

Module 2: Distances

Lecture 8: Earth Mover's Distance (EMD)

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The Lecture Contains:

- General case
- EMD general formulation
- Match distance
- Edit distance

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## General case

- Feature values need to be positive
  - Done by translating the entire feature range
- Feature values in  $x$  and  $y$  need to add up to the same mass
  - Normalization may not always be suitable

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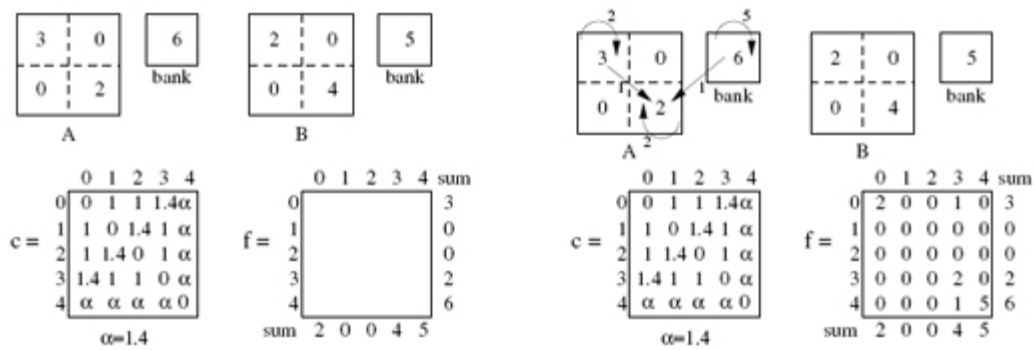
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## General case

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- Feature values in  $x$  and  $y$  need to add up to the same mass
  - Normalization may not always be suitable
- Special "bank" region
- Bank of  $x$  is sum of feature values of  $y$  and vice versa
- Parameter  $\alpha$ : ground distance from ordinary regions to bank
- Bank to bank distance is 0
- $\mathbf{C}$  and  $\mathbf{F}$  become matrices of size  $(m+1) \times (n+1)$
- Metric?

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- $C$  and  $F$  become matrices of size  $(m + 1) \times (n + 1)$
- Metric? Yes, if ground distance is a metric



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EMD general formulation

$$EMD(x, y, \alpha) = \min_F \sum_{i=1}^{m+1} \sum_{j=1}^{n+1} (c_{ij} f_{ij})$$

$$\forall i, j, f_{ij} \geq 0; \forall i, \sum_{j=1}^{n+1} f_{ij} = x_i; \forall j, \sum_{i=1}^{m+1} f_{ij} = y_j$$

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## Match distance

- A special form of earth mover's distance
- Only one dimension in ground distance
- Between histograms or vectors adding up to same total mass
- Ground distance between bins  $i$  and  $j$  is  $c_{ij} = |i - j|$

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## Edit distance

- Also known as **Levenshtein distance**
- Used for words and strings
- Minimum number of edit operations required to transform word  $x$  to word  $y$
- Edit operations on characters are
  - Insertion
  - Deletion
  - Substitution
- Insertions and deletions can be treated as substitutions using gap
- Example: each edit operation costs 1

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- Edit distance is 2
- Metric?

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