The Lecture Contains:

- Definition of vector space
  - Examples
- Dimensionality
- Definition of metric space

Definition of vector space

- Informally, a collection of vectors that can be added together and scaled by a scalar
- Vector space $\mathbf{V}$ over field F defines two operations
  - Vector addition: $\mathbf{V} \times \mathbf{V} \rightarrow \mathbf{V}$, denoted as $\mathbf{x} + \mathbf{y}$
  - Scalar multiplication: $F \times \mathbf{V} \rightarrow \mathbf{V}$, denoted as $c\mathbf{x}$
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- 8 properties of addition and multiplication operations:
  - Commutativity of vector addition: $\mathbf{x} + \mathbf{y} = \mathbf{y} + \mathbf{x}$
  - Associativity of vector addition: $(\mathbf{x} + \mathbf{y}) + \mathbf{z} = \mathbf{x} + (\mathbf{y} + \mathbf{z})$
  - Additive identity: $\exists \mathbf{0}$ (zero vector), s.t. $\forall \mathbf{x},\ \mathbf{x} + \mathbf{0} = \mathbf{x}$
  - Additive inverse: $\forall \mathbf{x},\ \exists(-\mathbf{x})$, s.t. $\mathbf{x} + (-\mathbf{x}) = \mathbf{0}$
  - Multiplicative identity: $\exists 1$ (multiplicative identity of F), s.t. $\forall, x 1\, x = x$
  - Associativity of scalar multiplication: $(c_1 c_2)\mathbf{x} = c_1(c_2\mathbf{x})$
  - Distributivity of scalar sums: $(c_1 + c_2)\mathbf{x} = c_1\mathbf{x} + c_2\mathbf{x}$
  - Distributivity of vector sums: $c(\mathbf{x} + \mathbf{y}) = c\mathbf{x} + c\mathbf{y}$

Examples

- $\mathcal{R}^n$
- $\mathcal{R}^\infty$
- $m \times n$ matrices
- Subspaces: Non-empty subset that is closed under the vector addition and the scalar multiplication operations

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Module 1: Basics and Background

Lecture 5: Vector and Metric Spaces

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Dimensionality

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- The span of a set of vectors is the vector space generated by their linear combinations
- A basis of a vector space $\mathbf{V}$ is a set of vectors that are linearly independent and that spans $\mathbf{V}$
- The cardinality of the basis of a vector space $\mathbf{V}$, i.e., the number of linearly independent vectors needed to span $\mathbf{V}$ is called its dimensionality
- The bases (i.e., the basis vectors) may vary, but their cardinality remains the same

Definition of metric space

- A set of elements $\mathbf{M}$ with a distance function $d : \mathbf{M} \times \mathbf{M} \rightarrow \mathcal{R}$ defined between any two elements of the set
- 4 properties of d:
  - Non-negativity: $\forall x, y,\ d(x, y) \geq 0$
  - Identity: $\forall x, y,\ d(x, y) = 0 \iff x = y$
  - Symmetry: $\forall x, y,\ d(x, y) = d(y, x)$
  - Triangular inequality: $\forall x, y, z,\ d(x, y) + d(y, z) \geq d(x, z)$

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- Pseudometric space: Condition 2 (identity) is relaxed
  - Example: Number of vertices and edges of a graph
- Quasimetric space: Condition 3 (symmetry) is relaxed
  - Example: Time to walk from plain A to hill B
- Semimetric space: Condition 4 (triangular inequality) is relaxed
  - Example: Air fares on certain routes