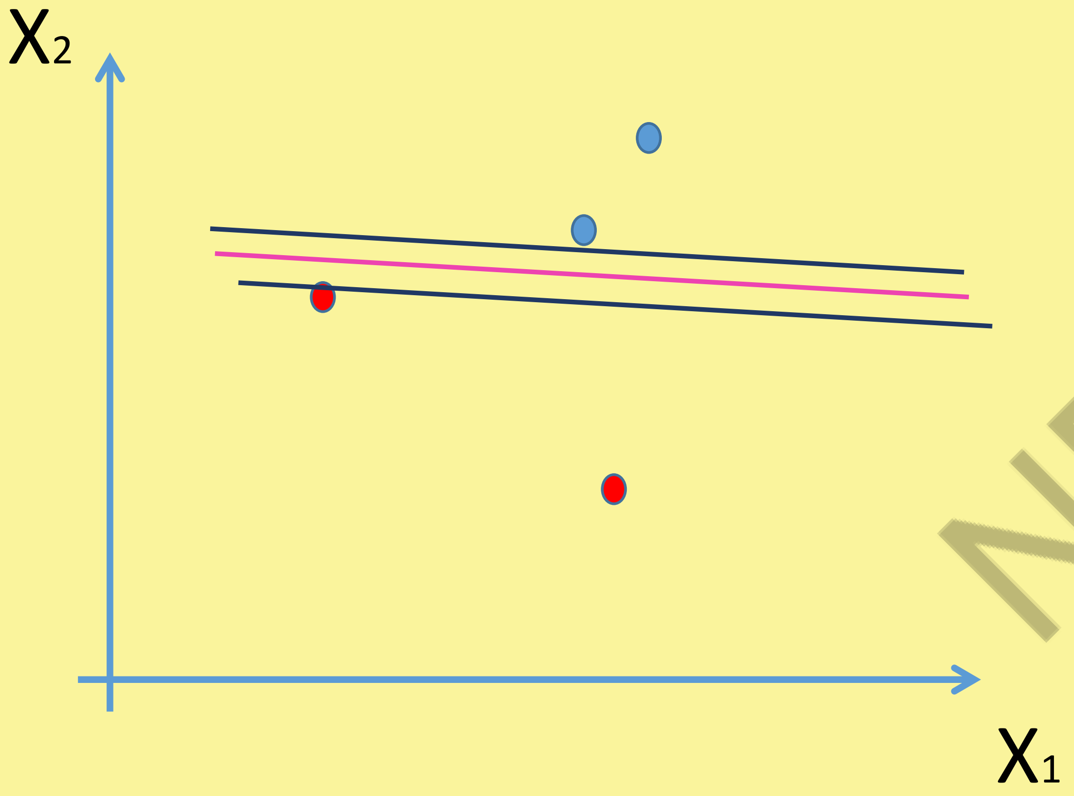
Support Vector Machine



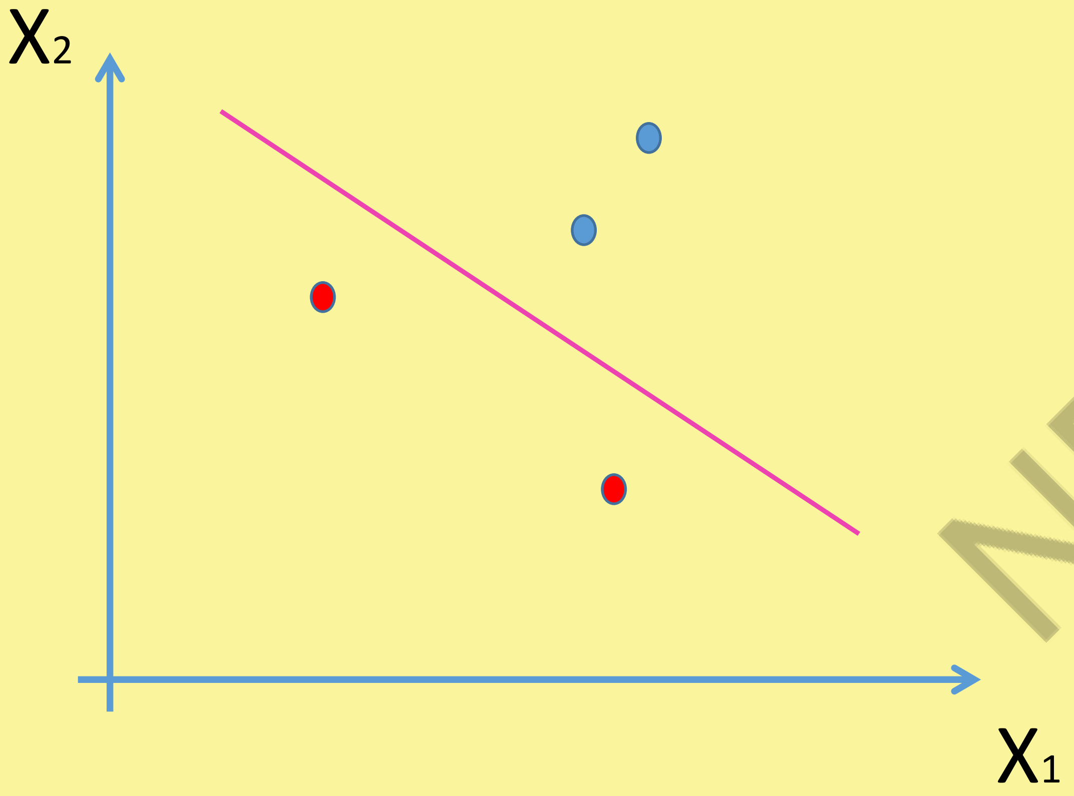
Support Vector Machine



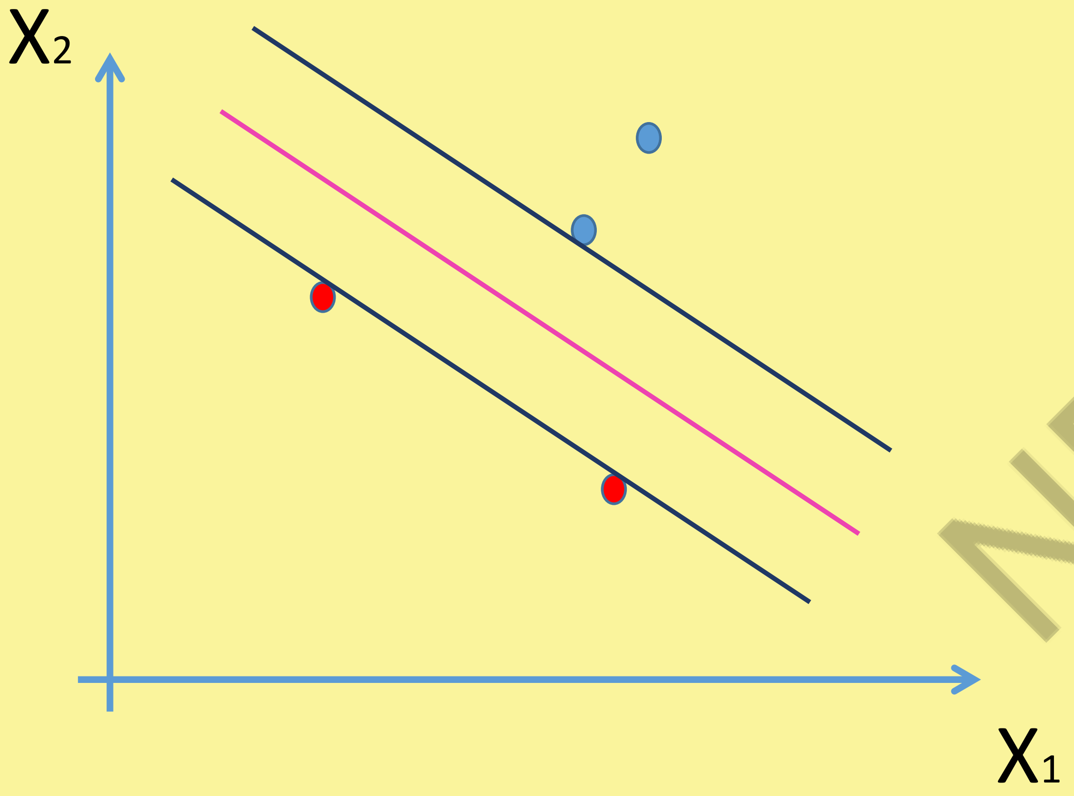
Support Vector Machine



Support Vector Machine



Support Vector Machine



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Course Name: Deep Learning

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Topic

Lecture 13: Linear Machine

CONCEPTS COVERED

Concepts Covered:

- Linear Classifier
- Support Vector Machine
- Linear Machine
- Multiclass Support Vector Machine

Multiclass Problem: Linear Machine

Linear Classifier & SVM

Multiclass Problem: Linear Machine

Multiclass Problem: Linear Machine

$$\begin{bmatrix} 0.2 & 0.6 & -1.0 & 0.8 \\ 1.5 & 0.9 & 3.1 & 0.1 \\ 0.5 & 1.1 & 0.7 & 0.0 \\ 2.1 & 0.3 & 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 45 \\ 110 \\ 21 \\ 16 \end{bmatrix} + \begin{bmatrix} 1.1 \\ 5.3 \\ -2.1 \\ 0.6 \end{bmatrix} \Rightarrow \begin{array}{cc} \begin{bmatrix} 67.9 \\ 238.5 \\ 156.1 \\ 140.3 \end{bmatrix} & \begin{array}{c} \textit{Cat score} \\ \textit{Bird score} \\ \textit{Dog score} \\ \textit{Car score} \end{array} \end{array}$$

$$f(X_i, W, b)$$

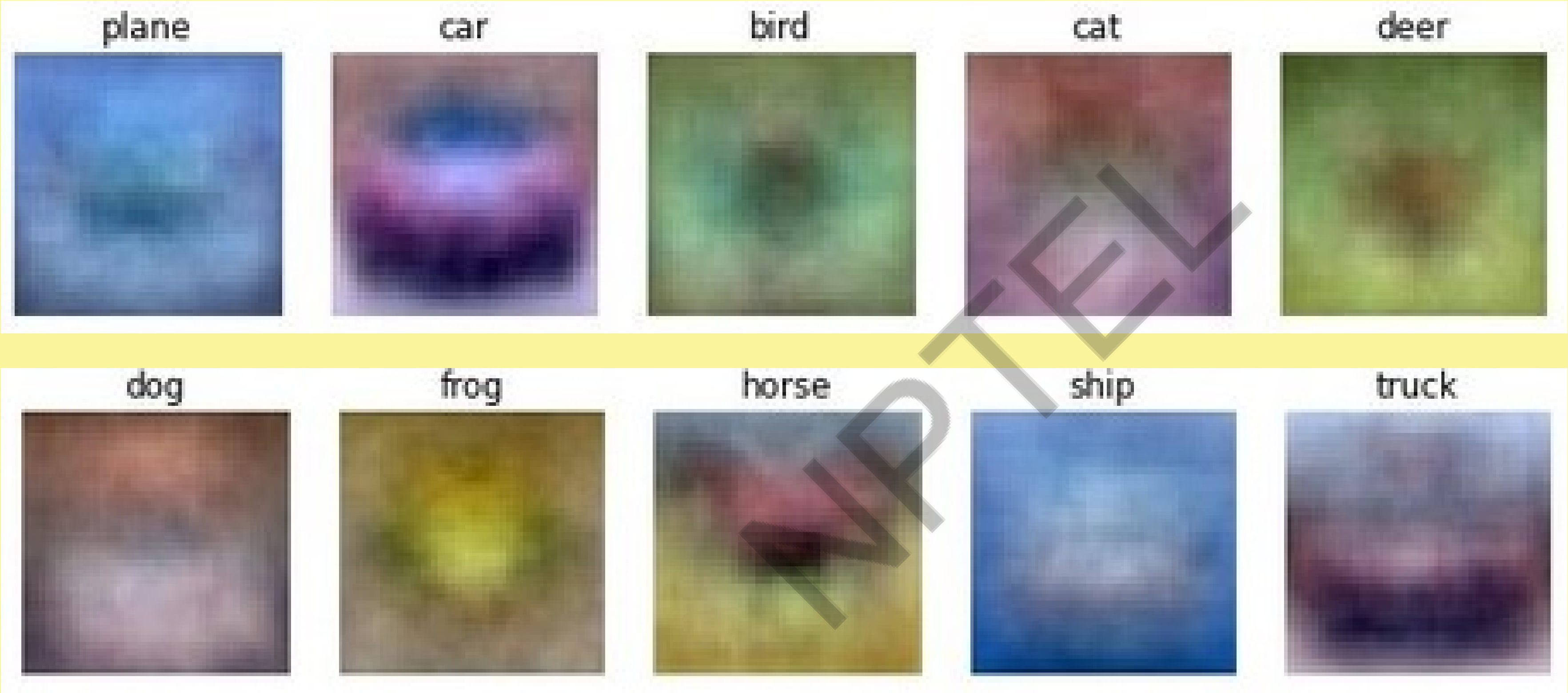
Interpretation

$$\begin{bmatrix} 0.2 & 0.6 & -1.0 & 0.8 \\ 1.5 & 0.9 & 3.1 & 0.1 \\ 0.5 & 1.1 & 0.7 & 0.0 \\ 2.1 & 0.3 & 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 45 \\ 110 \\ 21 \\ 16 \end{bmatrix} + \begin{bmatrix} 1.1 \\ 5.3 \\ -2.1 \\ 0.6 \end{bmatrix} \Rightarrow \begin{array}{cl} \begin{bmatrix} 67.9 \\ 238.5 \\ 156.1 \\ 140.3 \end{bmatrix} & \begin{array}{l} \textit{Cat score} \\ \textit{Bird score} \\ \textit{Dog score} \\ \textit{Car score} \end{array} \end{array}$$

$$f(X_i, W, b)$$

Interpretation

Multiclass Problem: Linear Machine



Source - http://cs231n.github.io

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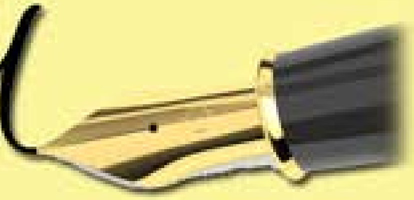

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Topic

Lecture 14: Linear Machine and Multiclass SVM

CONCEPTS COVERED

Concepts Covered:

- Linear Classifier
- Support Vector Machine
- Linear Machine
- Multiclass Support Vector Machine

Multiclass Problem: Linear Machine

Multiclass Problem: Linear Machine

$$f : R^D \rightarrow R^K$$

$$f(X_i, W, b) = WX_i + b = s$$

$$\begin{bmatrix} W_{11} & W_{12} & W_{13} & .. & W_{1D} \\ W_{21} & W_{22} & W_{23} & .. & W_{2D} \\ .. & .. & .. & .. & .. \\ W_{K1} & W_{K2} & W_{K3} & .. & W_{KD} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ .. \\ X_D \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ .. \\ b_K \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ .. \\ s_K \end{bmatrix}$$

Multiclass Problem: Linear Machine

$$\begin{bmatrix} 0.2 & 0.6 & -1.0 & 0.8 \\ 1.5 & 0.9 & 3.1 & 0.1 \\ 0.5 & 1.1 & 0.7 & 0.0 \\ 2.1 & 0.3 & 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 45 \\ 110 \\ 21 \\ 16 \end{bmatrix} + \begin{bmatrix} 1.1 \\ 5.3 \\ -2.1 \\ 0.6 \end{bmatrix} \Rightarrow \begin{array}{cc} \begin{bmatrix} 67.9 \\ 238.5 \\ 156.1 \\ 140.3 \end{bmatrix} & \begin{array}{l} \textit{Cat score} \\ \textit{Bird score} \\ \textit{Dog score} \\ \textit{Car score} \end{array} \end{array}$$

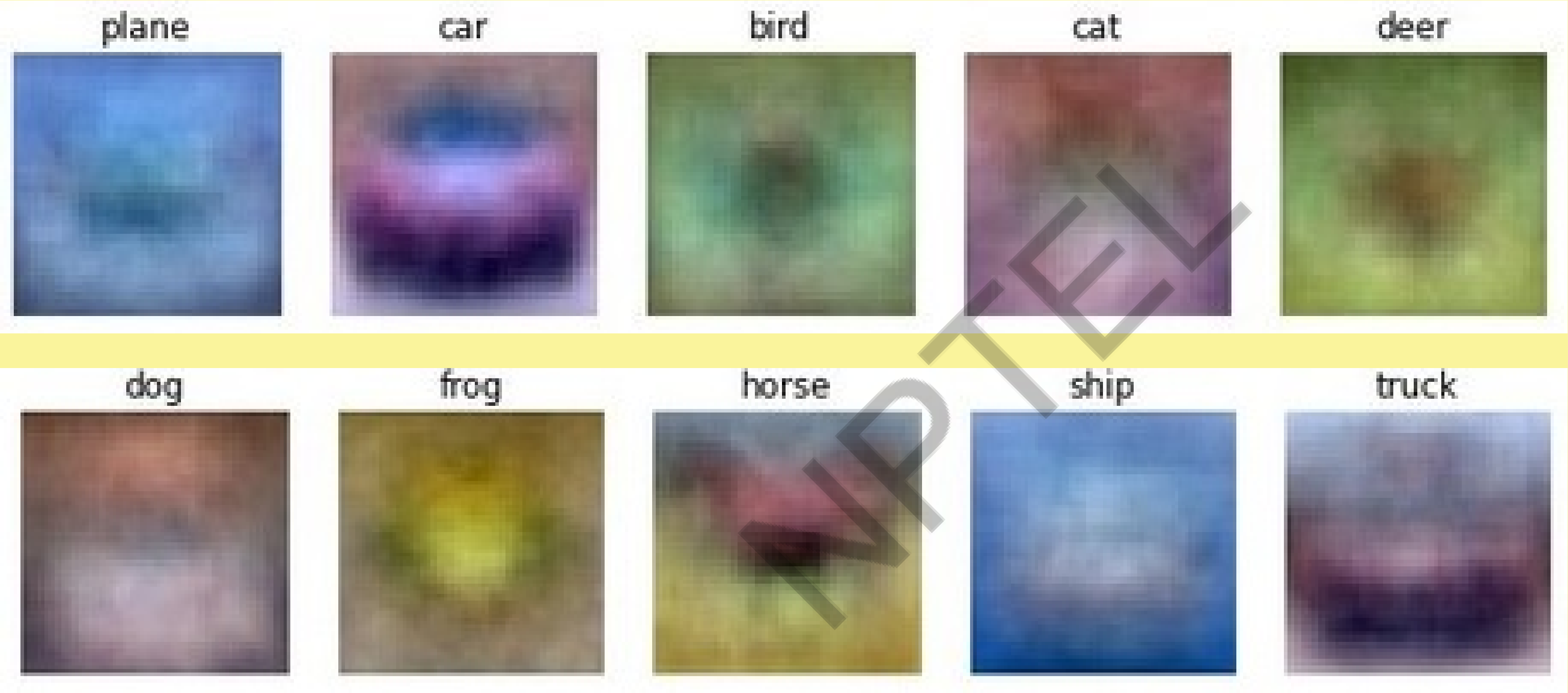
$$f(X_i, W, b)$$

Interpretation

$$\begin{bmatrix} 0.2 & 0.6 & -1.0 & 0.8 \\ 1.5 & 0.9 & 3.1 & 0.1 \\ 0.5 & 1.1 & 0.7 & 0.0 \\ 2.1 & 0.3 & 0.2 & 0.5 \end{bmatrix} \begin{bmatrix} 45 \\ 110 \\ 21 \\ 16 \end{bmatrix} + \begin{bmatrix} 1.1 \\ 5.3 \\ -2.1 \\ 0.6 \end{bmatrix} \Rightarrow \begin{array}{cl} \begin{bmatrix} 67.9 \\ 238.5 \\ 156.1 \\ 140.3 \end{bmatrix} & \begin{array}{l} \textit{Cat score} \\ \textit{Bird score} \\ \textit{Dog score} \\ \textit{Car score} \end{array} \end{array}$$

$$f(X_i, W, b)$$

Multiclass Problem: Linear Machine



Source - http://cs231n.github.io

Bias Trick

$$\begin{bmatrix} 0.2 & 0.6 & -1.0 & 0.8 & 1.1 \\ 1.5 & 0.9 & 3.1 & 0.1 & 5.3 \\ 0.5 & 1.1 & 0.7 & 0.0 & -2.1 \\ 2.1 & 0.3 & 0.2 & 0.5 & 0.1 \end{bmatrix} \begin{bmatrix} 45 \\ 110 \\ 21 \\ 16 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 67.9 \\ 238.5 \\ 156.1 \\ 140.3 \end{bmatrix} \begin{array}{l} \textit{Cat score} \\ \textit{Bird score} \\ \textit{Dog score} \\ \textit{Car score} \end{array}$$

$$f(X_i, W)$$

Multiclass SVM

$$\left.\begin{aligned} s_j &= f(X_i, W)_j \\ &= WX_i \end{aligned}\right\} \rightarrow \textit{Score for } j^{th} \textit{ Class of } i^{th} \textit{ Vector } (X_i, y_i)$$

$$s_{y_i} = f(X_i, W)_{y_i} \quad \rightarrow \textit{should be} \max \textit{imum}$$

$$s_{y_i} - s_j \geq \Delta$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

Loss Function: An Example

$$For\ some\ (X_i, y_i)\ where\ y_i = 2$$

$$s = (10\ \ 30\ \ -20\ \ 25)^t \qquad \Delta = 10$$

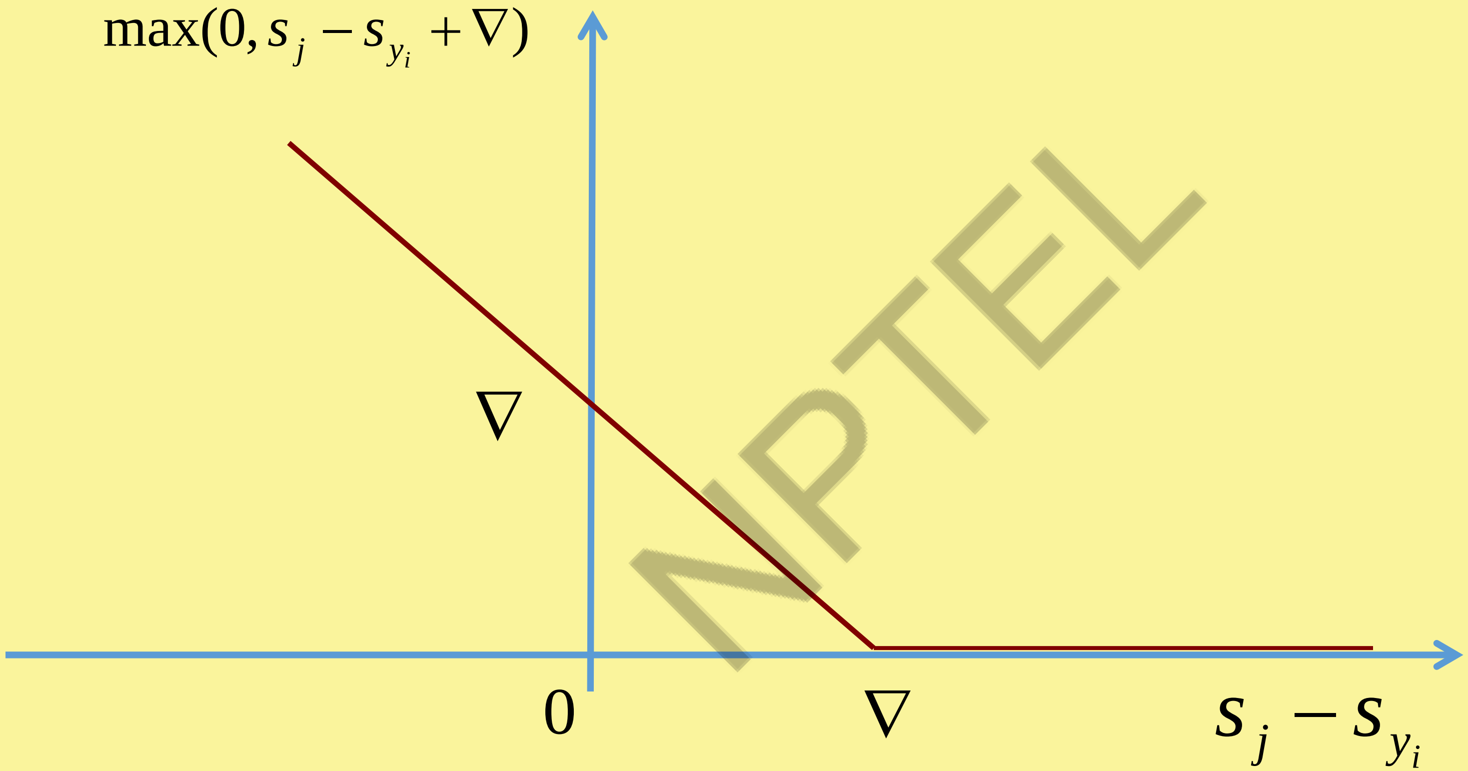
$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

$$= \max(0,10-30+10) + \max(0,-20-30+10) + \max(0,25-30+10)$$

$$= 0 + 0 + 15$$

$$= 15$$

Hinge Loss



Regularization

$$s_j - s_{y_i} = W_j^t X_i - W_{y_i}^t X_i$$

$$\text{Scaling } W \text{ by } \lambda : W \leftarrow \lambda W$$

$$\Downarrow$$

$$s_j - s_{y_i} \leftarrow \lambda\left(s_j - s_{y_i}\right)$$

Regularization

Include a regularization term $R(W)$

$$R(W) = \lambda \sum_k \sum_l W_{kl}^2$$

$$L = \frac{1}{N} \sum_i L_i + \lambda R(W)$$

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} [\max(0, f(X_i, W)_j - f(X_i, W)_{y_i} \quad + \nabla) + \lambda \sum_k \sum_l W_{kl}^2$$

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Topic

Lecture 15: Multiclass SVM Loss Function

CONCEPTS COVERED

Concepts Covered:

- Linear Machine
- Multiclass Support Vector Machine
- Multiclass SVM Loss Function
- Optimization

Multiclass Problem: Linear Machine

$$f : R^D \rightarrow R^K$$

$$f(X_i, W, b) = WX_i + b = s$$

$$\begin{bmatrix} W_{11} & W_{12} & W_{13} & .. & W_{1D} \\ W_{21} & W_{22} & W_{23} & .. & W_{2D} \\ .. & .. & .. & .. & .. \\ W_{K1} & W_{K2} & W_{K3} & .. & W_{KD} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ .. \\ X_D \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ .. \\ b_K \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ .. \\ s_K \end{bmatrix}$$

Multiclass SVM

$$\left.\begin{aligned} s_j &= f(X_i, W)_j \\ &= WX_i \end{aligned}\right\} \rightarrow Score\ \ for\ \ j^{th}\ Class\ of\ \ i^{th}\ Vector\,(X_i, y_i)$$

$$s_{y_i} = f(X_i, W)_{y_i} \quad \rightarrow \text{should be maximum}$$

$$s_{y_i} - s_j \geq \Delta$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

Loss Function: An Example

$$For\ some\ (X_i, y_i)\ where\ y_i = 2$$

$$s = (10\ \ 30\ \ -20\ \ 25)^t \qquad \Delta = 10$$

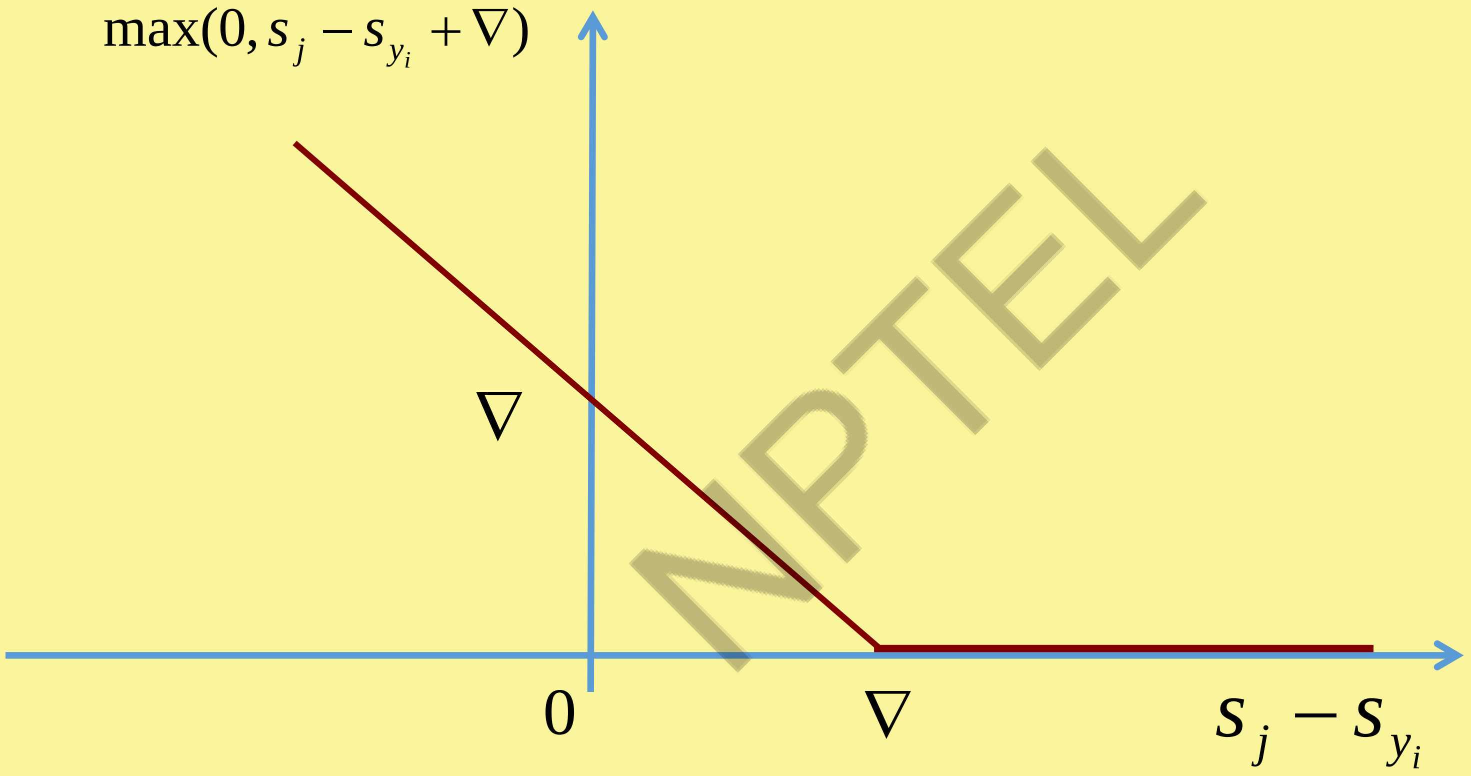
$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

$$= \max(0, 10 - 30 + 10) + \max(0, -20 - 30 + 10) + \max(0, 25 - 30 + 10)$$

$$= 0 + 0 + 15$$

$$= 15$$

Hinge Loss



Regularization

$$s_j - s_{y_i} = W_j^t X_i - W_{y_i}^t X_i$$

$$\text{Scaling } W \text{ by } \lambda : W \leftarrow \lambda W$$

$$\Downarrow$$

$$s_j - s_{y_i} \leftarrow \lambda\left(s_j - s_{y_i}\right)$$

Regularization

Include a regularization term *R(W)*

$$R(W) = \lambda \sum_k \sum_l W_{kl}^2$$

$$L = \frac{1}{N} \sum_i L_i + \lambda R(W)$$

$$L = \frac{1}{N} \sum_i \sum_{j \neq y_i} [\max(0, f(X_i, W)_j - f(X_i, W)_{y_i} + \nabla) + \lambda \sum_k \sum_l W_{kl}^2$$

Choice of Hyper Parameter

$$L = \frac{1}{N}\sum_i \sum_{j \neq y_i} [\max(0,\ f(X_i, W)_j - f(X_i, W)_{y_i} \ \ + \nabla) + \lambda \sum_k \sum_l W_{kl}^2$$

∇ and λ control the same tradeoff $\quad \Rightarrow \nabla = 1$

$$L = \frac{1}{N}\sum_i \sum_{j \neq y_i} [\max(0,\ f(X_i, W)_j - f(X_i, W)_{y_i} + 1) + \lambda \sum_k \sum_l W_{kl}^2$$

Binary SVM $\Rightarrow L_i = C \max(0, 1 - y_i W^t X_i) + R(W)$

Loss Function Visualization

Visualizing Loss Function

$$\text{Consider 3 Classes} \Rightarrow W = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}$$

$$3 \;\; 1\text{–dimensional points} \Rightarrow (X_1, 1), (X_2, 2) \text{ and } (X_3, 3)$$

Visualizing Loss Function

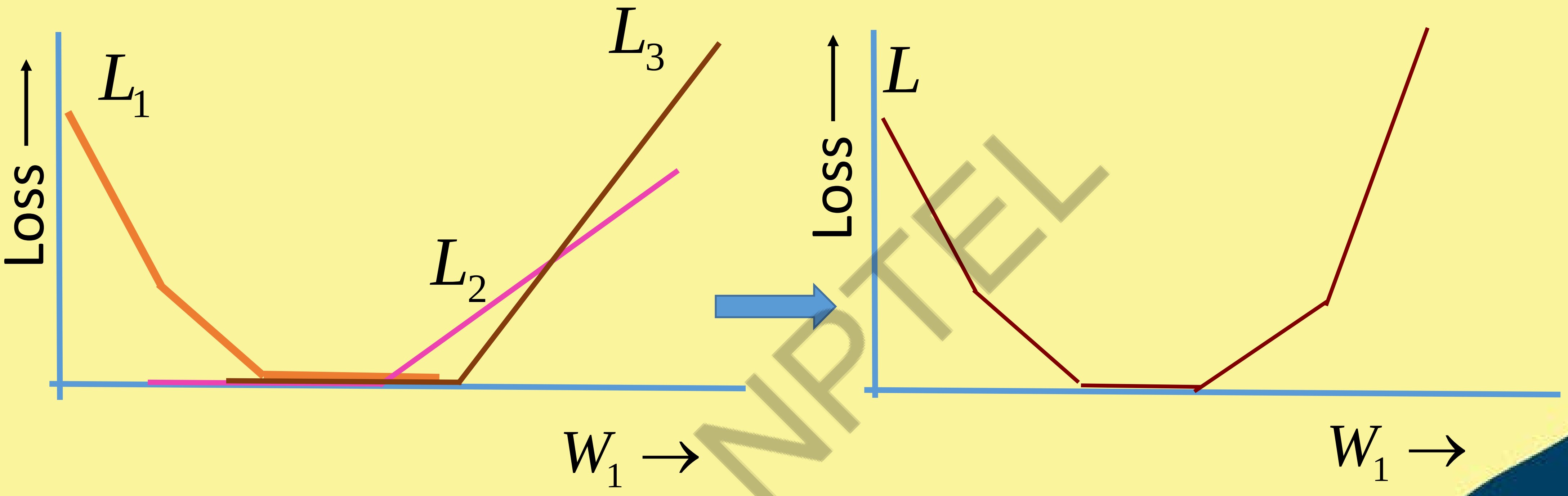
$$L_1 = \max(0, W_2^t X_1 - W_1^t X_1 + 1) + \max(0, W_3^t X_1 - W_1^t X_1 + 1)$$

$$L_2 = \max(0, W_1^t X_2 - W_2^t X_2 + 1) + \max(0, W_3^t X_2 - W_2^t X_2 + 1)$$

$$L_3 = \max(0, W_1^t X_3 - W_3^t X_3 + 1) + \max(0, W_2^t X_3 - W_3^t X_3 + 1)$$

$$L = \frac{1}{3}(L_1 + L_2 + L_3)$$

Visualizing Loss Function



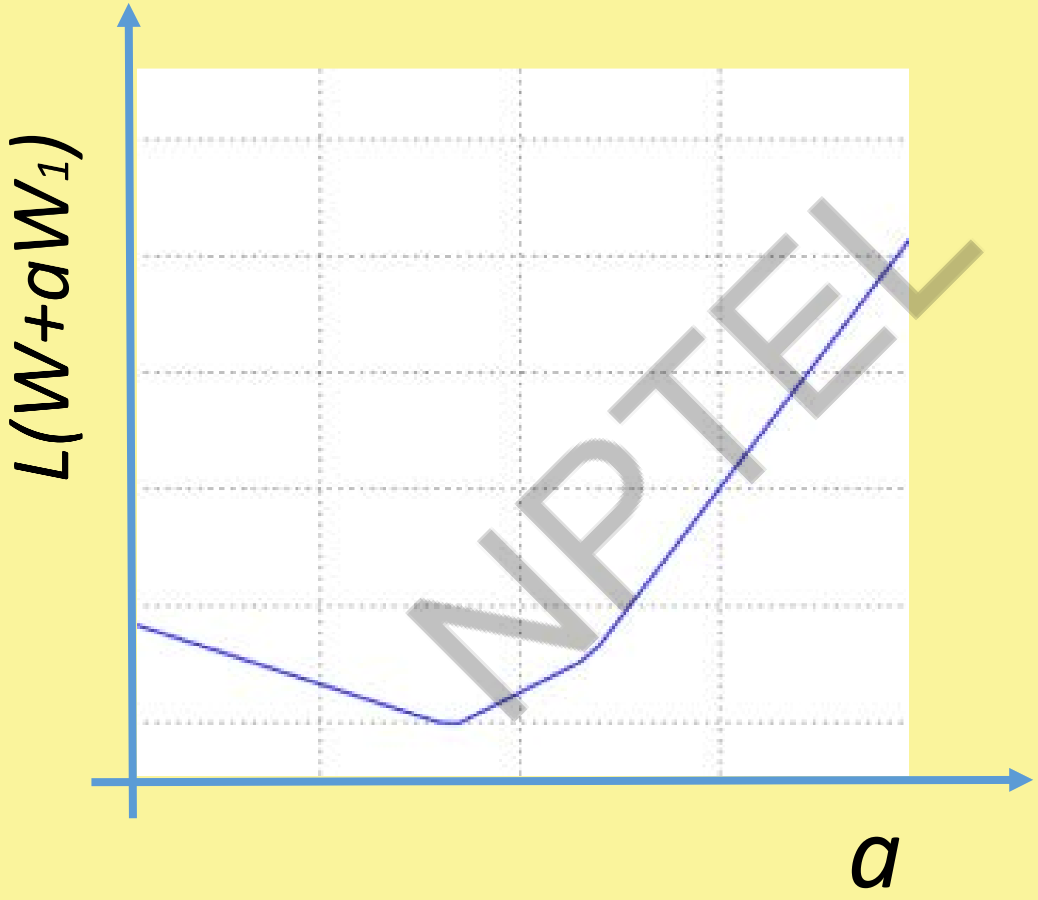
Visualizing Loss Function

- Take a random W (a single point in space)
- Take a random direction W_1
- Record the Loss along W_1

$$\Rightarrow L(W + aW_1)$$

Source - http://cs231n.github.io

Visualizing Loss Function



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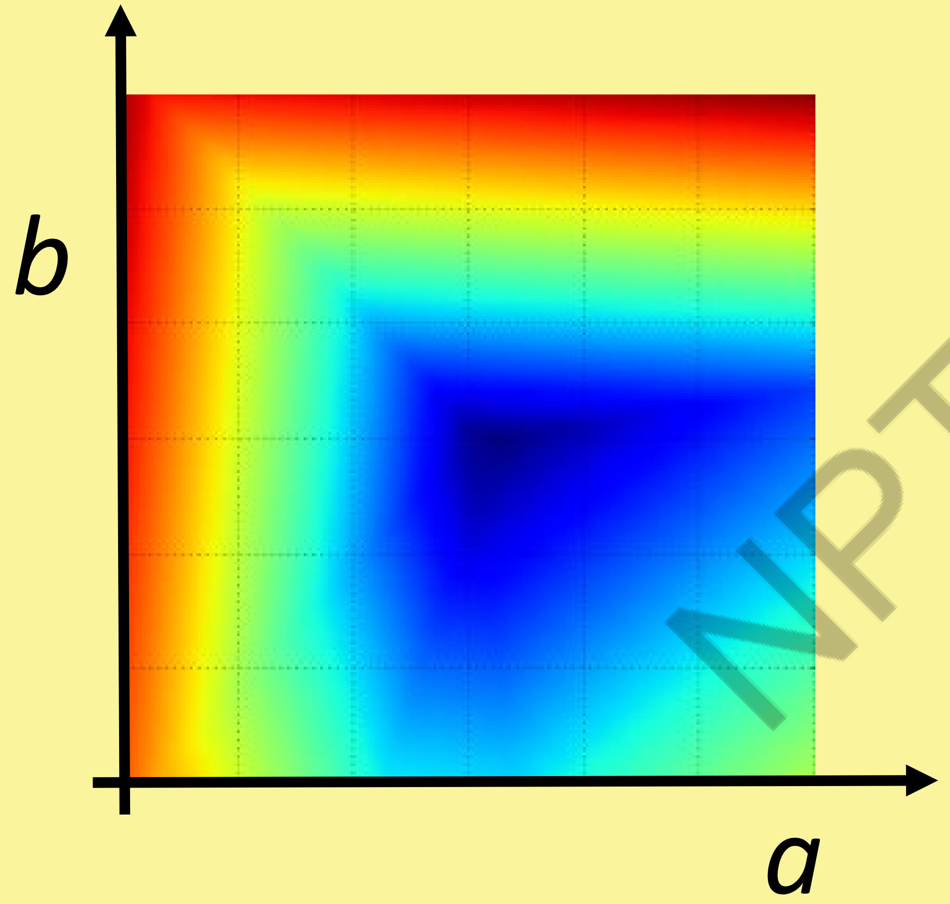
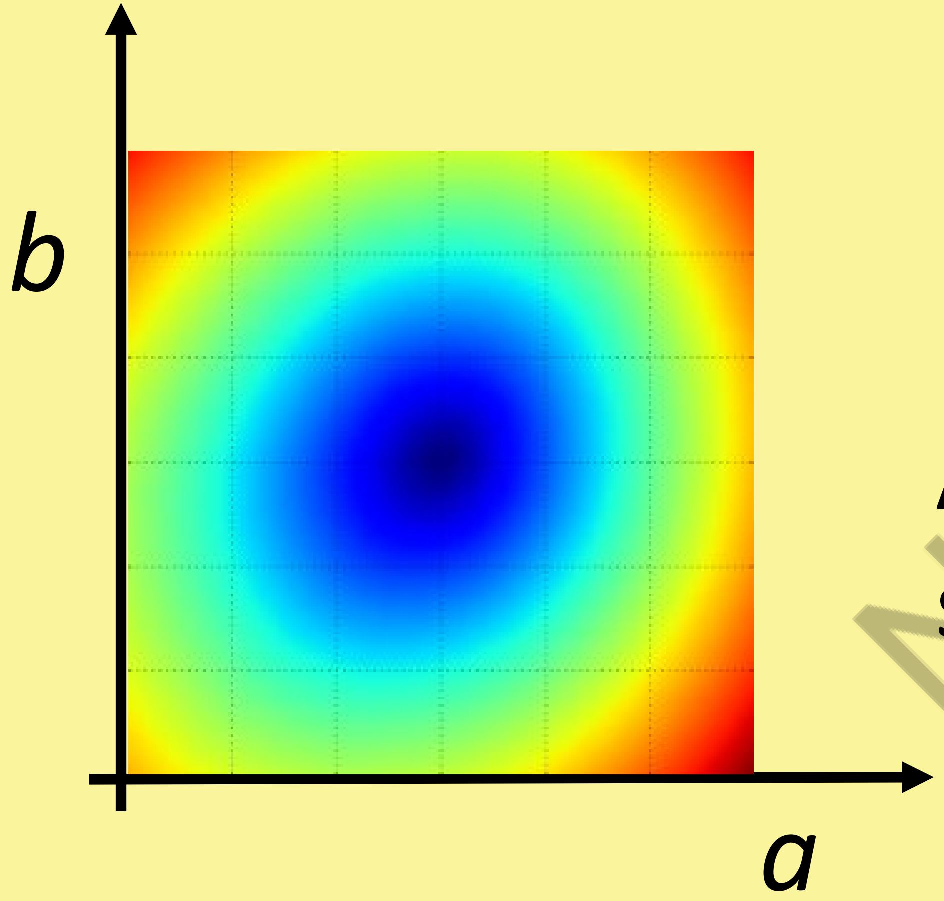
Visualizing Loss Function



$$L(W + aW_1 + bW_2)$$

Source - http://cs231n.github.io

Visualizing Loss Function



$$L(W + aW_1 + bW_2)$$

Averaged over multiple samples

Source - http://cs231n.github.io

Optimizing Loss Function

$$L = \frac{1}{N}\sum_{i}\sum_{j\neq y_i}\left[\max(0, W_j^t X_i - W_{y_i}^t X_i + \nabla\right] + \lambda\sum_{k}\sum_{l} W_{kl}^2$$

$$\nabla_{W_{y_i}} = -\frac{1}{N}\sum_{i}\sum_{j\neq y_i}[X_i \mid (W_j^t X_i - W_{y_i}^t X_i + \nabla > 0)] + \eta W_{y_i}$$

$$\nabla_{W_j} = \frac{1}{N}\sum_{i}\sum_{j\neq y_i}[X_i \mid (W_j^t X_i - W_{y_i}^t X_i + \nabla > 0)] + \xi W_j$$

Optimizing Loss Function

$$\nabla_{W_{y_i}} = -\frac{1}{N}\sum_i \sum_{j \neq y_i} [X_i \mid (W_j^t X_i - W_{y_i}^t X_i + \nabla > 0)] + \eta W_{y_i} \qquad \nabla_{W_j} = \frac{1}{N}\sum_i \sum_{j \neq y_i} [X_i \mid (W_j^t X_i - W_{y_i}^t X_i + \nabla > 0)] + \xi W_j$$

Gradient descent

$$W_{y_i}(k+1) \leftarrow (1-\eta) W_{y_i}(k) + \frac{1}{N}\sum_i \sum_{j \neq y_i} [X_i \mid (W_j^t X_i - W_{y_i}^t X_i + \nabla > 0)]$$

$$W_j(k+1) = (1-\xi) W_j(k) - \frac{1}{N}\sum_i \sum_{j \neq y_i} [X_i \mid (W_j^t X_i - W_{y_i}^t X_i + \nabla > 0)]$$

Source - http://cs231n.github.io

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Thank you