Module 7

Limit State of Serviceability

Lesson 17

Limit State of Serviceability

Instruction Objectives:

At the end of this lesson, the student should be able to:

- explain the need to check for the limit state of serviceability after designing the structures by limit state of collapse,
- differentiate between short- and long-term deflections,
- state the influencing factors to both short- and long-term deflections,
- select the preliminary dimensions of structures to satisfy the requirements as per IS 456,
- calculate the short- and long-term deflections of designed beams.

7.17.1 Introduction

Structures designed by limit state of collapse are of comparatively smaller sections than those designed employing working stress method. They, therefore, must be checked for deflection and width of cracks. Excessive deflection of a structure or part thereof adversely affects the appearance and efficiency of the structure, finishes or partitions. Excessive cracking of concrete also seriously affects the appearance and durability of the structure. Accordingly, cl. 35.1.1 of IS 456 stipulates that the designer should consider all relevant limit states to ensure an adequate degree of safety and serviceability. Clause 35.3 of IS 456 refers to the limit state of serviceability comprising deflection in cl. 35.3.1 and cracking in cl. 35.3.2. Concrete is said to be durable when it performs satisfactorily in the working environment during its anticipated exposure conditions during service. Clause 8 of IS 456 refers to the durability aspects of concrete. Stability of the structure against overturning and sliding (cl. 20 of IS 456), and fire resistance (cl. 21 of IS 456) are some of the other importance issues to be kept in mind while designing reinforced concrete structures.

This lesson discusses about the different aspects of deflection of beams and the requirements as per IS 456. In addition, lateral stability of beams is also taken up while selecting the preliminary dimensions of beams. Other requirements, however, are beyond the scope of this lesson.

7.17.2 Short- and Long-term Deflections

As evident from the names, short-term deflection refers to the immediate deflection after casting and application of partial or full service loads, while the long-term deflection occurs over a long period of time largely due to shrinkage and creep of the materials. The following factors influence the short-term deflection of structures:

(a) magnitude and distribution of live loads,
(b) span and type of end supports,
(c) cross-sectional area of the members,
(d) amount of steel reinforcement and the stress developed in the reinforcement,
(e) characteristic strengths of concrete and steel, and
(f) amount and extent of cracking.

The long-term deflection is almost two to three times of the short-term deflection. The following are the major factors influencing the long-term deflection of the structures.

(a) humidity and temperature ranges during curing,
(b) age of concrete at the time of loading, and
(c) type and size of aggregates, water-cement ratio, amount of compression reinforcement, size of members etc., which influence the creep and shrinkage of concrete.

7.17.3 Control of Deflection

Clause 23.2 of IS 456 stipulates the limiting deflections under two heads as given below:

(a) The maximum final deflection should not normally exceed span/250 due to all loads including the effects of temperatures, creep and shrinkage and measured from the as-cast level of the supports of floors, roof and all other horizontal members.

(b) The maximum deflection should not normally exceed the lesser of span/350 or 20 mm including the effects of temperature, creep and shrinkage occurring after erection of partitions and the application of finishes.

It is essential that both the requirements are to be fulfilled for every structure.

7.17.4 Selection of Preliminary Dimensions

The two requirements of the deflection are checked after designing the members. However, the structural design has to be revised if it fails to satisfy any one of the two or both the requirements. In order to avoid this, IS 456 recommends the guidelines to assume the initial dimensions of the members which will generally satisfy the deflection limits. Clause 23.2.1 stipulates different span to effective depth ratios and cl. 23.3 recommends limiting slenderness of beams, a relation of b and d of the members, to ensure lateral stability. They are given below:

(A) For the deflection requirements

Different basic values of span to effective depth ratios for three different support conditions are prescribed for spans up to 10 m, which should be modified under any or all of the four different situations: (i) for spans above 10 m, (ii) depending on the amount and the stress of tension steel reinforcement, (iii) depending on the amount of compression reinforcement, and (iv) for flanged beams. These are furnished in Table 7.1.

(B) For lateral stability

The lateral stability of beams depends upon the slenderness ratio and the support conditions. Accordingly cl. 23.3 of IS code stipulates the following:

(i) For simply supported and continuous beams, the clear distance between the lateral restraints shall not exceed the lesser of $60b$ or $250b^2/d$, where d is the effective depth and b is the breadth of the compression face midway between the lateral restraints.

(ii) For cantilever beams, the clear distance from the free end of the cantilever to the lateral restraint shall not exceed the lesser of $25b$ or $100b^2/d$.

Table 7.1 Span/depth ratios and modification factors

Sl. No.	Items	Cantilever	Simply supported	Continuous
1	Basic values of span to effective depth ratio for spans up to 10 m	7	20	26
2	Modification factors for spans > 10 m	Not applicable as deflection calculations are to be done.	Multiply values of row 1 by 10/span in metres.	
3	Modification factors depending on area and stress of steel	Multiply values of row 1 or 2 with the modification factor from Fig.4 of IS 456.		
4	Modification factors depending as area of compression steel	Further multiply the earlier respective value with that obtained from Fig.5 of IS 456.		
5	Modification factors for flanged beams	(i) Modify values of row 1 or 2 as per Fig.6 of IS 456. (ii) Further modify as per row 3 and/or 4 where		

		reinforcement percentage to be used on area of section equal to $b_f d$.

7.17.5 Calculation of Short-Term Deflection

Clause C-2 of Annex C of IS 456 prescribes the steps of calculating the short-term deflection. The code recommends the usual methods for elastic deflections using the short-term modulus of elasticity of concrete E_c and effective moment of inertia I_{eff} given by the following equation:

$$I_{eff} = \frac{I_r}{1.2 - (M_r / M)(z/d)(1 - x/d)(b_w/b)} \text{; but } I_r \le I_{eff} \le I_{gr}$$

(7.1)

where I_r = moment of inertia of the cracked section,

M_r = cracking moment equal to $(f_{cr} I_{gr})/y_t$, where f_{cr} is the modulus of rupture of concrete, I_{gr} is the moment of inertia of the gross section about the centroidal axis neglecting the reinforcement, and y_t is the distance from centroidal axis of gross section, neglecting the reinforcement, to extreme fibre in tension,

M = maximum moment under service loads,

z = lever arm,

x = depth of neutral axis,

d = effective depth,

b_w = breadth of web, and

b = breadth of compression face.

For continuous beams, however, the values of I_r, I_{gr} and M_r are to be modified by the following equation:

$$X_e = k_1 \left[\frac{X_1 + X_2}{2} \right] + (1 - k_1) X_o$$

(7.2)

where X_e = modified value of X,

X_1, X_2 = values of X at the supports,

X_o = value of X at mid span,

k_1 = coefficient given in Table 25 of IS 456 and in Table 7.2 here, and

X = value of I_r, I_{gr} or M_r as appropriate.

Table 7.2 Values of coefficient k_1

k_1	0.5 or less	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
k_2	0	0.03	0.08	0.16	0.30	0.50	0.73	0.91	0.97	1.0

Note: k_2 is given by $(M_1 + M_2)/(M_{F1} + M_{F2})$, where M_1 and M_2 = support moments, and M_{F1} and M_{F2} = fixed end moments.

7.17.6 Deflection due to Shrinkage

Clause C-3 of Annex C of IS 456 prescribes the method of calculating the deflection due to shrinkage α_{cs} from the following equation:

$$\alpha_{cs} = k_3 \psi_{cs} l^2 \tag{7.3}$$

where k_3 is a constant which is 0.5 for cantilevers, 0.125 for simply supported members, 0.086 for members continuous at one end, and 0.063 for fully continuous members; ψ_{cs} is shrinkage curvature equal to $k_4 \varepsilon_{cs}/D$ where ε_{cs} is the ultimate shrinkage strain of concrete. For ε_{cs}, cl. 6.2.4.1 of IS 456 recommends an approximate value of 0.0003 in the absence of test data.

$$\begin{aligned} k_4 &= 0.72(p_t - p_c)/\sqrt{p_t} \le 1.0, \text{ for } 0.25 \le p_t - p_c < 1.0 \\ &= 0.65(p_t - p_c)/\sqrt{p_t} \le 1.0, \text{ for } p_t - p_c \ge 1.0 \end{aligned} \tag{7.4}$$

where p_t = $100A_{st}/bd$ and p_c = $100A_{sc}/bd$, D is the total depth of the section, and l is the length of span.

7.17.7 Deflection Due to Creep

Clause C-4 of Annex C of IS 456 stipulates the following method of calculating deflection due to creep. The creep deflection due to permanent loads $\alpha_{cc(perm)}$ is obtained from the following equation:

$$\alpha_{cc(perm)} = \alpha_{1cc(perm)} - \alpha_{1(\mathrm{perm})} \tag{7.5}$$

where $\alpha_{1cc(perm)}$ = initial plus creep deflection due to permanent loads obtained using an elastic analysis with an effective modulus of elasticity,

$E_{ce} = E_c/(1+\theta)$, θ being the creep coefficient, and

$\alpha_{1(perm)}$ = short-term deflection due to permanent loads using E_c.

7.17.8 Numerical Problems

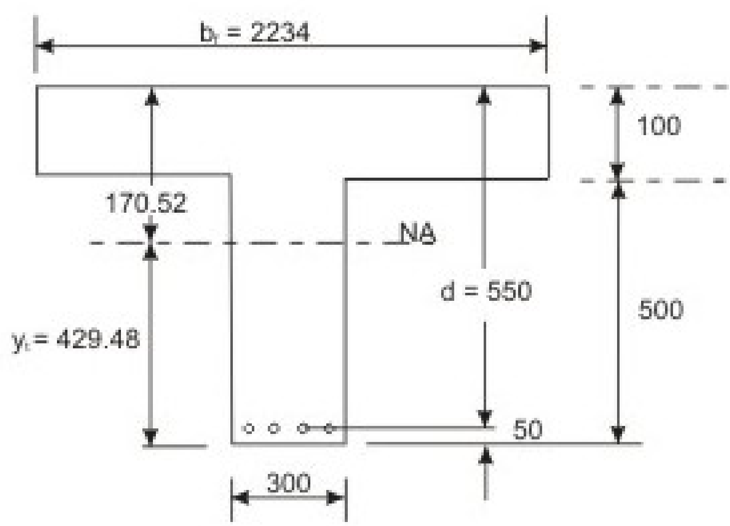

Fig. 7.17.1: Problem 1 (uncracked section)

Problem 1:

Figures 7.17.1 and 2 present the cross-section and the tensile steel of a simply supported *T*-beam of 8 m span using M 20 and Fe 415 subjected to dead load of 9.3 kN/m and imposed loads of 10.7 kN/m at service. Calculate the short- and long-term deflections and check the requirements of IS 456.

Solution 1:

Step 1: Properties of plain concrete section

Taking moment of the area about the bottom of the beam

$$y_t = \frac{(300)(600)(300) + (2234 - 300)(100)(550)}{(300)(600) + (2234 - 300)(100)} = 429.48 \text{ mm}$$

$$I_{gr} = \frac{300(429.48)^3}{3} + \frac{2234(170.52)^3}{3} - \frac{1934(70.52)^3}{3} = (11.384)\,(10)^9 \text{ mm}^4$$

This can also be computed from SP-16 as explained below:

Here, b_f/b_w = 7.45, D_f/D = 0.17. Using these values in chart 88 of SP-16, we get k_1 = 2.10.

$I_{gr} = k_1 b_w D^3/12 = (2.10)(300)(600)^3/12 = (11.384)(10)^9$ mm^4

Step 2: Properties of the cracked section (Fig.7.17.2)

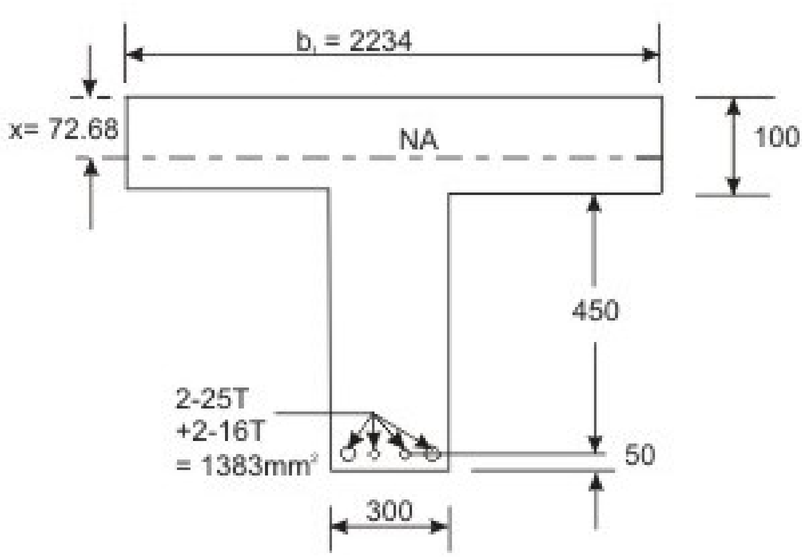

Fig. 7.17.2: Problem 1 (cracked section, $E_c = E_c$)

$f_{cr} = 0.7\sqrt{f_{ck}}$ (cl. 6.2.2 of IS 456) = 3.13 N/mm^2

$M_r = f_{cr} I_{gr} / y_t = 3.13(11.384)(10)^9/429.48 = 82.96$ kNm

$E_s = 200000$ N/mm^2

$E_c = 5000 \sqrt{f_{ck}}$ (cl. 6.2.3.1 of IS 456) = 22360.68 N/mm^2

$m = E_s/E_c = 8.94$

Taking moment of the compressive concrete and tensile steel about the neutral axis, we have (Fig.7.17.2)

$b_f x^2/2 = m A_{st} (d - x)$ gives $(2234)(x^2/2) = (8.94)(1383)(550 - x)$

or $x^2 + 11.07x - 6087.92 = 0$. Solving the equation, we get $x = 72.68$ mm.

z = lever arm = $d - x/3 = 525.77$ mm

$$I_r = \frac{2234(72.68)^3}{3} + 8.94(1383)(550 - 72.68)^2 = 3.106(10)^9 \text{ mm}^4$$

$M = wl^2/8 = (9.3 + 10.7)(8)(8)/8 = 160$ kNm

$$I_{eff} = \frac{I_r}{1.2 - \frac{M_r}{M}\frac{z}{d}(1 - \frac{x}{d})(\frac{b_w}{b})} \quad \text{.... (Eq. 7.1)}$$

$$= \frac{I_r}{1.2 - (\frac{82.96}{160})(\frac{525.77}{550})(1 - \frac{72.68}{550})(\frac{300}{2234})} = 0.875 I_r. \text{ But } I_r \le I_{eff} \le I_{gr}$$

So, $I_{eff} = I_r = 3.106(10)^9$ mm^4.

Step 3: Short-term deflection (sec. 7.17.5)

$E_c = 5000\sqrt{f_{ck}}$ (cl. 6.2.3.1 of IS 456) = 22360.68 N/mm^2

Short-term deflection = $(5/384)\, wl^4/E_c I_{eff}$

= $(5)(20)(8)^4(10^{12})/(384)(22360.68)(3.106)(10^9)$ = 15.358 mm

(1)

Step 4: Deflection due to shrinkage (sec. 7.17.6)

$$k_4 = 0.72(p_t - p_c)/\sqrt{p_t} = 0.72(0.84)\sqrt{0.84} = 0.6599$$

$$\psi_{cs} = k_4\, \varepsilon_{cs} / D = (0.6599)(0.0003)/600 = 3.2995(10)^{-7}$$

k_3 = 0.125 (from sec. 7.17.6)

$\alpha_{cs} = k_3\, \psi_{cs}\, l^2$ (Eq. 7.3) = $(0.125)(3.2995)(10)^{-7}(64)(10^6)$ = 2.64 mm
(2)

Step 5: Deflection due to creep (sec. 7.17.7)

Equation 7.5 reveals that the deflection due to creep $\alpha_{cc(perm)}$ can be obtained after calculating $\alpha_{1cc(perm)}$ and $\alpha_{1(perm)}$. We calculate $\alpha_{1cc(perm)}$ in the next step.

Step 5a: Calculation of $\alpha_{1cc(perm)}$

Assuming the age of concrete at loading as 28 days, cl. 6.2.5.1 of IS 456 gives θ = 1.6. So, E_{cc} = E_c /(1 + θ) = 22360.68/(1 + 1.6) = 8600.2615 N/mm^2 and m = E_s /E_{cc} = 200000/8600.2615 = 23.255

Step 5b: Properties of cracked section

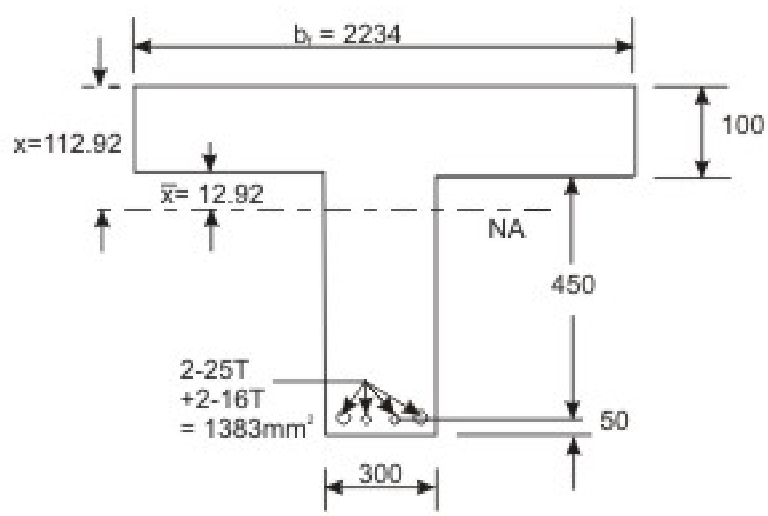

Fig. 7.17.3: Problem 1 (cracked section, E_c=E_{cc})

Taking moment of compressive concrete and tensile steel about the neutral axis (assuming at a distance of $\bar{x}$ from the bottom of the flange as shown in Fig.7.17.3):

$$2234(100)(50 + \bar{x}) = (23.255)(1383)(450 - \bar{x}) \text{ or } \quad \bar{x} = 12.92 \text{ mm}$$

which gives x = 112.92 mm. Accordingly, z = lever arm = $d - x/3$ = 512.36 mm.

$$I_r = 2234(100)^3/12 + 2234(100)(62.92)^2 + 23.255(1383)(550 - 112.92)^2$$

$$+ 300(12.92)^3/3 \qquad = 7.214(10^9) \text{ mm}^4$$

M_r = 82.96 kNm (see Step 2)

$M = w_{perm}\, l^2/8 = 9.3(8)(8)/8 = 74.4$ kNm.

$$I_{eff} = \frac{I_r}{(1.2) - (\frac{82.96}{74.4})(\frac{512.36}{550})(1 - \frac{112.92}{550})(\frac{300}{2234})} = 0.918\ I_r$$

However, to satisfy $I_r \le I_{eff} \le I_{gr}$, I_{eff} should be equal to I_{gr}. So, $I_{eff} = I_{gr} = 11.384(10^9)$. For the value of I_{gr} please see Step 1.

Step 5c: Calculation of $\alpha_{1cc(perm)}$

$\alpha_{1cc(perm)}$ = $5wl^4/384(E_{cc})(I_{eff})$ = $5(9.3)(8)^4(10)^{12}/384(8600.2615)(11.384)(10^9)$

= 5.066 mm (3)

Step 5d: Calculation of $\alpha_{1(perm)}$

$\alpha_{1(perm)}$ = $5wl^4/384(E_c)(I_{eff})$ = $5(9.3)(8)^4(10)^{12}/384(22360.68)(11.384)(10^9)$

= 1.948 mm (4)

Step 5e: Calculation of deflection due to creep

$$\alpha_{cc(perm)} = \alpha_{1cc(perm)} - \alpha_{1(perm)}$$

$= 5.066 - 1.948 = 3.118$ mm

(5)

It is important to note that the deflection due to creep $\alpha_{cc(perm)}$ can be obtained even without computing $\alpha_{1cc(perm)}$. The relationship of $\alpha_{cc(perm)}$ and $\alpha_{1(perm)}$ is given below.

$$\alpha_{cc(perm)} = \alpha_{1cc(perm)} - \alpha_{1(perm)}$$

$$= \{5wl^4/384(E_c)(I_{eff})\}\{(E_c/E_{cc}) - 1\} = \alpha_{1(perm)}(\theta)$$

Hence, the deflection due to creep, for this problem is:

$$\alpha_{cc(perm)} = \alpha_{1(perm)}(\theta) = 1.948(1.6) = 3.116 \text{ mm}$$

Step 6: Checking of the requirements of IS 456

The two requirements regarding the control of deflection are given in sec. 7.17.3. They are checked in the following:

Step 6a: Checking of the first requirement

The maximum allowable deflection = 8000/250 = 32 mm

The actual final deflection due to all loads

= 15.358 (see Eq.1 of Step 3) + 2.64 (see Eq.2 of Step 4)

+ 3.118 (see Eq.5 of Step 5e) = 21.116 mm < 32 mm. Hence, o.k.

Step 6b: Checking of the second requirement

The maximum allowable deflection is the lesser of span/350 or 20 mm. Here, span/350 = 22.86 mm. So, the maximum allowable deflection = 20 mm. The actual final deflection = 1.948 (see Eq.4 of Step 5d) + 2.64 (see Eq.2 of Step 4) + 3.118 (see Eq.5 of step 5e) = 7.706 mm < 20 mm. Hence, o.k.

Thus, both the requirements of cl.23.2 of IS 456 and as given in sec. 7.17.3 are satisfied.

7.17.9 Practice Questions and Problems with Answers

Q.1: Why is it essential to check the structures, designed by the limit state of collapse, by the limit state of serviceability?

A.1: See sec. 7.17.1.

Q.2: Explain short- and long-term deflections and the respective influencing factors of them.

A.2: See sec. 7.17.2.

Q.3: State the stipulations of IS 456 regarding the control of deflection.

A.3: See sec. 7.17.3.

Q.4: How would you select the preliminary dimensions of structures to satisfy (i) the deflection requirements, and (ii) the lateral stability ?

A.4: See secs. 7.17.4 A for (i) and B for (ii).

Q.5: Check the preliminary cross-sectional dimensions of Problem 1 of sec. 7.17.8 (Fig.7.17.1) if they satisfy the requirements of control of deflection. The spacing of the beam is 3.5 m c/c. Other data are the same as those of Problem 1 of sec. 7.17.8.

A.5:

Step 1: Check for the effective width

$b_f = l_o/6 + b_w + 6D_f$ or spacing of the beam, whichever is less.

Here, $b_f = (8000/6) + 300 + 6(100) = 2234 < 3500$. Hence, $b_f = 2234$ mm is o.k.

Step 2: Check for span to effective depth ratio

(i) As per row 1 of Table 7.1, the basic value of span to effective depth ratio is 20.

(ii) As per row 2 of Table 7.1, the modification factor is 1 since the span 8 m < 10 m.

(iii) As per row 5 of Table 7.1, the modification factor for the flanged beam is to be obtained from Fig. 6 of IS 456 for which the ratio of web width to flange width

= 300/2234 = 0.134. Figure 6 of IS 456 gives the modification factor as 0.8. So, the revised span to effective depth ratio = 20(0.8) = 16.

(iv) Row 3 of Table 7.1 deals with the area and stress of tensile steel. At the preliminary stage these values are to be assumed. However, for this problem the area of steel is given as 1383 mm^2 (2-25T + 2-16T), for which $p_t = A_{st}(100)/b_f d$ = 1383(100)/(2234)(550) = 0.112.

f_s = 0.58 f_y (area of cross-section of steel required)/(area of cross-section of steel provided) = 0.58(415)(1) = 240.7 (assuming that the provided steel is the same as required, which is a rare case). Figure 4 of IS 456 gives the modification factor as 1.8. So, the revised span to effective depth ratio = 16(1.8) = 28.8.

(v) Row 4 is concerning the amount of compression steel. Here, compression steel is not there. So, the modification factor = 1.

Therefore, the final span to effective depth ratio = 28.8.

Accordingly, effective depth of the beam = 8000/28.8 = 277.8 mm < 550 mm.

Hence, the dimensions of the cross-section are satisfying the requirements.

7.17.9 References

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7.17.11 Test 17 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1: Explain short- and long-term deflections and the respective influencing factors of them.
(10 marks)

A.TQ.1: See sec. 7.17.2.

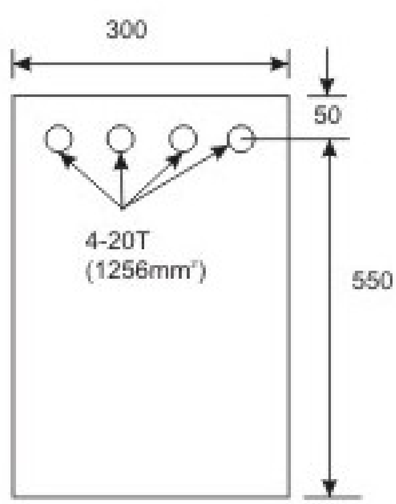

Fig. 7.17.4: TQ. 2 and TQ. 3

TQ.2: Check the preliminary dimensions of a singly reinforced rectangular cantilever beam of span 4 m (Fig.7.17.4) using M 20 and Fe 415. (15 marks)

A.TQ.2: (i) From row 1 of Table 7.1, the basic value of span to effective depth ratio is 7.

(ii) Modification factor for row 2 is 1 as this is a singly reinforced beam.

(iii) Assuming p_t as 0.6 and area of steel to be provided is the same as area of steel required, f_s = 0.58(415(1) = 240.7 N/mm^2. From Fig. 4 of IS 456, the modification factor = 1.18. Hence, the revised span to effective depth ratio is 7(1.18) = 8.26.

(iv) Modification factors for rows 4 and 5 are 1 as there is no compression steel and this being a rectangular beam. Hence, the preliminary effective depth needed = 4000/8.26 = 484.26 mm < 550 mm. Hence, o.k.

TQ.3: Determine the tensile steel of the cantilever beam of TQ 2 (Fig. 7.17.4) subjected to service imposed load of 11.5 kN/m using M 20 and Fe 415. Use Sp-16 for the design. Calculate short- and long-term deflections and check the requirements of IS 456 regarding the deflection. (25 marks)

A.TQ.3: Determination of tensile steel of the beam using SP-16:

Dead load of the beam = 0.3(0.6)(25) kN/m = 4.5 kN/m

Service imposed loads = 11.5 kN/m

Total service load = 16.0 kN/m

Factored load = 16(1.5) = 24 kN/m

M_u = 24(4)(4)/2 = 192 kNm

For this beam of total depth 600 mm, let us assume d = 550 mm.

M_u /bd^2 = 192/(0.3)(0.55)(0.55) = 2115.70 kN/m^2

Table 2 of SP-16 gives the corresponding p_t = 0.678 + 0.007(0.015)/0.02 = 0.683

Again, for M_u per metre run as 192/0.3 = 640 kNm/m, chart 15 of SP-16 gives p_t = 0.68 when d = 550 mm.

With p_t = 0.683, A_{st} = 0.683(300)(500)/100 = 1126.95 mm^2. Provide 4-20T to have 1256 mm^2. This gives provided p_t = 0.761%.

Calculation of deflection

Step 1: Properties of concrete section

$y_t = D/2 = 300$ mm, $I_{gr} = bD^3/12 = 300(600)^3/12 = 5.4(10^9)$ mm^4

Step 2: Properties of cracked section

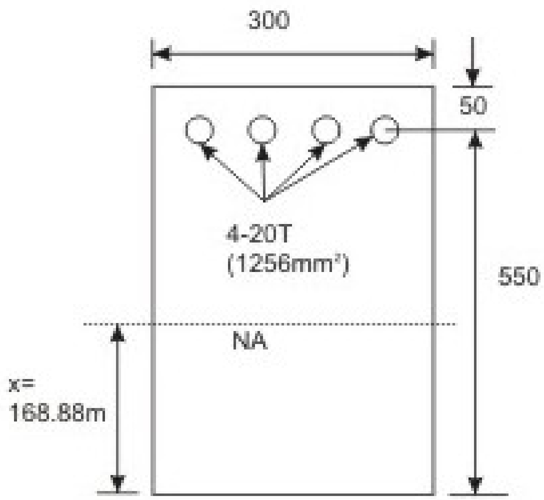

Fig. 7.17.5: TQ. 3 (cracked section, E_c=E_c)

$f_{cr} = 0.7\sqrt{20}$ (cl. 6.2.2 of IS 456) = 3.13 N/mm^2

y_t = 300 mm

$M_r = f_{cr}\, I_{gr} / y_t = 3.13(5.4)(10^9)/300 = 5.634(10^7)$ Nmm

E_s = 200000 N/mm^2

$E_c = 5000\sqrt{f_{ck}}$ (cl. 6.2.3.1 of IS 456) = 22360.68 N/mm^2

$m = E_s / E_c$ = 8.94

Taking moment of the compressive concrete and tensile steel about the neutral axis (Fig.7.17.5):

$300\, x^2/2 = (8.94)(1256)(550 - x)$ or $x^2 + 74.86\, x - 41171.68 = 0$

This gives x = 168.88 mm and $z = d - x/3$ = 550 – 168.88/3 = 493.71 mm.

I_r = 300(168.88)^3/3 + 8.94(1256)(550 – 168.88)^2 = 2.1126(10^9) mm^4

$M = wl^2/2$ = 20(4)(4)/2 = 160 kNm

$$I_{eff} = \frac{I_r}{(1.2) - (\frac{5.634}{16})(\frac{493.71}{550})(1 - \frac{168.88}{550})(1)} = 1.02\ I_r = 2.1548\,(10^9)\,\text{mm}^4$$

This satisfies $I_r \leq I_{eff} \leq I_{gr}$. So, I_{eff} = 2.1548(10^9) mm^4.

Step 3: Short-term deflection (sec. 7.17.5)

E_c = 22360.68 N/mm^2 (cl. 6.2.3.1 of IS 456)

Short-term deflection = $wl^4/8E_cI_{eff}$

= 20(4^4)(10^{12})/8(22360.68)(2.1548)(10^9) = 13.283 mm

So, short-term deflection = 13.283 mm

(1)

Step 4: Deflection due to shrinkage (sec. 7.17.6)

$$k_4 = 0.72(0.761)/\sqrt{0.761} = 0.664$$

$$\psi_{cs} = k_4\,\varepsilon_{cs}/D = (0.664)(0.0003)/600 = 3.32(10)^{-7}$$

k_3 = 0.5 (from sec. 7.17.6)

$\alpha_{cs} = k_3\,\psi_{cs}\,l^2$ = (0.5)(3.32)(10)$^{-7}$(16)(10^6) = 2.656 mm

(2)

Step 5: Deflection due to creep (sec. 7.17.7)

Step 5a: Calculation of $\alpha_{1cc(perm)}$

Assuming the age of concrete at loading as 28 days, cl. 6.2.5.1 of IS 456 gives

$\theta = 1.6$

So, $\quad E_{cc} = E_c/(1 + \theta) = 8600.2615 \text{ N/mm}^2$

$m = E_s/E_{cc} = 200000/8600.2615 = 23.255$

Step 5b: Properties of cracked section

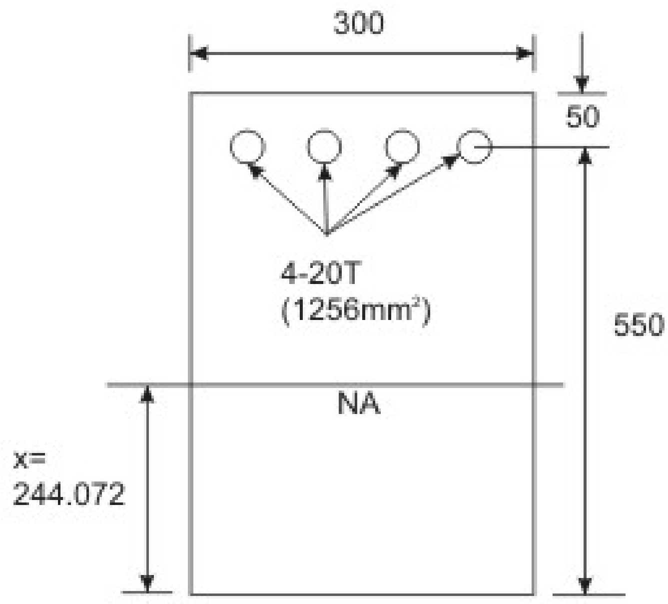

Fig. 7.17.6: TQ. 3 (cracked section, E_c=E_{cc})

From Fig.7.17.6, taking moment of compressive concrete and tensile steel about the neutral axis, we have:

$300\, x^2/2 = (23.255)(1256)(550 - x)$

or $\quad x^2 + 194.72\, x - 107097.03 = 0$

solving we get $x = 244.072$ mm

$z = d - x/3 = 468.643$ mm

$I_r = 300(244.072)^3/3 + (23.255)(1256)(550 - 468.643)^2$

$= 1.6473(10)^9 \text{ mm}^4$

M_r = 5.634(10^7) Nmm (see Step 2)

M = $w_{perm}\, l^2/2$ = 4.5(4^2)/2 = 36 kNm

$$I_{eff} = \frac{I_r}{1.2 - (\frac{5.634}{3.6})(\frac{468.643}{550})(1 - \frac{244.072}{550})(1)} = 2.1786\ I_r = 3.5888(10^9)\,\text{mm}^4$$

Since this satisfies $I_r \le I_{eff} \le I_{gr}$, we have, I_{eff} = 3.5888(10^9) mm^4. For the value of I_{gr} please see Step 1.

Step 5c: Calculation of $\alpha_{1cc(perm)}$

$\alpha_{1cc(perm)}$ = $(w_{perm})(l^4)/(8E_{cc}\, I_{eff})$ = $4.5(4)^4(10)^{12}/8(8600.2615)(3.5888)(10^9)$

= 4.665 mm

(3)

Step 5d: Calculation of $\alpha_{1(perm)}$

$\alpha_{1(perm)}$ = $(w_{perm})(l^4)/(8E_c\, I_{eff})$ = $4.5(4)^4(10)^{12}/8(22360.68)(3.5888)(10^9)$

= 1.794 mm

(4)

Step 5e: Calculation of deflection due to creep

$\alpha_{cc(perm)}$ = $\alpha_{1cc(perm)}$ - $\alpha_{1(perm)}$

= 4.665 – 1.794 = 2.871 mm

(5)

Moreover: $\alpha_{cc(perm)}$ = $\alpha_{1cc(perm)}$ (θ) gives $\alpha_{cc(perm)}$ = 1.794(1.6) = 2.874 mm.

Step 6: Checking of the two requirements of IS 456

Step 6a: First requirement

Maximum allowable deflection = 4000/250 = 16 mm

The actual deflection = 13.283 (Eq.1 of Step 3) + 2.656 (Eq.2 of Step 4)

+ 2.871 (Eq.5 of Step 5e) = 18.81 > Allowable 16 mm.

Step 6b: Second requirement

The allowable deflection is lesser of span/350 or 20 mm. Here, span/350 = 11.428 mm is the allowable deflection. The actual deflection = 1.794 (Eq.4 of Step 5d) + 2.656 (Eq.2 of Step 4) + 2.871 (Eq.5 of step 5e) = 7.321 mm < 11.428 mm.

Remarks:

Though the second requirement is satisfying, the first requirement is not satisfying. However the extra deflection is only 2.81 mm, which can be made up by giving camber instead of revising the section.

7.17.12 Summary of this Lesson

This lesson illustrates the importance of checking the structures for the limit state of serviceability after designing by the limit state of collapse. The short- and long-term deflections along with their respective influencing factors are explained. The code requirements for the control of deflection and the necessary guidelines for the selection of dimensions of cross-section are stated. Numerical examples, solved as illustrative example and given in the practice problem and test will help the students in understanding the calculations clearly for their application in the design problems.