

## Department of Civil Engineering IIT Madras



*Photograph of the  
Meenakshi  
Sundareswarar  
Temple, Madurai,  
Tamil Nadu*

## Fracture Mechanics



**Modern Construction Materials – Lecture 9  
Prof. Ravindra Gettu  
IIT Madras**

## Why is Fracture Important ?

**Brittle fracture can cause sudden catastrophic failures.**



## Why is Fracture Important ?



Aloha Airlines Boeing 737-297, operating from Hilo to Honolulu, at Kahului Airport on April 28, 1988 after its fuselage was torn away during the flight.

Failure was attributed to fatigue fracture of the panels forming the fuselage in the section weakened by the rivets.

## Why is Fracture Important ?

**Brittle fracture can cause sudden catastrophic failures.**



Failure of the Liberty ships and T2 tankers built during World War II attributed to the fracture of the weld due to cold weather and fatigue.

## Why is Fracture Important ?

**Brittle fracture can cause sudden catastrophic failures.**



Cause of Moonie to Brisbane pipeline incident that led to the spilling of 1.9 million litres of crude oil into a mangrove lined canal flowing into the Brisbane river (Australia) in 2003.

Failure of pipelines can cause spills that can damage ecosystems and lead to big financial losses.

## Why is Fracture Important ?

**Brittle fracture can cause sudden catastrophic failures.**



Cracks suddenly appeared on two of three steel girders of the Hoan bridge in Milwaukee (USA) in 2000. Triaxial constraint due to bracing system led to brittle crack propagation.

## Why is Fracture Important ?



Visible crack in the flange

Cracks in the web plate at the joint with the lower lateral bracing system



## Why is Fracture Important ?

**Brittle fracture can cause sudden catastrophic failures.**



Collapse of the Wu-Shi bridge during the Chi Chi earthquake in Taiwan (1999).

Failure of the concrete piers due to the lateral movement caused by the fault which crossed the bridge



### **Why Fracture Mechanics?**

- Conventional design procedures based only on some maximum stress criterion are not adequate under all circumstances.
- Fracture mechanics determines failure based on the interaction between the *applied stress*, the *crack* (or *flaw*) and material parameters (e.g., *fracture toughness*).
- Instead of the magnitude of stress or strain, fracture mechanics is concerned primarily with the distribution of stresses and displacements in the vicinity of a crack tip.

### **Why Fracture Mechanics?**

- Fracture mechanics is particularly applicable to the failure of brittle materials but under certain circumstances to other materials as well.
- The difference between the theoretical material fracture (or cohesive) strength calculated from the interatomic bonding energy and the values actually measured is enormous, perhaps two or three orders of magnitude.

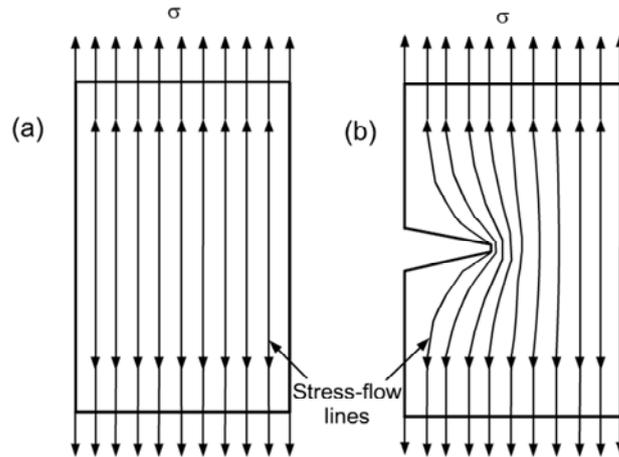
## **Why Fracture Mechanics?**

- Griffith, in 1920, concluded that any real material has flaws, microcracks or other defects that would have the effect of concentrating the stress sufficiently to reach the theoretical fracture stress in highly localized regions. Cracks would grow under an applied stress until failure occurred.

## Background

### Stress Concentration

Can be visualized as the concentration of stress-flow lines due to a geometrical discontinuity in the continuum

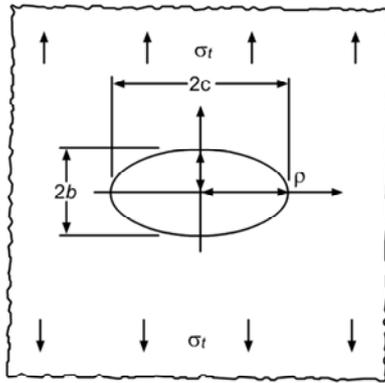


Shah and Ahmad

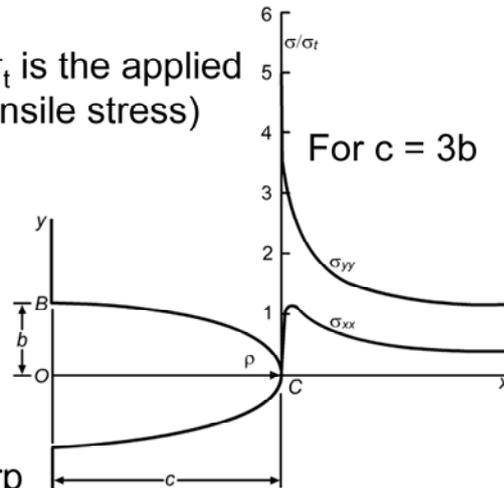
## Stress Concentration

A defect or a crack leads to stress concentration.

The stress concentration increases as the radius of the tip decreases.



( $\sigma_t$  is the applied  
tensile stress)

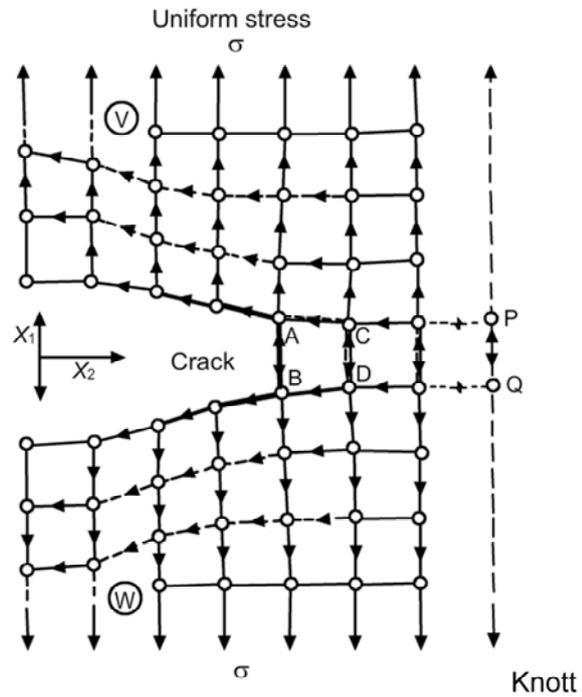


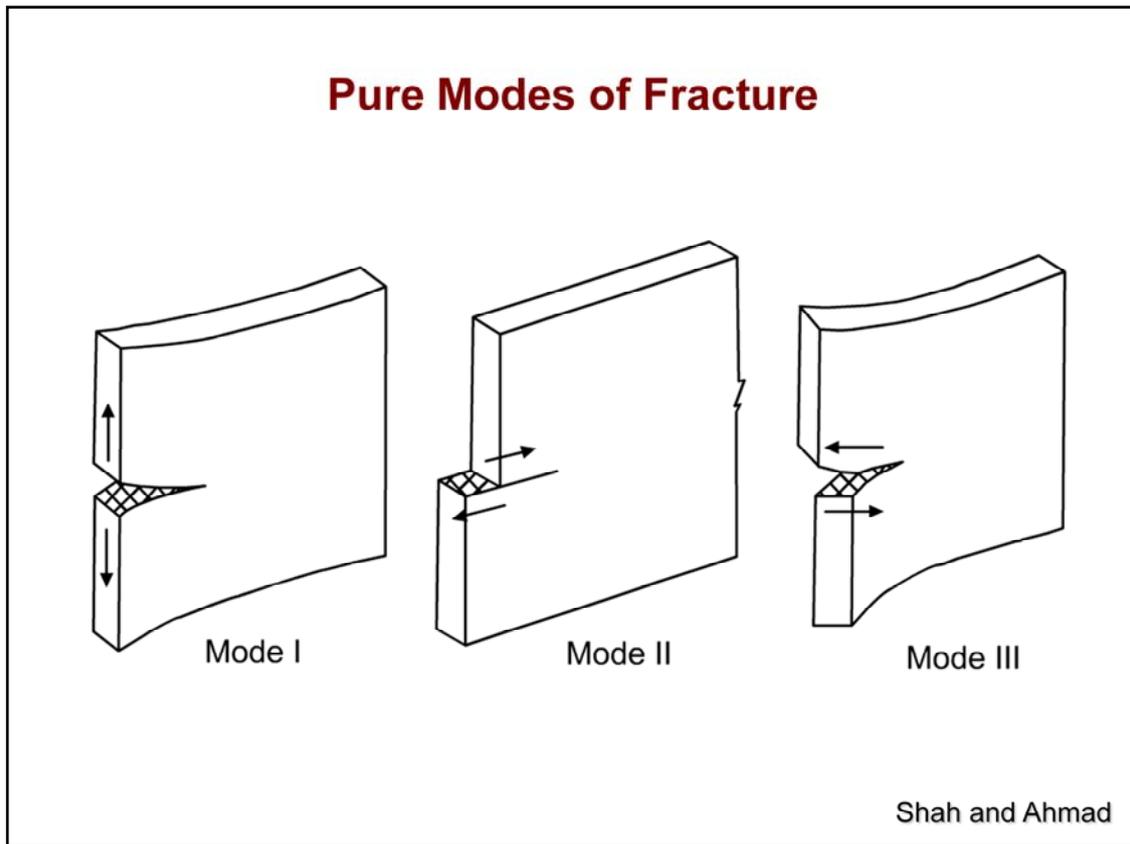
When  $c \gg \rho$ , that is, for a sharp  
"crack", there is a singularity at the tip.

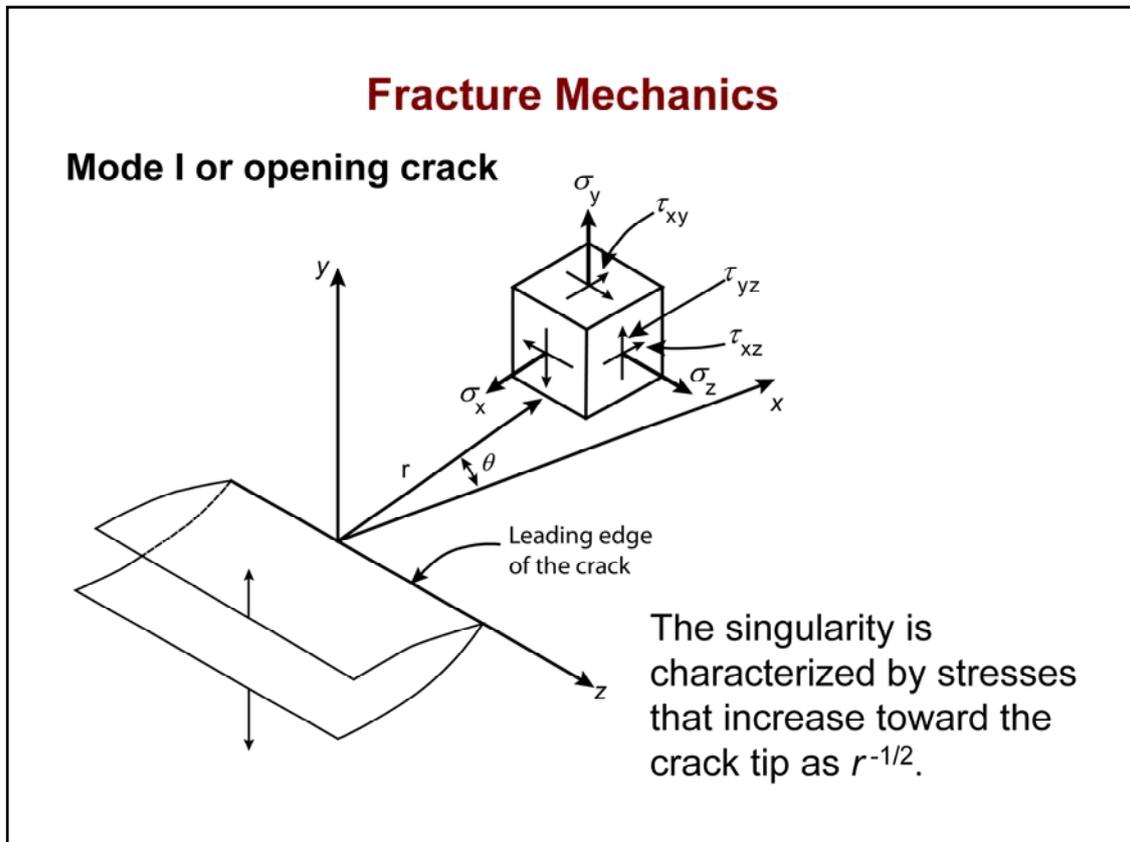
Young et al.

## Stress Concentration

**Schematic loading of  
atomic bonds near a  
crack-tip**







## Fracture Mechanics

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{K_1}{(2\pi r)^{1/2}} \begin{Bmatrix} \cos\left(\frac{\theta}{2}\right) \left[ 1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] \\ \cos\left(\frac{\theta}{2}\right) \left[ 1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right) \right] \\ \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right) \end{Bmatrix}$$

**$K_1$  is the stress intensity factor**

$$\sigma_z = \mu(\sigma_x + \sigma_y) \quad \tau_{xz} = \tau_{yz} = 0$$

$\mu$  is the Poisson's ratio

Mindess & Young

## Stress Intensity Factor

- $K_I$  may be considered as a single-parameter description of the stress and displacement fields near the crack tip.
- Its calculation considers a linear elastic material that is both isotropic and homogenous. Although these are incorrect for most materials, it is generally assumed that the approximations involved in the application of linear elastic fracture mechanics to many cases are reasonable.
- $K_I$  has the dimension of stress  $\times$  (length)<sup>1/2</sup> : MPa-m<sup>1/2</sup>

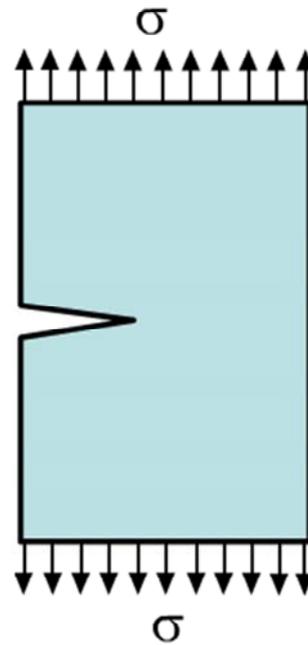
## Fracture Mechanics

**Mode I or opening crack**

**Stress intensity factor:**

$$K_I = \sigma F \sqrt{\pi a}$$

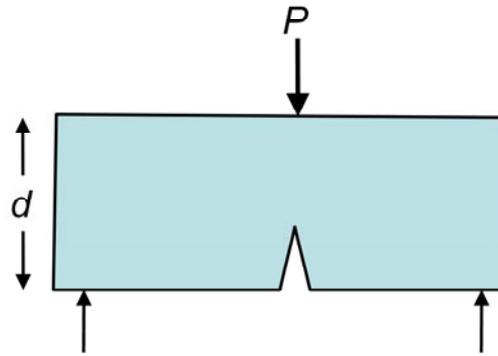
where  $\sigma$  is the applied (nominal or far-field) stress,  $a$  = crack length,  $F$  is a function of geometry and crack length.



## Fracture Mechanics

Or, in another form,

$$K_I = \frac{P}{bd} \left\{ \sqrt{d} f(\alpha) \right\}$$



where  $P$  is the applied load,  $d$  = depth of specimen or structure,  $b$  = out-of-plane thickness,  $\alpha$  ( $= a/d$ ) = relative crack length,  $f(\alpha)$  is a function that depends on the span/depth ratio.

Shah and Ahmad

## **Linear Elastic Fracture Mechanics (LEFM)**

The main features of LEFM are:

- The fracture criterion involves only one material parameter, which is related to the near-tip stress field and the energy of the structure
- The stresses near the crack-tip have an  $r^{-1/2}$  singularity (and become infinite at the crack-tip)
- During fracture, the entire body remains elastic and energy is dissipated only at the crack-tip; i.e., fracture occurs at a point

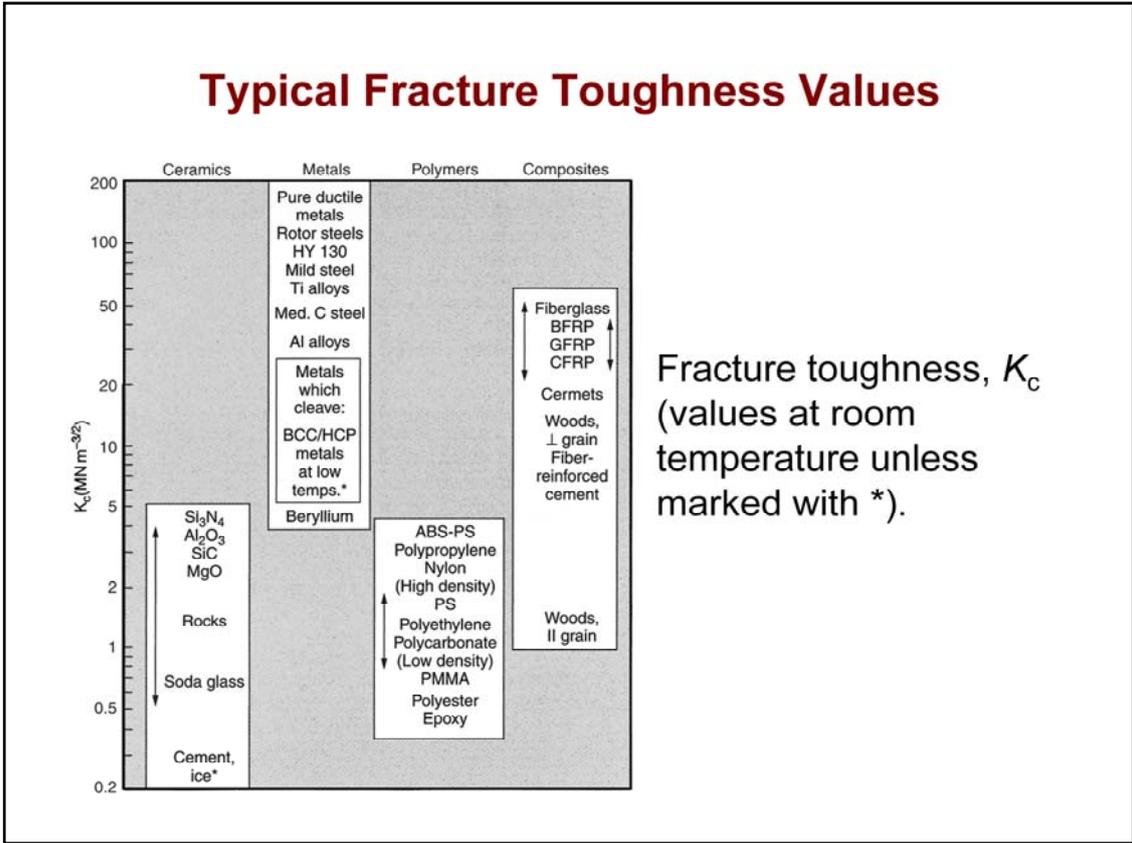
## Linear Elastic Fracture Mechanics (LEFM)

The crack propagates when:

$$K_I \geq K_{Ic}$$

where  $K_{Ic}$  is the critical stress intensity factor or fracture toughness (It is essentially a material property).

## Typical Fracture Toughness Values



Fracture toughness,  $K_c$   
(values at room temperature unless marked with \*).

### Typical Fracture Toughness Values

Material	$K_{Ic}$ (MPa-m <sup>1/2</sup> )
Medium carbon steel	55
Pressure vessel steel	210
Hardened steel	20
Aluminium	20-30
Titanium	75
Copper	110
Lead	20
Glass	0.8
Hardened cement paste	0.6
Concrete	1
Nylon	3.5

Ashby and Jones

## **Linear Elastic Fracture Mechanics (LEFM)**

*Another way of considering fracture involves the energy release rate.*

Crack extension (i.e., fracture) occurs when the energy available for crack growth is sufficient to overcome the resistance of the material. The material resistance may include the surface energy, plastic work, or other type of energy dissipation associated with a propagating crack.

The energy release rate,  $G$ , is defined as the rate of change in potential energy with crack area (for a linear elastic material).

## **Linear Elastic Fracture Mechanics (LEFM)**

**Fracture occurs when (Irwin, 1956):**

$$G \geq G_c$$

**where  $G_c$  is the critical energy release rate or fracture energy.**

## **Linear Elastic Fracture Mechanics (LEFM)**

*Relationship between  $K_I$  and  $G$*

$K_I$  characterises the stress and displacement fields near the crack-tip (i.e., local parameter).

$G$  quantifies the net change in potential energy due to an increment in the crack extension (i.e., describes global behaviour).

## Linear Elastic Fracture Mechanics (LEFM)

For linear elastic materials,  $K_I$  and  $G$  are uniquely related (Irwin, 1957):

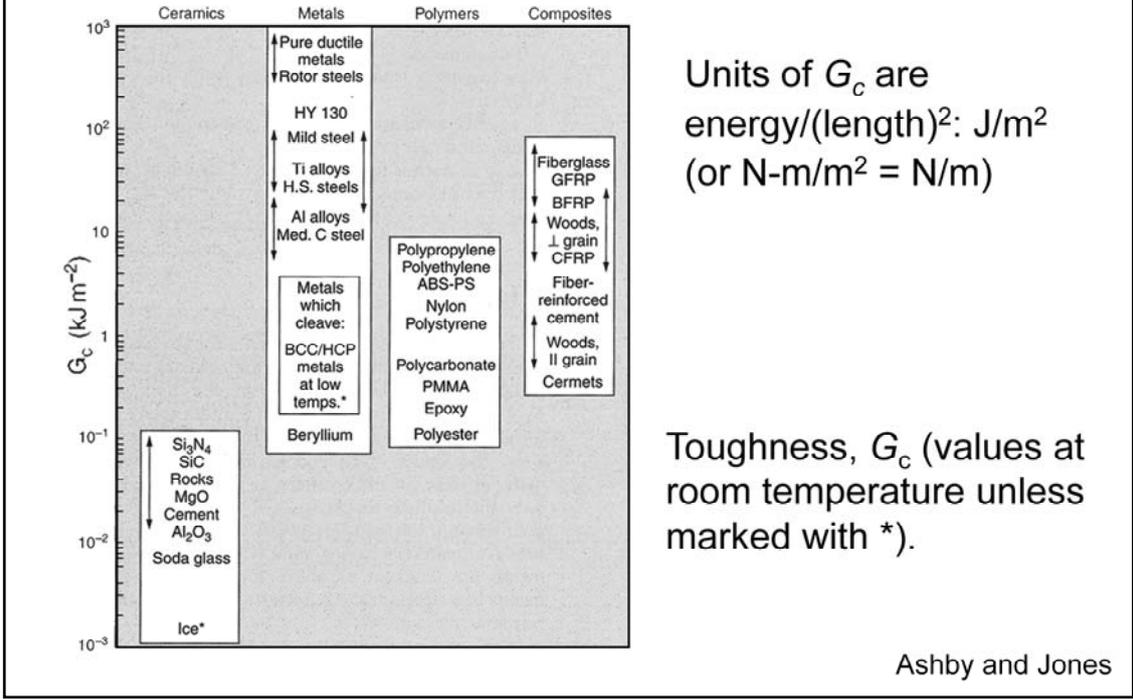
$$G = \frac{K_I^2}{E'}$$

where

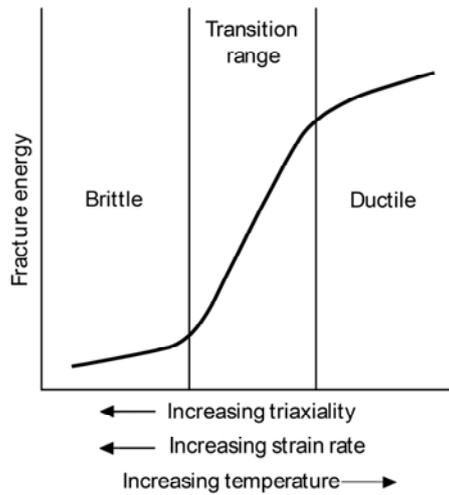
$$E' = E \quad \text{for plane stress}$$

$$E' = \frac{E}{(1-\nu^2)} \quad \text{for plane strain}$$

## Typical Fracture Energy Values



## Brittle-Ductile Transition



Fracture energy increases with:

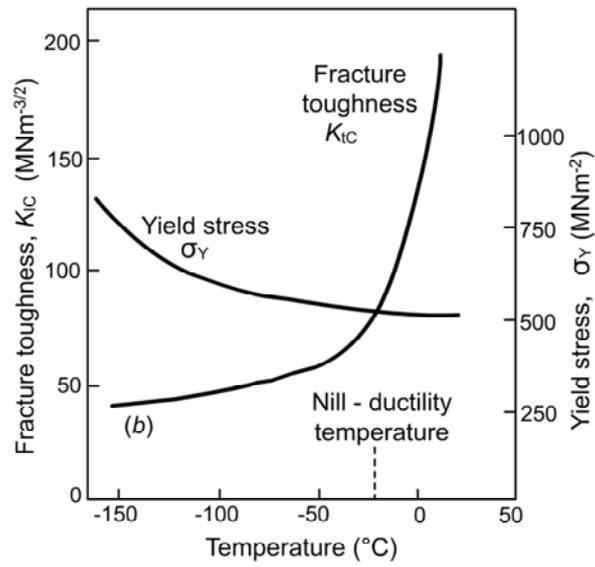
- An increase in temperature
- A decrease in loading rate
- A decrease in triaxiality

When the fracture energy is lower, the tendency for the failure to be brittle is higher.

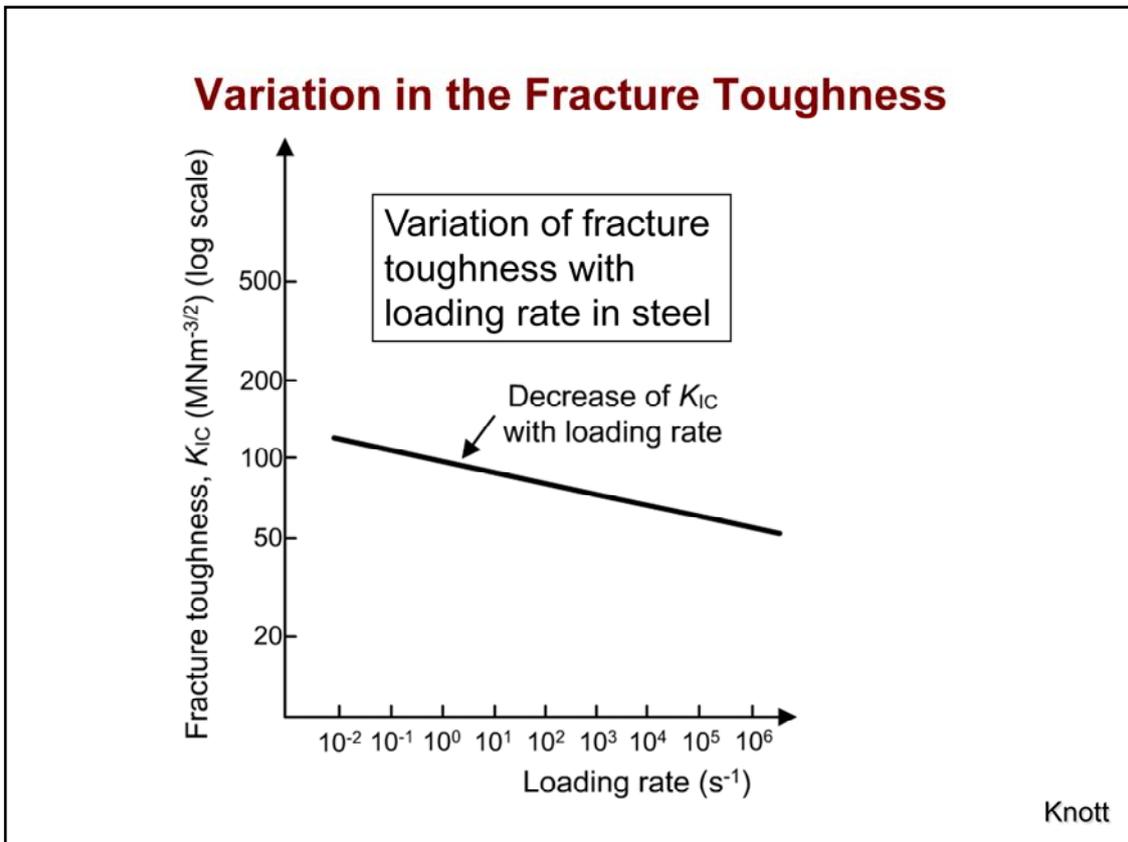
When the fracture energy is higher, there is a higher resistance to brittle failure and the tendency for yield (ductile failure) is higher.

Young et al.

## Variation in the Fracture Toughness



Variation of fracture toughness (curve b) with temperature for a low alloy structural steel



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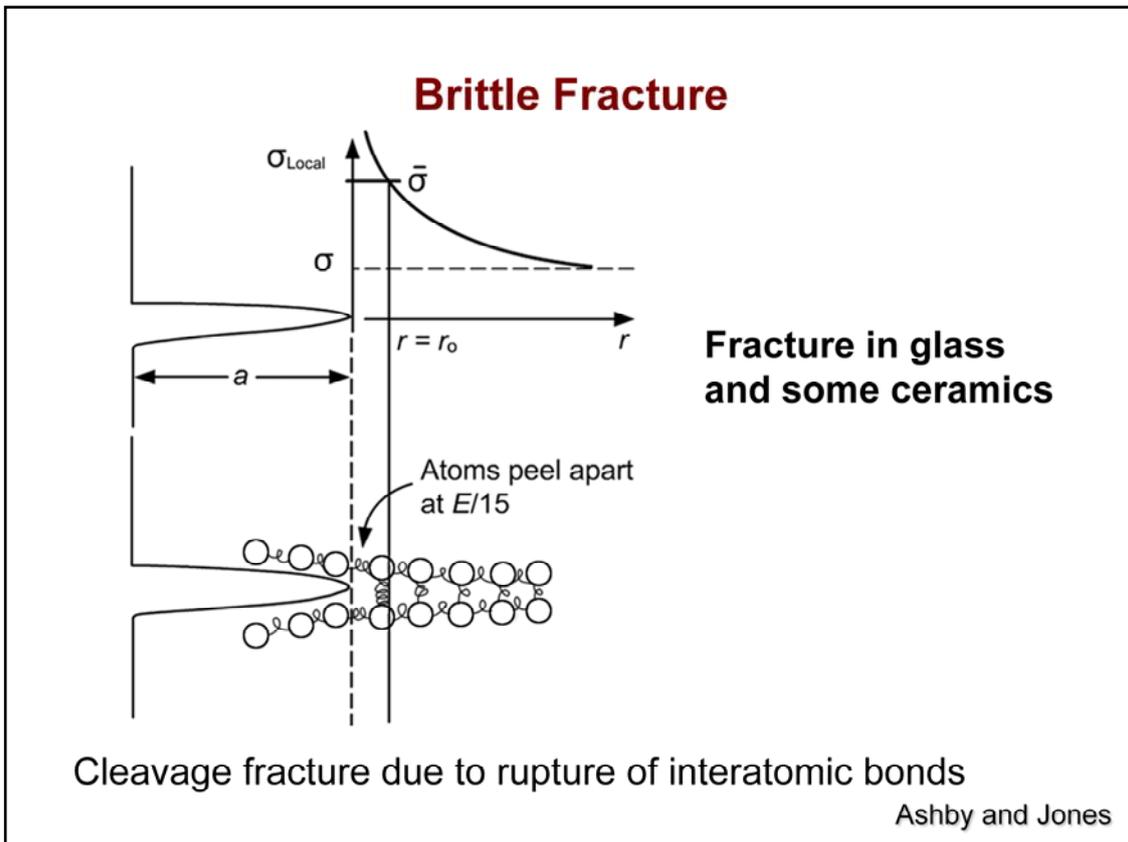


*Cracked clay bed  
of a pond*

**Fracture Mechanics**

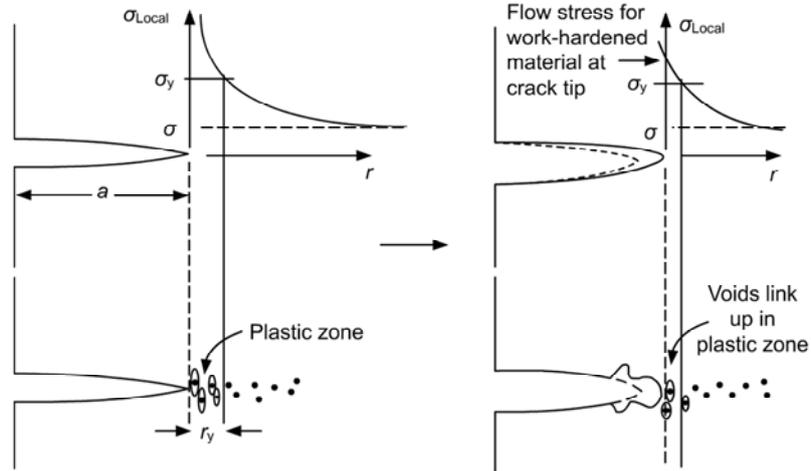


**Modern Construction Materials – Lecture 9 continued  
Prof. Ravindra Gettu  
IIT Madras**



## Elasto-Plastic Fracture

Fracture in ductile metals (i.e., metals that can undergo large plastic deformation)



Plastic zone formation, ductile tearing and crack blunting

Ashby and Jones

## Elasto-Plastic Fracture

When the stress close to the crack-tip reaches the yield stress ( $\sigma_y$ ) at some distance  $r_y$  from the tip, plastic deformation occurs. Over the distance  $r_y$ , it can be considered that  $\sigma_{\text{Local}} = \sigma_y$ .

The width of the plastic zone,  $r_y = \frac{K_I^2}{2\pi\sigma_y^2}$  for a state of plane stress.

*Materials with higher yield strength have a smaller plastic zone; i.e., less ductile failure.*

## Elasto-Plastic Fracture

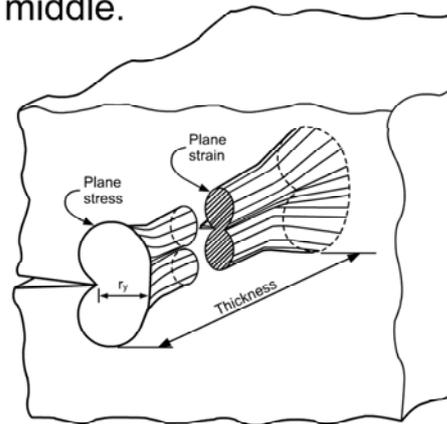
Most metals contain inclusions around which plastic flow occurs resulting in elongated cavities. These cavities link up to cause *ductile tearing*. This *blunts* the crack, lowering the stress concentration. Therefore,  $\sigma_{\text{Local}}$  decreases but is sufficient to cause work hardening of the material near the crack tip.

Ashby and Jones

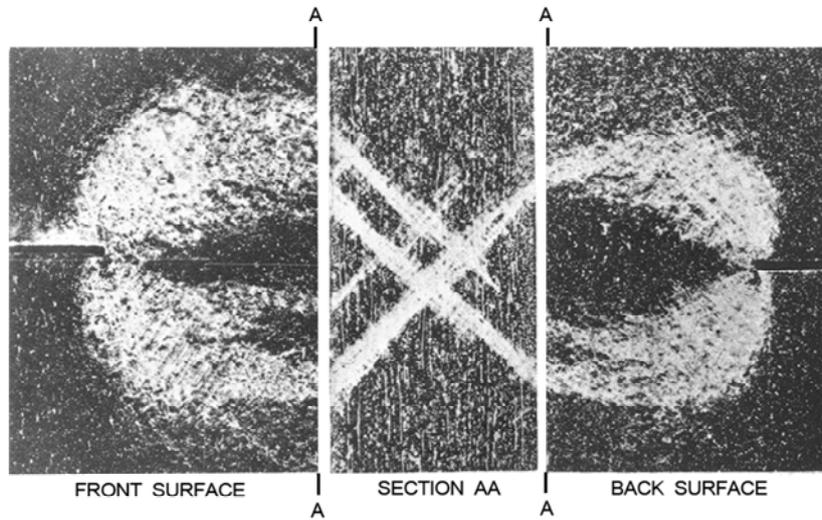
## Elasto-Plastic Fracture

Under plane strain conditions, the restraint elevates the stress required to produce yielding. Therefore, the plastic zone for plain strain is much smaller than the plain stress zone. Consequently, for a thick element, the plastic zone is larger at the surface than in the middle.

**Schematic of Mode I plastic zone varying from plane stress at the lateral surfaces to plane strain at the mid-section**



## Elasto-Plastic Fracture



Appearance of plane stress plastic deformation at the front surface, a normal section and the back surface of a silicon iron fracture specimen

Kanninen and Popelar

## **Fracture in Polymers**

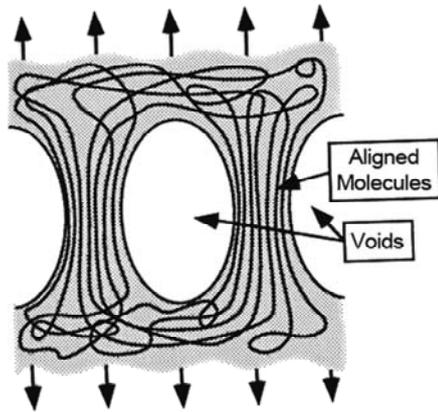
As in metals, fracture and yielding are competing failure mechanisms. During failure, polymers exhibit either shear yielding or crazing.

Shear yielding resembles plastic flow in metals, with molecules sliding with respect to each other in a ductile manner.

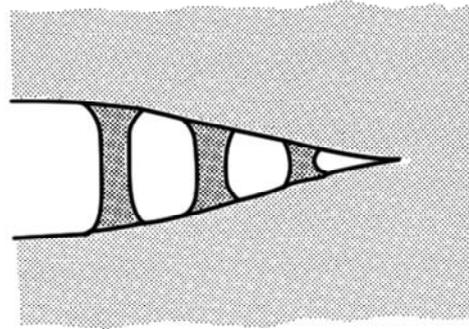
Crazing is a localised deformation that leads to cavitation and strains in the order of 100%. On the macroscopic level, crazing appears as a stress-whitened region that forms perpendicular to the maximum principal stress.

## Fracture in Polymers

At high strains, molecular chains form aligned packets called fibrils that carry very high stresses. Microvoids form between the fibrils.



Mechanism of crazing in homogeneous glassy polymers

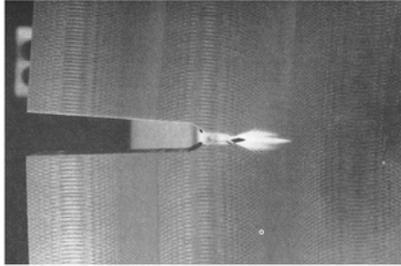


Fibrils in the crack-tip craze zone

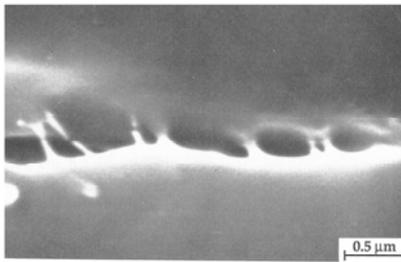
Anderson

## Fracture in Polymers

Fracture occurs when the fibrils rupture.

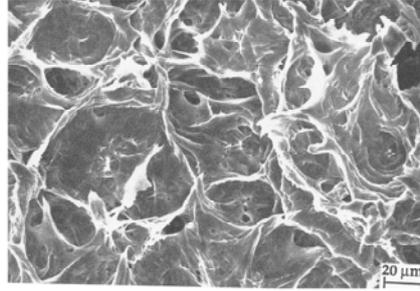


Stress-whitened zone ahead  
of crack-tip indicating crazing



Craze zone in polypropylene

## Fracture in Polymers

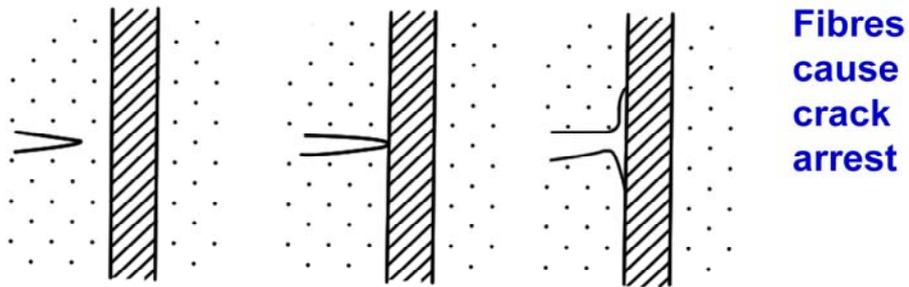


**Fracture surface of craze crack growth in  
polypropylene**

Anderson

## Fracture in Composites

Fracture in wood and other materials reinforced with fibres and inclusions.

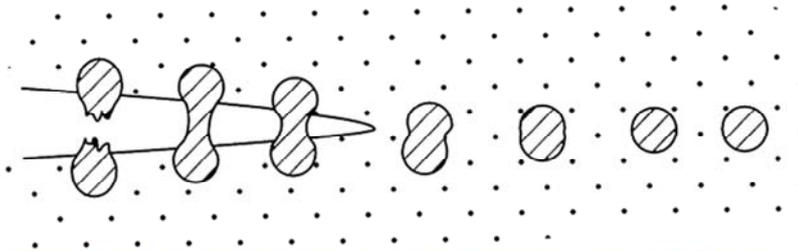


The toughness of polymers is enormously increased by reinforcing them with fibres that act as crack arresters. They deflect the crack and blunt the crack tip.

Ashby and Jones

## Fracture in Composites

The load needed to propagate the crack is higher; fibres and inclusions increase fracture energy

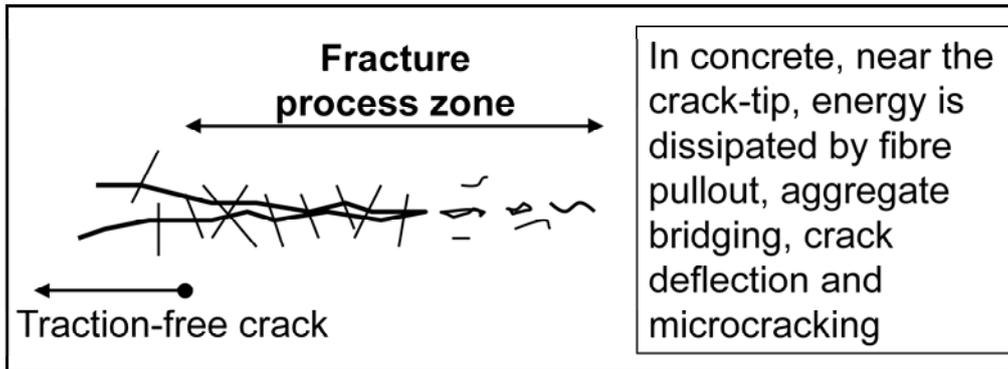


### Inclusions toughen polymers by bridging

Rubber toughened polymers derive their toughness from the bridging action of the small rubber particles that act as springs, tending to close the crack.

Ashby and Jones

## Fracture in Concrete



The size of the fracture process zone that occurs ahead of a propagating crack determines the toughness of concrete.

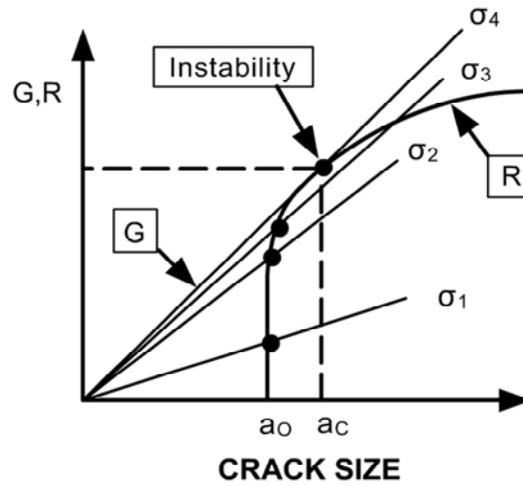
Since the toughening mechanisms are weak in concrete, rocks and some ceramics, they are called quasi-brittle materials.

## **Nonlinear Fracture Mechanics: R-curve**

### ***R*-curve**

The effect of the toughening mechanisms ahead of the crack tip is sometimes represented by a rising resistance curve or *R*-curve, where the critical energy release rate, is not constant, but changes with the crack extension.

## Nonlinear Fracture Mechanics: R-curve



As the applied stress increases, the energy release rate  $G$  changes. Until the  $G$  function becomes tangential to the  $R$ -curve, the crack extension is stable.

## Nonlinear Fracture Mechanics: R-curve

*Condition for stable crack growth:*

$$G = R$$

$$\frac{dG}{da} \leq \frac{dR}{da}$$

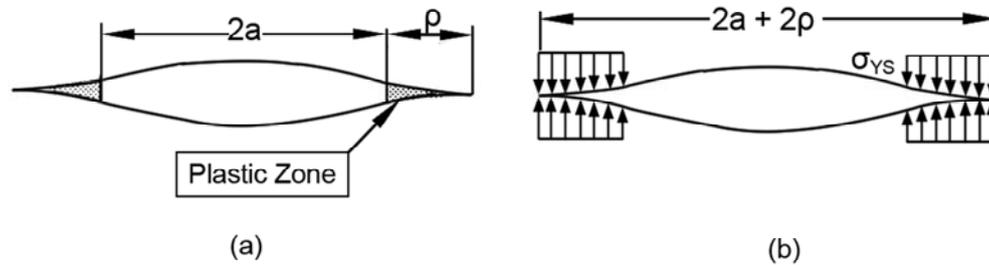
*Unstable crack growth (failure) occurs when:*

$$\frac{dG}{da} > \frac{dR}{da}$$

Anderson

## Nonlinear Fracture Mechanics: Cohesive crack

### *The Dugdale-Barenblatt Model*



The plastic zone over a length  $\rho$  ahead of the traction-free crack of length  $2a$  is modeled as a crack of length  $2a+2\rho$  with closure stresses equal to the yield strength  $\sigma_{YS}$  over a certain length near the crack-tip.

Anderson

## **Nonlinear Fracture Mechanics: Cohesive crack**

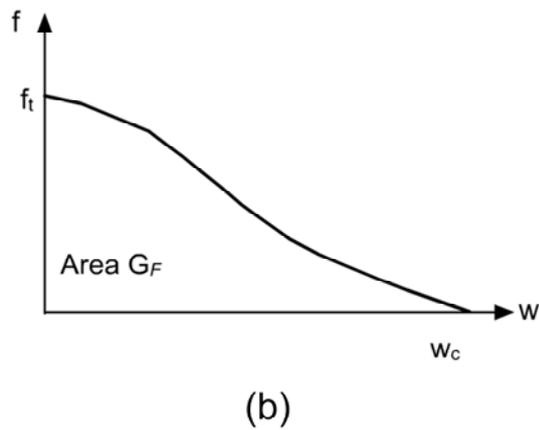
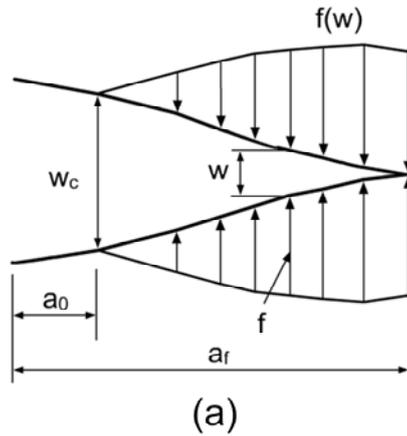
### ***The Dugdale-Barenblatt Model***

This represents the elastoplastic behaviour by superimposing two elastic solutions: a through crack (or traction-free crack) under the applied remote stress and a through crack with closure stresses at the tip.

## Nonlinear Fracture Mechanics: Cohesive crack

### Fictitious crack model (Hillerborg, 1976)

The effect of the fracture process zone is represented by a cohesive zone.



## Nonlinear Fracture Mechanics: Cohesive crack

### Fracture criteria:

$$\begin{array}{l} \text{for } w = 0, f = f_t \\ \text{for } w = w_c, f = 0 \end{array} \quad \int_0^{w_c} f(w) dw = G_F$$

where  $w$  is the crack opening or width,  $w_c$  is the critical crack opening,  $f_t$  is the tensile strength,  $f(w)$  are the cohesive stresses and  $G_F$  is the fracture energy

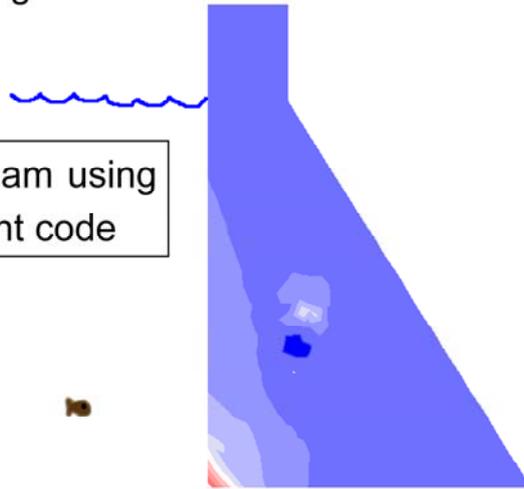
This can be used in finite element analysis or any other analytical technique to represent the tensile cracking response of concrete, rock, ceramics, etc.

## Application of Fracture Mechanics

Fracture mechanics is often used in the framework of the finite element method to simulate the failure of a structure or body through crack propagation.

Analysis of cracking in a dam using  
the FRANC2D finite element code

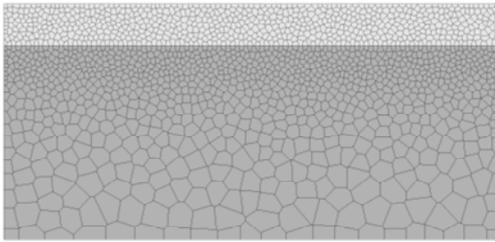
*See link for the animation*



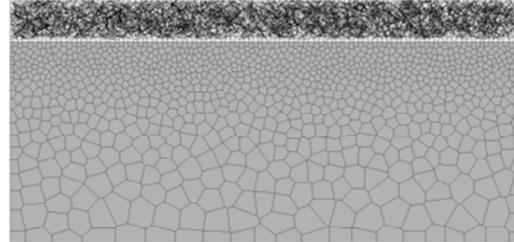
[simscience.org/cracks/](http://simscience.org/cracks/)

## Application of Fracture Mechanics

Cracking in pavement overlay systems (over concrete substrates) analysed using FEMLAB finite element software. Shrinkage cracking due to drying during 110 days of exposure.



**Unreinforced overlay**



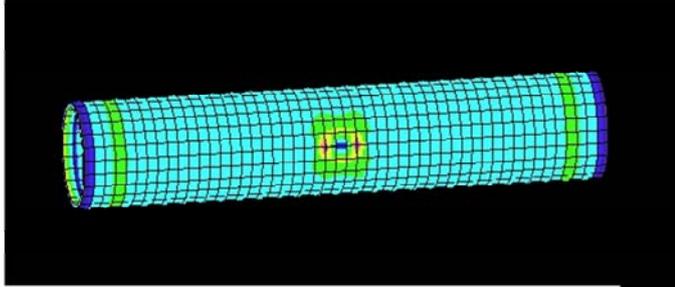
**Fibre reinforced overlay**

*See link for the animations*

[cee.engr.ucdavis.edu/faculty/bolander/overlayAnime.html](http://cee.engr.ucdavis.edu/faculty/bolander/overlayAnime.html)

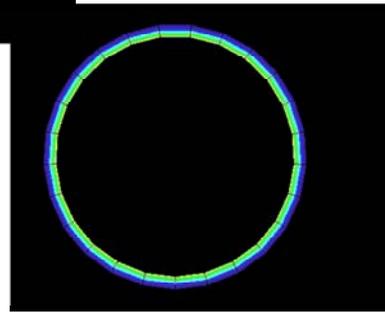
## Application of Fracture Mechanics

### Failure of pipe due to excessive internal pressure



**Longitudinal view**

*See link for the animations*



**Side view**

[www.madisongroup.com/Services/Failure/casepipe/pipeanalysis.html](http://www.madisongroup.com/Services/Failure/casepipe/pipeanalysis.html)

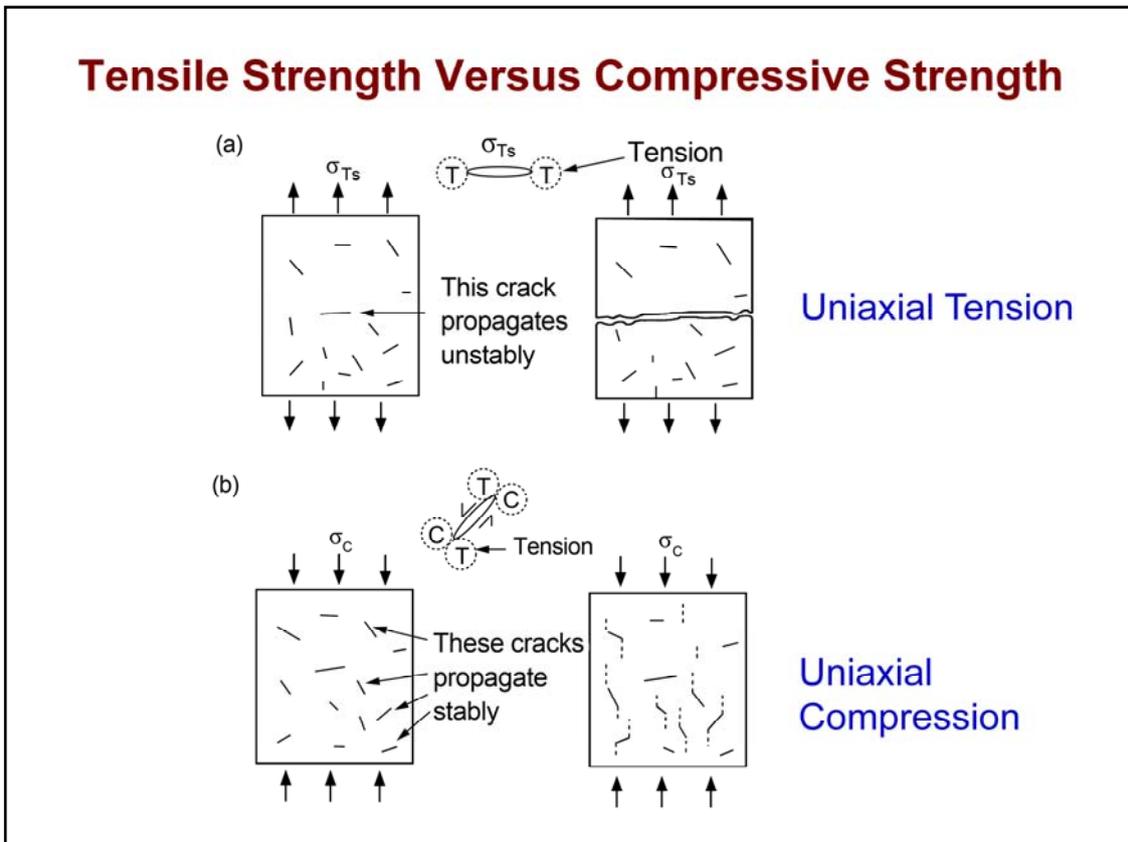


*Traditional house  
of the Kulu-Manali  
Valley, Naggar,  
Himachal Pradesh*

## ***Probabilistic Fracture of Brittle Materials***

### **Defect-Sensitivity**

- Materials such as glass, ceramics, rigid polymers and concrete have low fracture toughness making them vulnerable to the presence of crack-like defects. Such *defect-sensitive* materials are prone to brittle failure well before they can yield.
- Moreover, many of these materials always contain cracks and flaws. Consequently, their tensile strengths are low.
- Concrete can have flaws in the order of 5-10 mm; brick and stone can have defects of the order of 2 mm. Engineered ceramics have smaller defects, in the order of 60  $\mu\text{m}$ .



## **Tensile Strength Versus Compressive Strength**

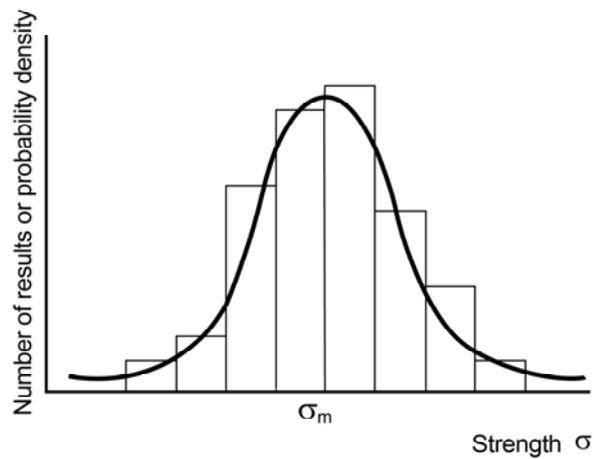
- a) Under (uniaxial) tension, failure generally occurs by the propagation of the longest most favourably oriented flaw or crack.
  
- b) Under compression, cracks propagate stably as they change orientation. Moreover many cracks have to propagate and join to cause failure. Consequently, failure is not dependent on the characteristics of any one crack but the average.

The compressive strength of brittle materials is usually about 10-15 times the tensile strength. Also, the variability of the strength is lower in compression than in tension.

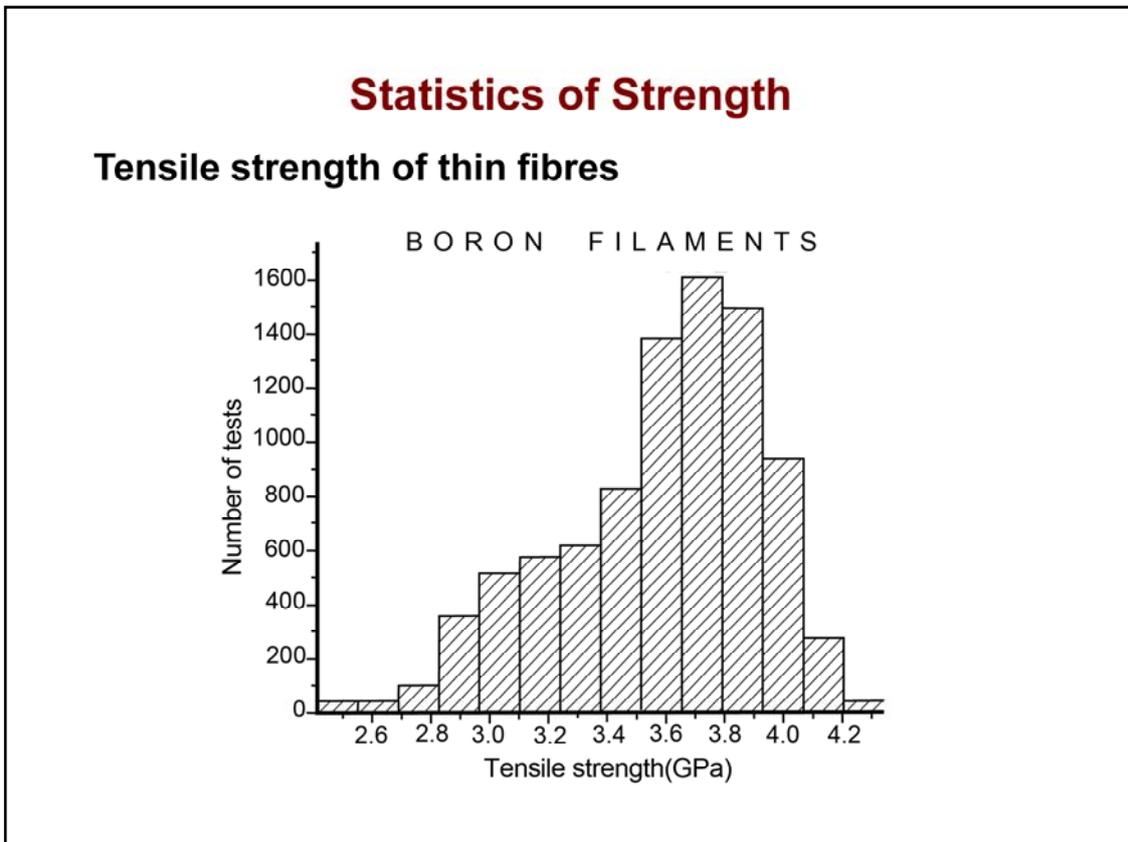
Ashby and Jones

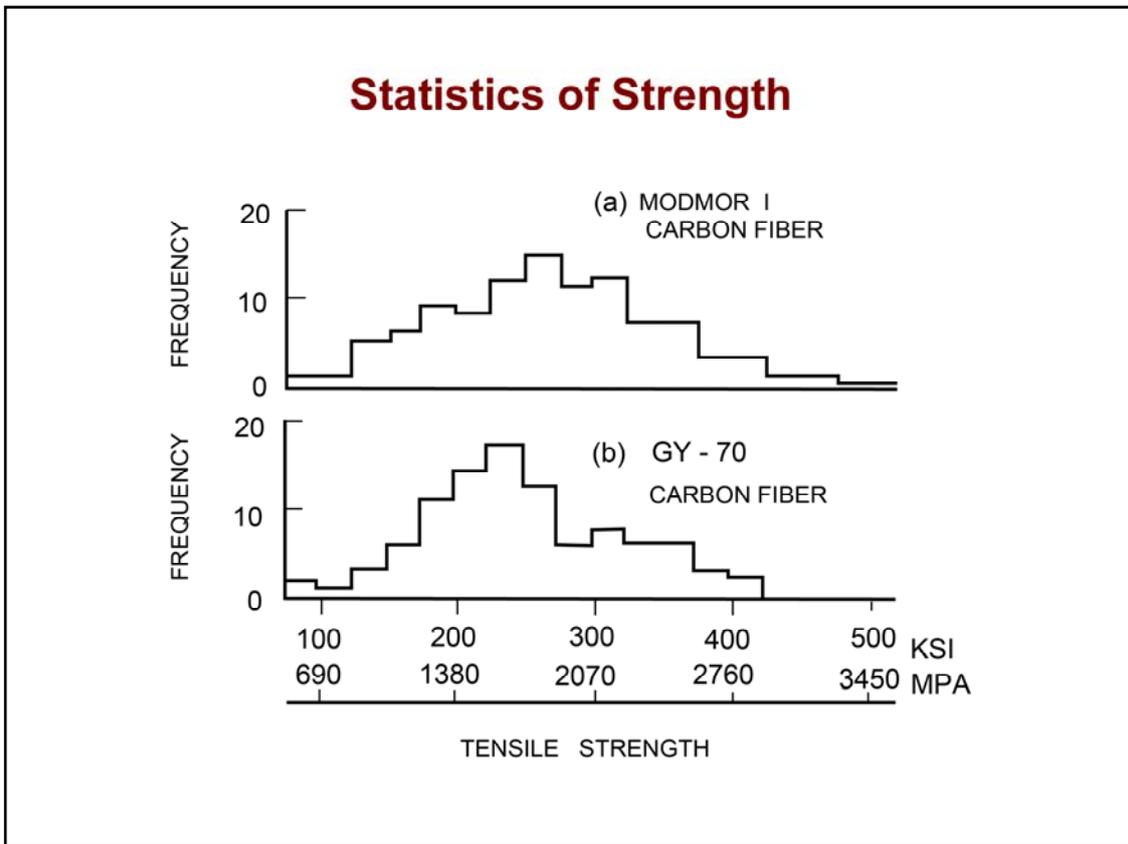
## Statistics of Strength

Since failure in a brittle solid is governed by the presence of defects and cracks, and these flaws can vary in size and orientation within the body and from specimen to another, there is a statistical variation in the strength.



Instead of defining a single “tensile strength”, it is more appropriate to consider a probability that the specimen or body has a certain tensile strength.

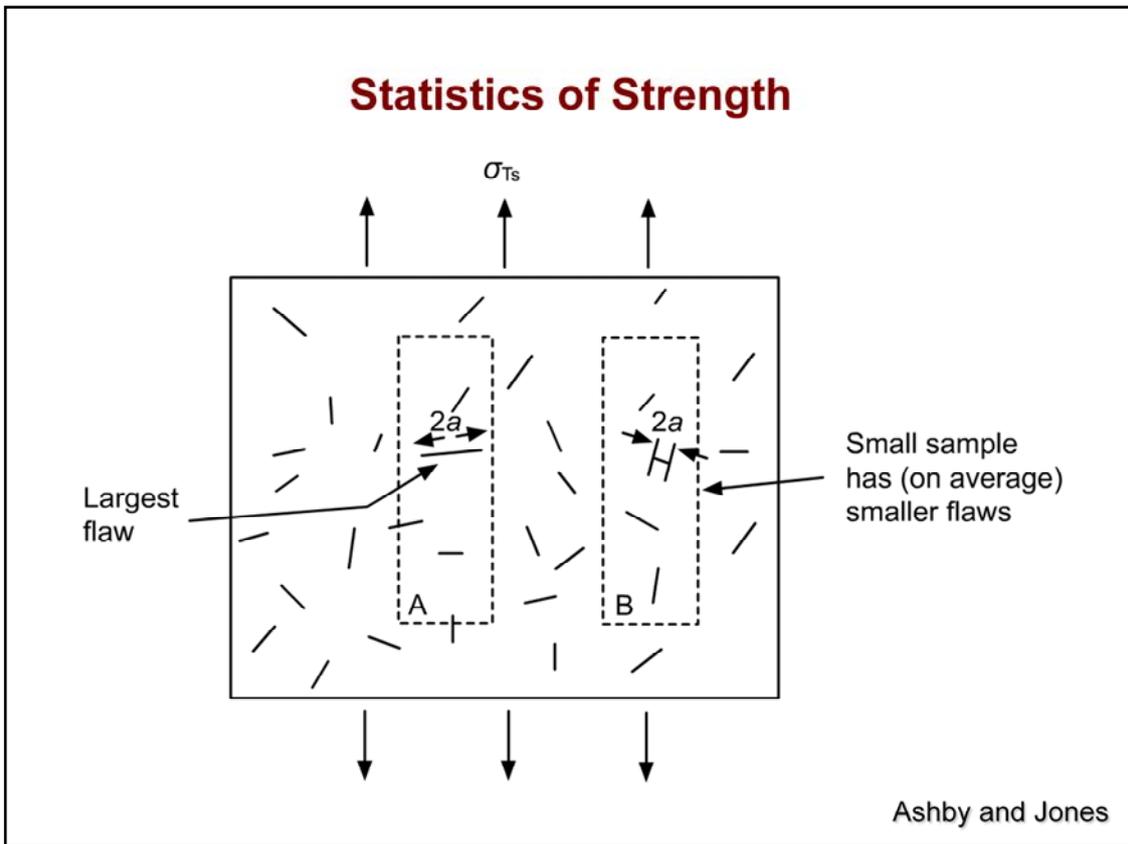




## **Statistics of Strength**

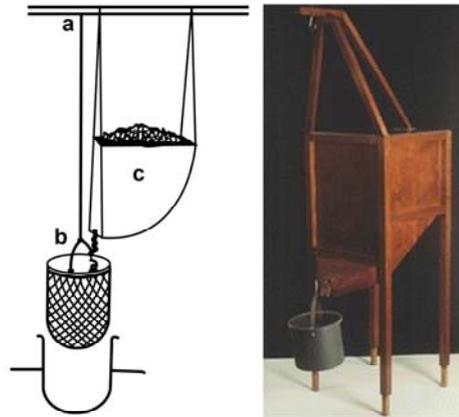
### **Volume or Size Dependence of Strength**

- The distribution of flaw or crack lengths is related to the volume of material being considered.
- A larger sample of the material is more likely to contain a larger flaw than a smaller sample.
- Therefore, the (tensile) strength of a larger or longer specimen is generally lower than that of smaller or shorter specimen, when the failure is brittle.



## Statistics of Strength

### Tensile Tests of Wires by Leonardo da Vinci (c. 1500)



Experimental setup consisting of hanging a basket by the wire, and slowly adding sand to the basket until the wire broke. The sand was weighed and the failure stress was calculated.

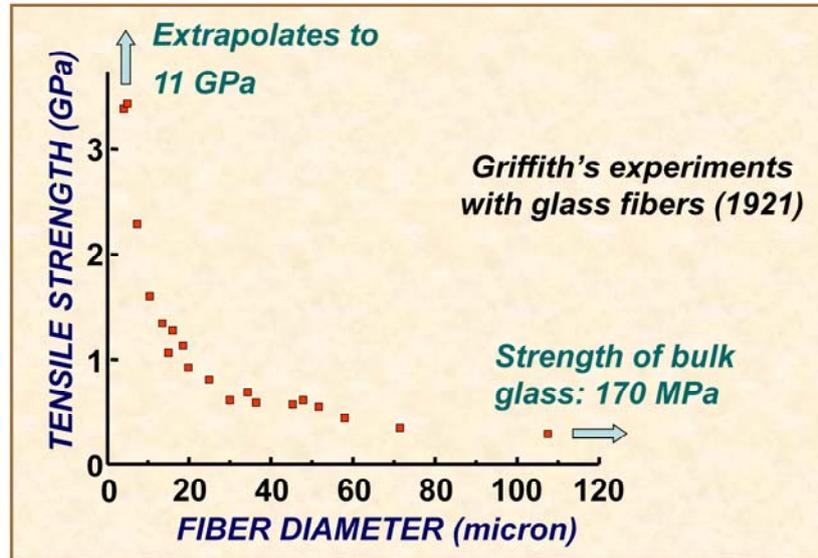
### **Statistics of Strength**

- Leonardo da Vinci found that longer wires were weaker than shorter wires.
- This observation conflicted with classical material mechanics theory where the length is irrelevant. Consequently, it was attributed to an error in da Vinci's notes and ignored !
- da Vinci's observation is, however, in accordance with the weakest link theory, where a chain is said to have the strength of its weakest link.
- Accordingly, a wire would have the strength of its weakest section. A longer wire has a higher likelihood (probability) of having a weaker section (i.e., with a longer flaw) than a shorter one.

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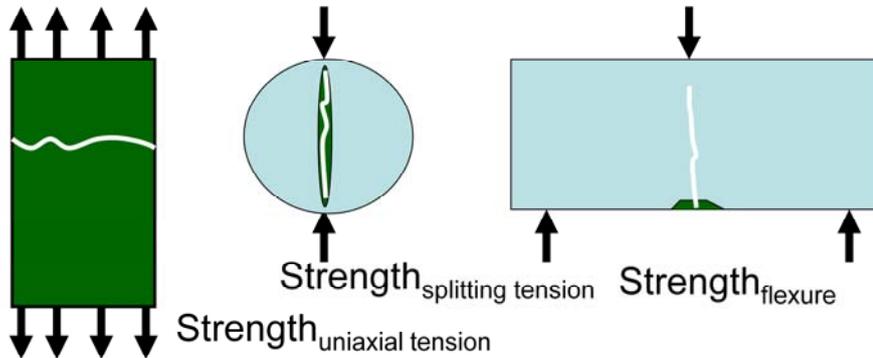
## Statistics of Strength

Strength Decreases with Fibre Diameter in Tensile Tests of Glass



## Statistics of Strength

Tensile Strength Depends on the Critically Stressed Volume



**$\text{Strength}_{\text{uniaxial tension}} < \text{Strength}_{\text{splitting tension}} < \text{Strength}_{\text{flexure}}$**   
(for concrete,  $f_{\text{spl. tens.}} = 1.10-1.15 \times f_{\text{un. tens.}}$  &  
 $f_{\text{flex.}} = 1.5-2.0 \times f_{\text{un. tens.}}$ )

### **Statistics of Strength**

- In all three loading cases, the failure occurs due to tensile cracking, and the maximum stress (i.e., tensile strength) occurs just after crack initiation.
- However, the volume of the material taking the peak stress is different in each case; i.e., the volume of material where the crack can initiate differs. Only the flaw size within this volume matters.
- The strength is higher when this volume is smaller, independent of the total volume of the specimen.

## Statistics of Strength

### Weibull Model

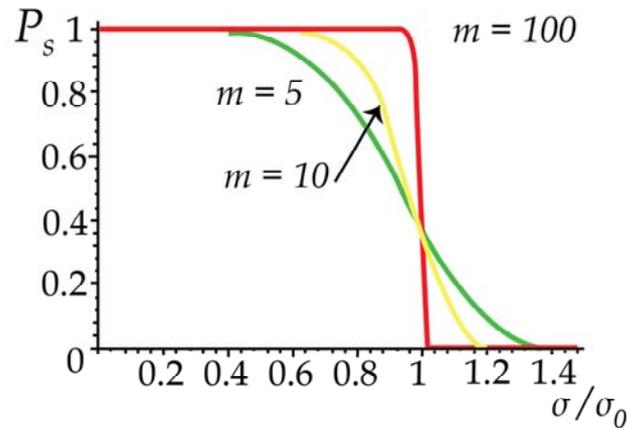
In this model, the survival probability of a sample of volume  $V_0$  subjected to a tensile stress  $\sigma$  is given as:

$$P_s(V_0) = \exp\left\{-\left(\frac{\sigma}{\sigma_0}\right)^m\right\}$$

where  $\sigma_0$  and  $m$  are parameters that depend on the material.

Lower the value of  $m$ , greater is the variability of the strength. It is called the Weibull modulus.

### Statistics of Strength



For brick and concrete,  $m$  is about 5; for engineered ceramics, it is about 10; and for steel, it is about 100.

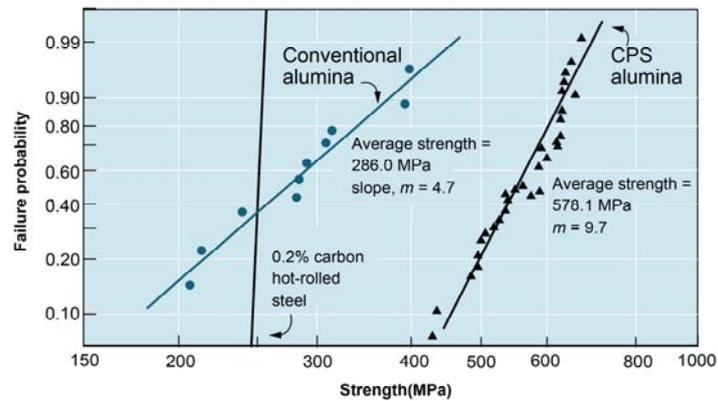
A material with a Weibull modulus,  $m$ , of about 100 can be treated as having a single well-defined tensile strength.

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## Statistics of Strength

### Weibull Model

The parameters  $\sigma_0$  and  $m$  can be determined from tests.



Log-log plots of the failure probability as a function of strength for 0.2% plain carbon steel, a conventional alumina ceramic and specially processed alumina with a controlled particle size.

## Statistics of Strength

### Weibull Model: Dependence on Stress and Volume

If  $V_0$  is the volume of the tested samples of a material with parameters  $\sigma_0$  and  $m$ , the survival probability of a sample of any other volume  $V$  is given by:

$$\ln P_s(V) = -\frac{V}{V_0} \left( \frac{\sigma}{\sigma_0} \right)^m$$

This is the final design equation that gives the dependence of the survival probability on both applied stress and critically stressed volume.

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