Module 3

Limit State of Collapse - Flexure (Theories and Examples)

Lesson 7

Numerical Problems on Singly Reinforced Rectangular Beams (Continued)

Instructional Objectives:

At the end of this lesson, the student should be able to:

- Apply the principles to analyse a given cross-section of a beam with specific reinforcement to determine its moment of resistance.

3.7.1 Introduction

This lesson explains the determination of moment of resistance of given singly reinforced rectangular beam sections with the help of illustrative analysis type of problem. The numerical problem is solved by (i) direct computation method, (ii) using charts of SP-16 and (iii) using tables of SP-16. Step by step solutions illustrate the procedure clearly.

3.7.2 Analysis Type of Problems

It may be required to estimate the moment of resistance and hence the service load of a beam already designed earlier with specific dimensions of b, d and D and amount of steel reinforcement A_{st}. The grades of concrete and steel are also known. In such a situation, the designer has to find out first if the beam is under-reinforced or over-reinforced. The following are the steps to be followed for such problems.

3.7.2.1 $x_{u,\ max}$

The maximum depth of the neutral axis $x_{u,\ max}$ is determined from Table 3.2 of Lesson 5 using the known value of f_y.

3.7.2.2 x_u

The depth of the neutral axis for the particular beam is determined from Eq. 3.16 of Lesson 5 employing the known values of f_y, f_{ck}, b and A_{st}.

3.7.2.3 M_u and service imposed loads

The moment of resistance M_u is calculated for the three different cases as follows:

(a) If $x_u < x_{u,\ max}$, the beam is under-reinforced and M_u is obtained from Eq. 3.22 of Lesson 5.

(b) If $x_u = x_{u,\ max}$, the M_u is obtained from Eq. 3.24 of Lesson 5.

(c) If $x_u > x_{u,\,max}$, the beam is over-reinforced for which x_u is taken as $x_{u,\,max}$ and then M_u is calculated from Eq. 3.24 of Lesson 5, using $x_u = x_{u,\,max}$.

With the known value of M_u, which is the factored moment, the total factored load can be obtained from the boundary condition of the beam. The total service imposed loads is then determined dividing the total factored load by partial safety factor for loads (= 1.5). The service imposed loads are then obtained by subtracting the dead load of the beam from the total service loads.

3.7.3 Analysis Problems 3.2 and 3.3

Determine the service imposed loads of two simply supported beam of same effective span of 8 m (Figs. 3.7.1 and 2) and same cross-sectional dimensions, but having two different amounts of reinforcement. Both the beams are made of M 20 and Fe 415.

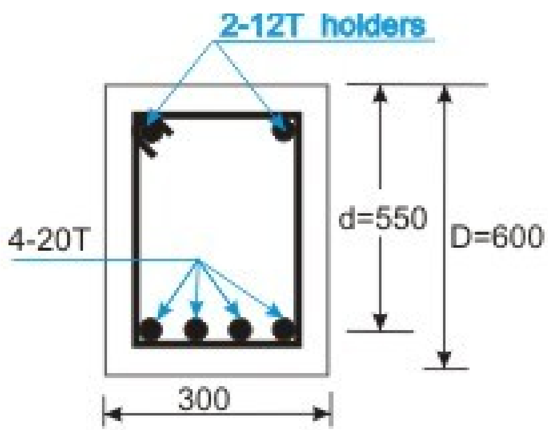

Fig. 3.7.1: Analysis Problem 3.2

3.7.4 Solution by Direct Computation Method - Problem 3.2

Given data: b = 300 mm, d = 550 mm, D = 600 mm, A_{st} = 1256 mm^2 (4-20 T), L_{eff} = 8 m and boundary condition = simply supported (Fig. 3.7.1).

3.7.4.1 $x_{u,\,max}$

From Table 3.2 of Lesson 5, we get $x_{u,\,max} = 0.479\, d$ = 263.45 mm

3.7.4.2 x_u

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\, b\ f_{ck}}$$
(3.16)

$$= \frac{0.87\ (415)\ (1256)}{0.36\ (300)\ (20)}$$

= 209.94385 mm < $x_{u,\,max}$ = (263.45 mm)

Hence, the beam is under-reinforced.

3.7.4.3 M_u and service imposed loads

For $x_u < x_{u,\,max}$, we have

$M_u = 0.87\ f_y\ A_{st}\ (d - 0.42\ x_u)$
(3.22)

= 0.87 (415) (1256) {550 - 0.42(209.94385)}

= 209.4272 kNm

Total factor load $F_d = \frac{8\ M_u}{L^2_{eff}} = \frac{8\ (209.4272)}{8\ (8)}$

= 26.1784 kN/m

Total service load = $\frac{F_d}{1.5} = \frac{26.1784}{1.5} = 17.452266$ kN/m

Dead load of the beam = 0.3 (0.6) (25) = 4.5 kN/m

Hence, service imposed loads = (17.452266 - 4.5) kN/m

= 12.952266 kN/m

3.7.5 Solution by Direct Computation Method - Problem 3.3

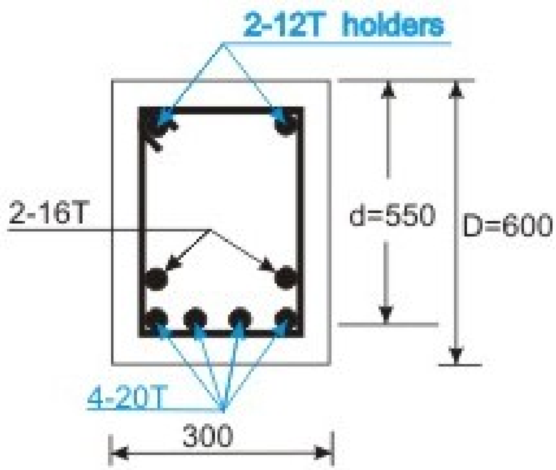

Fig. 3.7.2: Analysis Problem 3.3

Given data: b = 300 mm, d = 550 mm, D = 600 mm, A_{st} = 1658 mm^2 (4-20 T + 2-16 T), L_{eff} = 8 m and boundary conditions = simply supported (Fig. 3.7.2)

3.7.5.1 $x_{u,\,max}$

From Table 3.2 of Lesson 5, we get $x_{u,\,max} = 0.479\,d$ = 263.45 mm

3.7.5.2 x_u

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\,b\ f_{ck}} \tag{3.16}$$

$$= \frac{0.87\ (415)\ (1658)}{0.36\ (300)\ (20)}$$

= 277.13924 mm > $x_{u,\,max}$ = (263.45 mm)

Hence, the beam is over-reinforced.

3.7.5.3 M_u and service imposed loads

For $x_u > x_{u,max}$, we have

$$M_u = 0.36\ \frac{x_{u,\max}}{d}(1-0.42\,\frac{x_{u,\max}}{d})\,b\,d^2\ f_{ck} \tag{3.24}$$

$= 0.36 (0.479) \{1 - 0.42(0.479)\} (300) (550) (550) (20)$ Nmm

$= 250.01356$ kNm

If we use Eq. 3.22 using $x_u = x_{u,max}$, for M_u

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_{u,max}) \quad (3.22)$$

Then, $(M_u)_{steel} = 0.87 (415) (1658) \{550 - 0.42 (263.45)\}$ Nmm

$= 263.00468$ kNm $> (M_u)_{concrete}$ $(= 250.01356$ kNm$)$

The higher M_u as obtained from steel is not true because the entire amount of steel (1658 mm^2) cannot yield due to over-reinforcing. Prior to that, concrete fails at 250.01356 kNm. However, we can get the same of M_u as obtained from Eq. 3.24 of Lesson 5, if we can find out how much A_{st} is needed to have x_u = 263.45 mm. From Eq. 3.16 of Lesson 5, we can write:

$$(A_{st})_{needed\ for\ x_u = 263.45} = \frac{0.36\, b\, f_{ck}\, x_u}{0.87\, f_y}$$

$$= \frac{0.36(300)(20)(263.45)}{0.87(415)}$$

$$= 1576.1027 \text{ mm}^2$$

If we use this value for A_{st} in Eq. 3.22 of Lesson 5, we get

$$(M_u) = 0.87(415)(1576.1027)\{550 - 0.42(263.45)\}$$

$= 250.0135$ (same as obtained from Eq. 3.24).

From the factored moment M_u = 250.01356 kNm, we have:

Total factored load $= F_d = \dfrac{8\, M_u}{L_{eff}^2} = \dfrac{8(250.01356)}{8(8)} = 31.251695$ kN/m

Total service load $= \dfrac{31.251695}{1.5} = 20.834463$ kN/m

Now, Dead load of the beam = 0.3 (0.6) (25) = 4.5 kN/m

Hence, service imposed loads = 20.834463 - 4.5 = 16.334463 kN/m

3.7.6 Solution by Design Chart - Problems 3.2 and 3.3

For the two problems with known *b, d, D,* A_{st}, grade of concrete and grade of steel, chart 14 of SP-16 is applicable. From the effective depth *d* and percentage of reinforcement $p_t \; (= \frac{A_{st}\, 100}{b\, d})$, the chart is used to find M_u per metre width. Multiplying M_u per meter width with *b*, we get M_u for the beam. After that, the service imposed load is calculated using the relation

$$\text{Service imposed load} = \frac{8\, M_u}{(1.5)\, L^2} - \text{Dead load}$$

(3.27)

The results of the two problems are furnished in Table 3.7.

Table 3.7 Results of Problems 3.2 and 3.3 (Chart of SP-16)

Prob-lem	A_{st} (mm^2) p_t (%)	*d, b* (mm)	M_u (kNm/m) (Chart 14)	M_u (kNm)	$\frac{8 M_u}{1.5 L^2}$ (kN/m)	Dead load (kN/m)	Service impose d loads (kN/m)	Remarks
3.2	1256 0.7612	550, 300	700	210	17.5	4.5	13	M_u per m of 700 is well within the chart, hence under-reinforced.
3.3	1658 1.2058181	550, 300	820	246	20.5	4.5	16	Maximum M_u = 820, so over-reinforced.

3.7.7 Solution by Design Tables - Problems 3.2 and 3.3

Table 2 of SP-16 presents the value of reinforcement percentage for different combinations of f_y and (M_u/bd^2) for M-20. Here, from the known values of p and f_y, the corresponding values of M_u/bd^2 are determined. These in turn give M_u of the beam. Then the service imposed load can be obtained using Eq. 3.27 as explained in the earlier section (sec. 3.7.6). The results of the two problems are presented in Table 3.8.

Table 3.8 Results of Problems 3.2 and 3.3 (Table of SP-16)

Problem	A_{st} (mm^2) p_t (%)	d, b (mm)	f_y (N/mm^2)	$\frac{M_u}{b\,d^2}$ (N/mm^2) from Table 2	M_u (kNm)	$\frac{8M_u}{1.5\,L^2}$	Dead load (kN/m)	Service imposed loads (kN/m)	Remarks
3.2	1256 0.7612	550, 300	415	2.3105	209.67784	17.473155	4.5	12.973155	*
3.3	1658 1.2058181	500, 300	415	2.76	250.47	20.8725	4.5	16.3725	**

* Linear interpolated $\frac{M_u}{b\,d^2} \;=\; 2.30 + \frac{(0.7612 - 0.757)}{(0.765 - 0.757)}\;(2.32 - 2.30) \;=\; 2.3105$

** p_t = 1.2058181 is not admissible, i.e. over-reinforced. So at p_t = 0.955, M_u/bd^2 =2.76.

3.7.8 Comparison of Results of Three Methods

The values of M_u and service imposed loads of the under-reinforced and over-reinforced problems (Problems 3.2 and 3.3), computed by three methods, are presented in Table 3.9.

Table 3.9 Comparison of results of Problems 3.2 and 3.3

Problem	M_u (kNm)			Service imposed loads (kN/m)		
	Direct computation	Chart of SP-16	Table of SP-16	Direct computation	Chart of SP-16	Table of SP-16
3.2	209.4272	210	209.67784	12.9522	13	12.973155
3.3	250.01356	246	250.47	16.334463	16	16.3725

3.7.9 Practice Questions and Problems with Answers

Q.1: Determine the moments of resistance M_u and service imposed loads on a simply supported beam of effective span 10.0 m with b = 300mm, d = 500 mm, D = 550 mm and grades of concrete and steel are M20 and Fe500, respectively for the two different cases employing (a) direct computation method and (b) using charts and tables of SP-16: (i) when A_{st} is minimum acceptable and (ii) when A_{st} is maximum acceptable (Fig. 3.7.3).

A.1: Given data: b = 300 mm, d = 500 mm, D = 550 mm, L_{eff} = 10.0 m, f_{ck} = 20 N/mm^2 and f_y = 500 N/mm^2.

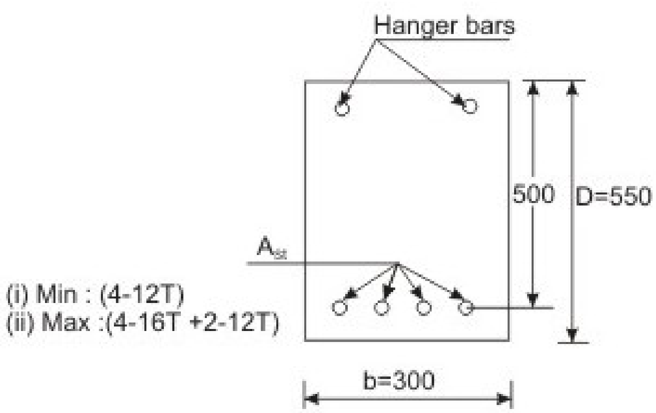

Fig. 3.7.3: Problem of Q 1

(a) Direct computation method:

Case (i) When A_{st} is minimum acceptable (Eq. 3.26 of Lesson 6)

$$\text{Minimum } A_s = \frac{0.85\, b\, d}{f_y}$$

(3.26)

So, $A_s = \frac{0.5\,(300)\,(500)}{500}$ mm^2 = 255 mm^2. Providing 4-12T gives A_{st} = 452 mm^2.

Equation 3.16 of Lesson 5 gives the depth of the neutral axis x_u:

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\,b\ f_{ck}} = \frac{0.87\,(500)\,(452)}{0.36\,(300)\,(20)} = 91.03 \text{ mm}$$

Table 3.2 of Lesson 5 gives $x_{u,\,max}$ = 0.46(500) = 230 mm.

The beam is, therefore, under-reinforced (as $x_u < x_{u,\,max}$).

Equation 3.22 of Lesson 5 gives the M_u as follows:

$$M_u = 0.87\ f_y\ A_{st}\ (d - 0.42\ x_u) \qquad (3.22)$$

= 0.87(500) (452) {500 - 0.42(91.03)} Nmm

= 90.79 kNm

Total factored load = $\frac{8\,M_u}{L_{eff}^2} = \frac{8\,(90.79)}{100} = 7.26$ kN/m

The dead load of the beam = 0.3 (0.55) (25) = 4.125 kN/m

So, the service imposed loads = {(Total factored load)/(Load factor)} - (Dead load)

= 7.26/1.5 - 4.125 = 0.715 kN/m

This shows that the beam can carry maximum service imposed loads, 17 per cent of its dead load only, when the acceptable minimum tensile reinforcement is 452 mm^2 (4 bars of 12 mm diameter).

Case (ii) when A_{st} is maximum acceptable:

To ensure ductile failure, it is essential that the acceptable maximum tensile reinforcement should be between 75 and 80 per cent of $p_{t,\,lim}$ and not as given in clause 26.5.1.1.(b), i.e. 0.04 *bD*. Thus, here the maximum acceptable p_t should be between 0.57 and 0.61 per cent (as $p_{t,\,lim}$ = 0.76 from Table 3.1 of Lesson 5). However, let us start with 0.76 per cent as the span is relatively large and keeping in mind that while selecting the bar diameter, it may get reduced to some extent.

So, $(A_{st})_{max}$ = 0.76 (300) (500)/100 = 1140 mm^2

Selecting 4-16 and 2-12 mm diameter bars, we get A_{st} = 1030 mm^2 when p_t becomes 0.67 per cent. So, the maximum acceptable tensile reinforcement is 1030 mm^2.

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\,b\ f_{ck}} = \frac{0.87\,(500)\,(1030)}{0.36\,(300)\,(20)} = 207.43\ \text{mm} \quad \text{(Eq. 3.16 of Lesson 5)}$$

x_u < $x_{u,\,max}$ (as $x_{u,\,max}$ = 230 mm; see Case (i) of this problem).

Therefore, M_u is obtained from Eq. 3.22 of Lesson 5 as,

$$M_u = 0.87\ f_y\ A_{st}\ (d - 0.42\ x_u) \quad (3.22)$$

$$= 0.87(500)\ (1030)\ \{500 - 0.42(207.43)\}\ \text{Nmm}$$

$$= 184.99\ \text{kNm}$$

$$\text{Total factored load} = \frac{8\,M_u}{L^2_{eff}} = \frac{8\,(184.99)}{100} = 14.8\ \text{kN/m}$$

With dead load = 4.125 kN/m (see Case (i) of this problem), we have:

Service imposed load = 14.8/1.5 - 4.125 = 5.74 kN/m

This beam, therefore, is in a position to carry service imposed loads of 5.74 kN/m, about 40% higher than its own dead load.

(b) Using chart and tables of SP-16:

Tables 3.10 and 3.11 present the results using charts and tables respectively of SP-16. For the benefit of the reader the different steps are given below separately for the use of chart and table respectively for the minimum acceptable reinforcement.

The steps using chart of SP-16 are given below:

Step 1: With the given f_{ck}, f_y and d, choose the particular chart. Here, for f_{ck} = 20 N/mm^2, f_y = 500 N/mm^2 and d = 500 mm, the needed chart no. is 17.

Step 2: Chart 17 shows minimum p_t = 0.13% which gives A_{st} = 195 mmm^2

Step 3: Provide bars of 4-12 mm diameter, which give p_t = 0.301%

Step 4: For p_t = 0.301, Chart 17 shows M_u = 300 kNm per metre width, which gives M_u = 300 (0.3) = 90 kNm for the beam.

Step 5: The service imposed loads are calculated as follows:

Service imposed loads = $\dfrac{8\,M_u}{L_{eff}^2\ (1.5)} - 4.125 = 0.675$ kN/m, using the dead load of the beam as 4.125 kN/m (see case (i) of this problem).

Step 6: The capacity of the beam is to carry 0.675 kN/m which is (0.675/4.125) 100 = 16.36%.

Table 3.10 Results of Q.1 using chart of SP-16

Sl. No.	Chart No. Min/Max	Minimum/ Maximum p_t and A_{st} (%, mm^2)	Provided bars (No, mm diameter)	(p_t) provided (%)	M_u (kNm/m, kNm)	Service imposed loads (kN/m)	Service imposed loads in (%)
1	17 (Minimum)	0.13, 195	4-12	0.301	300, 90	0.675	16.36
2	17 (Maximum)	0.76, 1140	4-16 + 2-12	0.67	610, 183	5.635	136.61

Similar calculations are done for the maximum acceptable reinforcement. The steps are given below:

Step 1: With the given f_{ck} = 20 N/mm^2, f_y = 500 N/mm^2, Table 2 of SP-16 is selected.

Step 2: $(p_t)_{min}$ = 0.07 from Table 2, which gives A_{st} = 105 mmm^2

Step 3: Provide bars of 4-12 mm diameter, which give p_t = 0.301%

Step 4: For $(p_t)_{provided}$, we get (M_u/bd^2) from Table 2 of SP-16 by linear interpolation as follows:

$$\frac{M_u}{b\,d^2} = 1.20 + \frac{(1.25-1.2)(0.301-0.298)}{(0.312-0.298)} = 1.204$$

Hence, $M_u = 1.204\ (300)\ (500)^2 = 90.3$ kNm

Step 5: Same as Step 5 while using chart of SP-16.

Step 6: Same as Step 6 while using chart of SP-16.

Table 3.11 Results of Q.1 using table of SP-16

Sl. No.	Table No. Min/Max	Minimum/ Maximum p_t and A_{st} (%, mm^2)	Provided bars (No, mm diameter)	(p_t) provided (%)	$\frac{M_u}{b\,d^2}$ (kNm/m, kNm)	Service imposed loads (kN/m)	Service imposed loads in (%)
1	2 (Minimum)	0.07, 105	4-12	0.301	1.204, 90.3	0.691	16.75
2	2 (Maximum)	0.755, 1132	4-16 + 2-12	0.67	2.42, 181.5	5.555	134.66

3.7.10 References

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2. Limit State Design of Reinforced Concrete, 2nd Edition, by P.C.Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2002.
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3.7.11 Test 7 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

TQ.1: Determine the moment of resistance for the beams shown in Figs. 3.7.4 and 3.7.5 using M 20 and Fe 250 by direct computation and using charts and tables of SP-16.

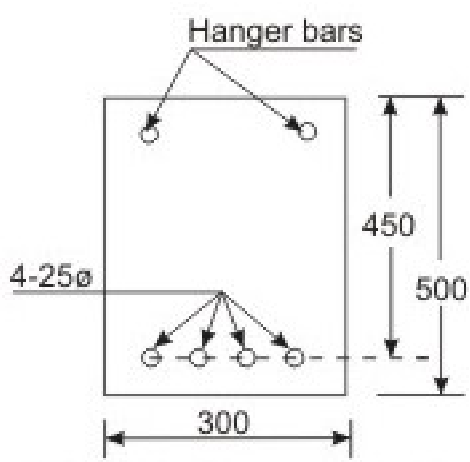

Fig. 3.7.4: Problem of Test 7, TQ. 1A

A.TQ.1: Case A: TQ.1 A of Fig. 3.7.4

(i) Direct computation method

A_{st} = 1963 mm^2 (4-25 ϕ)

$x_{u,\max}$ = 0.53 (450) = 238.5 mm (Table 3.2 of Lesson 5 gives 0.53)

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\ b\ f_{ck}} = \frac{0.87\ (250)\ (1963)}{0.36\ (300)\ (20)} = 197.66$$ (See Eq. 3.16 of Lesson 5)

So, $x_u < x_{u,\max}$ shows that it is under-reinforced section for which M_u is obtained from Eq. 3.22 of Lesson 5

M_u = 0.87 f_y A_{st} (d - 0.42 x_u)
(3.22)

= 0.87(250) (1963) {450 - 0.42(197.66)}

= 156.68 kNm

(ii) Chart of SP-16

A_{st} = 1963 mm^2

$$p_t = \frac{A_{st}\ (100)}{b\ d} = \frac{1963\ (100)}{300\ (450)} = 1.45$$

From chart 11 of SP-16, when p_t = 1.45, d = 450, we get

M_u per metre width = 522 kNm/m

M_u = 522 (0.3) = 156.6 kNm

(iii) Table of SP-16

Table 2 of SP-16 is for M-20 and Fe250. At p_t = 1.451, we get

$$\frac{M_u}{b\,d^2} = 2.58 \text{ N/mm}^2$$

So, M_u = 2.58 (300) (450) (450) (10^{-6}) kNm = 156.74 kNm

The three values of M_u are close to each other.

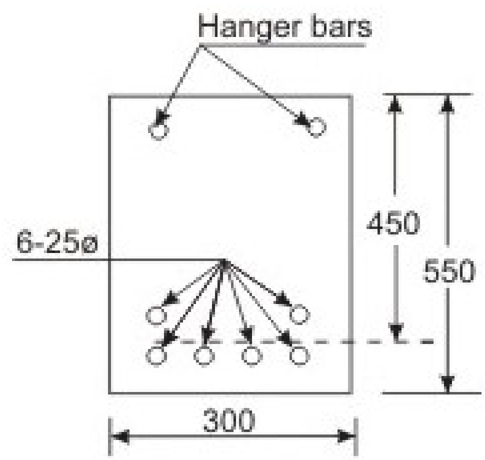

Fig. 3.7.5: Problem of Test 7, TQ. 1B

A.TQ.1: Case B: TQ.1 B of Fig. 3.7.5

(i) Direct computation method

A_{st} = 1963 + 981 = 2944 mm^2 (6-25 ϕ)

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\ b\ f_{ck}} = \frac{0.87\ (250)(2944)}{0.36\ (300)\ (20)} = 296.44$$ mm (See Eq. 3.16 of Lesson 5)

$x_{u,\max}$ = 0.53 (450) = 238.5 mm (Table 3.2 of Lesson 5 gives 0.53)

So, $x_u > x_{u,\ max}$ and the beam is over-reinforced. In such a situation, we take $x_u = x_{u,\ max}$ = 238.5 mm. The M_u will be calculated from Eq. 3.24 of Lesson 5.

$$M_u = M_{u,\lim} = 0.36\ \frac{x_{u,\max}}{d}\ (1 - 0.42\ \frac{x_{u,\max}}{d})\ b\,d^2\ f_{ck} \quad (3.24)$$

= 0.36 (0.53) {1 - 0.42 (0.53)} (300) (450)^2 (20) Nmm

= 180.22 kNm

(ii) Chart of SP-16

$$p_t = \frac{2944\,(100)}{300\,(450)} = 2.18.$$ In chart 11 (for M 20 and Fe 250), maximum admissible p_t is 1.75 and for this p_t when d = 450, M_u = 600 kNm/m.

So, M_u = 600 (0.3) = 180 kNm

(iii) Table of SP-16

Table 2 (for M 20 and Fe 250) has the maximum p_t = 1.76 and at that value, (M_u/bd^2) = 2.98. This gives

M_u = 2.98 (300) $(450)^2$ (10^{-6}) = 181.03 kNm. Here also the three values of M_u are close to each other.

3.7.12 Summary of this Lesson

This lesson explains the use of equations derived in Lesson 5 for the analysis type of problems. The three methods (i) direct computation, (ii) use of charts of SP-16 and (iii) use of tables of SP-16 are illustrated through the step by step solutions of numerical problems. Their results are compared to show the closeness of them.