

Module 14

Tension Members

Lesson 37 Numerical Problems

Instructional Objectives:

At the end of this lesson, the student should be able to:

- select the category of the problem if it is a liquid retaining structure or a structure having no contact with the liquid,
- identify the appropriate equations to be applied for the solution of a particular problem belonging to specific category,
- select the permissible stresses in concrete and steel depending on the category of the problems,
- apply the appropriate equations for the solution of the problems,
- solve the problems by establishing the equations from the first principle,
- revise the sections, if needed, to satisfy the requirements as per IS codes.

14.37.1 Introduction

The requirements for the design of tension structures / members are discussed in Lesson 36. The stipulations of Indian Standard Codes are also mentioned. Tension structures / members are classified depending on (i) if they are liquid retaining or not having any contact with the liquid, (ii) if the tensile stresses are developed due to pure axial force, pure moment or combined effects of axial force and moment, (iii) if the tension is developed on the liquid face or on the remote face and (iv) if the depth of the member is less than, equal to or greater than 225 mm when the tension is developed in the remote face. Accordingly, the permissible stresses of concrete and steel are different. The equations are also different based on if the section is assumed to be cracked or uncracked. All these issues are explained in Lesson 36.

This lesson explains the applications of the established equations for the different cases as mentioned above through several numerical problems. The problems are solved in step-by-step mostly utilising the equations. However, a few problems are solved from the first principle for better understanding. Understanding the solutions of illustrative examples and solutions of the practice and test problems will give confidence in applying the appropriate equations and selecting the permissible stresses for analysing and designing tension structures or members using working stress method and following the stipulations of Indian Standard Codes.

14.37.2 Numerical Problems

Problem 1. Determine the dimensions and area of tension steel of a reinforced concrete rectangular tie member of a truss for carrying a direct tensile force $F_t = 470$ kN using M 25 and Fe 415.

Solution 1. The theory of such tensile members made of reinforced concrete is discussed in sec.14.36.5 of Lesson 36. We have Eqs. 14.1 and 14.2 to determine the required parameters. The modular ratio m of M 25 concrete is $93.33/8.5 = 10.98$, f_{st} of Fe 415 is 230 N/mm^2 and $f_{td} \leq 3.2 \text{ N/mm}^2$ of M 25 concrete (Table 13.1 of Lesson 34).

Step 1. Determination of A_{st}

From Eq. 14.1 of Lesson 36, we have: $A_{st} = F_t / f_{st}$ assuming that f_{st} , the actual stress will reach the value of $\sigma_{st} = 230 \text{ N/mm}^2$, which gives $A_{st} = 470,000/230 = 2043.48 \text{ mm}^2$. Provide 4 bars of 20 mm diameter and 4 bars of 16 mm diameter ($= 2060 \text{ mm}^2$).

Step 2. Determination of width and depth of the rectangular tie member

From Eq. 14.2 of Lesson 36, we have: $A_c = (F_t / f_{td}) - m A_{st}$
or $A_g = (F_t / f_{td}) - (m - 1)A_{st}$, where A_g = gross area of the tie member. Substituting the values of $F_t = 470$ kN, $f_{td} = \sigma_{td} = 3.2 \text{ N/mm}^2$, $m = 10.98$ and $A_{st} = 2060 \text{ mm}^2$, we get $A_c = 126316.2 \text{ mm}^2$. Provide $300 \text{ mm} \times 425 \text{ mm}$ ($= 127500 \text{ mm}^2$) as the width and depth of the tie member.

Step 3. Checking for the tensile stress in concrete

From Eq. 14.2 of Lesson 36, we have:

$$f_{td} = F_t / (A_c + m A_{st}) = F_t / \{A_g + (m - 1)A_{st}\} \\ = 3.17 \text{ N/mm}^2 < 3.2 \text{ N/mm}^2. \text{ Hence, ok}$$

Therefore, the tie member of $300 \text{ mm} \times 425 \text{ mm}$ with 4 bars of 20 mm diameter and 4 bars of 16 mm diameter is the solution.

Problem 2. Design a bunker wall of 250 mm depth for resisting a moment of 33 kNm using M 20 and Fe 415.

Solution 2. Let us assume the section as cracked. The necessary equations of a cracked section subjected to pure flexure are given in sec.14.36.6B. For M 20 grade of concrete, $m = 93.33/7 = 13.33$ and $k_b = 93.33 / (\sigma_{st} + 93.33)$, as per Eq. 13.3 of Lesson 36. Here, $k_b = 0.288$, using

$\sigma_{st} = 230 \text{ N/mm}^2$ and assume $d = 225 \text{ mm}$ from the given value of $D = 250 \text{ mm}$.

Step 1. Determination of preliminary area of steel

The lever arm $= jd = (1 - k_b / 3) d = 0.904 (225) = 203.4 \text{ mm}$.
Therefore, from Eq. 13.8 of Lesson 34, preliminary $A_{st} = M / \{\sigma_{st} (\text{lever arm})\} = 33 (10^6) / (230) (203.4) = 705.399 \text{ mm}^2$. Let us provide 12 mm diameter bars @ 160 mm c/c to have $707 \text{ mm}^2/\text{m}$.

Step 2. Determination of depth of neutral axis

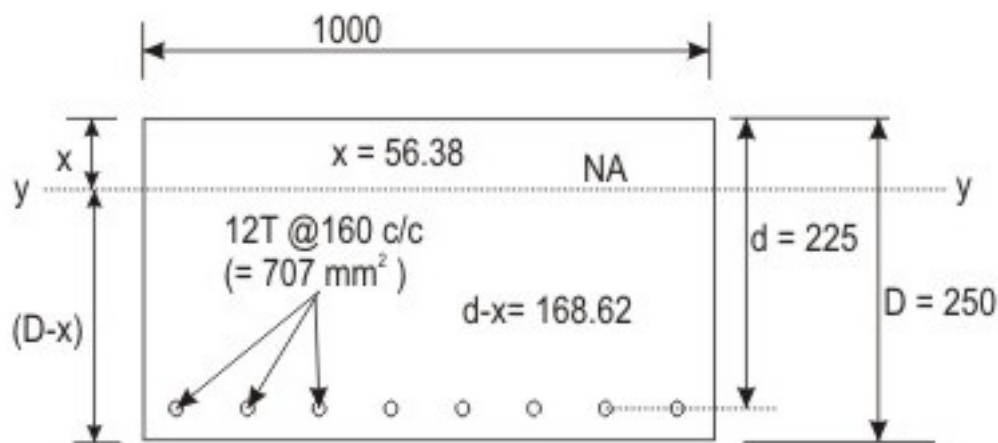


Fig. 14.37.1: Problem 2 - assumed section

The depth of neutral axis can be determined from Eq. 13.16 of Lesson 34. Alternatively, we can determine x by taking moment of the area of concrete and steel, converting it into equivalent concrete. With reference to Fig. 14.37.1, taking moment of the two areas about the neutral axis, $1000 (x^2/2) = 13.33 (707) (225-x)$ or $x^2 + 18.84862 x - 4240.9395 = 0$, which gives $x = 56.38 \text{ mm}$. This gives the lever arm $= 225 - 56.38/3 = 206.21 \text{ mm}$.

Step 3. Determination of tensile stress of steel

Tensile stress of steel $f_{st} = M / \{A_{st} (\text{lever arm})\} = 33 (10^6) / \{(707) (206.21)\} = 226.35 \text{ N/mm}^2 < 230 \text{ N/mm}^2$. Hence, the section is ok.

Step 4. Checking of the maximum bending compression stress in concrete

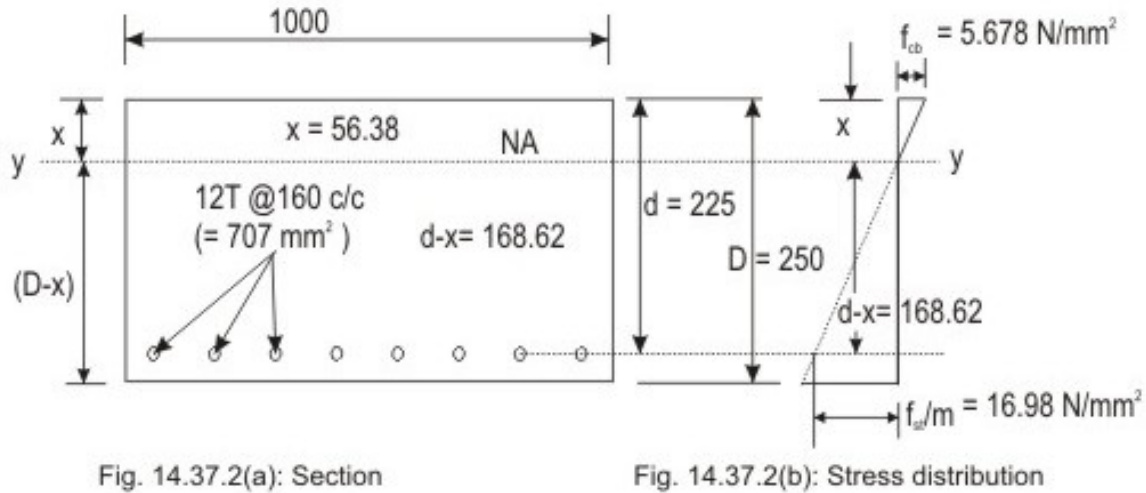


Fig. 14.37.2: Problem 2- Final section

Figure 14.37.2a shows the section. The maximum bending compression stress of concrete is determined either from the stress distribution diagram of Fig. 14.37.2b or after determining the moment of inertia of the section about the neutral axis. From the stress distribution diagram of Fig. 14.37.2b, we have: $f_{cb} = (f_{st} / m) \{x / (d-x)\} = (226.35 / 13.33) (56.38 / 168.62) = 5.678 \text{ N/mm}^2 < 7 \text{ N/mm}^2$.

The moment of inertia about the neutral axis is:

$$I_{yy} = 1000 (56.38)^3 / 3 + (13.33) (707) (168.62)^2 = 0.3277 (10^9) \text{ mm}^4.$$

Then, $f_{cb} = 33 (10^6) (56.38) / \{0.3277 (10^9)\} = 5,678 \text{ N/mm}^2 < 7 \text{ N/mm}^2$. Hence, the section, as shown in Fig. 14.37.2a, is ok.

Problem 3. Determine the area of tensile steel of a singly-reinforced bunker wall of depth 250 mm subjected to $F_t = 50 \text{ kN/m}$ and $M = 35 \text{ kNm/m}$ at the horizontal level. Use M 20 and Fe 415 grade of steel.

Solution 3. In this problem, the eccentricity of the tension force F_t is $M/F_t = 35(10^3) / 50 = 700 \text{ mm}$, i.e., the tension force is acting outside the section (Figs. 14.36.6a and b). The governing equations of such section are given in sec. 14.36.7.B(ii), assuming the section as cracked. Given $D = 250 \text{ mm}$, so $d = 225 \text{ mm}$. The value of the modular ratio $m = 93.33 / 7 = 13.33$. For M 20 concrete $f_{cb} \leq 7 \text{ N/mm}^2$. Let us assume $j = 0.87$.

Step 1. Preliminary area of tension steel

From Eq. 14.27 of Lesson 36, we have:

$$A_{st} = (F_t / \sigma_{st}) \{1 + (e + 0.5 D - d) / jd\} \quad (14.27)$$

or $A_{st} = (5000/230) \{1 + (700 + 125 - 225) / (0.87) (225)\} = 883.72 \text{ mm}^2$.
Provide 12 mm diameter bars @ 120 mm c/c to have 942 mm².

Step 2. Determination of the depth of the neutral axis

From Eq. 14.23 of Lesson 36, we get

$$m(A_{st}) (d-x) (e+0.5 D - d) = 0.5 b x^2 (e + 0.5 D - x/3)$$

or $13.33 (942) (225 - x) (700+125-225) = 500 x^2 (700 +125-x/3)$

or $x^3 - 2475 x^2 - 45204.696 x + 1017105.6 = 0$

By Newton's method, we get $x = 56.171 \text{ mm}$.

Step 3. Determination of stresses of concrete and steel

From Eq. 14.22 of Lesson 36, we have:

$$\begin{aligned} f_{cb} &= F_t / \{m A_{st} (d-x)/x - (0.5 b x)\} \\ &= 50000 / \{(13.33) (942) (225 - 56.171)/56.171 - 0.5 (1000) (56.171)\} \\ &= 5.18 \text{ N/mm}^2 < 7.00 \text{ N/mm}^2 \end{aligned}$$

From Eq. 14.15 of Lesson 35, we have:

$$\begin{aligned} f_{st} &= m f_{cb} (d - x) / x = 13.33 (5.18) (225-56.171)/56.171 \\ &= 207.54 \text{ N/mm}^2 < 230 \text{ N/mm}^2. \end{aligned}$$

Hence, the design is ok.

Problem 4. Design an interior slab panel of dimensions 4000 mm x 4000 mm of a flat bottom water tank to support 3 m of water. Use M 20 concrete and Fe 415 steel.

Solution 4. This is a slab panel of liquid retaining structure. We have to determine the depth of the slab and area of steel by trial and error. Let us assume $D = 250 \text{ mm}$, and $d = 200 \text{ mm}$ giving a cover of 50 mm in the top liquid surface. The cover at the bottom surface is assumed to be 30 mm.

Step 1. Evaluation of loads

Loads of 3 m height of water =	30000 N/m ²
Loads of plastering (assumed) =	500 N/m ²
Weight of reinforced concrete slab (of depth = 250 mm) = 25000 (250) (10 ⁻³) =	6250 N/m ²
Total loads =	36750 N/m ²

Step 2. Calculation of bending moments

Table 26 of IS 456 gives the values of coefficients $\alpha_x = 0.032$ for the negative moment and $\alpha_y = 0.024$ for the positive moment. Thus, we have, negative moment = (0.032) (36750) (4) (4) = 18.816 kNm and positive moment = (0.024) (36750) (4) (4) = 14.112 kNm.

Step 3. Preliminary areas of steel

Assuming $f_{st} = \sigma_{st} = 150 \text{ N/mm}^2$, $k_b = 93.33/(150+93.33) = 0.38$,
Lever arm = $jd = (0.87) (200) = 174 \text{ mm}$ for the negative moment
and lever arm = $jd = 0.87(220) = 191.4 \text{ mm}$ for the positive moment
at mid-span, the areas of steel are as follows:

At the support = $M/\sigma_{st} jd = 18816000 / (150) (174) = 720.92 \text{ mm}^2$

At the mid-span = $M/\sigma_{st} jd = 14112000 / (150) (191.4) = 491.54 \text{ mm}^2$.

Provide 10 mm diameter bars @ 100 mm c/c to give 785 mm² at the support and 10 mm diameter bars @ 150 mm c/c at the mid-span to give 524 mm².

Step 4. Checking for stresses and moment of resistance of the section

(A) At the support

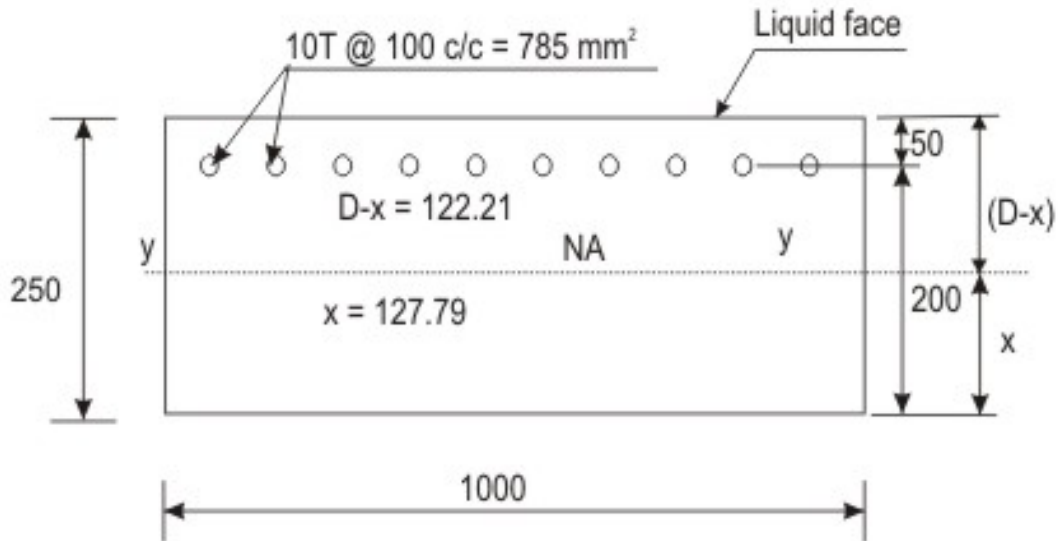


Fig. 14.37.3: Problem 4 - assumed section at support

Figure 14.37.3 shows the section at the support, which is assumed as uncracked. The depth of neutral axis x is determined by taking moment of the areas of concrete and steel about the bottom, which gives:

$$x = \{(1000) (250) (125) + (12.33) (785) (200)\} / \{(1000) (250) + (12.33) (785)\} = 127.79 \text{ mm}; D - x = 250 - 127.79 = 122.21 \text{ mm}.$$

The moment of inertia about the neutral axis is:

$$I_{yy} = (1000) (122.21)^3 / 3 + (1000) (127.79)^3 / 3 + (12.33) (785) (122.21 - 50)^2 = (1.35) (10^9) \text{ mm}^4$$

The moment of resistance of the section is determined using $f_{tb} = \sigma_{tb} = 1.7 \text{ N/mm}^2$, as given below:

$$M = f_{tb} (I_{yy}) / (D - x) = 1.7 (1.35) (10^9) / 122.21 = 18.7 \text{ kNm} < 18.816 \text{ kNm}.$$

Hence, the design has to be revised. Accordingly, the checking of stresses at the mid-span for positive moment is not carried out.

Step 5.

Revised Section

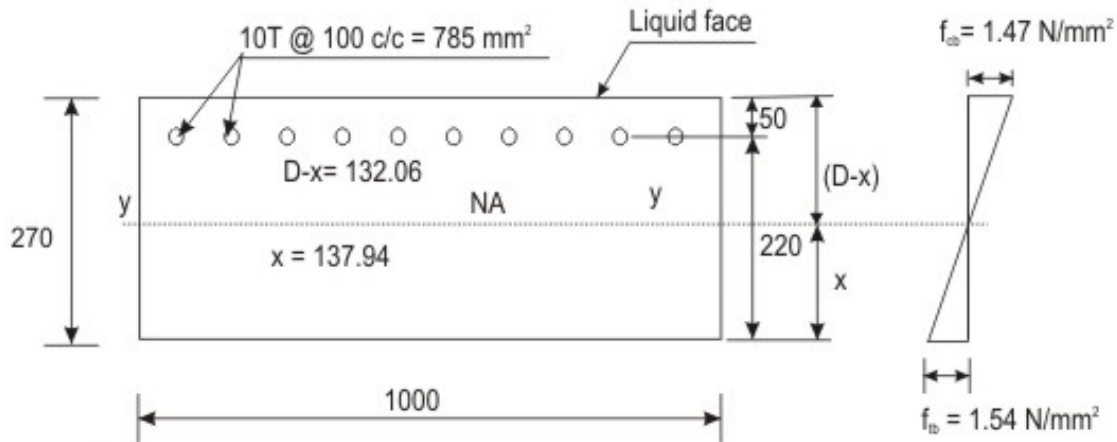


Fig. 14.37.4(a): Final section at support

Fig. 14.37.4(b): Stress distribution

Fig. 14.37.4: Problem 4 - final section at support and stress distribution

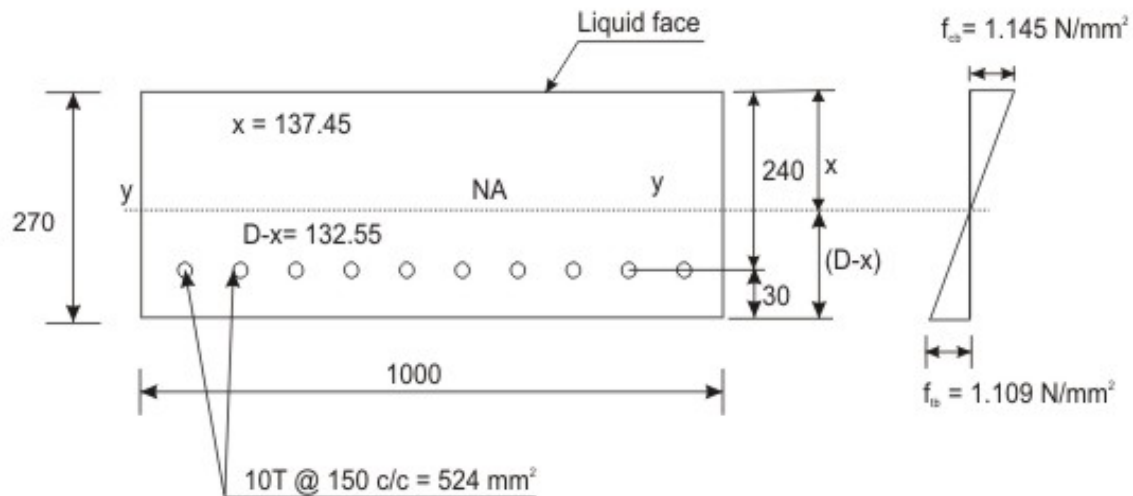


Fig. 14.37.5(a): Final section at mid-span

Fig. 14.37.5(b): Stress distribution

Fig. 14.37.5: Problem 4 - final section at mid-span and stress distribution

- (A) The depth of the section is increased to 270 mm. Accordingly, $d = 220$ mm for support section and $d = 240$ mm for mid-span section. With $j = 0.87$, the lever arm at the support section $= 0.87 (220) = 191.4$ mm and at the mid-span section $jd = 0.87 (240) = 208.8$ mm. We have to add the extra self-weight of slab.

Earlier total loads in Step 1	=	36750 N/m ²
Additional load of 20 mm thickness	=	500
(25000) (20) / 10 ³ = 500 N/m ²		

Total revised loads = 37250 N/m²

Revised negative moment = $(0.032) (37250) (16) = 19.072 \text{ kNm}$

Revised positive moment = $(0.024) (37250) (16) = 14.304 \text{ kNm}$

Revised negative steel = $(19072000) / (150) (191.4) = 664.298 \text{ mm}^2$

and Revised positive steel = $(14304000) / (150) (208.8) = 456.7 \text{ mm}^2$.

Provide 10 mm diameter bars @100 mm c/c ($= 785 \text{ mm}^2$) at the support and 10 mm diameter bars @ 150 mm c/c ($= 524 \text{ mm}^2$) at the mid-span, as shown in Figs. 14.37.4a and 14.37.5a, respectively.

Step 6. Checking of stresses and moment of resistance

(A) At the support (Fig. 14.37.4b)

The depth of neutral axis $x = \{(1000) (270) (135) + (112.33) (785) (220)\} / \{(1000) (270) + (12.33) (785)\} = 137.94 \text{ mm}$.

$(D-x) = 270 - 137.94 = 132.06 \text{ mm}$ and $(d-x) = 220 - 137.94 = 82.06 \text{ mm}$.

$$I_{yy} = \{(1000) (137.94)^3 / 3\} + \{(1000) (132.06)^3 / 3\} + \{(12.33) (785) (82.06)^2\} = 1.71 (10^9) \text{ mm}^4$$

Moment of resistance = $(1.7) (1.71) (10^3) / 132.06 = 22.01 \text{ kNm} > 19.072 \text{ kNm}$.

$$f_{tb} = 19.072 (10^6) (132.06) / (1.71) (10^9) = 1.47 \text{ N/mm}^2 < 1.7 \text{ N/mm}^2.$$

$$f_{cb} = 19.072 (10^6) (137.94) / (1.71) (10^9) = 1.54 \text{ N/mm}^2 < 7 \text{ N/mm}^2.$$

$$f_{st} = (12.33)(19.072) (10^6)(82.06) / (1.71)(10^9) = 12.2 \text{ N/mm}^2 < 230 \text{ N/mm}^2$$

Hence, the section at the support is ok.

(B) At the mid-span (Fig. 14.37.5b)

The depth of neutral axis $x = \{(1000) (270) (135) + (12.33) (524) (240)\} / \{(1000) (270) + (12.33) (524)\} = 137.45 \text{ mm}$, $D - x = 132.55 \text{ mm}$ and $d-x = 102.55 \text{ mm}$.

$$I_{yy} = \{(1000) (137.45)^3 / 3\} + \{(1000) (132.55)^3 / 3\} + \{(12.33) (524) (102.55)^2\} = 1.71 (10^9) \text{ mm}^4.$$

Moment of resistance of the section (when $f_{tb} = 1.7 \text{ N/mm}^2$) = $1.7 (1.71) (10^3) / (132.55) = 21.93 \text{ kNm} > 14.304 \text{ kNm}$.

When $f_{tb} = 1.7 \text{ N/mm}^2$, $f_{cb} = 1.7 (137.45) / (132.55) = 1.7642 \text{ N/mm}^2$.
The moment of resistance (when $f_{cb} = 1.7642 \text{ N/mm}^2$) = $1.7642 (1.71) (10^3) / (137.45) = 21.94 \text{ kNm} > 14.304 \text{ kNm}$.

For the positive moment of 14.304 kNm , $f_{tb} = (14.304) (132.55) (10^6) / (1.71) (10^9) = 1.109 \text{ N/mm}^2 < 1.7 \text{ N/mm}^2$ and $f_{cb} = (14.304) (137.45) (10^6) / (1.71) (10^9) = 1.145 \text{ N/mm}^2 < 7 \text{ N/mm}^2$

Problem 5. Determine the moment of resistance of the slab of Fig. 14.37.6 subjected to moment alone using M 20 and Fe 415.

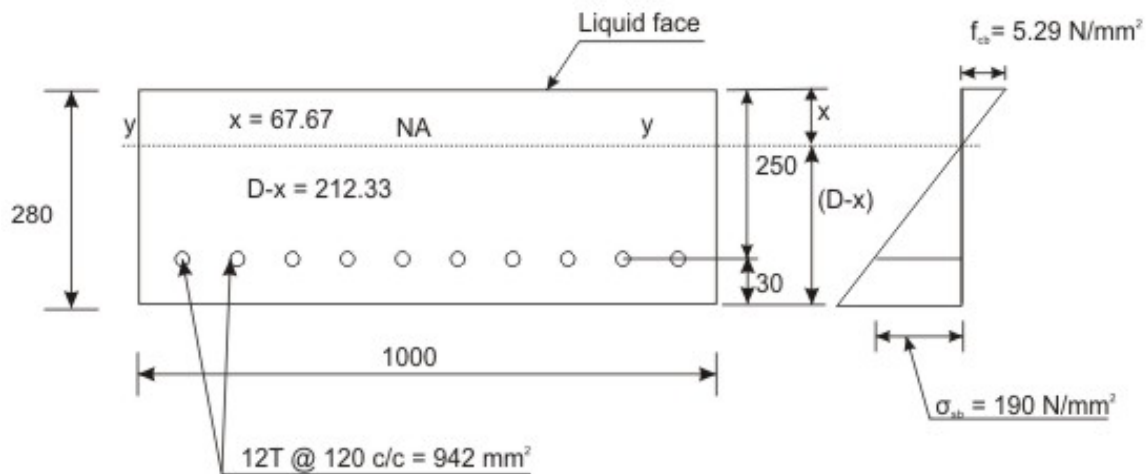


Fig. 14.37.6(a): Section

Fig. 14.37.6(b): Stress distribution

Fig. 14.37.6: Problem 5 - section and stress distribution

Solution 5. Since tension is on the remote face and the depth of the slab is more than 225 mm, we use cracked section. The value of $m = 93.33/7 = 13.33$.

Taking moment of area of concrete and steel about the top face.

or, $500 x^2 = (13.33) (942) (250 - x)$
 $x^2 + 25.11372 x - 6278.43 = 0$
 which gives $x = 67.67 \text{ mm}$. Accordingly, $d - x = 250 - 67.67 = 182.33 \text{ mm}$ and $D - x = 280 - 67.67 = 212.33 \text{ mm}$.

Moment of resistance from steel is determined using $f_{st} = \sigma_{st} = 190$ N/mm². Then $M = \sigma_{st} A_{st} (d - x/3) = 190(942) (250 - 67.67/3) (10^{-6}) = 40.71$ kNm.

When $f_{st} = \sigma_{st} = 190$ N/mm², $f_{cb} = 190 (67.67) / (182.33) (13.33) = 5.29$ N/mm² < 7 N/mm².

The moment of resistance from compression concrete = $0.5(67.67) (5.29) (1000) (250 - 67.67/3) = 40.707$ kNm.

Problem 6. Design the wall of a water tank using concrete of grade M 20 and steel of grade Fe 415, when subjected to $F_t = 60$ kN/m and $M = 25$ kNm/m.

Solution 6. Since both F_t and M are acting on the wall of a liquid retaining structure, the design must satisfy Eq. 14.8 of Lesson 36, i.e.,

$$(f_{td} / \sigma_{td}) + (f_{tb} / \sigma_{tb}) \leq 1.0 \quad (14.8)$$

where $\sigma_{td} = 1.2$ N/mm² and $\sigma_{tb} = 1.7$ N/mm² for M 20 concrete, as given in Table 14.1 of Lesson 36, when the tension is on the liquid face. The section is considered as uncracked.

Step 1. Preliminary values of d , D , A_{st1} for moment and A_{st2} axial force

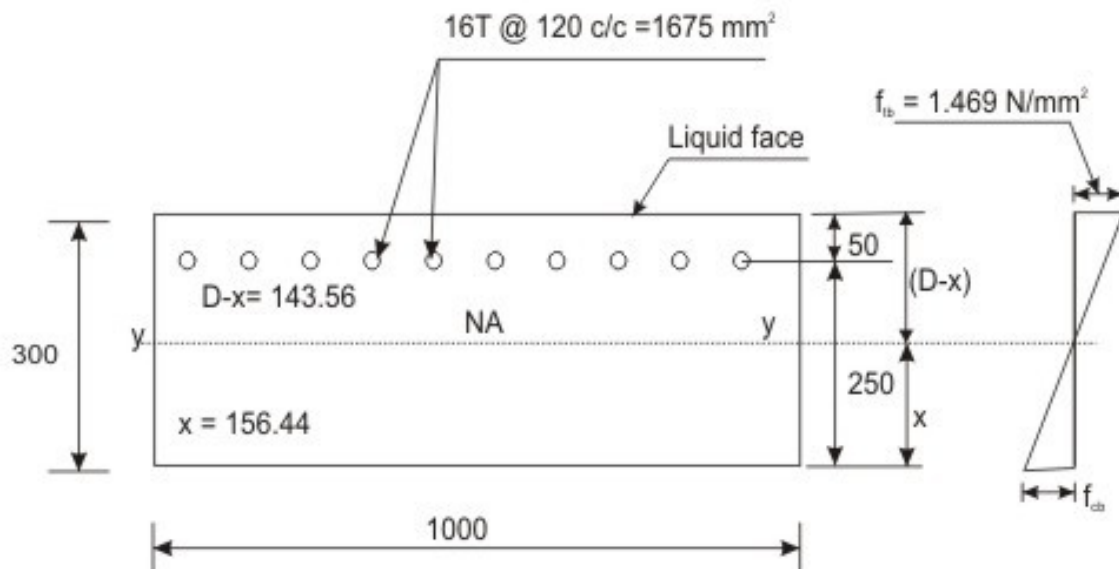


Fig. 14.37.7(a): Section (assumed)

Fig. 14.37.7(b): Stress distribution

Fig. 14.37.7: Problem 6 - assumed section and stress distribution

Assuming $\sigma_{st} = 150$ N/mm², we get $k_b = 93.33 / (150 + 93.33) = 0.38$ and $j = 1 - 0.38/3 = 0.87$. Assuming $p_t = 0.4$ per cent, we get from:

or, $M = A_{st1} \sigma_{st} j d = (p_t j \sigma_{st} / 100) (b d^2)$
 $d = \{100 M / \sigma_{st} p_t j b\}^{1/2} = \{(100) (25000 \ 000) / (150) (0.4) (0.87) (1000)\}^{1/2} = 218.8 \text{ mm.}$ Provide $d = 250 \text{ mm}$ with cover of 50 mm , $D = 300 \text{ mm}$.

A_{st1} for moment (0.4 per cent) $= 0.4 (300) (1000) / 100 = 1200 \text{ mm}^2$.
 A_{st2} for $F_t = 60000 / 150 = 400 \text{ mm}^2$.

Total $A_{st} = A_{st1} + A_{st2} = 1200 + 400 = 1600 \text{ mm}^2$. Provide 16 mm diameter bars @ 120 mm c/c to give 1675 mm^2 , as shown in Fig. 14.37.7a.

Step 2. Sectional properties

Figure 14.37.7a shows the section.

$x = \{1000 (300) (150) + (12.33) (1675) (250)\} / \{1000 (300) + (12.33) (1675)\} = 156.44 \text{ mm}$.

$D - x = 300 - 156.44 = 143.56 \text{ mm}$, and $d - x = 93.56 \text{ mm}$.

$I_{yy} = 1000 (143.56)^3 / 3 + 1000 (156.44)^3 / 3 + (12.33) (1675) (93.56)^2$
 $= 2.443 (10^9) \text{ mm}^4$.

$A = \text{area of the section} = 300 (1000) + (12.33) (1675) = 320652.75 \text{ mm}^2$.

Step 3. Values of stresses and checking of Eq. 14.8

$f_{tb} = (25000 \ 000) (143.56) / (2.443) (10^9) = 1.469 \text{ N/mm}^2 < 1.7 \text{ N/mm}^2$.

$f_{td} = 60000 / 320652.75 = 0.187 \text{ N/mm}^2 < 1.2 \text{ N/mm}^2$

$(f_{tb} / \sigma_{tb}) + (f_{td} / \sigma_{td}) = (1.469 / 1.7) + (0.187 / 1.2) = 1.02 > 1.0$.
Hence, the section has to be revised.

The revision can be done in two ways: (i) increasing A_{st1} to 0.5 per cent keeping the depth of the section $= 300 \text{ mm}$, and / or (ii) increasing the depth of the section by 20 mm . We try with the first one in the next step.

Step 4. Revised section

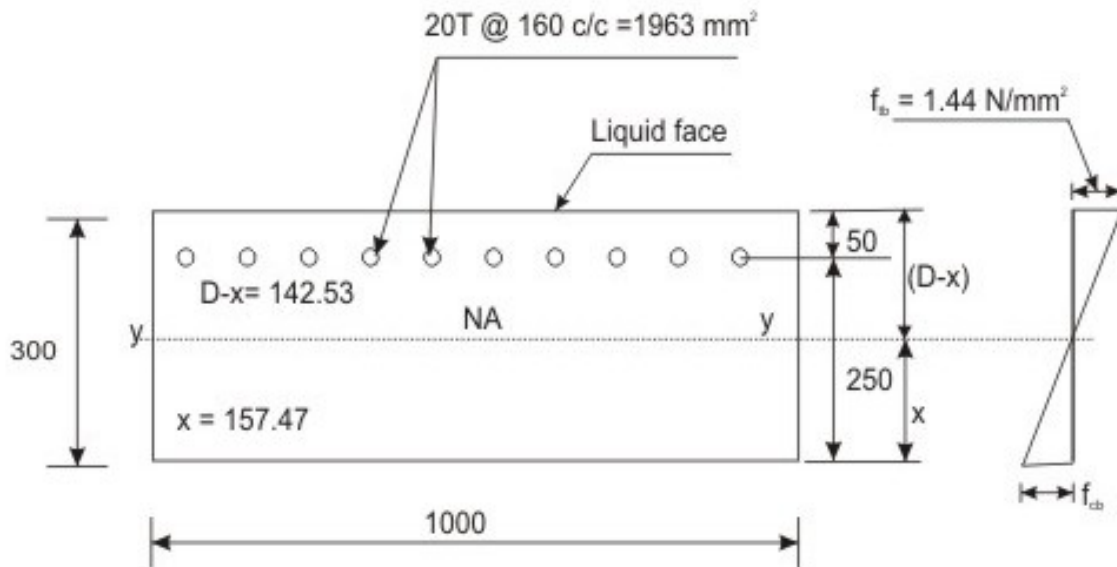


Fig. 14.37.8(a): Revised section

Fig. 14.37.8(b): Stress distribution

Fig. 14.37.8: Problem 6 - revised section

With D as 300 mm, $A_{st1} = 0.5 (300) (1000) / 100 = 1500 \text{ mm}^2$ and $A_{st2} = 400 \text{ mm}^2$ (as in earlier). So, $A_{st} = 1900 \text{ mm}^2$. Provide 20 mm diameter bars @ 160 mm c/c to give 1963 mm^2 as shown in Fig. 14.37.8a.

$$x = \{(1000) (300) (150) + (12.33) (1963) (250)\} / \{(1000) (300) + (12.33) (1963)\} = 157.47 \text{ mm.}$$

$$D - x = 300 - 157.47 = 142.53 \text{ mm and } d - x = 250 - 157.47 = 92.53 \text{ mm.}$$

$$I = 1000 (157.47)^3 / 3 + 1000 (142.53)^3 / 3 + (12.33) (1963) (92.53)^2 = 2.47(10^9) \text{ mm}^4$$

$$A = (300) (1000) + (12.33) (1963) = 324203.79 \text{ mm}^2.$$

$$f_{tb} = 25(10^6) (142.53) / 2.47(10^9) = 1.44 \text{ N/mm}^2$$

$$f_{td} = 6000 / 324203.79 = 0.185 \text{ N/mm}^2$$

$$(f_{tb} / \sigma_{tb}) + (f_{td} / \sigma_{td}) = (1.44 / 1.7) + (0.185 / 1.2) = 1.0012 > 1.0.$$

So, we further increase the depth of the concrete and repeat the calculations of Step 4 in Step 5.

Step 5.

Further revision

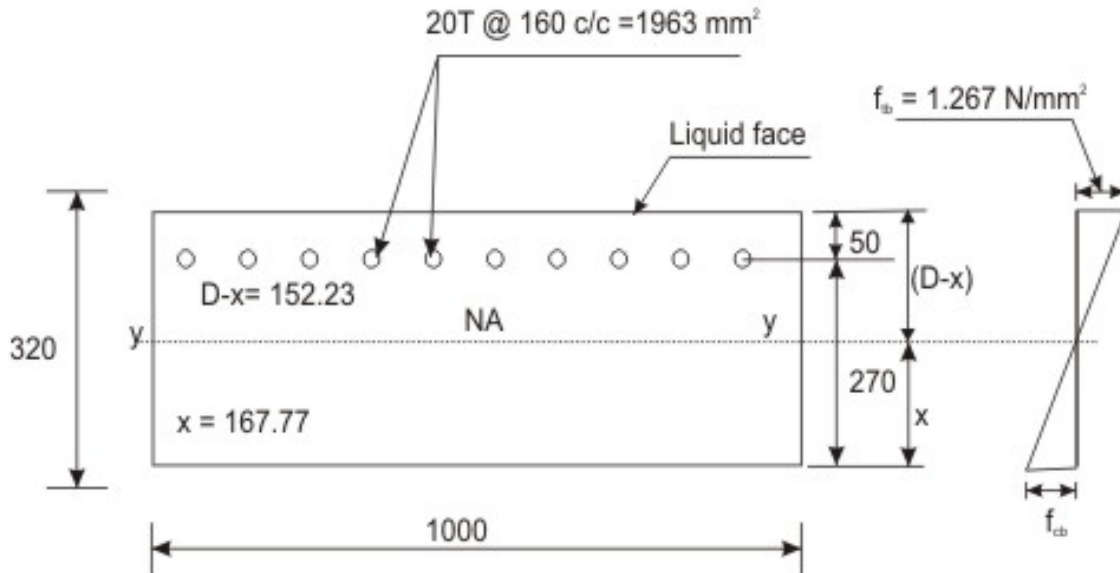


Fig. 14.37.9(a): Final section

Fig. 14.37.9(b): Stress distribution

Fig. 14.37.9: Problem 6 - final section and stress distribution

Now, the revised section has: $D = 320$ mm, $d = 270$ mm and $A_{st} = 1963$ mm² (20 mm diameter bars @ 160 mm c/c) as shown in Figs. 14.37.9a and b. The following are the results:

$$\begin{aligned}
 x &= 167.77 \text{ mm} \\
 D - x &= 152.23 \text{ mm} \\
 I &= (3.003) (10^9) \text{ mm}^4 \\
 A &= 344203.79 \text{ mm}^2 \\
 f_{tb} &= 1.267 \text{ N/mm}^2 \\
 f_{td} &= 0.174 \text{ N/mm}^2
 \end{aligned}$$

$$\text{and } (f_{tb} / \sigma_{tb}) + (f_{td} / \sigma_{td}) = 0.745 + 0.145 = 0.89 < 1.0$$

Hence, the revised section of Fig. 14.37.9a is ok.

Problem 7. Determine the area of tensile steel of the wall of a water tank of 250 mm depth and subjected to $M = 35$ kNm/m only. Use M 20 and Fe 415. Redesign, if needed.

Solution 7. Given data are $D = 250$ mm, $d = 225$ mm, M 20 and Fe 415. Since, the tension face is at a distance of 225 mm away, we consider $f_{st} \leq 190$ N/mm² and assume the section as cracked. We determine the preliminary area of steel first.

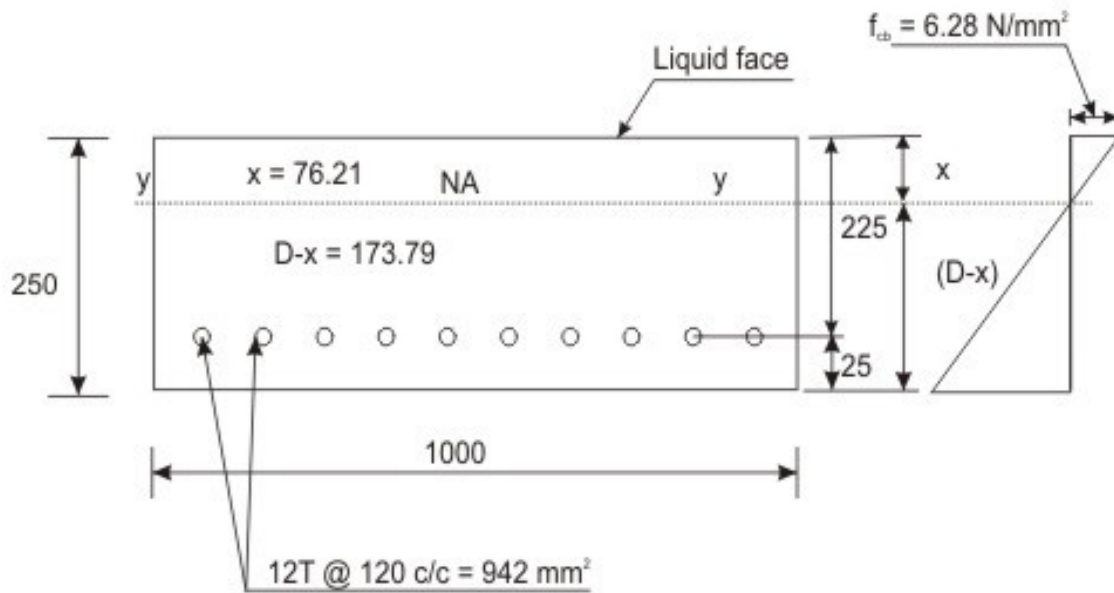
Step 1.**Determination of preliminary area of steel**

Fig. 14.37.10(a): Section

Fig. 14.37.10(b): Stress distribution

Fig. 14.37.10: Problem 7 - section and stress distribution

For M 20 concrete, $m = 93.33/7 = 13.33$, $k_b = 93.33 / (\sigma_{st} + 93.33) = 93.33 / (190 + 93.33) = 0.329$ and $j = 1 - k_b / 3 = 1 - 0.329/3 = 0.89$.

Preliminary $A_{st} = M / \sigma_{st} j d = 35(10^6) / (190) (0.89) (225) = 919.9 \text{ mm}^2$. Provide 12 mm diameter bars @ 120 mm c/c to have 942 mm^2 , as shown in Fig. 14.37.10a.

Step 2.**Properties of section**

Taking moment of the area about the top surface (Fig. 14.37.10a), we have:

$$500 x^2 = (13.33) (942) (225 - x)$$

or, $x^2 + 25.11372 x - 5650.587 = 0$. The solution is $x = 76.21 \text{ mm}$.

$$I_{yy} = 1000 (76.21)^3 / 3 + (13.33) (942) (225 - 76.21)^2 = 0.425 (10^9) \text{ mm}^4.$$

Step 3.**Checking for the stresses**

Compression stress in concrete in bending $f_{cb} = Mx/I_{yy} = 35 (10^6) (76.21) / (0.425) (10^9) = 6.28 \text{ N/mm}^2$. Tensile stress in steel $= f_{st} = m(f_{cb}) (d-x) / x = (13.33) (6.28)(148.79)/76.21 = 163.43 \text{ N/mm}^2 < 190 \text{ N/mm}^2$. Hence, the section is ok.

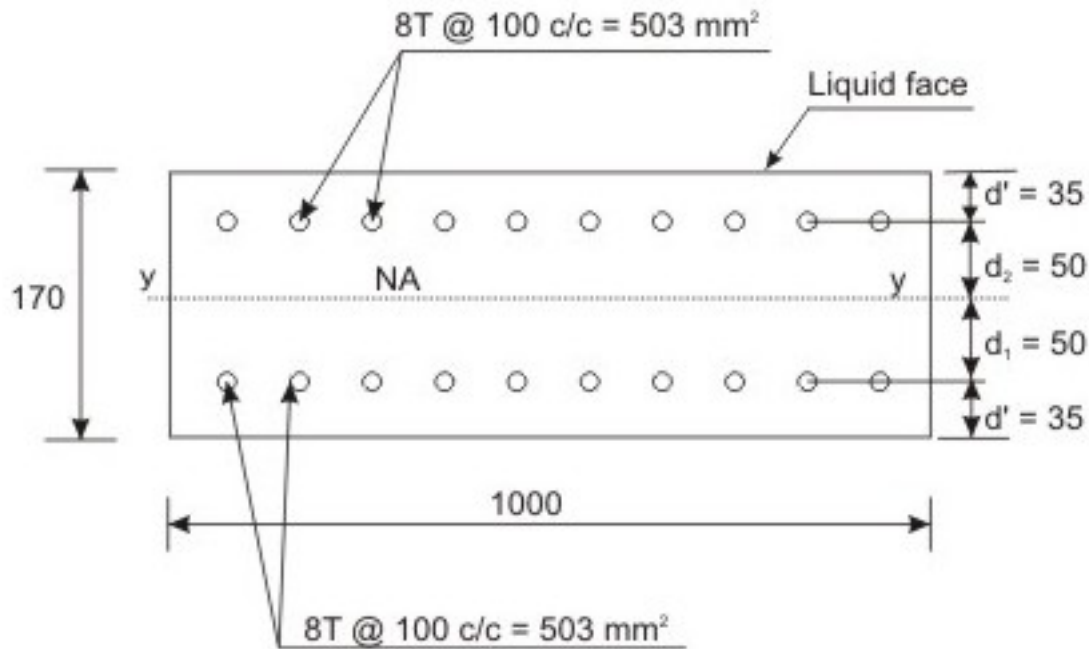


Fig. 14.37.11: Problem 8 - Section

Problem 8. Check the acceptability of the design of the base slab of a water tank as shown in Fig. 14.37.11, which is subjected to $F_t = 60 \text{ kN/m}$ and $M = 6 \text{ kNm/m}$. Assume the section as uncracked. Use M 20 and Fe 415.

Solution 8. For M 20 concrete $m = 93.33/7 = 13.33$. Equations 14.2 and 14.4 of Lesson 36 are used in determining f_{td} and f_{tb} , respectively. Since the section is symmetrically reinforced, the depth of the neutral axis = 85 mm.

$$I_{yy} = 1000 (170)^3 / 12 + (13.33) (503) (2) (50)^2 = 0.4404 (10^9) \text{ mm}^4$$

$$f_{td} = F_t / \{ (1000) (170) + (13.33) (2) (503) \} = 0.329 \text{ N/mm}^2$$

$$f_{tb} = 6 (10^6) (85) / (0.4404) (10^9) = 1.158 \text{ N/mm}^2$$

$$(f_{td} / \sigma_{td}) + (f_{tb} / \sigma_{tb}) = (0.329 / 1.2) + (1.158 / 1.7) = 0.224 + 0.681 = 0.905 < 1.0. \text{ Hence, the design is ok.}$$

14.37.3 Practice Questions and Problems with Answers

Q.1: Solve Problem 1 of sec.14.37.2 when Fe 250 is used.

A.1: Given data are $F_t = 470$ kN, concrete grade is M 25 and steel grade is Fe 250. Modular ratio $m = 10.98$ (see solution 1 of sec. 14.37.2). The values of f_{st} of Fe 250 ≤ 140 N/mm² and $f_{td} \leq 3.2$ N/mm².

Step 1. Determination of A_{st}

From Eq. 14.1 of Lesson 36, we have $A_{st} = F_t / \sigma_{st} = 470000 / 140 = 3357.14$ mm². Provide 6 bars of 25 mm diameter and 4 bars of 12 mm diameter to have $A_{st} = 2945 + 452 = 3397$ mm².

Step 2. Determination of width and depth of the section

From $A_g = (F_t / f_{td}) - (m - 1) A_{st}$, we have $A_g = 112972.94$ mm². Provide 300 mm \times 380 mm section.

Step 3. Checking for tensile stress of concrete

From Eq. 14.2 of Lesson 36, we have:

$f_{td} = F_t / (A_c + m A_{st}) = F_t / \{A_g + (m - 1) A_{st}\} = 3.18$ N/mm² < 3.2 N/mm². Hence, the section of 300 mm \times 380 mm with 6 bars of 25 mm diameter and 4 bars of 12 mm diameter is ok.

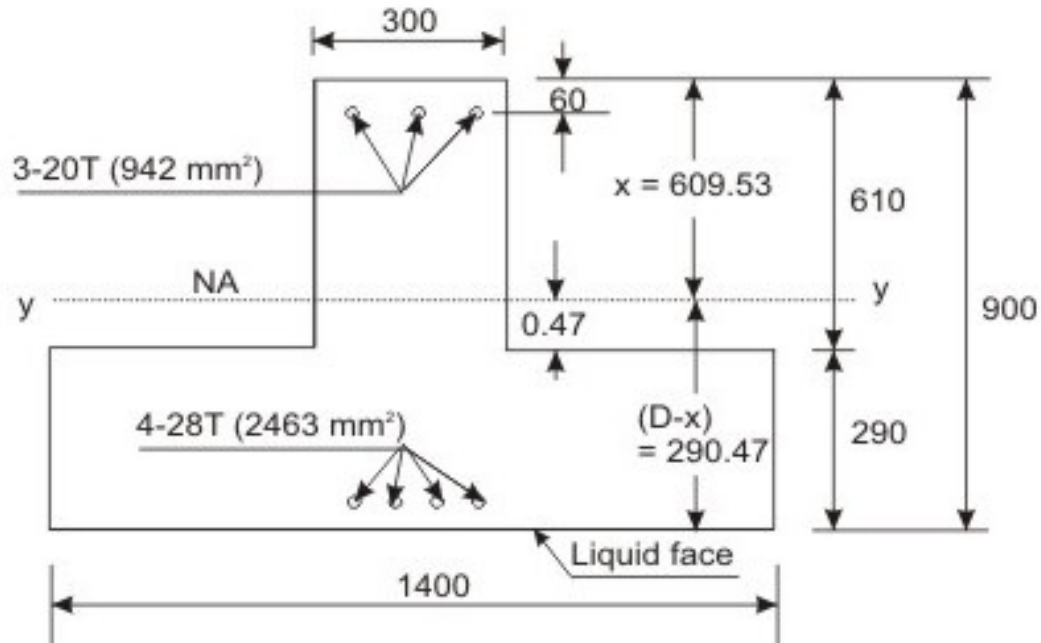


Fig. 14.37.12: Q.2

Q.2: Determine the moment of resistance of the section shown in Fig. 14.37.12. Use M 20 and Fe 415.

A.2: The section is considered uncracked as the liquid face is having tension. The modular ratio $m = 93.33 / 7 = 13.33$ and $f_{tb} \leq 1.7 \text{ N/mm}^2$.

Step 1. Sectional properties

The depth of the neutral axis, as shown in Fig. 14.37.12, is obtained by taking moment of the areas about the top, which gives:

$$x = \{(300)(900)(450) + (1100)(290)(610 + 145) + (12.33)(2463)(900 - 85) + (1.5)(12.33)(942)(60)\} / \{(300)(900) + (1100)(290) + (12.33)(2463) + (1.5)(12.33)(942)\} = 609.53 \text{ mm},$$

$$D - x = 900 - 609.53 = 290.47 \text{ mm}.$$

The moment of inertia about the neutral axis yy is:

$$I_{yy} = (300)(609.53)^3 / 3 + (300)(0.47)^3 / 3 + 1400(290)^3 / 12 + (1400)(290)(145 + 0.47)^2 + (12.33)(2463)(290.47 - 85)^2 + (1.5)(12.33)(942)(609.53 - 60)^2 = (40.626)(10^9) \text{ mm}^4.$$

$$\text{Moment of resistance} = (1.7)(40.626)(10^9) / 290.47 = 237.767 \text{ kNm/m}.$$

Q.3: Solve Problem 5 of sec. 14.37.2 (Fig. 14.37.6a) using Fe 250.

A.3: Given data are: $D = 280$ mm, $d = 250$ mm, $A_{st} = 12$ mm diameter bars @ 120 mm c/c = 942 mm^2 , M 20 and Fe 250 (Fig. 14.37.6a).

Modular ratio $m = 93.33 / 7 = 13.33$, $x = 67.67$ (as obtained in the solution of Problem 5), $\sigma_{st} = 125 \text{ N/mm}^2$ and $(d - x) = 250 - 67.67 = 182.33$ mm. Therefore, $M = \sigma_{st} A_{st} (d - x/3) = (125) (942) (250 - 67.67/3) = 26.78 \text{ kNm}$.

$f_{cb} = \sigma_{st} (67.67) / (13.13) (182.33) = 3.48 \text{ N/mm}^2$. The moment of resistance from compression concrete = $125 (942) (250 - 67.67/3) = 26.78 \text{ kNm} < 40.71 \text{ kNm}$ of the slab of Problem 5.

It is thus seen that though the slab of Problem 5 and this one have $x = 67.67$ mm but moment of resistance of this slab is much less when Fe 250 is used.

14.37.4 Reference

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14.37.5 Test 37 with Solutions

Maximum Marks = 50
minutes

Maximum Time = 30

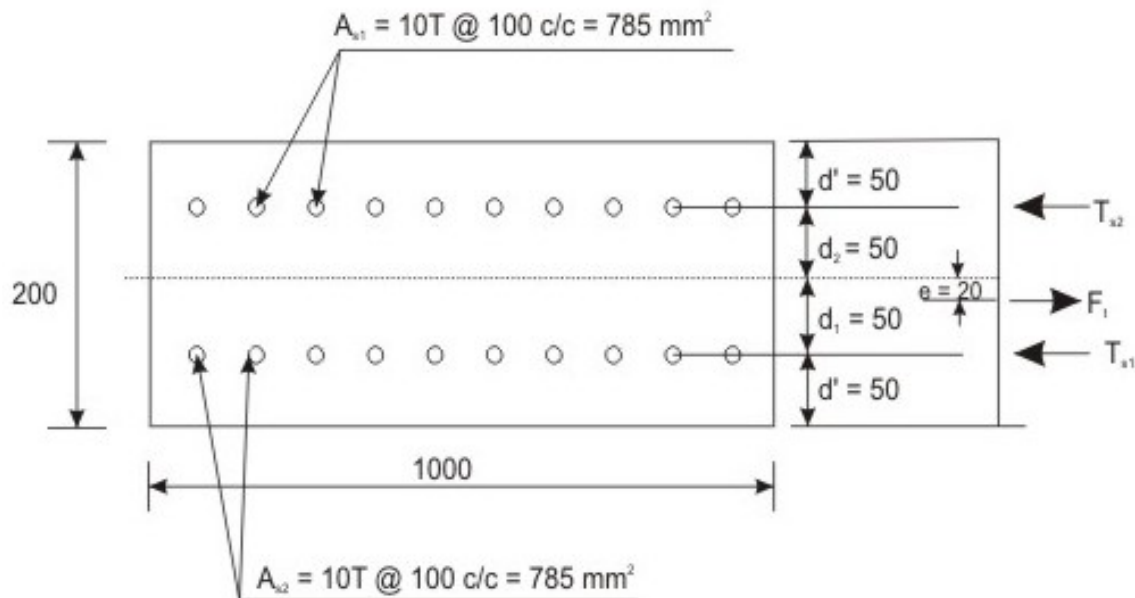


Fig. 14.37.13(a): Section

Fig. 14.37.13(b): Forces

Fig. 14.37.13: TQ.1 - section and forces

TQ.1: The base slab of the water tank, as shown in Fig. 14.37.13a, is subjected to $F_t = 60 \text{ kN/m}$ and $M = 1.2 \text{ kNm/m}$. Determine the stresses in steel and concrete. Use M 20 and Fe 415.

[20 marks]

A.TQ.1: Given $D = 200 \text{ mm}$, $A_{s1} = 785 \text{ mm}^2$, $A_{s2} = 785 \text{ mm}^2$, M 20 and Fe 415.

The eccentricity of $F_t = e = 1.2 (10^3) / 60 = 20$ mm, i.e., within the section. The whole section is in tension and the section is cracked. We have Eqs. 14.9 and 14.10 as

$$F_t = f_{st1} A_{s1} + f_{st2} A_{s2} \quad (14.9)$$

$$F_t (d_1 - e) - f_{st2} A_{s2} (d_1 + d_2) = 0 \quad (14.10)$$

From Eq. 14.10, we have: $f_{st2} = F_t (d_1 - e) / \{A_{s2} (d_1 + d_2)\} = (60000) (50 - 20) / (785) (100) = 22.93 \text{ N/mm}^2 < 150 \text{ N/mm}^2$.

Using the value of f_{st2} in Eq. 14.9, we have $f_{st1} = (F_t - f_{st2} A_{s2}) / A_{s1} = \{60000 - (22.93) (785)\} / 785 = 53.50 \text{ N/mm}^2 < 150 \text{ N/mm}^2$.

Modular ratio m for M20 = $93.33 / 7 = 13.33$.

$$f_{td} = F_t / \{1000(200) + (13.33 - 1) (785) (2)\} = 0.27 \text{ N/mm}^2$$

$f_{tb} = f_{st1} / m = 53.50 / 13.33 = 4.0 \text{ N/mm}^2 > 1.7 \text{ N/mm}^2$. It shows that the section is cracked.

TQ.2: Determine the area of tensile steel of a singly-reinforced bunker wall of depth 300 mm subjected to $F_t = 60 \text{ kN/m}$ and $M = 42 \text{ kNm/m}$ at the horizontal level. Use M 20 and Fe 415 grade of steel.

(30 marks)

A.TQ.2: In this problem, the eccentricity of the tensile force $F_t = M/F_t = 42000 / 60 = 700$ mm, i.e., e is outside the section. Given data are: $D = 300$ mm, $d = 275$ mm. For M 20 concrete, $m = 93.33/7 = 13.33$, $f_{cb} \leq 7 \text{ N/mm}^2$. Let us assume $j = 0.87$.

Step 1. Preliminary area of tension steel

From Eq. 14.27 of Lesson 36, we have: $A_{st} = (F_t / \sigma_{st}) \{1 + (e + 0.5 D - d) / jd\} = (60000 / 230) \{1 + (700 + 150 - 275) / 0.87 (275)\} = 887.83 \text{ mm}^2$. Provide 12 mm diameter bars @ 120 mm c/c to give 942 mm^2 .

Step 2. Depth of the neutral axis

Eq. 14.23 of Lesson 36 gives: $m (A_{st}) (d - x) (e + 0.5 D - d) = 0.5 b x^2 (e + 0.5 D - x/3)$, which gives: $(13.33) (942) (275 - x) (700 + 150 - 275) = 500 x^2 (700 + 150 - x/3)$

or $x^3 - 2550 x^2 - 43321.167 x + 11913320.93 = 0$. The solution of the equation is $x = 61.0265$ mm by trial and error.

Step 3. Determination of stresses of concrete and steel

Equation 14.22 gives: $f_{cb} = F_t / \{m A_{st} (d - x) / x - (0.5 b x)\} = 60000 / \{(13.33) (942) (275 - 61.0265) / 61.0265 - 500 (61.0265)\} = 4.44 \text{ N/mm}^2 < 7 \text{ N/mm}^2$.

Equation 14.15 gives: $f_{st} = m f_{cb} (d - x) / x = (13.33) (4.44) (275 - 61.0265) / 61.0265 = 207.52 \text{ N/mm}^2 < 230 \text{ N/mm}^2$. Hence, the solution is o.k.

14.37.6 Summary of this Lesson

This lesson illustrates the applications of the equations explained in Lesson 36 for the analysis and design of tension members employing working stress method as stipulated in IS Codes. These structures may be either liquid retaining structures or may not have any contact with the liquid. The permissible stresses in concrete and steel depend on several factors as explained in Lesson 36. The section may be considered cracked or uncracked. Such tension structures may be subjected to axial tension only, moment only or combinations of them. Illustrative examples cover most of the categories of problems. For better understanding, some of the problems are solved from the first principle instead of using the equations directly. Understanding the illustrative examples and solutions of the practice and test problems will help in applying the equations in analysing and designing tension structures as per the stipulations of IS Codes.