Module 18: Loop Optimizations

Lecture 36: Cycle Shrinking

The Lecture Contains:

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Cycle Shrinking ...

- Dependence cycle with distance > 1
- Transform a serial loop into two nested loops (outer serial and inner parallel)
- Consider the loop

```
for I = 1,n
A[i+k] = B[i] –1
B[i+k] = A[i] + C[i]
endfor
```

```
for i=1, n, k
forall j=1, i+k-1
A[j+k]=B[j]-1
B[j+k]=A[j]+C[j]
endforall
endfor
```

```
For I = 3,n
A[i]=B[i-2]-1
B[i]=A[i-3]*k
Endfor

For j =3, n, 2
forall I = j, j+1
A[i]=B[i-2]-1
B[i]=A[i-3]*k
endforall
Endfor
```

```
A3 = B1 -1
B3 = A0 * k
A4 = B2 -1
B4 = A1 * k
A5 = B3 -1
B5 = A2 * k
A6 = B4 -1
B6 = A3 * k
A7 = B5 -1
B7 = A4 * k
A8 = B6 -1
B8 = A5 * k
```

Cycle Shrinking in Distance Varying Loops

- The distance may not be constant
- Cycle may be reduced by the minimum distance

```
For I = 1,n
X[i]=Y[i]+Z[i]
Y[i+3]=X[i-4]*W[i]
Endfor
```

```
for j=1, n, 3
forall I = j, j+2
X[i]=Y[i]+Z[i]
Y[i+3]=X[i-4]*W[i]
endforall
endfor
```

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Loop Un-switching

- Removes loop independent conditionals from a loop
- Reduces frequency of execution of conditional statements
- Makes loop structure more complex

```
For I = 1, n
for j = 2, n
if T[i] > 0 then
A[i,j]=A[i,j-1]*T[i]+B[j]
else
A[I,j] = 0.0
endif
endfor
Endfor
```

```
for I = 1, n
if T[i]>0 then
for j=2,n
A[i,j]=A[i,j-1]*T[i]+B[j]
endfor
else
for j=2,n
A[I,j]=0.0
endfor
endif
endfor
```

Loop Peeling

- Used to handle wrap around variables
- Removes first or the last iteration of the loop into separate code
- Peeling can also be used to remove loop invariant code by executing it only in the first iteration (assuming n = 1)

```
for I = 1,n
A[i]=(x+y)*B[i]
endfor
```

```
A[1] = (t=x+y)*B[1]
for I = 2,n
A[i]=t*B[i]
endfor
```

Index Set Splitting

- Generalization of loop peeling
- Used to remove conditionals from the loops

```
For I = 1, 100
A[i]=B[i]+C[i]
if i>10 then
D[i]=A[i]+A[i-10]
endif
Endfor
```

```
for I = 1,10
A[i]=B[i]+C[i]
endfor
for I = 11 to 100
A[i]=B[i]+C[i]
D[i]=A[i]+A[i-10]
endfor
```

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Loop Fusion

- When two adjacent countable loops have the same loop limits they can sometimes be fused
- Reduces cost of test and branch
- Fusing loops which refer to the same data enhances temporal locality
  - It has significant impact on cache and virtual memory performance
- Loop fusion may increase size of the loop which can reduce instruction locality (noticeable with very small cache memories)
- Fusion is legal if all the dependence relations are preserved
- Before fusion all relations must flow from body1 to body2 (unless carried by an outer loop)

```
For I = 1,n
A[i]=B[i]+1
Endfor
For I = 1,n
C[i]=A[i]/2
Endfor
For I = 1,n
D[i]=1/C[i+1]
Endfor
S2→S5
S5→S8
```

```
For I = 1,n
A[i]=B[i]+1
C[i]=A[i]/2
D[i]=1/C[i+1]
Endfor
```

after fusion the second dependence is violated

```
For I = 1,n
A[i]=B[i]+1
C[i]=A[i]/2
Endfor
For I = 1,n
D[i]=1/C[i+1]
Endfor
```

```
for I = 1,99
A[i]=B[i]+1
Endfor
for I = 1,98
C[i]=A[i+1]*2
Endfor
```

```
A[1]=B[1]+1
for I = 2,99
A[i]=B[i]+1
Endfor
for I = 1,98
C[i]=A[i+1]* 2
Endfor
```

```
A[1]=B[1]+1
for j = 0,97
A[j+2]=B[j+2]+1
C[j+1]=A[j+2]*2
Endfor
```

Loop Fission

- A single loop may be broken into smaller loops (inverse of loop fusion)
- Used on machines which have very small instruction cache
- Improves memory locality
- Construct a statement level dependence graph of the body of the loop
  - Dependence relations carried by outer loop need not be preserved
  - Inner loops are treated as single nodes
  - If there are no cycles then loop fission can divide the loop into separate loops around each node
  - The loops are ordered in topological order of the dependence graph

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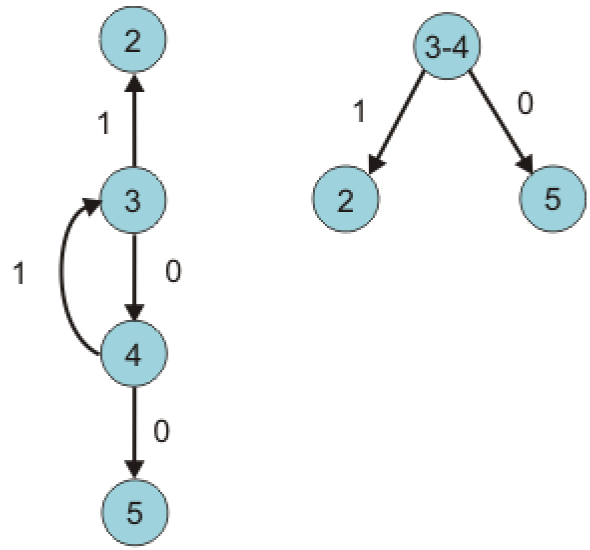
Module 18: Loop Optimizations

Lecture 36: Cycle Shrinking

For I = 1,n
A[i] = A[i] + B[i-1]
B[i] = C[i-1]*x + y
C[i] = 1/B[i]
D[i] = sqrt(C[i])
endfor



For ib = 0,n-1
B[ib+1] = C[ib]*x + y
C[ib+1] = 1/B[ib+1]
Endfor
For ib = 0,n-1
A[ib+1] = A[ib+1] + B[ib]
Endfor
For ib = 0,n-1
D[ib+1] = sqrt(C[i])
Endfor
I = n+1

Loop Reversal

- Compiler can decide to run a loop backward
- Always legal for parallel loops
- Illegal for sequential loop if it has loop carried dependence
- Allows loop fusion to proceed where it might otherwise fail

for I = 1,n	for i=n downto 1
A[i]=B[i]+1	A[i]=B[i]+1
C[i]=A[i]/2	C[i]=A[i]/2
endfor	D[i]=1/C[i+1]
for i=1,n	endfor
D[i]=1/C[i+1]	
endfor	

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Loop Skewing

- Normalization can change the shape of the iteration space
- It may affect the ability to interchange loops
- Consider following code
  for I = 2, n
  for j = I, n
  A[I,j] = 0.5 *(A[I,j-1]+A[i-1,j])
  endfor
  endfor
- Using un-normalized iteration vector the dependence distances are (0,1) and (1,0)

Iteration Space of The Loop

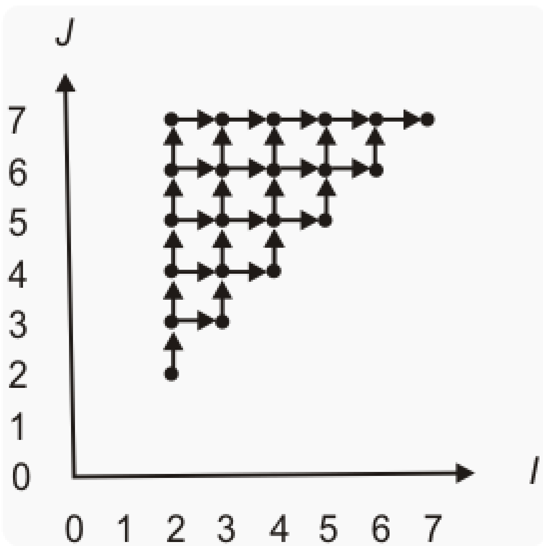
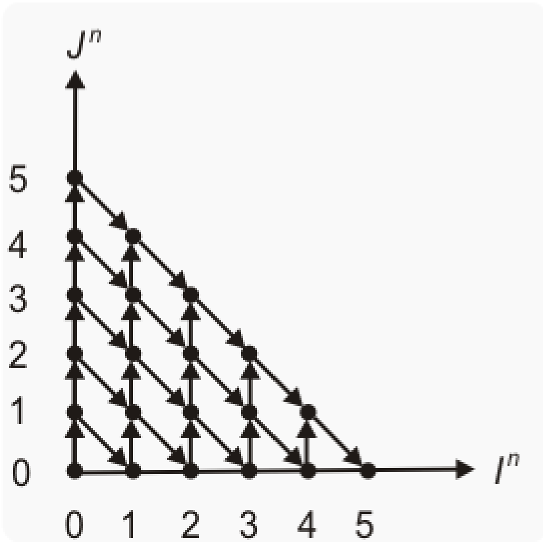


Figure Courtesy: High Performance Compilers
For Parallel Computing by Wolfe

- After interchange the code is
  for j = 2, n
  for i = 2, j
  A[I,,j] = 0.5 *(A[I,,j-1]+A[i-1,,j])
  endfor
  endfor

- Using normalized iteration vectors the shape of the iteration space changes as shown below



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The dependence distance are (0,1) and (1,-1)
This dependence prevents loop interchange

- If normalization can prevent interchange then un-normalization can enable loop interchange
- This is called loop skewing
- Skewing changes the iteration vector of each iteration by adding the outer loop index value to the inner loop index
    - (I,j) becomes (I, j+i)
    - A dependence relation from (i1, j1) to (i2, j2) will have distance (i1, j1) –(i2-j2) = (d1, d2)
    - After skewing the distance will change to (i1, j1+i1) –(i2, j2+i2) = (d1, d2+d1)
- In general loops can be skewed by a factor changing iteration label from (I,j) to (I, j+fi)
    - This changes distance from (d1, d2) to (d1, d2+fd1)
    - F can also be negative
- Choosing whether to skew and the factor by which to skew depends upon the goal to enable other transformations

Example

- Interchange following loop using skewing
```
for I = 2, n
for j = 2, m
A[I,j] = 0.5 * (A[i-1, j-1]+A[i-1, j+1])
endfor
Endfor
```
- The two dependence distances are (1,1) and (1,-1)
- The second one prevents the interchange
- Skewing the loop would change the dependence distance to (1,2) and (1,0) allowing the interchange
- The compiler must generate the correct limits using FM method

The Final Code After Skewing is

```
For js = 2, n+m-2
for is = max(0, js-m+2), min(n-2, js)
I = is+2
j = js-is+2
A[I,j] = 0.5 * (A[i-1, j-1] + A[i-1, j+1])
endfor
endfor
```

Loop Blocking or Strip Mining

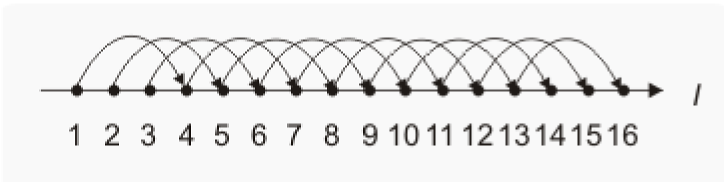
- Creates doubly nested loops out of single loops
- Organizes computation into chunks of approximately equal sizes
- Used to overcome size limitations of caches and local memory

```
for I = 1,n
A[i] = B[i] + C[i]
endfor
```

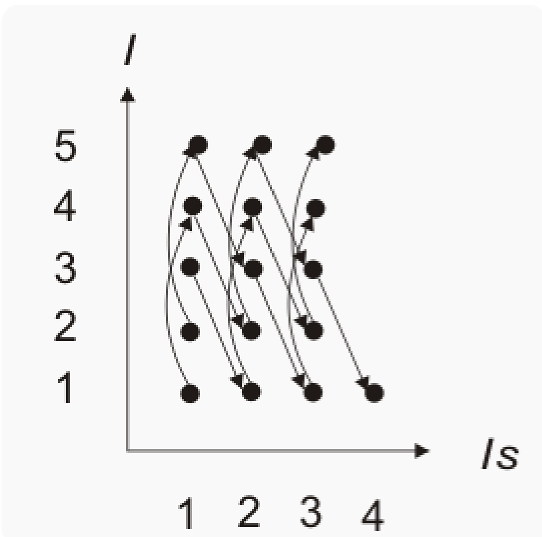
```
for j = 1, n, k
for I = j, min(j+k, n)
A[i]=B[i]+C[i]
endfor
endfor
```

Example

```
for I = 1, 16
A[i+3]=A[i]+B[i]
endfor
```




```
for It = 1, 16, 5
for i=it, min(16, it+4)
A[i+3]=A[i]+B[i]
endfor
endfor
```



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Loop Tiling

- Similar to strip mining (Strip mining works for single loops)
- Loop tiling is used for nested loops
- Tiling boundaries are parallel to the iteration space axes and not to iteration space boundaries
- The eventual goal is to interchange tile loops outward and element loops inward
- Tiling is characterized by tile size ts and a tile offset to (0 = to < ts)
- Each tile starts an iteration i such that i mod ts = to
- Each tile iterates from tn-ts+to to (tn+1)-ts+to-1 where tn is tile number
- The compiler must determine the minimum and maximum tile numbers
- The compiler must ensure that element loop does not execute outside its original iteration space
- The general formula for tiling for a loop such as
  for I = lo, hi
  is
  for it = floor((lo-to)/ts)*ts+to, floor((hi-to)/ts)*ts+to, ts
  for I = max(lo,it), min(hi,it+ts-1)

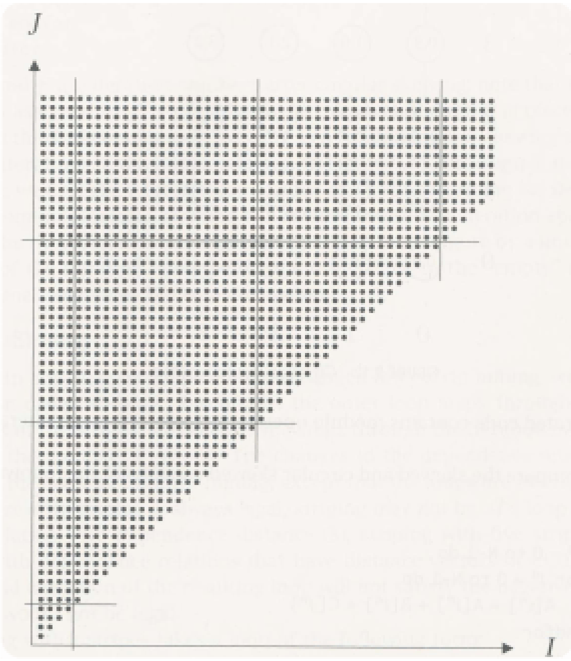
Tile following loops with a tile size of 20 and an offset of 5

```
For I = 1, 50
for j = i, 60
A[I,j] = A[I,j]+1
endfor
endfor
```

just applying the formula produces following loop

```
For It = -15, 45, 20
for i = max(1,it), min(50,it+19)
for jt=floor((i-5)/20)*20+5, 45, 20
for j=max(I,jt), min(60, jt+19)
A[I,j] = A[I,j]+1
endfor
endfor
endfor
endfor
```

The Tiled Iteration Space



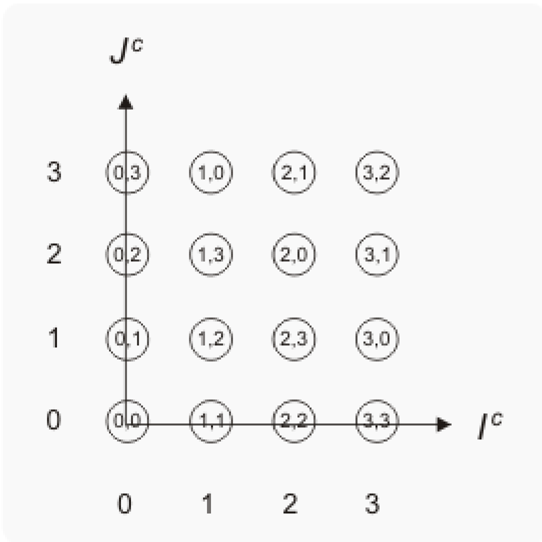
- Interchange jt loop with I loop
- Compiler finds the new lower limits for jt

```
For It = -15, 45, 20
for jt = max(-15,it), 45, 20
for i = max(I,it), min(50,it+19)
for j=max(I,jt), min(60, jt+19)
A[I,j] = A[I,j]+1
endfor
endfor
endfor
endfor
```

Circular Loop skewing

- A variation of loop skewing
- Skew the inner loop iterations such that they wrap around a cylinder
- The shape of the iteration space does not change but the relative positions change
- Backward dependencies with large distances make tiling unprofitable
- Circular loop skewing shortens backward dependencies

```
For I = 0, n-1
for j = 0, n-1
A[i] = A[i] + B[i] * C[i]
endfor
endfor
```



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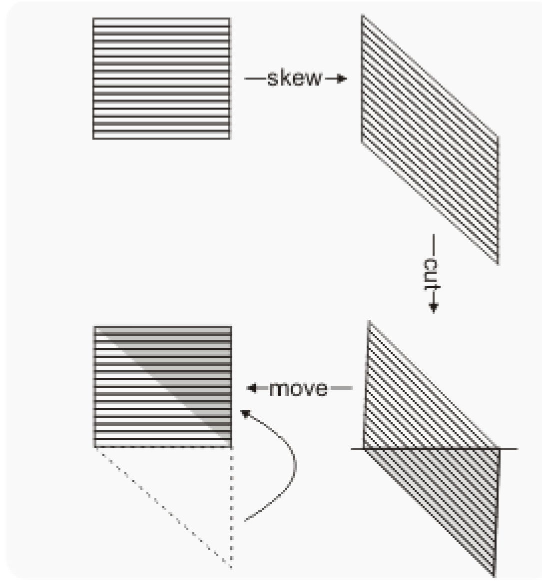
The circular loop skewing does not change the shape of the iteration space. It changes the iterations computed at each point.



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