

Module 4

Doubly Reinforced Beams – Theory and Problems

Lesson

8

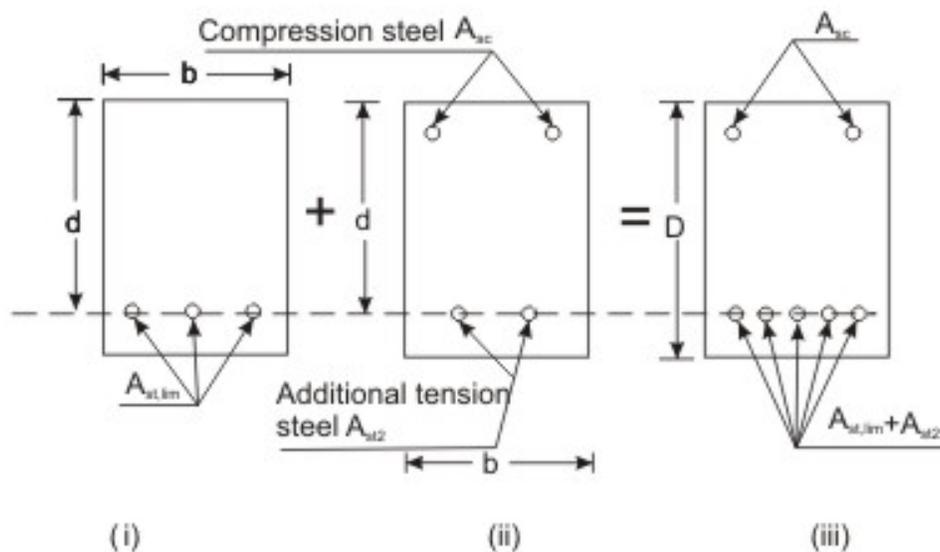
Doubly Reinforced
Beams – Theory

Instructional Objectives:

At the end of this lesson, the student should be able to:

- explain the situations when doubly reinforced beams are designed,
- name three cases other than doubly reinforced beams where compression reinforcement is provided,
- state the assumptions of analysis and design of doubly reinforced beams,
- derive the governing equations of doubly reinforced beams,
- calculate the values of f_{sc} from (i) d'/d and (ii) calculating the strain of the compression reinforcement,
- state the minimum and maximum amounts of A_{sc} and A_{st} in doubly reinforced beams,
- state the two types of numerical problems of doubly reinforced beams,
- name the two methods of solving the two types of problems, and
- write down the steps of the two methods for each of the two types of problems.

4.8.1 Introduction



For(i) » $M_{u1} = M_{u1m}$

For(ii) » $M_{u2} =$ Due to A_{sc} and A_{st2}

For(iii) » $M_u = M_{u1m} + M_{u2}$

Fig. 4.8.1: Doubly reinforced beam

Concrete has very good compressive strength and almost negligible tensile strength. Hence, steel reinforcement is used on the tensile side of concrete. Thus, singly reinforced beams reinforced on the tensile face are good both in compression and tension. However, these beams have their respective limiting moments of resistance with specified width, depth and grades of concrete and steel. The amount of steel reinforcement needed is known as $A_{st,lim}$. Problem will arise, therefore, if such a section is subjected to bending moment greater than its limiting moment of resistance as a singly reinforced section.

There are two ways to solve the problem. First, we may increase the depth of the beam, which may not be feasible in many situations. In those cases, it is possible to increase both the compressive and tensile forces of the beam by providing steel reinforcement in compression face and additional reinforcement in tension face of the beam without increasing the depth (Fig. 4.8.1). The total compressive force of such beams comprises (i) force due to concrete in compression and (ii) force due to steel in compression. The tensile force also has two components: (i) the first provided by $A_{st,lim}$ which is equal to the compressive force of concrete in compression. The second part is due to the additional steel in tension - its force will be equal to the compressive force of steel in compression. Such reinforced concrete beams having steel reinforcement both on tensile and compressive faces are known as doubly reinforced beams.

Doubly reinforced beams, therefore, have moment of resistance more than the singly reinforced beams of the same depth for particular grades of steel and concrete. In many practical situations, architectural or functional requirements may restrict the overall depth of the beams. However, other than in doubly reinforced beams compression steel reinforcement is provided when:

- (i) some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone or vice versa.
- (ii) the ductility requirement has to be followed.
- (iii) the reduction of long term deflection is needed.

It may be noted that even in so called singly reinforced beams there would be longitudinal hanger bars in compression zone for locating and fixing stirrups.

4.8.2 Assumptions

- (i) The assumptions of sec. 3.4.2 of Lesson 4 are also applicable here.
- (ii) Provision of compression steel ensures ductile failure and hence, the limitations of x/d ratios need not be strictly followed here.

- (iii) The stress-strain relationship of steel in compression is the same as that in tension. So, the yield stress of steel in compression is $0.87 f_y$.

4.8.3 Basic Principle

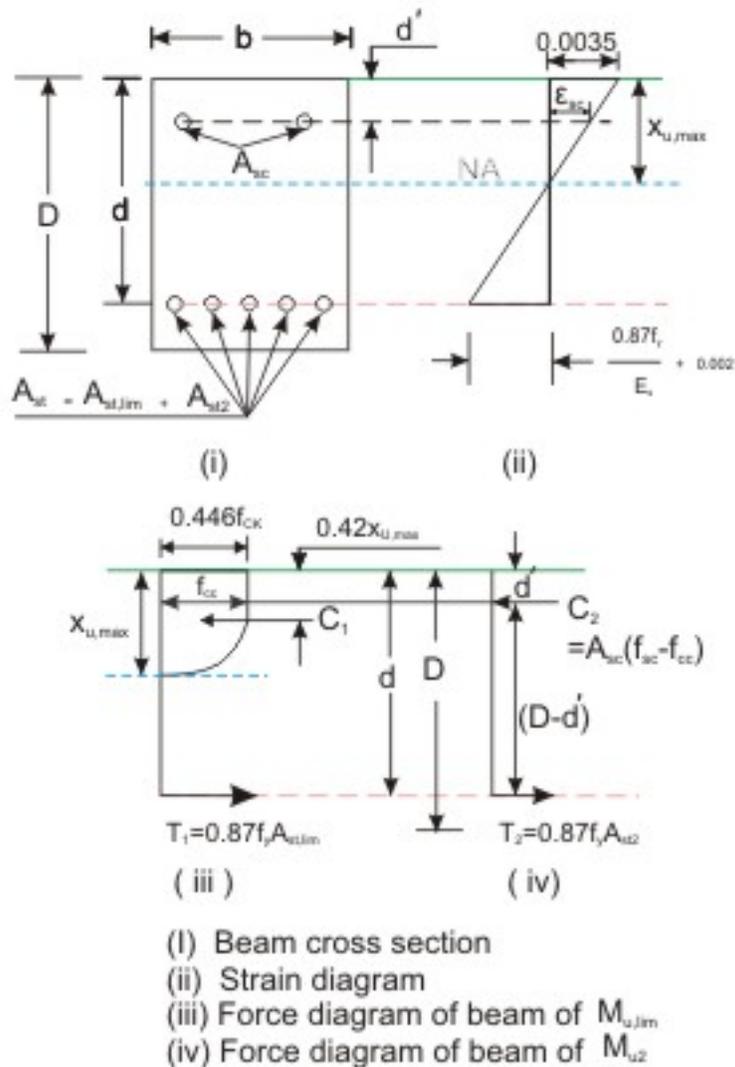


Fig. 4.8.2: Stress, strain and force diagrams of doubly reinforced beam

As mentioned in sec. 4.8.1, the moment of resistance M_u of the doubly reinforced beam consists of (i) $M_{u,lim}$ of singly reinforced beam and (ii) M_{u2} because of equal and opposite compression and tension forces (C_2 and T_2) due to additional steel reinforcement on compression and tension faces of the beam (Figs. 4.8.1 and 2). Thus, the moment of resistance M_u of a doubly reinforced beam is

$$M_U = M_{U,lim} + M_{U2} \quad (4.1)$$

The $M_{U,lim}$ is as given in Eq. 3.24 of Lesson 5, i.e.,

$$M_{U,lim} = 0.36 \left(\frac{x_{u,max}}{d} \right) \left(1 - 0.42 \frac{x_{u,max}}{d} \right) b d^2 f_{ck} \quad (4.2)$$

Also, $M_{U,lim}$ can be written from Eq. 3.22 of Lesson 5, using $x_u = x_{u,max}$, i.e.,

$$\begin{aligned} M_{U,lim} &= 0.87 A_{st,lim} f_y (d - 0.42 x_{u,max}) \\ &= 0.87 p_{t,lim} \left(1 - 0.42 \frac{x_{u,max}}{d} \right) b d^2 f_y \end{aligned} \quad (4.3)$$

The additional moment M_{U2} can be expressed in two ways (Fig. 4.8.2): considering (i) the compressive force C_2 due to compression steel and (ii) the tensile force T_2 due to additional steel on tension face. In both the equations, the lever arm is $(d - d')$. Thus, we have

$$M_{u2} = A_{sc} (f_{sc} - f_{cc}) (d - d') \quad (4.4)$$

$$M_{u2} = A_{st2} (0.87 f_y) (d - d') \quad (4.5)$$

where A_{sc} = area of compression steel reinforcement

f_{sc} = stress in compression steel reinforcement

f_{cc} = compressive stress in concrete at the level of centroid of compression steel reinforcement

A_{st2} = area of additional steel reinforcement

Since the additional compressive force C_2 is equal to the additional tensile force T_2 , we have

$$A_{sc} (f_{sc} - f_{cc}) = A_{st2} (0.87 f_y) \quad (4.6)$$

Any two of the three equations (Eqs. 4.4 - 4.6) can be employed to determine A_{sc} and A_{st2} .

The total tensile reinforcement A_{st} is then obtained from:

$$A_{st} = A_{st1} + A_{st2} \quad (4.7)$$

$$\text{where } A_{st1} = p_{t,\text{lim}} \frac{b d}{100} = \frac{M_{u,\text{lim}}}{0.87 f_y (d - 0.42 x_{u,\text{max}})} \quad (4.8)$$

4.8.4 Determination of f_{sc} and f_{cc}

It is seen that the values of f_{sc} and f_{cc} should be known before calculating A_{sc} . The following procedure may be followed to determine the value of f_{sc} and f_{cc} for the design type of problems (and not for analysing a given section). For the design problem the depth of the neutral axis may be taken as $x_{u,\text{max}}$ as shown in Fig. 4.8.2. From Fig. 4.8.2, the strain at the level of compression steel reinforcement ε_{sc} may be written as

$$\varepsilon_{sc} = 0.0035 \left(1 - \frac{d'}{x_{u,\text{max}}} \right) \quad (4.9)$$

The stress in compression steel f_{sc} is corresponding to the strain ε_{sc} of Eq. 4.9 and is determined for (a) mild steel and (b) cold worked bars Fe 415 and 500 as given below:

(a) Mild steel Fe 250

The strain at the design yield stress of 217.39 N/mm^2 ($f_d = 0.87 f_y$) is 0.0010869 ($= 217.39/E_s$). The f_{sc} is determined from the idealized stress-strain diagram of mild steel (Fig. 1.2.3 of Lesson 2 or Fig. 23B of IS 456) after computing the value of ε_{sc} from Eq. 4.9 as follows:

- (i) If the computed value of $\varepsilon_{sc} \leq 0.0010869$, $f_{sc} = \varepsilon_{sc} E_s = 2 (10^5) \varepsilon_{sc}$
- (ii) If the computed value of $\varepsilon_{sc} > 0.0010869$, $f_{sc} = 217.39 \text{ N/mm}^2$.

(b) Cold worked bars Fe 415 and Fe 500

The stress-strain diagram of these bars is given in Fig. 1.2.4 of Lesson 2 and in Fig. 23A of IS 456. It shows that stress is proportional to strain up to a stress of $0.8 f_y$. The stress-strain curve for the design purpose is obtained by substituting f_{yd} for f_y in the figure up to $0.8 f_{yd}$. Thereafter, from $0.8 f_{yd}$ to f_{yd} , Table A of SP-16 gives the values of total strains and design stresses for Fe 415 and Fe 500. Table 4.1 presents these values as a ready reference here.

Table 4.1 Values of f_{sc} and ϵ_{sc}

Stress level	Fe 415		Fe 500	
	Strain ϵ_{sc}	Stress f_{sc} (N/mm ²)	Strain ϵ_{sc}	Stress f_{sc} (N/mm ²)
0.80 f_{yd}	0.00144	288.7	0.00174	347.8
0.85 f_{yd}	0.00163	306.7	0.00195	369.6
0.90 f_{yd}	0.00192	324.8	0.00226	391.3
0.95 f_{yd}	0.00241	342.8	0.00277	413.0
0.975 f_{yd}	0.00276	351.8	0.00312	423.9
1.0 f_{yd}	0.00380	360.9	0.00417	434.8

Linear interpolation may be done for intermediate values.

The above procedure has been much simplified for the cold worked bars by presenting the values of f_{sc} of compression steel in doubly reinforced beams for different values of d'/d only taking the practical aspects into consideration. In most of the doubly reinforced beams, d'/d has been found to be between 0.05 and 0.2. Accordingly, values of f_{sc} can be computed from Table 4.1 after determining the value of ϵ_{sc} from Eq. 4.9 for known values of d'/d as 0.05, 0.10, 0.15 and 0.2. Table F of SP-16 presents these values of f_{sc} for four values of d'/d (0.05, 0.10, 0.15 and 0.2) of Fe 415 and Fe 500. Table 4.2 below, however, includes Fe 250 also whose f_{sc} values are computed as laid down in sec. 4.8.4(a) (i) and (ii) along with those of Fe 415 and Fe 500. This table is very useful and easy to determine the f_{sc} from the given value of d'/d . The table also includes strain values at yield which are explained below:

(i) The strain at yield of Fe 250 =

$$\frac{\text{Design Yield Stress}}{E_s} = \frac{250}{1.15 (200000)} = 0.0010869$$

Here, there is only elastic component of the strain without any inelastic strain.

(ii) The strain at yield of Fe 415 = Inelastic Strain + $\frac{\text{Design Yield Stress}}{E_s}$

$$= 0.002 + \frac{415}{1.15 (200000)} = 0.0038043$$

(iii) The strain at yield of Fe 500 = $0.002 + \frac{500}{1.15 (200000)} = 0.0041739$

Table 4.2 Values of f_{sc} for different values of d'/d

f_y (N/mm ²)	d'/d				Strain at yield
	0.05	0.10	0.15	0.20	
250	217.4	217.4	217.4	217.4	0.0010869
415	355	353	342	329	0.0038043
500	412	412	395	370	0.0041739

4.8.5 Minimum and maximum steel

4.8.5.1 In compression

There is no stipulation in IS 456 regarding the minimum compression steel in doubly reinforced beams. However, hangers and other bars provided up to 0.2% of the whole area of cross section may be necessary for creep and shrinkage of concrete. Accordingly, these bars are not considered as compression reinforcement. From the practical aspects of consideration, therefore, the minimum steel as compression reinforcement should be at least 0.4% of the area of concrete in compression or 0.2% of the whole cross-sectional area of the beam so that the doubly reinforced beam can take care of the extra loads in addition to resisting the effects of creep and shrinkage of concrete.

The maximum compression steel shall not exceed 4 per cent of the whole area of cross-section of the beam as given in cl. 26.5.1.2 of IS 456.

4.8.5.2 In tension

As stipulated in cl. 26.5.1.1(a) and (b) of IS 456, the minimum amount of tensile reinforcement shall be at least $(0.85 bd/f_y)$ and the maximum area of tension reinforcement shall not exceed $(0.04 bD)$.

It has been discussed in sec. 3.6.2.3 of Lesson 6 that the singly reinforced beams shall have A_{st} normally not exceeding 75 to 80% of $A_{st,lim}$ so that x_u remains less than $x_{u,max}$ with a view to ensuring ductile failure. However, in the case of doubly reinforced beams, the ductile failure is ensured with the presence of compression steel. Thus, the depth of the neutral axis may be taken as $x_{u,max}$ if the beam is over-reinforced. Accordingly, the A_{st1} part of tension steel can go up to $A_{st,lim}$ and the additional tension steel A_{st2} is provided for the additional moment $M_u - M_{u,lim}$. The quantities of A_{st1} and A_{st2} together form the total A_{st} , which shall not exceed $0.04 bD$.

4.8.6 Types of problems and steps of solution

Similar to the singly reinforced beams, the doubly reinforced beams have two types of problems: (i) design type and (ii) analysis type. The different steps of solutions of these problems are taken up separately.

4.8.6.1 Design type of problems

In the design type of problems, the given data are b , d , D , grades of concrete and steel. The designer has to determine A_{sc} and A_{st} of the beam from the given factored moment. These problems can be solved by two ways: (i) use of the equations developed for the doubly reinforced beams, named here as direct computation method, (ii) use of charts and tables of SP-16.

(a) Direct computation method

Step 1: To determine $M_{u, lim}$ and $A_{st, lim}$ from Eqs. 4.2 and 4.8, respectively.

Step 2: To determine M_{u2} , A_{sc} , A_{st2} and A_{st} from Eqs. 4.1, 4.4, 4.6 and 4.7, respectively.

Step 3: To check for minimum and maximum reinforcement in compression and tension as explained in sec. 4.8.5.

Step 4: To select the number and diameter of bars from known values of A_{sc} and A_{st} .

(b) Use of SP table

Tables 45 to 56 present the p_t and p_c of doubly reinforced sections for $d'/d = 0.05, 0.10, 0.15$ and 0.2 for different f_{ck} and f_y values against M_u/bd^2 . The values of p_t and p_c are obtained directly selecting the proper table with known values of M_u/bd^2 and d'/d .

4.8.6.2 Analysis type of problems

In the analysis type of problems, the data given are b , d , d' , D , f_{ck} , f_y , A_{sc} and A_{st} . It is required to determine the moment of resistance M_u of such beams. These problems can be solved: (i) by direct computation method and (ii) by using tables of SP-16.

(a) Direct computation method

Step 1: To check if the beam is under-reinforced or over-reinforced.

First, $x_{u,max}$ is determined assuming it has reached limiting stage using $\frac{x_{u,max}}{d}$ coefficients as given in cl. 38.1, Note of IS 456. The strain of tensile steel ϵ_{st} is computed from $\epsilon_{st} = \frac{\epsilon_c (d - x_{u,max})}{x_{u,max}}$ and is checked if ϵ_{st} has reached the yield strain of steel:

$$\epsilon_{st \text{ at yield}} = \frac{f_y}{1.15 (E)} + 0.002$$

The beam is under-reinforced or over-reinforced if ϵ_{st} is less than or more than the yield strain.

Step 2: To determine $M_{u,lim}$ from Eq. 4.2 and $A_{st,lim}$ from the $\rho_{t,lim}$ given in Table 3.1 of Lesson 5.

Step 3: To determine A_{st2} and A_{sc} from Eqs. 4.7 and 4.6, respectively.

Step 4: To determine M_{u2} and M_u from Eqs. 4.4 and 4.1, respectively.

(b) Use of tables of SP-16

As mentioned earlier Tables 45 to 56 are needed for the doubly reinforced beams. First, the needed parameters d'/d , ρ_t and ρ_c are calculated. Thereafter, M_u/bd^2 is computed in two stages: first, using d'/d and ρ_t and then using d'/d and ρ_c . The lower value of M_u is the moment of resistance of the beam.

4.8.7 Practice Questions and Problems with Answers

Q.1: When do we go for doubly reinforced beams ?

A.1: The depth of the beams may be restricted for architectural and/or functional requirements. Doubly reinforced beams are designed if such beams of restricted depth are required to resist moment more than its $M_{u,lim}$.

Q.2: Name three situations other than doubly reinforced beams, where the compression reinforcement is provided.

A.2: Compression reinforcement is provided when:

- (i) Some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone,

- (ii) the ductility requirement has to be satisfied,
- (iii) the reduction of long term deflection is needed.

Q.3: State the assumptions of the analysis and design of doubly reinforced beams.

A.3: See sec. 4.8.2 (i), (ii) and (iii).

Q.4: Derive the governing equations of a doubly reinforced beam.

A.4: See sec. 4.8.3

Q.5: How do you determine f_{sc} of mild steel and cold worked bars and f_{cc} ?

A.5: See sec. 4.8.4

Q.6: State the minimum and maximum amounts of A_{sc} and A_{st} in doubly reinforced beams.

A.6: See sec. 4.8.5

Q.7: State the two types of problems of doubly reinforced beams specifying the given data and the values to be determined in the two types of problems.

A.7: The two types of problems are:

- (i) Design type of problems and
- (ii) Analysis type of problems

(i) Design type of problems:

The given data are b, d, D, f_{ck}, f_y and M_u . It is required to determine A_{sc} and A_{st} .

(ii) Analysis type of problems:

The given data are $b, d, D, f_{ck}, f_y, A_{sc}$ and A_{st} . It is required to determine the M_u of the beam.

Q.8: Name the two methods of solving the two types of problems.

A.8: The two methods of solving the two types of problems are:

- (i) Direct computation method, and
- (ii) Use of tables of SP-16.

Q.9: Write down the steps of the solution by the two methods of each of the two types of problems.

A.9: (A) For the design type of problems:

(i) See sec. 4.8.6.1(a) for the steps of direct computation method, and

(ii) See sec. 4.8.6.1(b) for the steps of using the tables of SP-16

(B) For the analysis type of problems:

(i) See sec. 4.8.6.2 (a) for the steps of direct computation method, and

(ii) See sec. 4.8.6.2 (b) for the steps of using the tables of SP-16.

4.8.8 References

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4.8.9 Test 8 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1: Derive the governing equations of a doubly reinforced beam. (10 marks)

A.TQ.1: See sec. 4.8.3

TQ.2: State the two types of problems of doubly reinforced beams specifying the given data and the values to be determined in the two type of problems. (8 marks)

A.TQ.2: The two types of problems are:

- (i) Design type of problems and
- (ii) Analysis type of problems

(i) Design type of problems:

The given data are b, d, D, f_{ck}, f_y and M_u . It is required to determine A_{sc} and A_{st} .

(ii) Analysis type of problems:

The given data are $b, d, D, f_{ck}, f_y, A_{sc}$ and A_{st} . It is required to determine the M_u of the beam.

TQ.3: Write down the steps of the solution by the two methods of each of the two types of problems. (8 marks)

A.TQ.3: (A) For the design type of problems:

- (i) See sec. 4.8.6.1(a) for the steps of direct computation method, and
- (ii) See sec. 4.8.6.1(b) for the steps of using the tables of SP-16

(B) For the analysis type of problems:

(i) See sec. 4.8.6.2 (a) for the steps of direct computation method, and

(ii) See sec. 4.8.6.2 (b) for the steps of using the tables of SP-16.

TQ.4: How do you determine f_{sc} of mild steel and cold worked bars and f_{cc} ?
(8 marks)

A.TQ.4: See sec. 4.8.4

TQ.5: State the assumptions of the analysis and design of doubly reinforced beams.

(8 marks)

A.TQ.5: See sec. 4.8.2 (i), (ii) and (iii).

TQ.6: Name three situations other than doubly reinforced beams, where the compression reinforcement is provided.

(8 marks)

A.TQ.6: Compression reinforcement is provided when:

- (i) Some sections of a continuous beam with moving loads undergo change of sign of the bending moment which makes compression zone as tension zone,
- (ii) the ductility requirement has to be satisfied,
- (iii) the reduction of long term deflection is needed.

4.8.10 Summary of this Lesson

Lesson 8 derives the governing equations of the doubly reinforced beams explaining different assumptions and situations when they are needed. The methods of determination of compressive stress in steel are illustrated. The two types of problems and the different steps of solution of them by two different methods are explained.