Module 5

Flanged Beams – Theory and Numerical Problems

Lesson 11

Flanged Beams – Numerical Problems

Instructional Objectives:

At the end of this lesson, the student should be able to:

- identify the two types of numerical problems – analysis and design types,
- apply the formulations to analyse the capacity of a flanged beam,
- determine the limiting moment of resistance quickly with the help of tables of SP-16.

5.11.1 Introduction

Lesson 10 illustrates the governing equations of flanged beams. It is now necessary to apply them for the solution of numerical problems. Two types of numerical problems are possible: (i) Analysis and (ii) Design types. This lesson explains the application of the theory of flanged beams for the analysis type of problems. Moreover, use of tables of SP-16 has been illustrated to determine the limiting moment of resistance of sections quickly for the three grades of steel. Besides mentioning the different steps of the solution, numerical examples are also taken up to explain their step-by-step solutions.

5.11.2 Analysis Type of Problems

The dimensions of the beam b_f, b_w, D_f, d, D, grades of concrete and steel and the amount of steel A_{st} are given. It is required to determine the moment of resistance of the beam.

Step 1: To determine the depth of the neutral axis x_u

The depth of the neutral axis is determined from the equation of equilibrium $C = T$. However, the expression of C depends on the location of neutral axis, D_f /d and D_f /x_u parameters. Therefore, it is required to assume first that the x_u is in the flange. If this is not the case, the next step is to assume x_u in the web and the computed value of x_u will indicate if the beam is under-reinforced, balanced or over-reinforced.

Other steps:

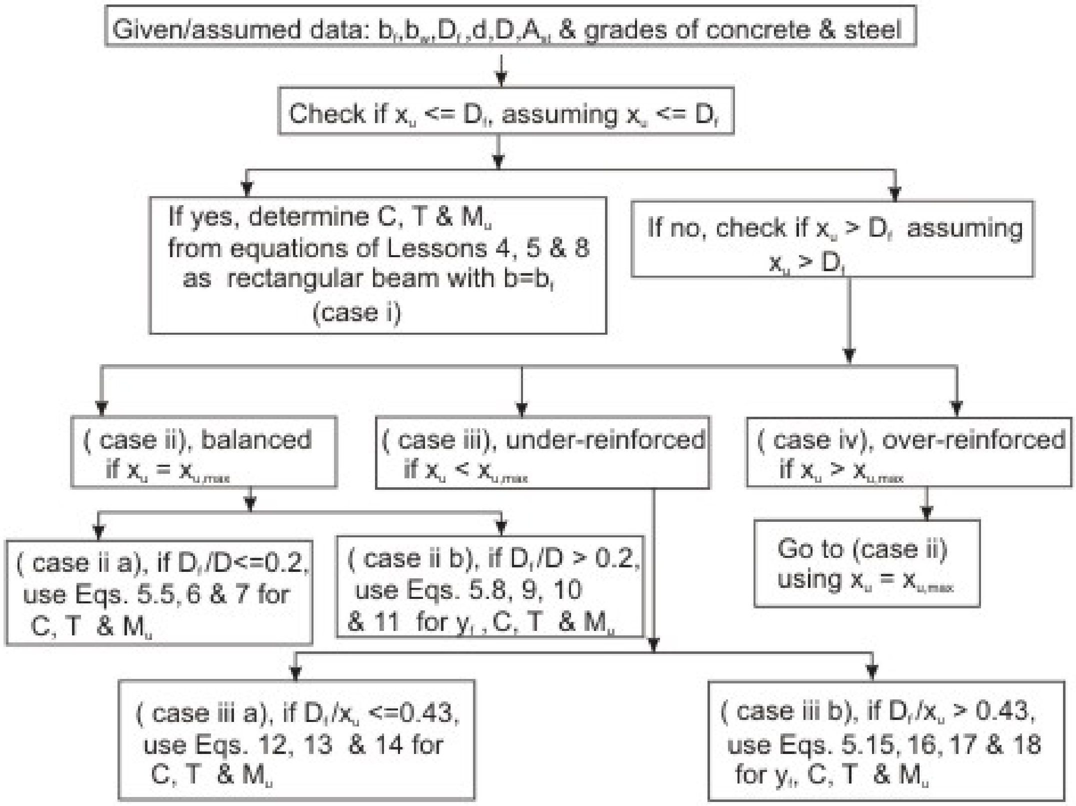

Fig. 5.11.1: Steps of solution of analysis type of problems

After knowing if the section is under-reinforced, balanced or over-reinforced, the respective parameter D_f/d or D_f/x_u is computed for the under-reinforced, balanced or over-reinforced beam. The respective expressions of C, T and M_u, as established in Lesson 10, are then employed to determine their values. Figure 5.11.1 illustrates the steps to be followed.

5.11.3 Numerical Problems (Analysis Type)

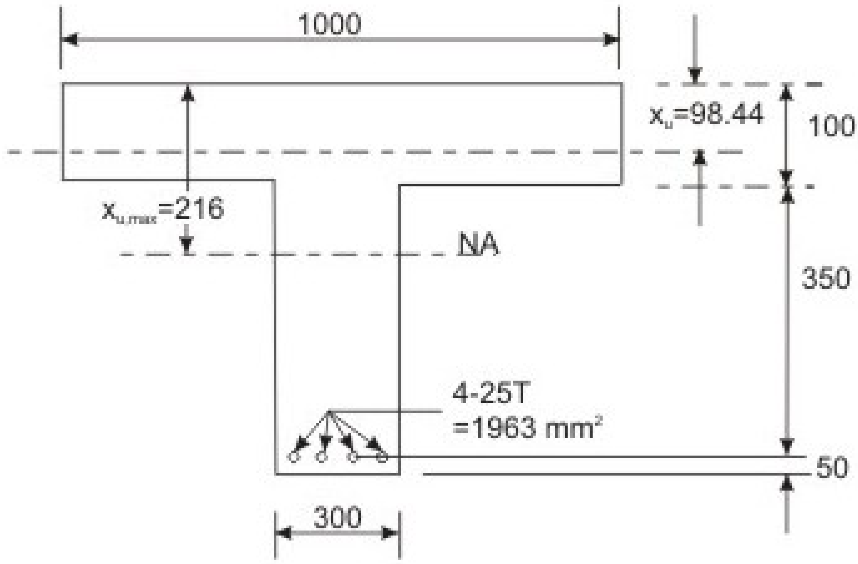

Fig. 5.11.2: Example 1, case (i)

Ex.1: Determine the moment of resistance of the *T*-beam of Fig. 5.11.2. Given data: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm, d = 450 mm and A_{st} = 1963 mm^2 (4- 25 T). Use M 20 and Fe 415.

Step 1: To determine the depth of the neutral axis x_u

Assuming x_u in the flange and equating total compressive and tensile forces from the expressions of C and T (Eq. 3.16 of Lesson 5) as the *T*-beam can be treated as rectangular beam of width b_f and effective depth d, we get:

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\ b_f\ f_{ck}} = \frac{0.87\,(415)\,(1963)}{0.36\,(1000)\,(20)} = 98.44\ \text{mm} < 100\,\text{mm}$$

So, the assumption of x_u in the flange is correct.

$x_{u,\,max}$ for the balanced rectangular beam = 0.48 d = 0.48 (450) = 216 mm.

It is under-reinforced since x_u < $x_{u,max}$.

Step 2: To determine C, T and M_u

From Eqs. 3.9 (using $b = b_f$) and 3.14 of Lesson 4 for C and T and Eq. 3.23 of Lesson 5 for M_u, we have:

$C = 0.36\ b_f\ x_u\ f_{ck}$ (3.9)

$= 0.36\ (1000)\ (98.44)\ (20) = 708.77$ kN

$T = 0.87\ f_y\ A_{st}$
(3.14)

$= 0.87\ (415)\ (1963) = 708.74$ kN

$$M_u = 0.87 f_y A_{st}\, d\, (1 - \frac{A_{st}\ f_y}{f_{ck}\ b_f\ d}) \tag{3.23}$$

$$= 0.87\ (415)\ (1963)\ (450)\ \{1 - \frac{(1963)\ (415)}{(20)\ (1000)\ (450)}\} = 290.06 \text{ kNm}$$

This problem belongs to the case (i) and is explained in sec. 5.10.4.1 of Lesson 10.

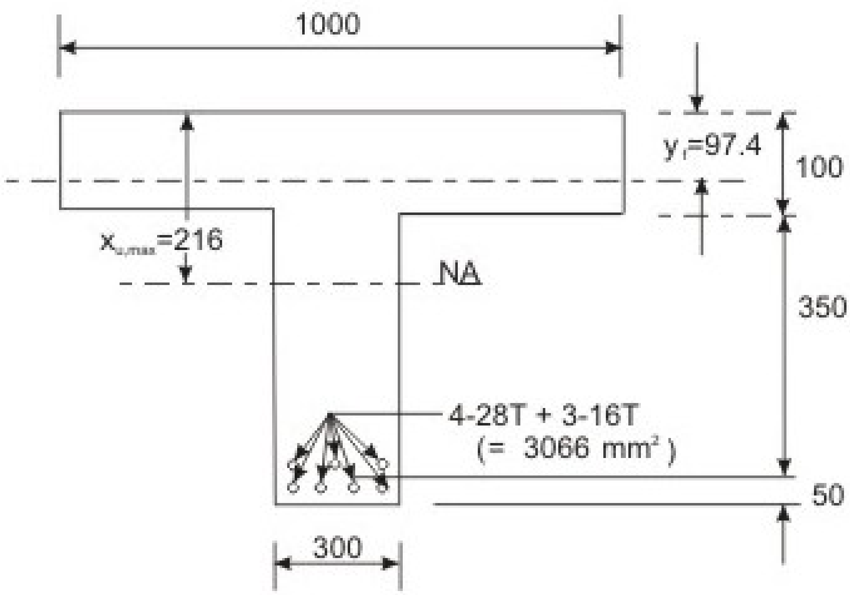

Fig. 5.11.3: Example 2, case (ii b)

Ex.2: Determine $A_{st,lim}$ and $M_{u,lim}$ of the flanged beam of Fig. 5.11.3. Given data are: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine D_f/d ratio

For the limiting case $x_u = x_{u,max} = 0.48\ (450) = 216$ mm $> D_f$. The ratio D_f/d is computed.

D_f/d = 100/450 = 0.222 > 0.2

Hence, it is a problem of case (ii b) and discussed in sec. 5.10.4.2 b of Lesson 10.

Step 2: Computations of y_f, C and T

First, we have to compute y_f from Eq.5.8 of Lesson 10 and then employ Eqs. 5.9, 10 and 11 of Lesson 10 to determine C, T and Mu, respectively.

$y_f = 0.15\, x_{u,max} + 0.65\, D_f = 0.15\,(216) + 0.65\,(100) = 97.4$ mm. (from Eq. 5.8)

$C = 0.36\, f_{ck}\, b_w\, x_{u,max} + 0.45\, f_{ck}\,(b_f - b_w)\, y_f$ (5.9)

$= 0.36\,(20)\,(300)\,(216) + 0.45\,(20)\,(1000 - 300)\,(97.4) = 1{,}080.18$ kN.

$T = 0.87\, f_y\, A_{st} = 0.87\,(415)\, A_{st}$ (5.10)

Equating C and T, we have

$$A_{st} = \frac{(1080.18)\,(1000)\ \text{N}}{0.87\,(415)\ \text{N/mm}^2} = 2{,}991.77\ \text{mm}^2$$

Provide 4-28 T (2463 mm^2) + 3-16 T (603 mm^2) = 3,066 mm^2

Step 3: Computation of M_u

$$M_{u,\,lim} = 0.36\left(\frac{x_{u,max}}{d}\right)\left\{1 - 0.42\left(\frac{x_{u,max}}{d}\right)\right\} f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\,(b_f - b_w)\, y_f\,(d - y_f/2) \qquad (5.11)$$

$$= 0.36\,(0.48)\,\{1 - 0.42\,(0.48)\}\,(20)\,(300)\,(450)^2 + 0.45\,(20)\,(1000 - 300)\,(97.4)\,(450 - 97.4/2) = 413.87 \text{ kNm}$$

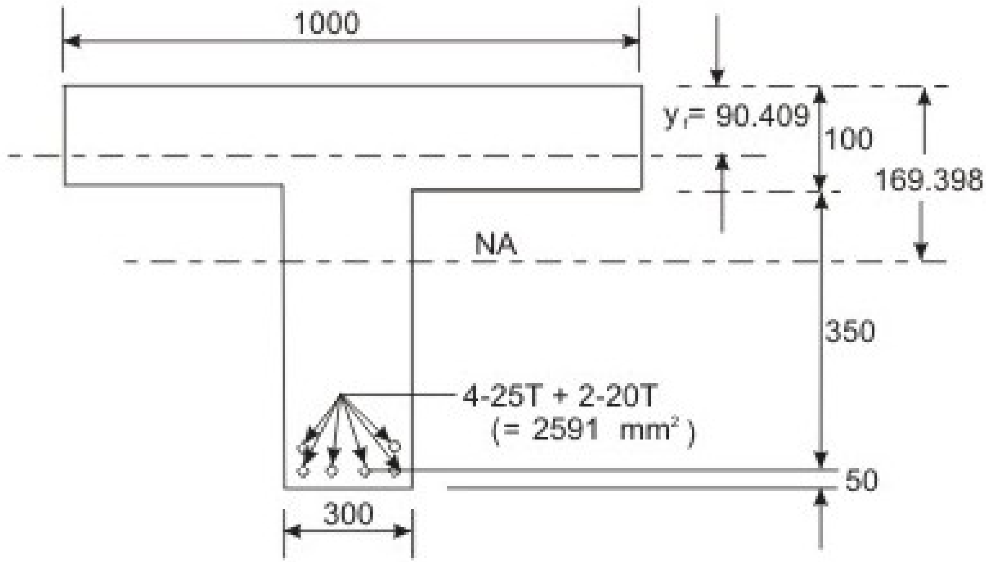

Fig. 5.11.4: Example 3, case (iii b)

Ex.3: Determine the moment of resistance of the beam of Fig. 5.11.4 when A_{st} = 2,591 mm^2 (4- 25 T and 2- 20 T). Other parameters are the same as those of Ex.1: b_f = 1,000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine x_u

Assuming x_u to be in the flange and the beam is under-reinforced, we have from Eq. 3.16 of Lesson 5:

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\, b_f\ f_{ck}} = \frac{0.87\,(415)\,(2591)}{0.36\,(1000)\,(20)} = 129.93\ \text{mm} > 100\ \text{mm}$$

Since $x_u > D_f$, the neutral axis is in web. Here, D_f/d = 100/450 = 0.222 > 0.2. So, we have to substitute the term y_f from Eq. 5.15 of Lesson 10, assuming D_f / x_u > 0.43 in the equation of $C = T$ from Eqs. 5.16 and 17 of sec. 5.10.4.3 b of Lesson 10. Accordingly, we get:

$$0.36\ f_{ck}\ b_w\ x_u + 0.45\ f_{ck}\,(b_f - b_w)\,y_f = 0.87\ f_y\,A_{st}$$

or $$0.36\,(20)\,(300)\,(x_u) + 0.45\,(20)\,(1000 - 300)\,\{0.15\,x_u + 0.65\,(100)\}$$

$$= 0.87\,(415)\,(2591)$$

or $$x_u = 169.398\ \text{mm} < 216\ \text{mm}\ (x_{u,max} = 0.48\,x_u = 216\ \text{mm})$$

So, the section is under-reinforced.

Step 2: To determine M_u

$$D_f/x_u = 100/169.398 = 0.590 > 0.43$$

This is the problem of case (iii b) of sec. 5.10.4.3 b. The corresponding equations are Eq. 5.15 of Lesson 10 for y_f and Eqs. 5.16 to 18 of Lesson 10 for C, T and M_u, respectively. From Eq. 5.15 of Lesson 10, we have:

$$y_f = 0.15\, x_u + 0.65\, D_f = 0.15\,(169.398) + 0.65\,(100) = 90.409 \text{ mm}$$

From Eq. 5.18 of Lesson 10, we have

$$M_u = 0.36(x_u/d)\{1 - 0.42(x_u/d)\} f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}(b_f - b_w)\, y_f\,(d - y_f/2)$$

or $$M_u = 0.36\,(169.398/450)\,\{1 - 0.42\,(169.398/450)\}\,(20)\,(300)\,(450)\,(450)$$

$$+\ 0.45\,(20)\,(1000 - 300)\,(90.409)\,(450 - 90.409/2)$$

$$= 138.62 + 230.56 = 369.18 \text{ kNm.}$$

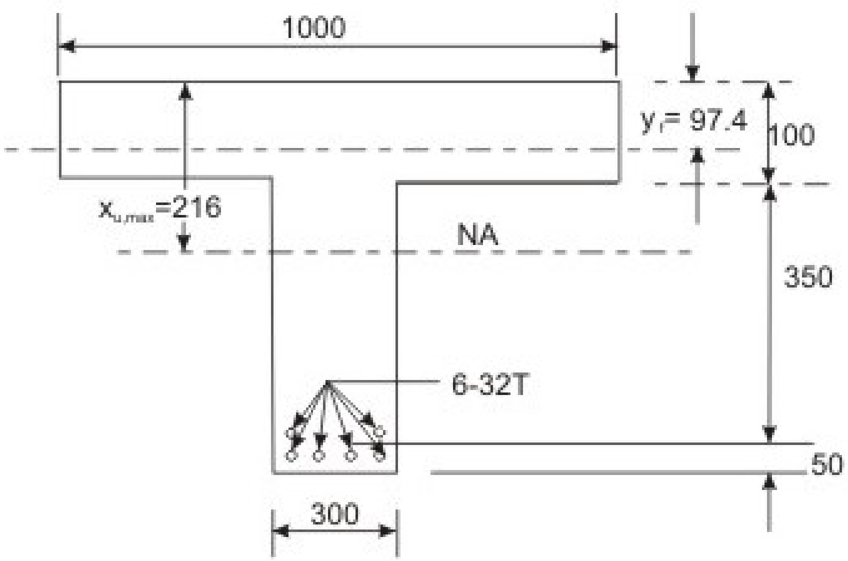

Fig. 5.11.5: Example 4, case (iv b)

Ex.4: Determine the moment of resistance of the flanged beam of Fig. 5.11.5 with A_{st} = 4,825 mm^2 (6- 32 T). Other parameters and data are the same as those of Ex.1: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm. Use M 20 and Fe 415.

Step 1: To determine x_u

Assuming x_u in the flange of under-reinforced rectangular beam we have from Eq. 3.16 of Lesson 5:

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\, b_f\ f_{ck}} = \frac{0.87\,(415)\,(4825)}{0.36\,(1000)\,(20)} = 241.95\ \text{mm} > D_f$$

Here, D_f/d = 100/450 = 0.222 > 0.2. So, we have to determine y_f from Eq. 5.15 and equating C and T from Eqs. 5.16 and 17 of Lesson 10.

$$y_f = 0.15\, x_u + 0.65\ D_f \qquad (5.15)$$

$$0.36\ f_{ck}\ b_w\ x_u + 0.45\ f_{ck}\,(b_f - b_w)\, y_f = 0.87\ f_y\, A_{st} \qquad (5.16 \text{ and } 5.17)$$

or $\quad 0.36\,(20)\,(300)\,(x_u) + 0.45\,(20)\,(1000 - 300)\,\{0.15\, x_u + 0.65\,(100)\}$

$\quad = 0.87\,(415)\,(4825)$

or $\quad 2160\, x_u + 945\, x_u = -\ 409500 + 1742066$

or $\quad x_u = 1332566/3105 = 429.17$ mm

$x_{u,max} = 0.48\,(450) = 216$ mm

Since $x_u > x_{u,max}$, the beam is over-reinforced. Accordingly.

$x_u = x_{u,\,max} = 216$ mm.

Step 2: To determine M_u

This problem belongs to case (iv b), explained in sec.5.10.4.4 b of Lesson 10. So, we can determine Mu from Eq. 5.11 of Lesson 10.

$$M_u = 0.36(x_{u,\,max}/d)\{1 - 0.42(x_{u,\,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\ f_{ck}(b_f - b_w)\, y_f\,(d - y_f/2) \qquad (5.11)$$

where $y_f = 0.15\, x_{u,\,max} + 0.65\ D_f = 97.4$ mm (5.8)

From Eq. 5.11, employing the value of y_f = 97.4 mm, we get:

$M_u = 0.36\,(0.48)\,\{1 - 0.42\,(0.48)\}\,(20)\,(300)\,(450)\,(450)$

$$+ 0.45 (20) (1000 - 300) (97.4) (450 - 97.4/2)$$

$$= 167.63 + 246.24 = 413.87 \text{ kNm}$$

It is seen that this over-reinforced beam has the same M_u as that of the balanced beam of Example 2.

5.11.4 Summary of Results of Examples 1-4

The results of four problems (Exs. 1-4) are given in Table 5.1 below. All the examples are having the common data except A_{st}.

Table 5.1 Results of Examples 1-4 (Figs. 5.11.2 – 5.11.5)

Ex. No.	A_{st} (mm^2)	Case	Section No.	M_u (kNm)	Remarks
1	1,963	(i)	5.10.4.1	290.06	x_u = 98.44 mm < $x_{u,\,max}$ (= 216 mm), $x_u < D_f$ (= 100 mm), Under-reinforced, (NA in the flange).
2	3,066	(ii b)	5.10.4.2 (b)	413.87	$x_u = x_{u,\,max}$ = 216 mm, D_f/d = 0.222 > 0.2, Balanced, (NA in web).
3	2,591	(iii b)	5.10.4.3 (b)	369.18	x_u = 169.398 mm < $x_{u,\,max}$(= 216 mm), D_f/x_u = 0.59 > 0.43, Under-reinforced, (NA in the web).
4	4,825	(iv b)	5.10.4.4 (b)	413.87	x_u = 241.95 mm > $x_{u,\,max}$ (= 216 mm), D_f/d = 0.222 > 0.2, Over-reinforced, (NA in web).

It is clear from the above table (Table 5.1), that Ex.4 is an over-reinforced flanged beam. The moment of resistance of this beam is the same as that of balanced beam of Ex.2. Additional reinforcement of 1,759 mm^2 (= 4,825 mm^2 – 3,066 mm^2) does not improve the M_u of the over-reinforced beam. It rather prevents the beam from tension failure. That is why over-reinforced beams are to be avoided. However, if the M_u has to be increased beyond 413.87 kNm, the flanged beam may be doubly reinforced.

5.11.5 Use of SP-16 for the Analysis Type of Problems

Using the two governing parameters (b_f/b_w) and (D_f/d), the $M_{u,lim}$ of balanced flanged beams can be determined from Tables 57-59 of SP-16 for the three grades of steel (250, 415 and 500). The value of the moment coefficient $M_{u,lim}/b_w d^2 f_{ck}$ of Ex.2, as obtained from SP-16, is presented in Table 5.2 making linear interpolation for both the parameters, wherever needed. $M_{u,lim}$ is then calculated from the moment coefficient.

Table 5.2 $M_{u,lim}$ of Example 2 using Table 58 of SP-16

Parameters: (i) b_f/b_w = 1000/300 = 3.33
(ii) D_f/d = 100/450 = 0.222

$(M_{u,lim}/b_w d^2 f_{ck})$ in N/mm²			
D_f/d	b_f/b_w		
	3	4	3.33
0.22	0.309	0.395	
0.23	0.314	0.402	
0.222	0.31*	0.3964*	0.339*

* by linear interpolation

So, from Table 5.2, $$\frac{M_{u,\lim}}{b_w\, d^2\, f_{ck}} = 0.339$$

$M_{u,lim}$ = $0.339\, b_w\, d^2\, f_{ck}$ = 0.339 (300) (450) (450) (20) 10^{-6} = 411.88 kNm

$M_{u,lim}$ as obtained from SP-16 is close to the earlier computed value of $M_{u,lim}$ = 413.87 kNm (see Table 5.1).

5.11.6 Practice Questions and Problems with Answers

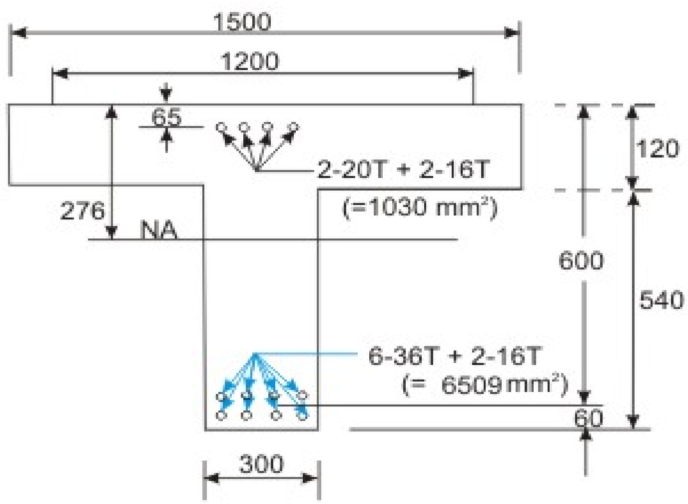

Fig. 5.11.6: Q. 1

Q.1: Determine the moment of resistance of the simply supported doubly reinforced flanged beam (isolated) of span 9 m as shown in Fig. 5.11.6. Assume M 30 concrete and Fe 500 steel.

A.1: Solution of Q.1:

Effective width b_f = $\dfrac{l_o}{(l_o/b)+4} + b_w = \dfrac{9000}{(9000/1500)+4} + 300 = 1200 \text{ mm}$

Step 1: To determine the depth of the neutral axis

Assuming neutral axis to be in the flange and writing the equation $C = T$, we have:

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_f\, x_u + (f_{sc}\, A_{sc} - f_{cc}\, A_{sc})$$

Here, d'/d = 65/600 = 0.108 = 0.1 (say). We, therefore, have f_{sc} = 353 N/mm^2.

From the above equation, we have:

$$x_u = \frac{0.87\,(500)\,(6509) - \{(353)\,(1030) - 0.446\,(30)\,(1030)\}}{0.36\,(30)\,(1200)} = 191.48 \text{ mm} > 120 \text{ mm}$$

So, the neutral axis is in web.

D_f/d = 120/600 = 0.2

Assuming D_f/x_u < 0.43, and Equating $C = T$

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_w\, x_u + 0.446\, f_{ck}\, (b_f - b_w)\, D_f + (f_{sc} - f_{cc})\, A_{sc}$$

$$x_u = \frac{0.87\,(500)\,(6509) - 1030\{353 - 0.446\,(30)\} - 0.446\,(30)\,(1200 - 300)\,(120)}{0.36\,(30)\,(300)}$$

$= 319.92 > 276 \text{ mm } (x_{u,max} = 276 \text{ mm})$

So, $x_u = x_{u,max}$ = 276 mm (over-reinforced beam).

D_f/x_u = 120/276 = 0.4347 > 0.43

Let us assume D_f/x_u > 0.43. Now, equating $C = T$ with y_f as the depth of flange having constant stress of 0.446 f_{ck}. So, we have:

$$y_f = 0.15\, x_u + 0.65\, D_f = 0.15\, x_u + 78$$

$$0.36\, f_{ck}\, b_w\, x_u + 0.446\, f_{ck}\, (b_f - b_w)\, y_f + A_{sc}\, (f_{sc} - f_{cc}) = 0.87\, f_y\, A_{st}$$

$$0.36\,(30)\,(300)\,x_u + 0.446\,(30)\,(900)\,(0.15\,x_u + 78)$$

$$= 0.87\,(500)\,(6509) - 1030\,\{353 - 0.446\,(30)\}$$

or $\quad x_u = 305.63 \text{ mm} > x_{u,max}. \quad (x_{u,max} = 276 \text{ mm})$

The beam is over-reinforced. Hence, $x_u = x_{u,max} = 276$ mm. This is a problem of case (iv), and we, therefore, consider the case (ii) to find out the moment of resistance in two parts: first for the balanced singly reinforced beam and then for the additional moment due to compression steel.

Step 2: Determination of $x_{u,lim}$ for singly reinforced flanged beam

Here, $D_f/d = 120/600 = 0.2$, so y_f is not needed. This is a problem of case (ii a) of sec. 5.10.4.2 of Lesson 10. Employing Eq. 5.7 of Lesson 10, we have:

$$M_{u,lim} = 0.36\,(x_{u,max}/d)\,\{1 - 0.42\,(x_{u,max}/d)\}\,f_{ck}\,b_w\,d^2$$

$$+ 0.45\,f_{ck}\,(b_f - b_w)\,D_f\,(d - D_f/2)$$

$$= 0.36(0.46)\,\{1 - 0.42(0.46)\}\,(30)\,(300)\,(600)\,(600) + 0.45(30)\,(900)\,(120)\,(540)$$

$$= 1{,}220.20 \text{ kNm}$$

$$A_{st,\lim} = \frac{M_{u,\lim}}{0.87\,f_y\,d\,\{1 - 0.42\,(x_{u,max}/d)\}}$$

$$= \frac{(1220.20)\,(10^6)}{(0.87)\,(500)\,(600)\,(0.8068)} = 5{,}794.6152 \text{ mm}^2$$

Step 3: Determination of M_{u2}

Total $A_{st} = 6{,}509$ mm^2, $A_{st,lim} = 5{,}794.62$ mm^2

$A_{st2} = 714.38$ mm^2 and $A_{sc} = 1{,}030$ mm^2

It is important to find out how much of the total A_{sc} and A_{st2} are required effectively. From the equilibrium of C and T forces due to additional steel (compressive and tensile), we have:

$$(A_{st2})\,(0.87)\,(f_y) = (A_{sc})\,(f_{sc})$$

If we assume $A_{sc} = 1{,}030$ mm^2

$$A_{st2} = \frac{1030\,(353)}{0.87\,(500)} = 835.84 \text{ mm}^2 > 714.38 \text{ mm}^2$$, (714.38 mm^2 is the total A_{st2} provided). So, this is not possible.

Now, using A_{st2} = 714.38 mm^2 , we get A_{sc} from the above equation.

$$A_{sc} = \frac{(714.38)\,(0.87)\,(500)}{353} = 880.326 < 1{,}030 \text{ mm}^2$$, (1,030 mm^2 is the total A_{sc} provided).

$$M_{u2} = A_{sc}\, f_{sc}\,(d - d') = (880.326)\,(353)\,(600 - 60) = 167.807 \text{ kNm}$$

Total moment of resistance = $M_{u,lim}$ + M_{u2} = 1,220.20 + 167.81 = 1,388.01 kNm

Total A_{st} required = $A_{st,lim}$ + A_{st2} = 5,794.62 + 714.38 = 6,509.00 mm^2 , (provided A_{st} = 6,509 mm^2)

A_{sc} required = 880.326 mm^2 (provided 1,030 mm^2).

5.11.7 References

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5.11.8 Test 11 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1: Determine $M_{u,lim}$ of the flanged beam of Ex. 2 (Fig. 5.11.3) with the help of SP-16 using (a) M 20 and Fe 250, (b) M 20 and Fe 500 and (c) compare the results with the $M_{u,lim}$ of Ex. 2 from Table 5.2 when grades of concrete and steel are M 20 and Fe 415, respectively. Other data are: b_f = 1000 mm, D_f = 100 mm, b_w = 300 mm, cover = 50 mm and d = 450 mm.

(10 X 3 = 30 marks)

A.TQ.1: From the results of Ex. 2 of sec. 5.11.5 (Table 5.2), we have:

Parameters: (i) b_f/b_w = 1000/300 = 3.33
(ii) D_f/d = 100/450 = 0.222

For part (a): When Fe 250 is used, the corresponding table is Table 57 of SP-16. The computations are presented in Table 5.3 below:

Table 5.3 ($M_{u,lim}/b_w d^2 f_{ck}$) in N/mm^2 Of TQ.1 (PART a for M 20 and Fe 250)

($M_{u,lim}/b_w d^2 f_{ck}$) in N/mm^2			
D_f/d	b_f/b_w		
	3	4	3.33
0.22	0.324	0.411	
0.23	0.330	0.421	
0.222	0.3252*	0.413*	0.354174*

- by linear interpolation

$M_{u,lim}/b_w d^2 f_{ck}$ = 0.354174 = 0.354 (say)

So, $M_{u,lim}$ = (0.354) (300) (450) (450) (20) N mm = 430.11 kNm

For part (b): When Fe 500 is used, the corresponding table is Table 59 of SP-16. The computations are presented in Table 5.4 below:

Table 5.4 ($M_{u,lim}/b_w d^2 f_{ck}$) in N/mm^2 Of TQ.1 (PART b for M 20 and Fe 500)

($M_{u,lim}/b_w d^2 f_{ck}$) in N/mm^2			
D_f/d	b_f/b_w		
	3	4	3.33
0.22	0.302	0.386	
0.23	0.306	0.393	
0.222	0.3028*	0.3874*	0.330718*

* by linear interpolation

$M_{u,lim}/b_w d^2 f_{ck}$ = 0.330718 = 0.3307 (say)

So, $M_{u,lim}$ = (0.3307) (300) (450) (450) (20) mm = 401.8 kNm

For part (c): Comparison of results of this problem with that of Table 5.2 (M 20 and Fe 415) is given below in Table 5.5.

Table 5.5 Comparison of results of $M_{u,lim}$

Sl. No.	Grade of Steel	$M_{u,lim}$ (kNm)
1	Fe 250	430.11
2	Fe 415	411.88
3	Fe 500	401.80

It is seen that $M_{u,lim}$ of the beam decreases with higher grade of steel for a particular grade of concrete.

TQ.2: With the aid of SP-16, determine separately the limiting moments of resistance and the limiting areas of steel of the simply supported isolated, singly reinforced and balanced flanged beam of Q.1 as shown in Fig. 5.11.6 if the span = 9 m. Use M 30 concrete and three grades of steel, Fe 250, Fe 415 and Fe 500, respectively. Compare the results obtained above with that of Q.1 of sec. 5.11.6, when balanced.

(15 + 5 = 20 marks)

A.TQ.2: From the results of Q.1 sec. 5.11.6, we have:

Parameters: (i) b_f/b_w = 1200/300 = 4.0

(ii) D_f/d = 120/600 = 0.2

For Fe 250, Fe 415 and Fe 500, corresponding tables are Table 57, 58 and 59, respectively of SP-16. The computations are done accordingly. After computing the limiting moments of resistance, the limiting areas of steel are determined as explained below. Finally, the results are presented in Table 5.6 below:

$$A_{st,\lim} = \frac{M_{u,\lim}}{0.87\, f_y\, d\, \{1 - 0.42\, (x_{u,max} / d\,)\}}$$

Table 5.6 Values of $M_{u,lim}$ in N/mm^2 Of TQ.2

Grade of Fe / Q.1 of sec. 5.11.6	($M_{u,lim}/b_w\ d^2\ f_{ck}$) (N/mm^2)	$M_{u,lim}$ (kNm)	$A_{st,lim}$ (mm^2)
Fe 250	0.39	1, 263.60	12,455.32
Fe 415	0.379	1, 227.96	7,099.78
Fe 500	0.372	1, 205.28	5,723.76
Q.1 of sec. 5.11.6 (Fe 415)		1, 220.20	5,794.62

The maximum area of steel allowed is .04 $b\ D$ = (.04) (300) (660) = 7,920 mm^2 . Hence, Fe 250 is not possible in this case.

5.11.9 Summary of this Lesson

This lesson mentions about the two types of numerical problems (i) analysis and (ii) design types. In addition to explaining the steps involved in solving the analysis type of numerical problems, several examples of analysis type of problems are illustrated explaining all steps of the solutions both by direct computation method and employing SP-16. Solutions of practice and test problems will give readers the confidence in applying the theory explained in Lesson 10 in solving the numerical problems.