

# Module 5

## Flanged Beams – Theory and Numerical Problems

# Lesson 10

## Flanged Beams – Theory

## Instructional Objectives:

At the end of this lesson, the student should be able to:

- identify the regions where the beam shall be designed as a flanged and where it will be rectangular in normal slab beam construction,
- define the effective and actual widths of flanged beams,
- state the requirements so that the slab part is effectively coupled with the flanged beam,
- write the expressions of effective widths of  $T$  and  $L$ -beams both for continuous and isolated cases,
- derive the expressions of  $C$ ,  $T$  and  $M_u$  for four different cases depending on the location of the neutral axis and depth of the flange.

### 5.10.1 Introduction

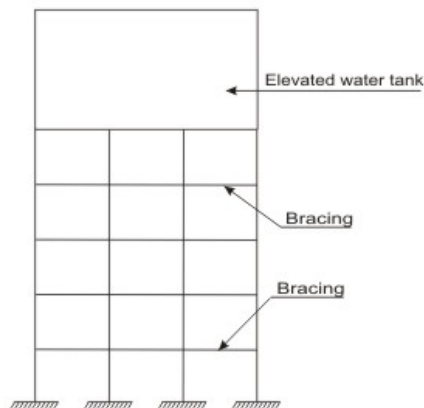


Fig. 5.10.1(a): Bracings of elevated water tank

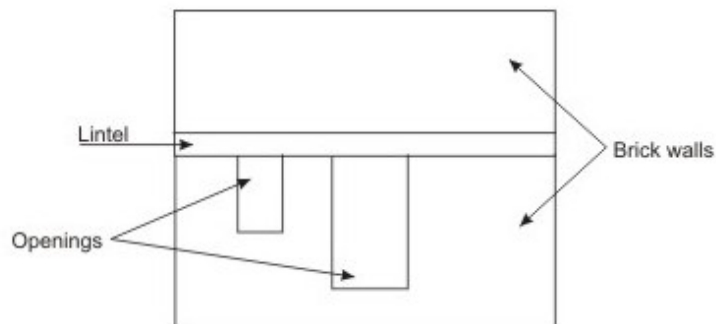
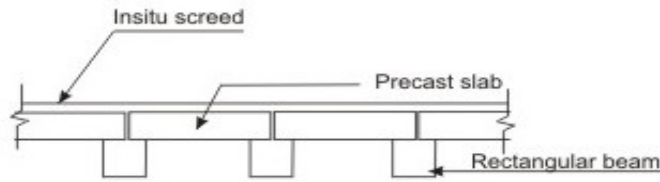
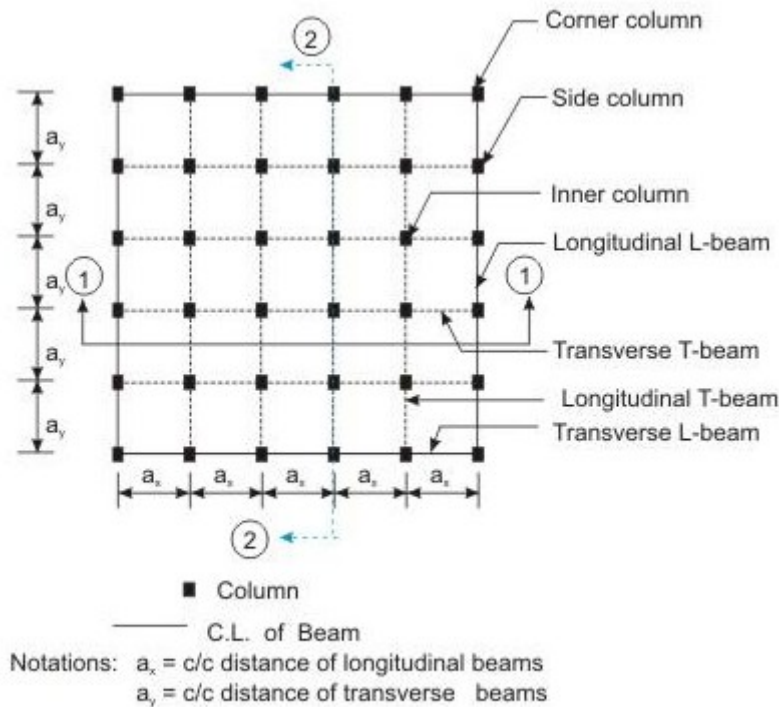


Fig. 5.10.1 (b) Lintels over openings (without effective chajjas )

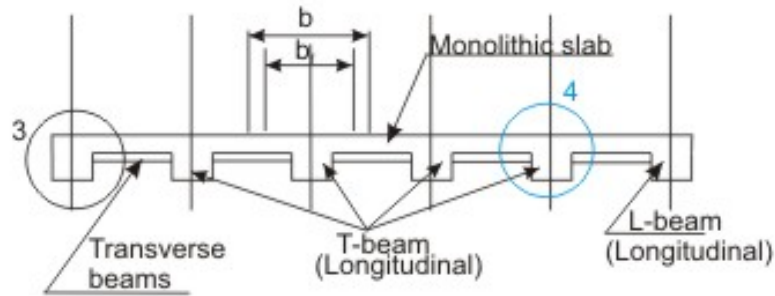


**Fig. 5.10.1(c) Precast slab on rectangular beams**

Reinforced concrete slabs used in floors, roofs and decks are mostly cast monolithic from the bottom of the beam to the top of the slab. Such rectangular beams having slab on top are different from others having either no slab (bracings of elevated tanks, lintels etc.) or having disconnected slabs as in some pre-cast systems (Figs. 5.10.1 a, b and c). Due to monolithic casting, beams and a part of the slab act together. Under the action of positive bending moment, i.e., between the supports of a continuous beam, the slab, up to a certain width greater than the width of the beam, forms the top part of the beam. Such beams having slab on top of the rectangular rib are designated as the flanged beams - either *T* or *L* type depending on whether the slab is on both sides or on one side of the beam (Figs. 5.10.2 a to e). Over the supports of a continuous beam, the bending moment is negative and the slab, therefore, is in tension while a part of the rectangular beam (rib) is in compression. The continuous beam at support is thus equivalent to a rectangular beam (Figs. 5.10.2 a, c, f and g).



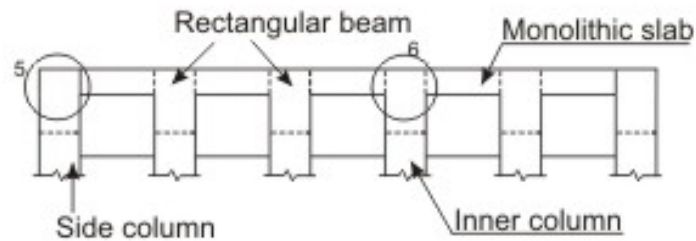
**Fig. 5.10.2 (a): Key plan**



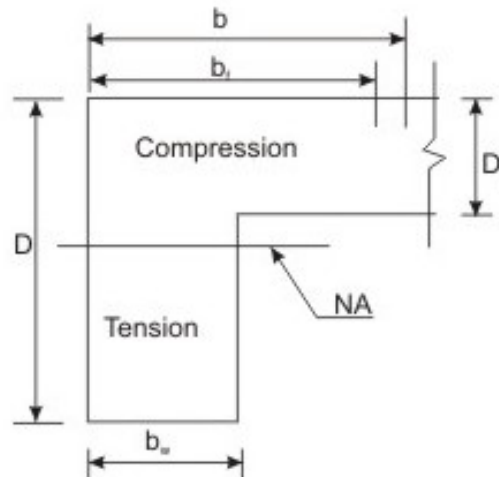
Notations:

$b$  = Actual width of flange  
 $b_f$  = Effective width of flange

**Fig. 5.10.2 (b): Section 1-1**



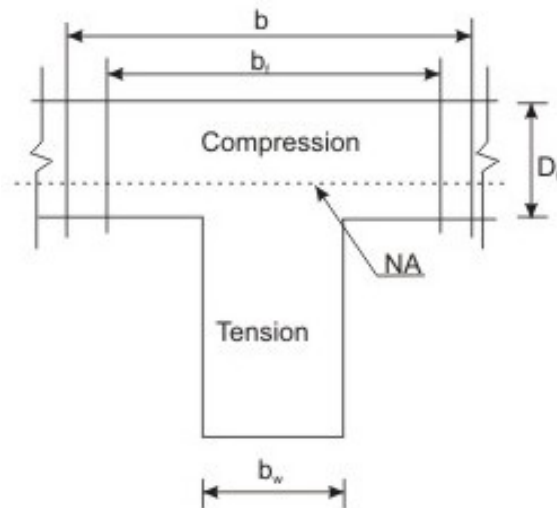
**Fig. 5.10.2(c): Section 2-2**



Notations:

$b$  = Actual width of flange  
 $b_f$  = Effective width of flange  
 $b_w$  = Width of web  
 $D$  = Depth of flange  
 $D_f$  = Depth of flange  
 $NA$  = Neutral axis

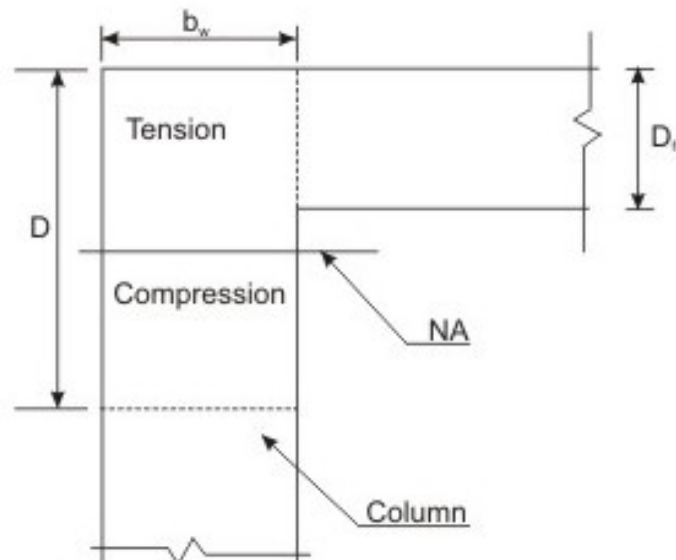
**Fig. 5.10.2 (d): Detail at 3 ( L-beam)**



Notations:

- $b$  = Actual width of flange
- $b_f$  = Effective width of flange
- $b_w$  = Width of web
- $D_f$  = Depth of flange
- NA = Neutral axis

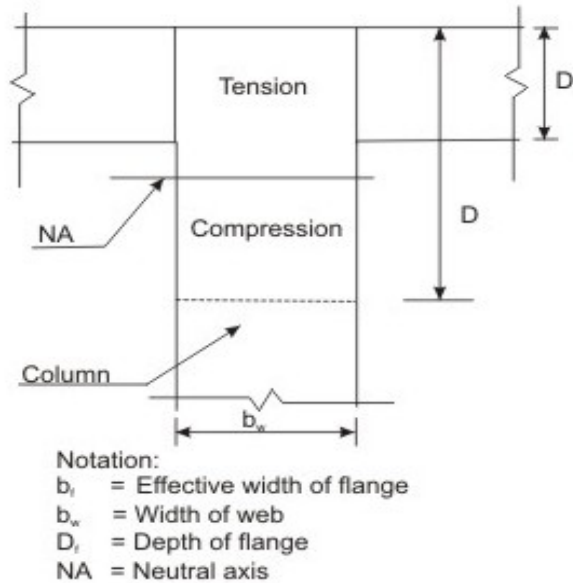
**Fig.5.10.2 (e): Detail at 4 (T-beam)**



Notation:

- $b_f$  = Effective width of flange
- $b_w$  = Width of web
- $D_f$  = Depth of flange
- NA = Neutral axis

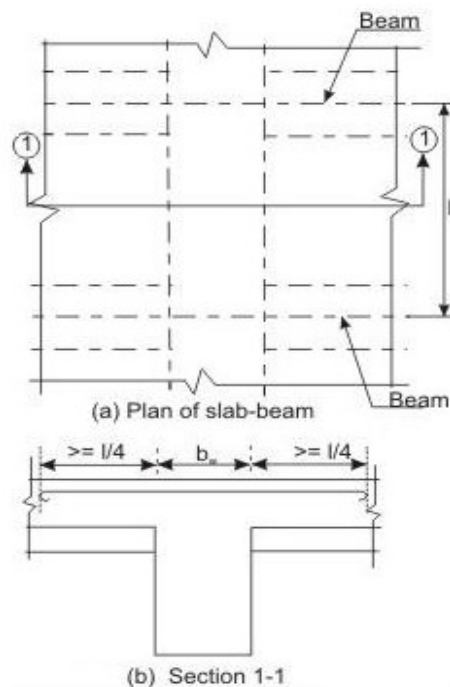
**Fig. 5.10.2 (f): Detail at 5 (rectangular beam)**



**Fig. 5.10.2 (g): Detail at 6 (rectangular beam)**

The actual width of the flange is the spacing of the beam, which is the same as the distance between the middle points of the adjacent spans of the slab, as shown in Fig. 5.10.2 b. However, in a flanged beam, a part of the width less than the actual width, is effective to be considered as a part of the beam. This width of the slab is designated as the effective width of the flange.

## 5.10.2 Effective Width

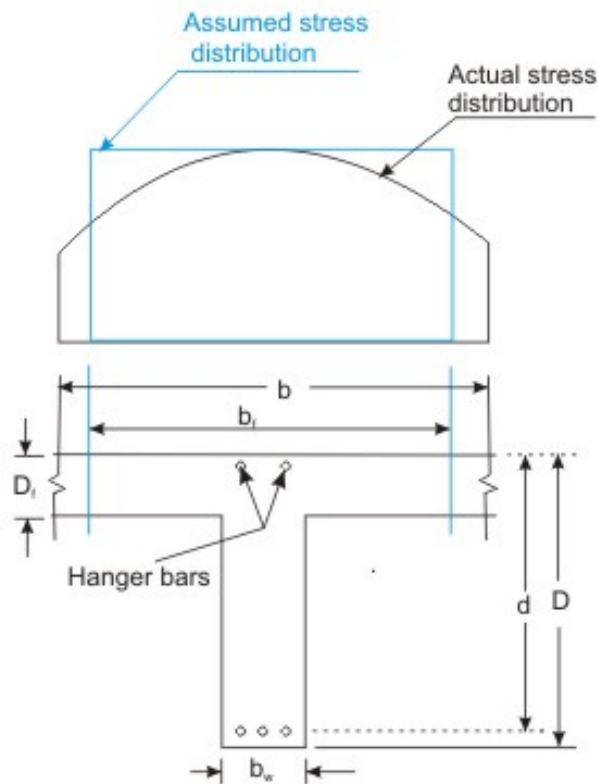


**Fig. 5.10.3: Transverse reinforcement of flange of T-beam**

### 5.10.2.1 IS code requirements

The following requirements (cl. 23.1.1 of IS 456) are to be satisfied to ensure the combined action of the part of the slab and the rib (rectangular part of the beam).

- (a) The slab and the rectangular beam shall be cast integrally or they shall be effectively bonded in any other manner.
- (b) Slabs must be provided with the transverse reinforcement of at least 60 per cent of the main reinforcement at the mid span of the slab if the main reinforcement of the slab is parallel to the transverse beam (Figs. 5.10.3 a and b).



**Fig. 5.10.4:** Variation of compressive stress

The variation of compressive stress (Fig. 5.10.4) along the actual width of the flange shows that the compressive stress is more in the flange just above the rib than the same at some distance away from it. The nature of variation is complex and, therefore, the concept of effective width has been introduced. The effective width is a convenient hypothetical width of the flange over which the compressive stress is assumed to be uniform to give the same compressive



force as it would have been in case of the actual width with the true variation of compressive stress.

#### 5.10.2.2 IS code specifications

Clause 23.1.2 of IS 456 specifies the following effective widths of  $T$  and  $L$ -beams:

(a) For  $T$ -beams, the lesser of

(i)  $b_f = l_o/6 + b_w + 6 D_f$

(ii)  $b_f =$  Actual width of the flange

(b) For isolated  $T$ -beams, the lesser of

(i)  $b_f = \frac{l_o}{(l_o/b) + 4} + b_w$

(ii)  $b_f =$  Actual width of the flange

(c) For  $L$ -beams, the lesser of

(i)  $b_f = l_o/12 + b_w + 3 D_f$

(ii)  $b_f =$  Actual width of the flange

(d) For isolated  $L$ -beams, the lesser of

(i)  $b_f = \frac{0.5 l_o}{(l_o/b) + 4} + b_w$

(ii)  $b_f =$  Actual width of the flange

where  $b_f =$  effective width of the flange,

$l_o =$  distance between points of zero moments in the beam, which is the effective span for simply supported beams and 0.7 times the effective span for continuous beams and frames,

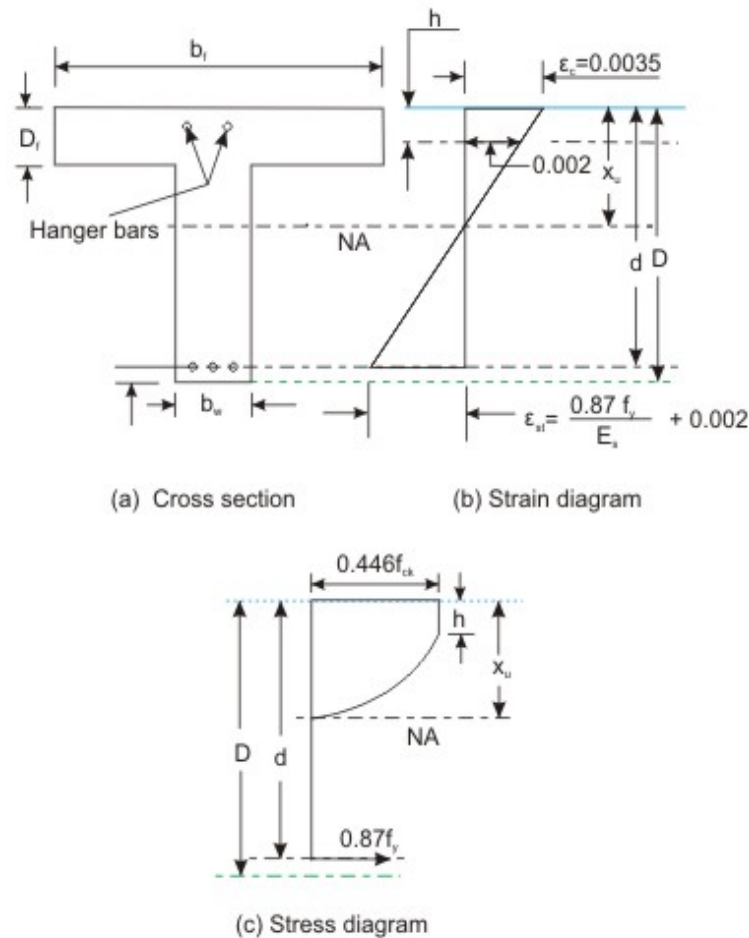
$b_w =$  breadth of the web,

$D_f =$  thickness of the flange,

and  $b =$  actual width of the flange.

### 5.10.3 Four Different Cases

The neutral axis of a flanged beam may be either in the flange or in the web depending on the physical dimensions of the effective width of flange  $b_f$ , effective width of web  $b_w$ , thickness of flange  $D_f$  and effective depth of flanged beam  $d$  (Fig. 5.10.4). The flanged beam may be considered as a rectangular beam of width  $b_f$  and effective depth  $d$  if the neutral axis is in the flange as the concrete in tension is ignored. However, if the neutral axis is in the web, the compression is taken by the flange and a part of the web.



**Fig. 5.10.5:** A typical T-beam section

All the assumptions made in sec. 3.4.2 of Lesson 4 are also applicable for the flanged beams. As explained in Lesson 4, the compressive stress remains constant between the strains of 0.002 and 0.0035. It is important to find the depth  $h$  of the beam where the strain is 0.002 (Fig. 5.10.5 b). If it is located in the web, the whole of flange will be under the constant stress level of  $0.446 f_{ck}$ . The

following gives the relation of  $D_f$  and  $d$  to facilitate the determination of the depth  $h$  where the strain will be 0.002.

From the strain diagram of Fig. 5.10.5 b:

$$\frac{0.002}{0.0035} = \frac{x_u - h}{x_u}$$

or 
$$\frac{h}{x_u} = \frac{3}{7} = 0.43$$
  
(5.1)

when  $x_u = x_{u, \max}$ , we get

$$h = \frac{3}{7} x_{u, \max} = 0.227 d, 0.205 d \text{ and } 0.197 d, \text{ for Fe 250, Fe 415 and Fe 500, respectively. In general, we can adopt, say}$$

$$h/d = 0.2 \quad (5.2)$$

The same relation is obtained below from the values of strains of concrete and steel of Fig. 5.10.5 b.

$$\frac{\varepsilon_{st}}{\varepsilon_c} = \frac{d - x_u}{x_u}$$

or 
$$\frac{d}{x_u} = \frac{\varepsilon_{st} + \varepsilon_c}{\varepsilon_c} \quad (5.3)$$

Dividing Eq. 5.1 by Eq. 5.3

$$\frac{h}{d} = \frac{0.0015}{\varepsilon_{st} + 0.0035} \quad (5.4)$$

Using  $\varepsilon_{st} = (0.87 f_y / E_s) + 0.002$  in Eq. 5.4, we get  $h/d = 0.227, 0.205$  and  $0.197$  for Fe 250, Fe 415 and Fe 500 respectively, and we can adopt  $h/d = 0.2$  (as in Eq. 5.2).

Thus, we get the same Eq. 5.2 from Eq. 5.4,

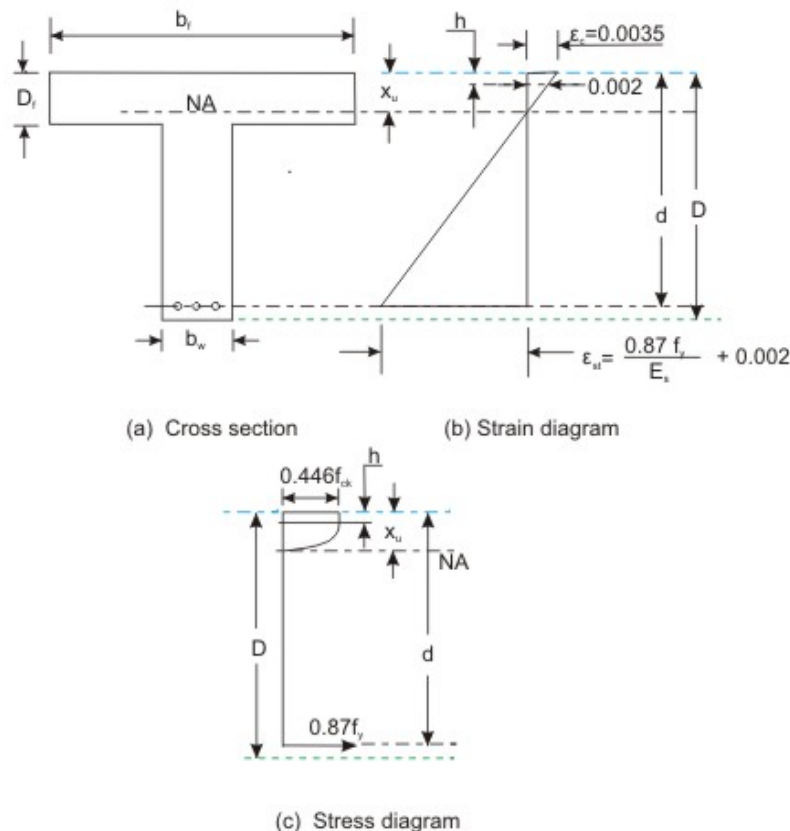
$$h/d = 0.2 \quad (5.2)$$

It is now clear that the three values of  $h$  are around  $0.2 d$  for the three grades of steel. The maximum value of  $h$  may be  $D_f$ , at the bottom of the flange where the strain will be 0.002, if  $D_f/d = 0.2$ . This reveals that the thickness of the flange may be considered small if  $D_f/d$  does not exceed 0.2 and in that case, the position of the fibre of 0.002 strain will be in the web and the entire flange will be under a constant compressive stress of  $0.446 f_{ck}$ .

On the other hand, if  $D_f$  is  $> 0.2 d$ , the position of the fibre of 0.002 strain will be in the flange. In that case, a part of the slab will have the constant stress of  $0.446 f_{ck}$  where the strain will be more than 0.002.

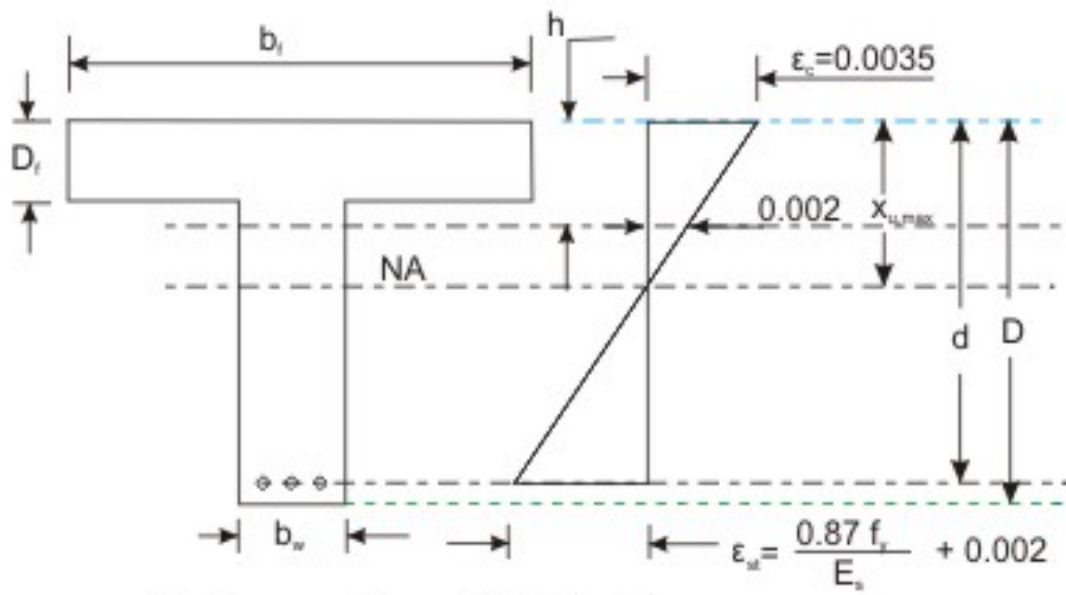
Thus, in the balanced and over-reinforced flanged beams (when  $x_u = x_{u, \max}$ ), the ratio of  $D_f/d$  is important to determine if the rectangular stress block is for the full depth of the flange (when  $D_f/d$  does not exceed 0.2) or for a part of the flange (when  $D_f/d > 0.2$ ). Similarly, for the under-reinforced flanged beams, the ratio of  $D_f/x_u$  is considered in place of  $D_f/d$ . If  $D_f/x_u$  does not exceed 0.43 (see Eq. 5.1), the constant stress block is for the full depth of the flange. If  $D_f/x_u > 0.43$ , the constant stress block is for a part of the depth of the flange.

Based on the above discussion, the four cases of flanged beams are as follows:

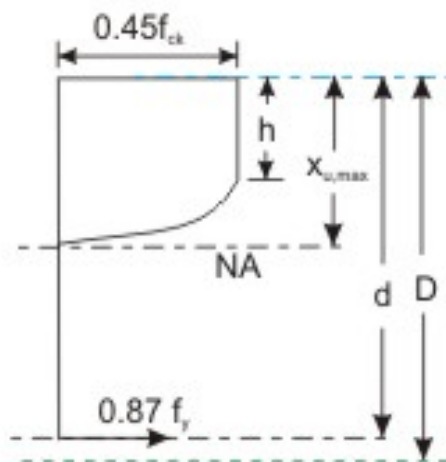


**Fig. 5.10.6:** T-beam, case (i), when  $x_u < D_f$

- (i) Neutral axis is in the flange ( $x_u < D_f$ ), (Fig. 5.10.6 a to c)



(a) Cross section (b) Strain diagram



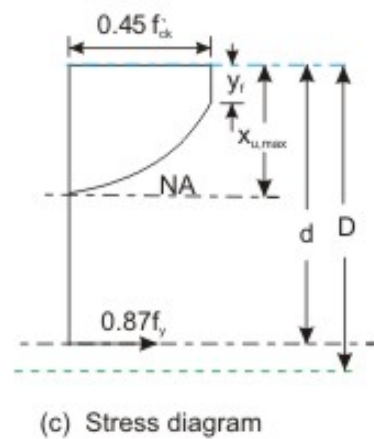
(c) Stress diagram

**Fig. 5.10.7:** T-beam, case (ii a), when  $D_f/d \leq 0.2$  and balanced  $x_{u,max} > D_f$

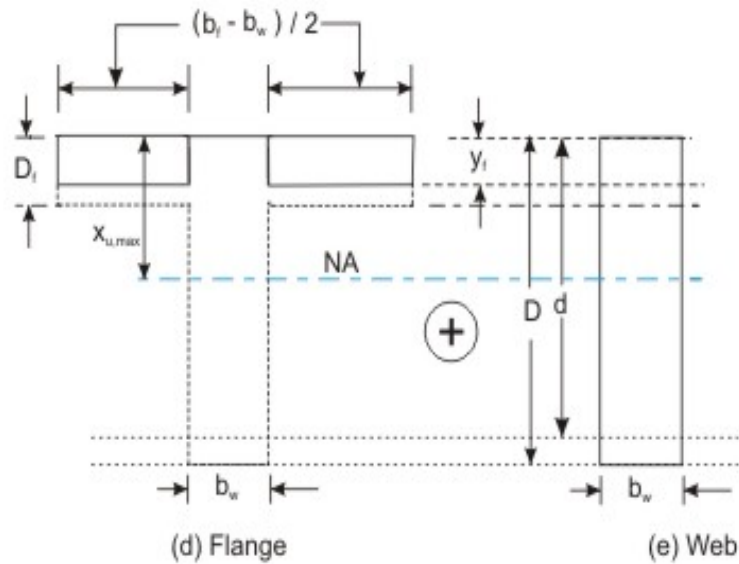


(a) Cross section

(b) Strain diagram



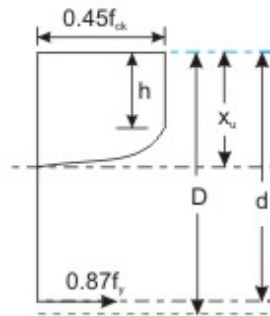
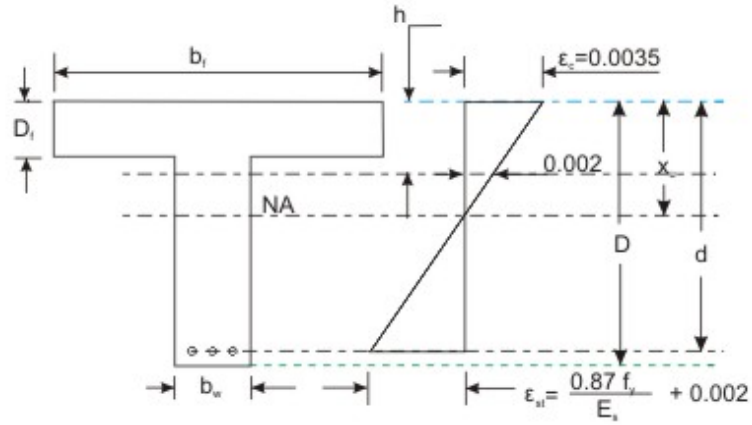
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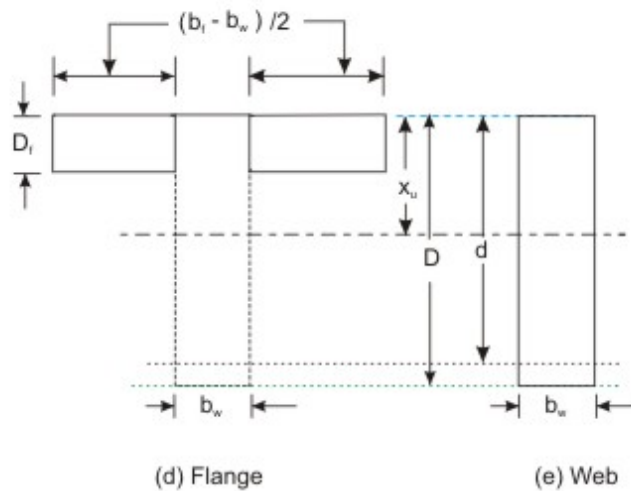
**Fig. 5.10.8:** T-beam, case (ii b), when  $D_f/d > 0.2$  and balanced  $x_{u,max} > D_f$

- (ii) Neutral axis is in the web and the section is balanced ( $x_u = x_{u,max} > D_f$ ), (Figs. 5.10.7 and 8 a to e)

It has two situations: (a) when  $D_f/d$  does not exceed 0.2, the constant stress block is for the entire depth of the flange (Fig. 5.10.7), and (b) when  $D_f/d > 0.2$ , the constant stress block is for a part of the depth of flange (Fig. 5.10.8).

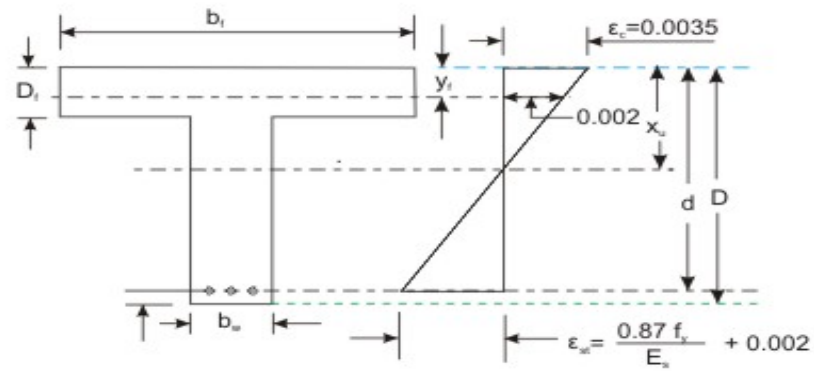


**Fig. 5.10.9:** T-beam, case (iii a), when  $D_f/x_u \leq 0.43$  and under-reinforced  $x_u > D_f$

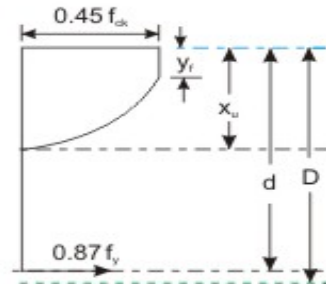


**Fig. 5.10.9:** T-beam, case (iii a), when  $D_f/x_u \leq 0.43$  and under-reinforced  $x_u > D_f$



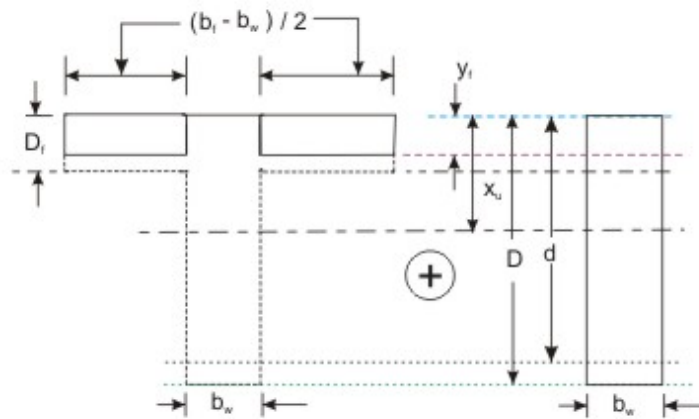


(a) Cross section (b) Strain diagram



(c) Stress diagram

**Fig. 5.10.10: T-beam, case (iii b),**  
when  $D_f/x_u > 0.43$  and under-reinforced  
 $x_u > D_f$



(d) Flange

(e) Web

**Fig. 5.10.10: T-beam, case (iii b),** when  $D_f/x_u > 0.43$  and  
under-reinforced  $x_u > D_f$

- (iii) Neutral axis is in the web and the section is under-reinforced ( $x_{u,max} > x_u > D_f$ ), (Figs. 5.10.9 and 10 a to e)

This has two situations: (a) when  $D_f/x_u$  does not exceed 0.43, the full depth of flange is having the constant stress (Fig. 5.10.9), and (b) when  $D_f/x_u > 0.43$ , the constant stress is for a part of the depth of flange (Fig. 5.10.10).

(iv) Neutral axis is in the web and the section is over-reinforced ( $x_u > x_{u,max} > D_f$ ), (Figs. 5.10.7 and 8 a to e)

As mentioned earlier, the value of  $x_u$  is then taken as  $x_{u,max}$  when  $x_u > x_{u,max}$ . Therefore, this case also will have two situations depending on  $D_f/d$  not exceeding 0.2 or  $> 0.2$  as in (ii) above. The governing equations of the four different cases are now taken up.

## 5.10.4 Governing Equations

The following equations are only for the singly reinforced  $T$ -beams. Additional terms involving  $M_{u,lim}$ ,  $M_{u2}$ ,  $A_{sc}$ ,  $A_{st1}$  and  $A_{st2}$  are to be included from Eqs. 4.1 to 4.8 of sec. 4.8.3 of Lesson 8 depending on the particular case. Applications of these terms are explained through the solutions of numerical problems of doubly reinforced  $T$ -beams in Lessons 11 and 12.

### 5.10.4.1 Case (i): When the neutral axis is in the flange ( $x_u < D_f$ ), (Figs. 5.10.6 a to c)

Concrete below the neutral axis is in tension and is ignored. The steel reinforcement takes the tensile force (Fig. 5.10.6). Therefore,  $T$  and  $L$ -beams are considered as rectangular beams of width  $b_f$  and effective depth  $d$ . All the equations of singly and doubly reinforced rectangular beams derived in Lessons 4 to 5 and 8 respectively, are also applicable here.

### 5.10.4.2 Case (ii): When the neutral axis is in the web and the section is balanced ( $x_{u,max} > D_f$ ), (Figs. 5.10.7 and 8 a to e)

(a) When  $D_f/d$  does not exceed 0.2, (Figs. 5.10.7 a to e)

As explained in sec. 5.10.3, the depth of the rectangular portion of the stress block (of constant stress  $= 0.446 f_{ck}$ ) in this case is greater than  $D_f$  (Figs. 5.10.7 a, b and c). The section is split into two parts: (i) rectangular web of width  $b_w$  and effective depth  $d$ , and (ii) flange of width  $(b_f - b_w)$  and depth  $D_f$  (Figs. 5.10.7 d and e).

Total compressive force = Compressive force of rectangular beam of width  $b_w$  and depth  $d$  + Compressive force of rectangular flange of width  $(b_f - b_w)$  and depth  $D_f$ .

Thus, total compressive force

$$C = 0.36 f_{ck} b_w x_{u, max} + 0.45 f_{ck} (b_f - b_w) D_f \quad (5.5)$$

(Assuming the constant stress of concrete in the flange as  $0.45 f_{ck}$  in place of  $0.446 f_{ck}$ , as per G-2.2 of IS 456), and the tensile force

$$T = 0.87 f_y A_{st} \quad (5.6)$$

The lever arm of the rectangular beam (web part) is  $(d - 0.42 x_{u, max})$  and the same for the flanged part is  $(d - 0.5 D_f)$ .

So, the total moment = Moment due to rectangular web part + Moment due to rectangular flange part

$$\text{or } M_u = 0.36 f_{ck} b_w x_{u, max} (d - 0.42 x_{u, max}) + 0.45 f_{ck} (b_f - b_w) D_f (d - D_f/2)$$

$$\text{or } M_u = 0.36(x_{u, max}/d)\{1 - 0.42(x_{u, max}/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) D_f(d - D_f/2) \quad (5.7)$$

Equation 5.7 is given in G-2.2 of IS 456.

**(b) When  $D_f/d > 0.2$ , (Figs. 5.10.8 a to e)**

In this case, the depth of rectangular portion of stress block is within the flange (Figs. 5.10.8 a, b and c). It is assumed that this depth of constant stress ( $0.45 f_{ck}$ ) is  $y_f$ , where

$$y_f = 0.15 x_{u, max} + 0.65 D_f, \text{ but not greater than } D_f \quad (5.8)$$

The above expression of  $y_f$  is derived in sec. 5.10.4.5.

As in the previous case (ii a), when  $D_f/d$  does not exceed 0.2, equations of  $C$ ,  $T$  and  $M_u$  are obtained from Eqs. 5.5, 6 and 7 by changing  $D_f$  to  $y_f$ . Thus, we have (Figs. 5.10.8 d and e)

$$C = 0.36 f_{ck} b_w x_{u, max} + 0.45 f_{ck} (b_f - b_w) y_f \quad (5.9)$$

$$T = 0.87 f_y A_{st} \quad (5.10)$$

The lever arm of the rectangular beam (web part) is  $(d - 0.42 x_{u, max})$  and the same for the flange part is  $(d - 0.5 y_f)$ . Accordingly, the expression of  $M_u$  is as follows:

$$M_u = 0.36(x_{u, max}/d)\{1 - 0.42(x_{u, max}/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) y_f(d - y_f/2) \quad (5.11)$$

#### 5.10.4.3 Case (iii): When the neutral axis is in the web and the section is under-reinforced ( $x_u > D_f$ ), (Figs. 5.10.9 and 10 a to e)

(a) When  $D_f / x_u$  does not exceed 0.43, (Figs. 5.10.9 a to e)

Since  $D_f$  does not exceed  $0.43 x_u$  and  $h$  (depth of fibre where the strain is 0.002) is at a depth of  $0.43 x_u$ , the entire flange will be under a constant stress of  $0.45 f_{ck}$  (Figs. 5.10.9 a, b and c). The equations of  $C$ ,  $T$  and  $M_u$  can be written in the same manner as in sec. 5.10.4.2, case (ii a). The final forms of the equations are obtained from Eqs. 5.5, 6 and 7 by replacing  $x_{u, max}$  by  $x_u$ . Thus, we have (Figs. 5.10.9 d and e)

$$C = 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) D_f \quad (5.12)$$

$$T = 0.87 f_y A_{st} \quad (5.13)$$

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) D_f(d - D_f/2) \quad (5.14)$$

(b) When  $D_f / x_u > 0.43$ , (Figs. 5.10.10 a to e)

Since  $D_f > 0.43 x_u$  and  $h$  (depth of fibre where the strain is 0.002) is at a depth of  $0.43 x_u$ , the part of the flange having the constant stress of  $0.45 f_{ck}$  is assumed as  $y_f$  (Fig. 5.10.10 a, b and c). The expressions of  $y_f$ ,  $C$ ,  $T$  and  $M_u$  can be written from Eqs. 5.8, 9, 10 and 11 of sec. 5.10.4.2, case (ii b), by replacing  $x_{u, max}$  by  $x_u$ . Thus, we have (Fig. 5.10.10 d and e)

$$y_f = 0.15 x_u + 0.65 D_f, \text{ but not greater than } D_f \quad (5.15)$$

$$C = 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f \quad (5.16)$$

$$T = 0.87 f_y A_{st} \quad (5.17)$$

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) y_f(d - y_f/2) \quad (5.18)$$

#### 5.10.4.4 Case (iv): When the neutral axis is in the web and the section is over-reinforced ( $x_u > D_f$ ), (Figs. 5.10.7 and 8 a to e)

For the over-reinforced beam, the depth of neutral axis  $x_u$  is more than  $x_{u, max}$  as in rectangular beams. However,  $x_u$  is restricted up to  $x_{u, max}$ . Therefore, the corresponding expressions of  $C$ ,  $T$  and  $M_u$  for the two situations (a) when  $D_f/d$  does not exceed 0.2 and (b) when  $D_f/d > 0.2$  are written from Eqs. 5.5 to 5.7 and 5.9 to 5.11, respectively of sec. 5.10.4.2 (Figs. 5.10.7 and 8). The expression of  $y_f$  for (b) is the same as that of Eq. 5.8.

(a) When  $D_f/d$  does not exceed 0.2 (Figs. 5.10.7 a to e)

The equations are:

$$C = 0.36 f_{ck} b_w x_{u, max} + 0.45 f_{ck} (b_f - b_w) D_f \quad (5.5)$$

$$T = 0.87 f_y A_{st} \quad (5.6)$$

$$M_u = 0.36(x_{u, max}/d)\{1 - 0.42(x_{u, max}/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) D_f(d - D_f/2) \quad (5.7)$$

(b) When  $D_f/d > 0.2$  (Figs. 5.10.8 a to e)

$$y_f = 0.15 x_{u, max} + 0.65 D_f, \text{ but not greater than } D_f \quad (5.8)$$

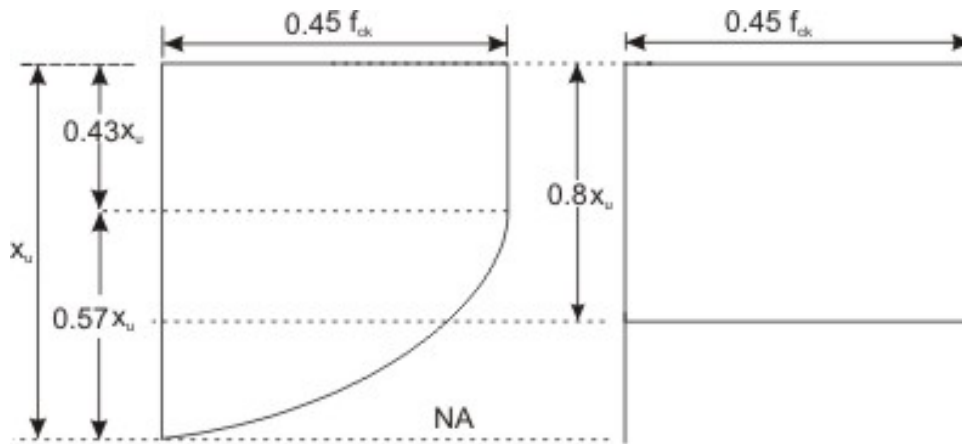
$$C = 0.36 f_{ck} b_w x_{u, max} + 0.45 f_{ck} (b_f - b_w) y_f \quad (5.9)$$

$$T = 0.87 f_y A_{st} \quad (5.10)$$

$$M_u = 0.36(x_{u, max}/d)\{1 - 0.42(x_{u, max}/d)\} f_{ck} b_w d^2 + 0.45 f_{ck}(b_f - b_w) y_f(d - y_f/2) \quad (5.11)$$

It is clear from the above that the over-reinforced beam will not have additional moment of resistance beyond that of the balanced one. Moreover, it will prevent steel failure. It is, therefore, recommended either to re-design or to go for doubly reinforced flanged beam than designing over-reinforced flanged beam.

#### 5.10.4.5 Derivation of the equation to determine $y_f$ , Eq. 5.8, Fig. 5.10.11



**Fig. 5.10.11:** (a) IS 456 stress block (b) Whitney's stress block

Whitney's stress block has been considered to derive Eq. 5.8. Figure 5.10.11 shows the two stress blocks of IS code and of Whitney.

$y_f$  = Depth of constant portion of the stress block when  $D_f/d > 0.2$ . As  $y_f$  is a function of  $x_u$  and  $D_f$  and let us assume

$$(5.19) \quad y_f = A x_u + B D_f$$

where  $A$  and  $B$  are to be determined from the following two conditions:

$$(5.20) \quad (i) \quad y_f = 0.43 x_u, \text{ when } D_f = 0.43 x_u$$

$$(5.21) \quad (ii) \quad y_f = 0.8 x_u, \text{ when } D_f = x_u$$

Using the conditions of Eqs. 5.20 and 21 in Eq. 5.19, we get  $A = 0.15$  and  $B = 0.65$ . Thus, we have

$$(5.8) \quad y_f = 0.15 x_u + 0.65 D_f$$

#### 5.10.5 Practice Questions and Problems with Answers

**Q.1:** Why do we consider most of the beams as  $T$  or  $L$ -beams between the supports and rectangular beams over the support of continuous span?

**A.1:** Sec. 5.10.1, first paragraph.

**Q.2:** Draw cross-section of a beam with top slab and show the actual width and effective width of the *T*-beam.

**A.2:** Fig. 5.10.2 b.

**Q.3:** State the requirements with figures as per IS 456 which ensure the combined action of the part of the slab and the rib of flanged beams.

**A.3:** Sec. 5.10.2.1(a) and (b), Figure 5.10.3 (a and b).

**Q.4:** Define “effective width” of flanged beams.

**A.4:** Effective width is an imaginary width of the flange over which the compressive stress is assumed to be uniform to give the same compressive force as it would have been in case of the actual width with the true variation of compressive stress (Fig. 5.10.4 of text).

**Q.5:** Write the expressions of effective widths of *T* and *L*-beams and isolated beams.

**A.5:** Sec. 5.10.2.2.

**Q.6:** Name the four different cases of flanged beams.

**A.6:** The four different cases are:

(i) When the neutral axis is in the flange ( $x_u < D_f$ ) (discussed in sec. 5.10.4.1).

(ii) When the neutral axis is in the web and the section is balanced ( $x_{u,max} > D_f$ ). It has two situations: (a) when  $D_f/d$  does not exceed 0.2 and (b) when  $D_f/d > 0.2$  (discussed in sec. 5.10.4.2).

(iii) When the neutral axis is in the web and the section is under-reinforced ( $x_{u,max} > x_u > D_f$ ). It has two situations: (a) when  $D_f/x_u$  does not exceed 0.43 and (b) when  $D_f/x_u > 0.43$  (discussed in sec. 5.10.4.3).

(iv) When the neutral axis is in the web and the section is over-reinforced ( $x_u > x_{u,max} > D_f$ ). It has two situations: (a) when  $D_f/d$  does not exceed 0.2 and (b) when  $D_f/d > 0.2$  (discussed in sec. 5.10.4.4).

**Q.7:** (a) Derive the following equation:

$$y_f = 0.15 x_{u,max} + 0.65 D_f$$

(b) State when this equation is to be used.

(c) What is the limiting value of  $y_f$  ?

**A.7:** (a) For derivation of the equation, see sec. 5.10.4.5.

(b) This equation gives the depth of flange over which the stress is constant at  $0.45 f_{ck}$  (i.e. strain is more than 0.002) when the neutral axis is in web. This occurs when  $D_f/d > 0.2$  for balanced beam and when  $D_f/x_u > 0.43$  for under-reinforced beams.

(c) Limiting value of  $y_f$  is  $D_f$ .

### 5.10.6 References

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### 5.10.7 Test 10 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

**TQ.1:** Why do we consider most of the beams as  $T$  or  $L$ - beams between the supports and rectangular beams over the support of continuous span?  
(5 marks)

**A.TQ.1:** Sec. 5.10.1, first paragraph.

**TQ.2:** Define “effective width” of flanged beams.  
(5 marks)

**A.TQ.2:** Effective width is a convenient hypothetical width of the flange over which the compressive stress is assumed to be uniform to give the same compressive force as it would have been in case of the actual width with the true variation of compressive stress (Fig. 5.10.4 of text).

**TQ.3:** State the requirements with figures as per IS 456 which ensure the combined action of the part of the slab and the rib of flanged beams.  
(10 marks)

**A.TQ.3:** Sec. 5.10.2.1(a) and (b), Figure 5.10.3 (a and b).

**TQ.4:** Write the expressions of effective widths of  $T$  and  $L$ -beams and isolated beams.  
(10 marks)

**A.TQ.4:** Sec. 5.10.2.2.

**TQ.5:** Name the four different cases of flanged beams.  
(10 marks)

**A.TQ.5:** The four different cases are

(i) When the neutral axis is in the flange ( $x_u < D_f$ ) (discussed in sec. 5.10.4.1).

(ii) When the neutral axis is in the web and the section is balanced. It has two situations: (a) when  $D_f/d$  does not exceed 0.2 and (b) when  $D_f/d > 0.2$  (discussed in sec. 5.10.4.2).

- (iii) When the neutral axis is in the web and the section is under-reinforced. It has two situations: (a) when  $D_f/x_u$  does not exceed 0.43 and (b) when  $D_f/x_u > 0.43$  (discussed in sec. 5.10.4.3).
- (iv) When the neutral axis is in the web and the section is over-reinforced. It has two situations: (a) when  $D_f/d$  does not exceed 0.2 and (b) when  $D_f/d > 0.2$  (discussed in sec. 5.10.4.4).

**TQ.6:** (a) Derive the following equation:

$$y_f = 0.15 x_{u,max} + 0.65 D_f$$

(b) State when this equation is to be used.

(c) What is the limiting value of  $y_f$ ?

(5 + 3 + 2 = 10 marks)

**A.TQ.6:** (a) For derivation of the equation, see sec. 5.10.4.5.

(b) This equation gives the depth of flange over which the stress is constant at  $0.45 f_{ck}$  (i.e. strain is more than 0.002) when the neutral axis is in web. This occurs when  $D_f/d > 0.2$  for balanced beam and when  $D_f/x_u > 0.43$  for under-reinforced beams.

(c) Limiting value of  $y_f$  is  $D_f$ .

## 5.10.8 Summary of this Lesson

This lesson illustrates the practical situations when slabs are cast integrally with the beams to form either *T* and *L*-beams or rectangular beams. The concept of effective width of the slab to form a part of the beam has been explained. The requirements as per IS 456 have been illustrated so that the considered part of the slab may become effective as a beam. Expressions of effective widths for different cases of *T* and *L*-beams are given. Four sets of governing equations for determining  $C$ ,  $T$  and  $M_u$  are derived for four different cases. These equations form the basis of analysis and design of singly and doubly reinforced *T* and *L*-beams.