Module
13

Working Stress Method

Lesson

34

Rectangular Beams under Flexure

Instructional Objectives:

At the end of this lesson, the student should be able to:

- explain the philosophy of the design by working stress method,
- explain the concepts of permissible stresses in concrete and steel and the factors of safety in this method,
- state the limit of increase of permissible stresses under specific loading considerations,
- state the assumptions for the design of flexural members employing working stress method,
- explain the concept of modular ratio,
- derive the governing equations of rectangular, balanced and under-reinforced (singly-reinforced) sections,
- name the two types of problems for the designer,
- write down the steps separately for the two types of problems in the working stress method,
- justify the need for doubly-reinforced beams, and
- derive the governing equations of the doubly-reinforced rectangular flexural members.

13.34.1 Introduction

Design of reinforced concrete structures started in the beginning of this century following purely empirical approach. Thereafter came the so called rigorous elastic theory where it is assumed that concrete is elastic and reinforcing steel bars and concrete act together elastically. The load-deflection relation is linear and both concrete and steel obey Hooke's law.

The method is designated as working stress method as the loads for the design of structures are the service loads or the working loads. The failure of the structure will occur at a much higher load. The ratio of the failure loads to the working loads is the factor of safety. Accordingly, the stresses of concrete and steel in a structure designed by the working stress method are not allowed to exceed some specified values of stresses known as permissible stresses. The permissible stresses are determined dividing the characteristic strength f_{ck} of the material by the respective factor of safety. The values of the factor of safety depend on the grade of the material and the type of stress. Thus, for concrete in bending compression, the permissible stress of grade M 20 is 7 N/mm^2, which is obtained by dividing the characteristic strength f_{ck} of M 20 concrete by a number 3 and then rationalising the value to 7. This permissible stress is designated by σ_{cbc}, the symbol σ stands for permissible stress and the letters *c*, *b* and *c* mean concrete in bending compression, respectively.

13.34.2 Permissible Stresses in Concrete

The permissible stress of concrete in direct tension is denoted by σ_{td}. The values of σ_{td} for member in direct tension for different grades of concrete are given in cl. B-2.1.1 of IS 456. However, for members in tension, full tension is to be taken by the reinforcement alone. Though full tension is taken by the reinforcement only, the actual tensile stress of concrete f_{td} in such members shall not exceed the respective permissible values of σ_{td} to prevent any crack. Table 13.1 presents the values of σ_{td} for selective grades of concrete as a ready reference. It may be worth noting that the factor of safety of concrete in direct tension is from 8.5 to 9.5.

The permissible stresses of concrete in bending compression σ_{cbc}, in direct compression σ_{cc} and the average bond for plain bars in tension τ_{bd} are given in Table 21 of IS 456 for different grades of concrete. However, Table 13.1 presents these values for selective grades of concrete, as a ready reference. The factors of safety of concrete in bending compression, direct compression and average bond for plain bars are 3, 4 and from 25 to 35, respectively. For plain bars in compression, the values of average bond stress are obtained by increasing the respective value in tension by 25 percent, as given in the note of Table 21 of IS 456. For deformed bars, the values of Table 21 are to be increased by sixty per cent, as stipulated in cl. B-2.1.2 of IS 456.

Table 13.1 Permissible stresses in concrete

Grade of concrete	Direct tension σ_{td} (N/mm^2)	Bending compression σ_{cbc} (N/mm^2)	Direct compression σ_{cc} (N/mm^2)	Average bond τ_{bd} for plain bars in tension (N/mm^2)
M 20	2.8	7.0	5.0	0.8
M 25	3.2	8.5	6.0	0.9
M 30	3.6	10.0	8.0	1.0

M 35	4.0	11.5	9.0	1.1
M 40	4.4	13.0	10.0	1.2

From the values of the permissible stresses and the respective characteristic strengths for different grades of concrete, it may be seen that the factors of safety of concrete in direct tension, bending compression, direct compression and average bond for plain bars in tension are from 8.5 to 9.5, 3, 4 and from 25 to 35, respectively.

13.34.3 Permissible Stresses in Steel Reinforcement

Permissible stresses in steel reinforcement for different grades of steel, diameters of bars and the types of stress in steel reinforcement are given in Table 22 of IS 456. Selective values of permissible stresses of steel of grade Fe 250 (mild steel) and Fe 415 (high yield strength deformed bars) in tension (σ_{st} and σ_{sh}) and compression in column (σ_{sc}) are furnished in Table 13.2 as a ready reference. It may be noted from the values of Table 13.2 that the factor of safety in steel for these stresses is about 1.8, much lower than concrete due to high quality control during the production of steel in the industry in comparison to preparing of concrete.

Table 13.2: Permissible stresses in steel reinforcement

Type of stress in steel reinforcement	Mild steel bars, Fe 250, (N/mm^2)	High yield strength deformed bars, Fe 415, (N/mm^2)
Tension σ_{st} or σ_{ss}		
(a) up to and including 20 mm diameter	140	230
(b) over 20 mm diameter	130	230
Compression in column bars σ_{sc}	130	190

13.34.4 Permissible Shear Stress in Concrete τ_c

Permissible shear stress in concrete in beams without any shear reinforcement depends on the grade of concrete and the percentage of main tensile reinforcement in beams. Table 23 of IS 456 furnishes the values of τ_c for wide range of percentage of tensile steel reinforcement for different grades concrete. Other relevant clauses regarding the permissible shear stress of concrete are given in cls.B-5.2.1.1, B-5.2.2 and B-5.2.3 of IS 456.

13.34.5 Increase in Permissible Stresses

Clause B-2.3 of IS 456 recommends the increase of permissible stresses of concrete and steel given in Tables 21 to 23 up to a limit of 33.33 per cent, where stresses due to wind (or earthquake), temperature and shrinkage effects are combined with those due to dead, live and impact loads.

13.34.6 Assumptions for Design of Members by Working Stress Method

As mentioned earlier, the working stress method is based on elastic theory, where the following assumptions are made, as specified in cl. B-1.3 of IS 456.

(a) Plane sections before bending remain plane after bending.

(b) Normally, concrete is not considered for taking the tensile stresses except otherwise specifically permitted. Therefore, all tensile stresses are taken up by reinforcement only.

(c) The stress-strain relationship of steel and concrete is a straight line under working loads.

(d) The modular ratio *m* has the value of $280/3\sigma_{cbc}$, where σ_{cbc} is the permissible compressive stress in concrete due to bending in N/mm^2. The values of σ_{cbc} are given in Table 21 of IS 456. The modular ratio is explained in the next section.

13.34.7 Modular Ratio *m*

In the elastic theory, structures having different materials are made equivalent to one common material. In the reinforced concrete structure, concrete and reinforcing steel are, therefore, converted into one material. This is done by transformation using the modular ratio *m* which is the ratio of modulus of elasticity of steel and concrete. Thus, $m = E_s/E_c$. where E_s is the modulus of elasticity of steel which is 200000 N/mm^2. However, concrete has different moduli, as it is not a perfectly elastic material.

The short-term modulus of concrete $E_c = 5000\sqrt{f_{ck}}$ in N/mm^2, where f_{ck} is the characteristic strength of concrete. However, the short-term modulus does not take into account the effects of creep, shrinkage and other long-term effects. Accordingly, the modular ratio *m* is not computed as $m = E_s/E_c$ = 200000/(5000

$\sqrt{f_{ck}}$). The value of m, as given in sec. 13.34.6 d, i.e., 280/3σ_{cbc}, partially takes into account long-term effects. This is also mentioned in the note of cl. B-1.3 of IS 456.

13.34.8 Flexural Members – Singly Reinforced Sections

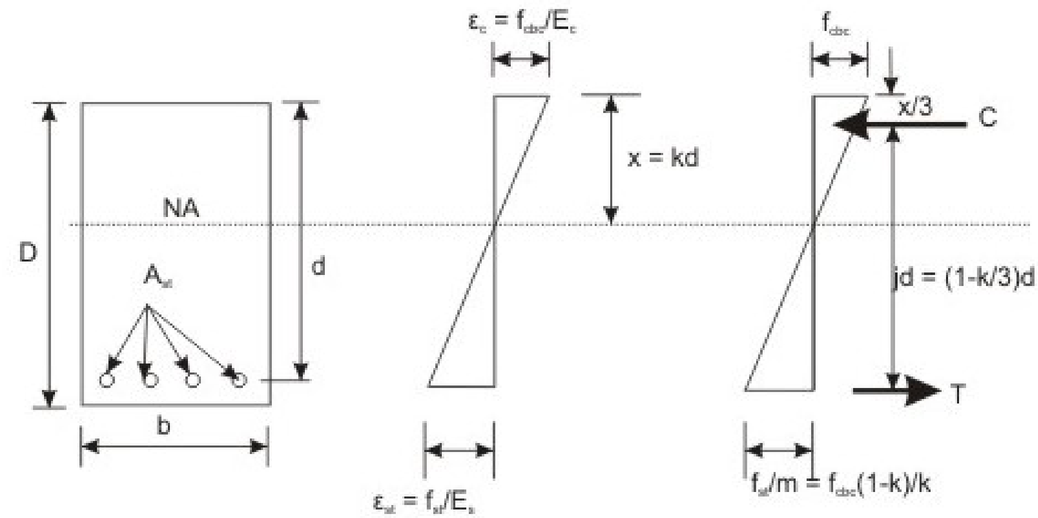

Fig. 13.34.1(a): Section   Fig. 13.34.1(b): Strain profile   Fig. 13.34.1(c): Stress distribution

Fig. 13.34.1: Singly reinforced rectangular beam

A simply supported beam subjected to two point loads shall have pure moment and no shear in the middle-third zone, as shown in Fig. 1.1.1 of Lesson 1.The cross-sections of the beam in this zone are under pure flexure. Figures 13.34.1a, b and c show the cross-section of a singly-reinforced beam, strain profile and stress distribution across the depth of the beam, respectively due to the loads applied on the beam. Most of the symbols are used in earlier lessons. The new symbols are explained below.

$x = kd$ = depth of the neutral axis, where k is a factor,

f_{cbc} = actual stress of concrete in bending compression at the top fibre which should not exceed the respective permissible stress of concrete in bending compression σ_{cbc},

f_{st} = actual stress of steel at the level of centroid of steel which should not exceed the respective permissible stress of steel in tension σ_{st},

jd = $d(1-k/3)$ = lever arm i.e., the distance between lines of action of total compressive and tensile forces C and T , respectively.

Figures 13.34.1b and c show linear strain profile and stress distribution, respectively. However, the value of the stress at the level of centroid of steel of Fig. 13.34.1 c is f_{st}/m due to the transformation of steel into equivalent concrete of area mA_{st}.

13.34.9 Balanced Section – Singly-Reinforced

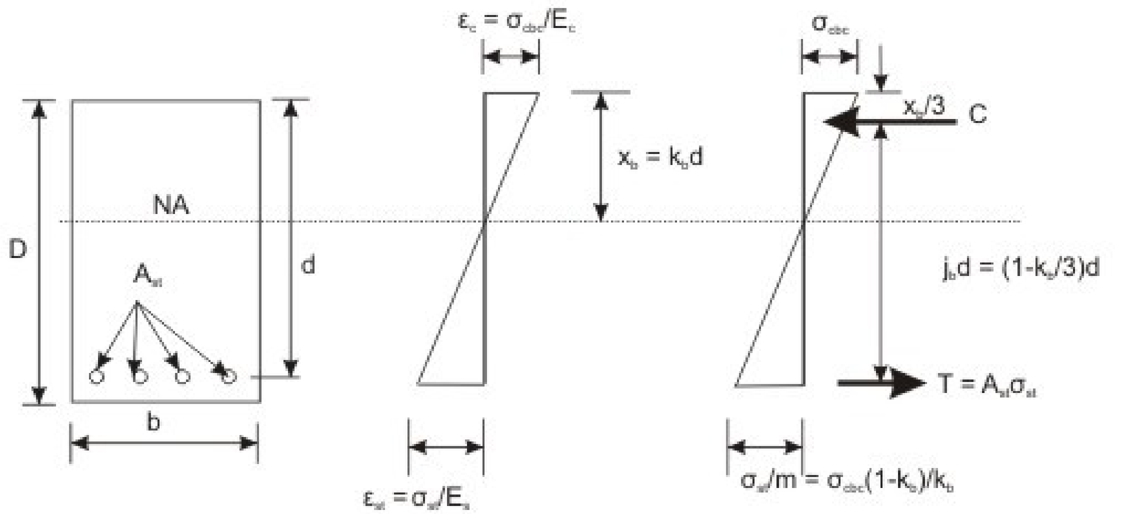

Fig. 13.34.2(a): Section   Fig. 13.34.2(b): Strain profile   Fig. 13.34.2(c): Stress distribution

Fig. 13.34.2: Singly-reinforced balanced section

In a balanced cross-section both f_{cbc} and f_{st} reach their respective permissible values of σ_{cbc} and σ_{st} at the same time as shown in Fig.13.34.2c. The depth of neutral axis is $x_b = k_b d$. From the stress distribution of Fig.13.34.2c, we have

$$\sigma_{st}/m = \sigma_{cbc}(1-k_b)/k_b \qquad (13.1)$$

An expression of k_b is obtained by substituting the expression of m as

$$m = 280 / 3\sigma_{cbc} \qquad (13.2)$$

into Eq. 13.1. This gives

$$k_b = 93.33 / (\sigma_{st} + 93.33) \qquad (13.3)$$

Equation 13.3 shows that the value of k_b for balanced section depends only on σ_{st}. It is independent of σ_{cbc}.

The lever arm, $j_b d = (1- k_b/3)\, d$ (13.4)

The expressions of total compressive and tensile forces, C and T are:

$C = (1/2)\ \sigma_{cbc}\ b\ x_b = (1/2)\ \sigma_{cbc}\ b\ k_b\ d$ (13.5)

$T = A_{st}\ \ \sigma_{st}$ (13.6)

The total compressive force is acting at a depth of $x_b/3$ from the top fibre of the section.

The moment of resistance of the balanced cross-section, M_b is obtained by taking moment of the total compressive force C about the centroid of steel or moment of the tensile force T about the line of action of the total compressive force C. Thus,

$$M_b = C(j_b d) = (1/2)\sigma_{cbc}\ k_b\, j_b (bd^2) \qquad (13.7)$$

or, $$M_b = T(j_b d) = \sigma_{st} A_{st}\, j_b d = \ (p_{t,bal}/100)\sigma_{st}\, j_b (bd^2) \qquad (13.8)$$

as $$A_{st} = p_{t,bal}(bd/100) \qquad (13.9)$$

where $p_{t,bal}$ = balanced percentage of steel
From Eqs.13.7 and 13.8, we can write

$M_b = R_b\ bd^2$ (13.10)

where $R_b = (1/2)\ \sigma_{cbc}\ k_b\ j_b = (p_{t,bal}/100)\ \sigma_{st}\ j_b$ (13.11)

and $j_b = 1 - (k_b/3)$ (13.12)

The expression of the balanced percentage of steel $p_{t,bal}$ is obtained by equating the total compressive force C to the tensile force T from Eqs. 13.5 and 13.6. This gives,

$A_{st}\ \sigma_{st} = (\sigma_{cbc}/2)\ b\ k_b\ d$, *which* gives: $p_{t,bal}\ (bd/100)\ \sigma_{st} = (\sigma_{cbc}/2)\ b\ k_b d$

or $p_{t,bal} = 50\ k_b(\sigma_{cbc}/\sigma_{st})$ (13.13)

It is always desirable, though may not be possible in most cases, to design the beam as balanced since the actual stresses of concrete f_{cbc} at the top compression fibre and steel at the centroid of steel should reach their respective permissible stresses σ_{cbc} and σ_{st} in this case only. The procedure of the design is given below.

Treating the design moment as the balanced moment of resistance M_b and assuming the width, b of the beam as 250 mm, 300 mm or 350 mm, the effective depth d is obtained from Eq.13.10. The required balanced area of steel A_{st} is then obtained from Eq. 13.9 getting the values of k_b from Eq. 13.3 and then $p_{t,bal}$ from Eq. 13.13.

The values of R_b, the moment of resistance factor M_b/bd^2 are obtained from Eq. 13.11 for different values of σ_{cbc} and σ_{st} (different grades of concrete and steel) and are presented in Table 13.3. Similarly, the values of balanced percentage of tensile reinforcement, $p_{t,\,bal}$ obtained form Eq. 13.13 for different grades of concrete and steel are given in Table 13.4.

Table 13.3 Moment of resistance factor R_b in N/mm^2 for balanced rectangular section.

σ_{cbc} (N/mm^2)	σ_{st} (N/mm^2)		
	140	230	275
7.0	1.21	0.91	0.81
8.5	1.47	1.11	0.99
10.0	1.73	1.30	1.16

Table 13.4 Percentage of tensile reinforcement $p_{t,bal}$ for singly-reinforced balanced section.

σ_{cbc} (N/mm^2)	σ_{st} (N/mm^2)		
	140	230	275
7.0	1.0	0.44	0.32
8.5	1.21	0.53	0.39
10.0	1.43	0.63	0.46

Values of R_b and $p_{t.bal}$ from Tables 13.3 and 13.4 reveal the following:

1. For given values of width and effective depth, b and d of a rectangular section, the balanced moment of resistance M_b increases with higher grade of concrete for a particular grade of steel. However, the balanced moment of resistance decreases with higher grade of steel for a particular grade of concrete.

2. The balanced percentage of steel, $p_{t.bal}$ increases with the increase of grade of concrete for a particular grade of steel for given values of width and effective depth of a rectangular section. On the other hand, the

balanced percentage of steel, $p_{t.bal}$ decreases with the increased grade of steel for a particular grade of concrete.

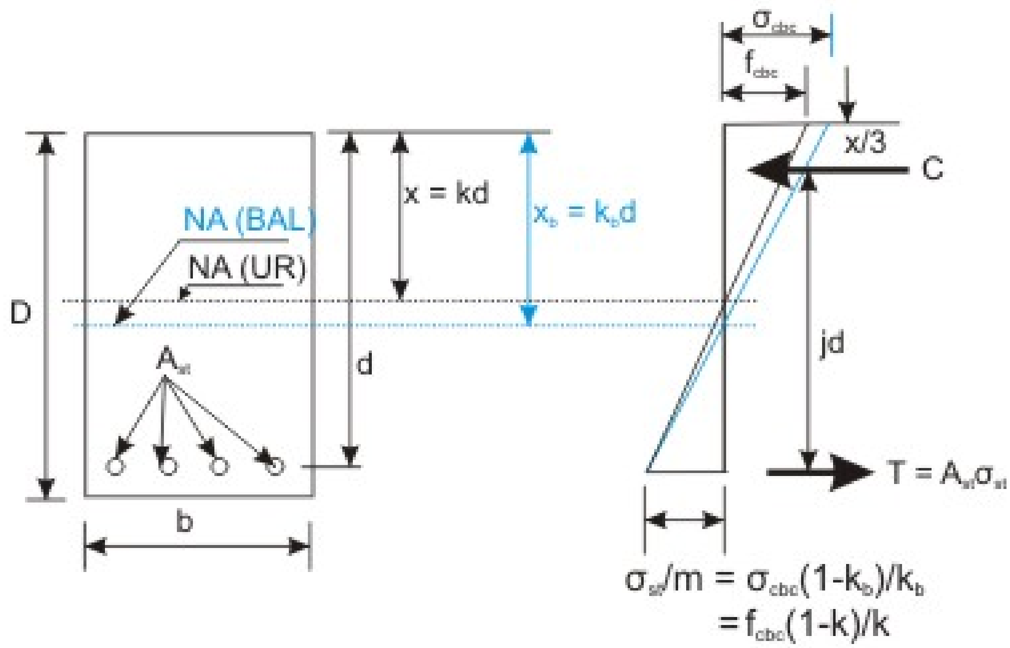

Fig. 13.34.3(a): Section

Fig. 13.34.3(b): Stress distribution

Fig. 13.34.3: Under-reinforced beam

As mentioned earlier, it may not be possible to design a balanced section since the area of steel required for the balanced condition is difficult to satisfy with available bar diameters. In such cases, it is essential that the beam should be provided with the steel less than the balanced steel so that the actual stress of steel in tension reaches the permissible value σ_{st} and the actual stress of concrete f_{cbc} is less than the permissible value. Such sections are designated as under-reinforced sections and moment of resistance shall be governed by the tensile stress of steel σ_{st}, which is known. The depth of the neutral axis will be less than the balanced depth of the neutral axis, as shown in Fig. 13.34.3b. The relevant equations for the design of under-reinforced section are established in the next section.

13.34.10 Under-reinforced Section -- Singly Reinforced

Figure 13.34.3a shows the cross-section where x (= kd) is the depth of the neutral axis. The depth of the neutral axis is determined by taking moment of the area of concrete in compression (= bx) and the transformed area of steel (= mA_{st}) about the neutral axis, which gives

$$b\,kd\,(kd/2) = m\,(p_t\,bd/100)\,(d-kd)$$

or $k^2 + (p_t\, mk/50) - (p_t\, m/50) = 0$ (13.14)

Equation 13.14 has two roots of *k* given by

$$k = -(p_t\, m/100) \pm \{(p_t\, m/100)^2 + (p_t\, m/50)\}^{1/2} \quad (13.15)$$

Since *k* cannot be negative, we have the positive root to be considered as

$$k = -(p_t m/100) + \{(p_t\, m/100)^2 + (p_t\, m/50)\}^{1/2} \quad (13.16)$$

The moment of resistance of the under-reinforced section is obtained from

$$M = T \text{ (lever arm)} = A_{st}\, \sigma_{st}\, d(1 - k/3) = (p_t\, bd/100)\, \sigma_{st}\, d\,(1 - k/3)$$

Therefore, we have:

$$M = (p_t/100)\sigma_{st}\,(1 - k/3)\, bd^2 \quad (13.17)$$

which can also be expressed as

$$M = R\ \ bd^2 \quad (13.18)$$

where

$$R = (p_t/100)\, \sigma_{st}\,(1 - k/3) \quad (13.19)$$

is the moment of resistance factor M/bd^2. The values of *R* are obtained for given values of p_t for different grades of steel and concrete. Tables 68 to 71 of SP-16 furnish the values of *R* for four grades concrete and five values of σ_{st}.

The actual stress of concrete at the top fibre f_{cbc} shall not reach σ_{cbc} in under- reinforced sections. The actual stress f_{cbc} is determined from the equation $C=T$ as explained below:

With reference to Fig. 13.34.3c, the compressive force *C* of concrete and tensile for *T* of steel are:

$$C = (1/2)\, f_{cbc}\ \ b\, kd \quad (13.20)$$

$$T = \sigma_{st}\ \ A_{st} \quad (13.6)$$

For the tensile force, the actual stress of steel f_{st} shall reach the value of σ_{st}. So, we are using Eq. 13.6, the same equation as for the balanced section. Equating C and T from Eqs. 13.20 and 13.6, we get

$$(1/2) f_{cbc}\ b\,kd = \sigma_{st} A_{st}$$

or, $f_{cbc} = 2\ \sigma_{st} A_{st} / b\,k\,d$

(13.21)

Expressing $A_{st} = p_t\,bd\,/100$

(13.22)

and using Eq. 13.22 in Eq. 13.21, we get

$$f_{cbc} = p_t\ \sigma_{st} / 50\ k$$

(13.23)

The two types of problems: (i) Analysis type and (ii) Design type are now taken up in the following sections.

13.34.11 Analysis Type of Problems

For the purpose of analysing a singly-reinforced beam where the working loads, area of steel, b and d of the cross section are given, the actual stresses of concrete at the top fibre and steel at the centroid of steel are to be determined in the following manner.

Step 1: To determine the depth of the neutral axis *kd* from Eq. 13.16.

Step 2: The beam is under-reinforced, balanced or over-reinforced, if k is less than, equal to or greater than k_b, to be obtained from Eq. 13.3.

Step 3: The actual compressive stress of concrete f_{cbc} and tensile stress of steel at the centroid of steel f_{st} are determined in the following manner for the three cases of Step 2.

Case (i) When $k < k_b$ (under-reinforced section)

From the moment equation, we have

$$M = A_{st}\ f_{st}\ d(1 - k/3)$$

or $f_{st} = M / \{A_{st}\ d(1 - k/3)\}$

(13.24)

where M is obtained from the given load, A_{st} and d are given, and k is determined in Step 1.

Equating $C = T$, we have: $(1/2)\, f_{cbc}\, b(kd) = A_{st}\, f_{st}$

or $f_{cbc} = (2\, A_{st}\, f_{st}) / (b\, kd)$ (13.25)

where A_{st}, b and d are given and k and f_{st} are determined in steps 1 and 2, respectively.

Case (ii) When $k = k_b$ (balanced section)

In the balanced section $f_{cbc} = \sigma_{cbc}$ and $f_{st} = \sigma_{st}$.

Case (iii) When $k > k_b$ (over-reinforced section)

Such beams are not to be used as in this case f_{cbc} shall reach σ_{cbc} while f_{st} shall not reach σ_{st}. These sections are to be redesigned either by increasing the depth of the beam or by providing compression reinforcement. Beams with compression and tension reinforcement are known as doubly-reinforced beam and is taken up in sec. 13.34.13.

13.34.12 Design Type of Problems

It has been explained earlier in sec.13.34.9 that it is difficult, though desirable, to design a balanced section, as the concrete and steel should attain their respective permissible values at the same time. Therefore, a practical design shall preferably be under-reinforced. However, the given data are the working load, span and support conditions of the beam. These data shall enable to get the design moment of the section. The breadth b, effective depth d and area of steel are to be determined in these problems. The steps to be followed are given below.

Step 1. Selection of preliminary dimension *b* and *D*

The width is normally taken as 250 mm, 300 mm or 350 mm so that the reinforcing bars can be accommodated without difficulty. The total depth of the beam may be supplied in architectural drawings or may be estimated from the span to depth ratios, as stiputed in cl. 23.2.1 of IS 456 for the control of deflection. However, these values should be revised, if needed.

Step 2. Determination of *d*

The effective depth of a balanced section is first obtained from Eq. 13.10, where the value of R_b is taken from Table 13.3 after finalising the grades of concrete and steel. This balanced effective and total depths of beam, as given in the architectural drawings or may be needed for the control of deflection, give some idea while finalising the depth of the beam, higher than the balanced depth, so that the beam becomes under-reinforced.

Step 3. Determination of A_{st}

First, the balanced percentage of steel $p_{t,bal}$ is taken from Table 13.4 for the selected grades of concrete and steel. The amount of increase of the depth of the beam of step 2 may also depend on the amount $p_{t,\ bal}$. In fact, there are sets of values of depth and the respective area of steel, to be selected judiciously on the basis of practical considerations.

Step 4. Checking for the stresses

After finalising the depth and area of steel (depth larger than the balanced depth and area of steel lower than the balanced area of steel), the section is to be analysed as an analysis type of problem, as explained in sec. 13.34.11.

Alternatively, appropriate table from Tables 63 to 71 of SP-16 may be used for getting several possible combinations quickly.

13.34.13 Doubly-Reinforced Beams

In some situations, it may not be possible to provide the required depth of the beam as singly-reinforced either due to restrictions given by the architects or due to functional requirements. In such cases, the designer has to satisfy the structural requirement with a depth less than that of a singly-reinforced section. This is done by providing steel reinforcing bars in the compression side along with the tensile steel in the tension side of the beam. These beams are designated as doubly-reinforced beams.

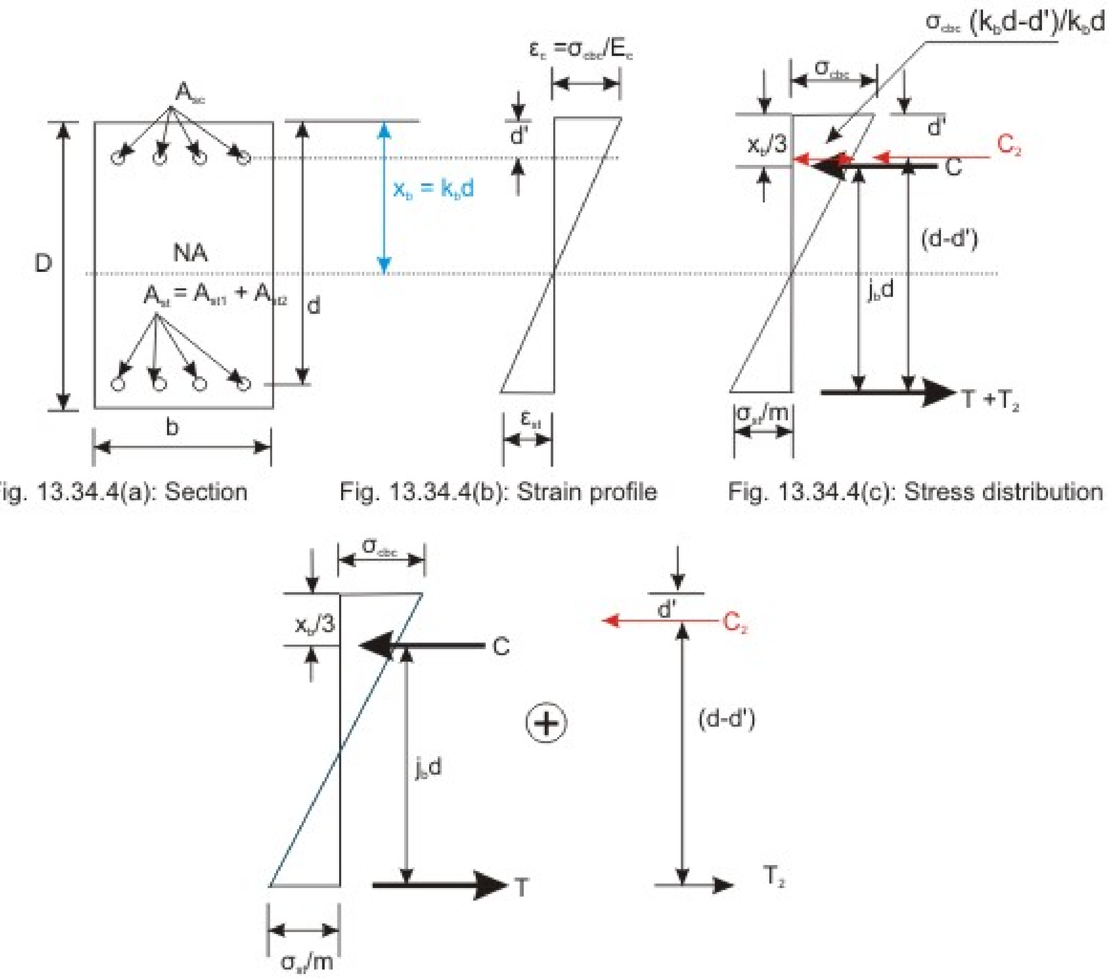

Fig. 13.34.4(a): Section Fig. 13.34.4(b): Strain profile Fig. 13.34.4(c): Stress distribution

Fig. 13.34.4(d): Singly-reinforced balanced stress distribution

Fig. 13.34.4(e): Additional forces

Fig. 13.34.4: Doubly-reinforced beam

Figures 13.34.4a to c show the cross-section, strain profile and stress distribution of a doubly-reinforced section. Since, the design moment is more than the balanced moment of resistance of the section, we have

$$M = M_b + M' \tag{13.26}$$

The additional moment M' is resisted by providing compression reinforcement A_{sc} (= $p_cbd/100$) and additional tensile reinforcement A_{st2}. The modular ratio of the compression steel is taken as 1.5 m, where m is the modular ratio as explained in sec. 13.34.7.

Figure 13.34.4c shows that the stress of concrete at the level of compression steel is σ_{cbc} $(k_bd - d') / k_dd$. Accordingly, the stress in the compression steel reinforcement is 1.5m σ_{cbc} $(k_bd - d') / k_bd$.

Figure 13.34.4d and e present separate stress distribution for the balanced beam (shown in Fig. 13.34.2c) and the compressive and tensile forces of compressive and tensile reinforcing bars C_2 and T_2, respectively. The expression of the additional moment M' is obtained by multiplying C_2 and T_2 with the lever arm (d - d'), where d' is the distance of the centroid of compression steel from the top fibre. We have, therefore,

$$C_2 = A_{sc}\,(1.5m - 1)\,\sigma_{cbc}\,(kd - d')\,/\,kd \tag{13.27}$$

$$T_2 = (p_t - p_{t,\,bal})\,(bd\,/\,100)\,\sigma_{st} \tag{13.28}$$

$$M' = C_2\,(d - d') = (p_c\,bd\,/100)\,(1.5m - 1)\;\sigma_{cbc}\,(kd - d')/\,kd\;\;(d - d')$$

or, $$M' = (p_c\,/\,100)(1.5m - 1)\,\sigma_{cbc}\,(1 - d'/\,kd)(1 - d'/d)bd^2 \tag{13.29}$$

also, $$M' = T_2\,(d - d')\,d\qquad = (p_t - p_{t,\,bal})\,(bd\,/\,100)\sigma_{st}\,(d - d')$$

or, $$M' = (p_t - p_{t,\,bal})\,/\,100\;\;\sigma_{st}\,(1 - d'/d)\,bd^2 \tag{13.30}$$

Equating $T_2 = C_2$ from Eqs. 13.28 and 13.27, we have

$$(p_t - p_{t,\,bal})\,\sigma_{st} = p_c\,(1.5m - 1)\,\sigma_{cbc}\,(1 - d'/\,kd) \tag{13.31}$$

The total moment M is obtained by adding M_{bal} and M', as given below:

$$M = M_{bal} + (p_t - p_{t,\,bal})\,/\,100\;\sigma_{st}\,(1 - d'/\,d)\,bd^2 \tag{13.32}$$

The total tensile reinforcement A_{st} has two components $A_{st1} + A_{st2}$ for M_{bal} and M', respectively. The equation of A_{st} is:

$$A_{st} = A_{st\,1} + A_{st\,2} \tag{13.33}$$

where $$A_{st1} = p_{t\,bal}\;(bd\,/\,100) \tag{13.34}$$

and $$A_{st\,2} = M'\,/\,\sigma_{st}\,(d - d') \tag{13.35}$$

The compression reinforcement A_{sc} is expressed as a ratio of additional tensile reinforcement A_{st2}, as given below:

$$(A_{sc} / A_{st2}) = \{p_c / (p_t - p_{t\,bal})\}$$

or, $$(A_{sc} / A_{st2}) = \sigma_{st} / \{\sigma_{cbc} (1.5m - 1) (1 - d' / kd)\} \quad (13.36)$$

Table M of SP-16 presents the values of A_{st} / A_{st2} for different values of d' / d and σ_{cbc} for two values of σ_{st} =140 N/mm^2 and 230 N/mm^2. Selective values are furnished in Table 13.5 as a ready reference. Tables 72 to 79 of SP-16 provide values of p_t and p_c for four values of d' / d against M/bd^2 for four grades of concrete and two grades of steel.

Table 13.5: Values of $A_{sc} / A_{st\,2}$

σ_{st} (N/mm^2)	σ_{cbc} (N/mm^2)	d' / d			
		0.05	0.10	0.15	0.20
140	7.0	1.20	1.40	1.68	2.11
	8.5	1.22	1.42	1.70	2.13
	10.0	1.23	1.44	1.72	2.15
230	7.0	2.09	2.65	3.60	5.54
	8.5	2.12	2.68	3.64	5.63
	10.0	2.14	2.71	3.68	5.76

13.34.14 Practice Questions and Problems with Answers

Q.1: Justify the name working stress method of design.

A.1: Paragraph 2 of Sec. 13.34.1

Q.2: How are the permissible stresses of concrete in direct tension, bending compression, direct compression and average bond for plain bars in tension related to the factor of safety in the working stress method of design?

A.2: Sec. 13.14.2

Q.3: How is the permissible stress of steel in tension related to the factor of safety in the working stress method of design?

A.3: Sec. 13.34.3

Q.4: Is it possible to increase the permissible stress? If yes, when is it done?

A.4: Sec. 13.34.5

Q.5: State the assumption for the design of members by working stress method.

A.5: Sec. 13.34.6

Q.6: Explain the concept of modular ratio.

A.6: Sec. 13.34.7

Q.7: Draw a cross-section of a singly-reinforced rectangular beam, the strain and stress distributions along the depth of the section.

A.7: Figs. 13.34.1a, b and c

Q.8: What do you mean by balanced rectangular beam? Establish the equations for determining the moment of resistance and percentage of tension steel in a balanced rectangular beam.

A.8: Sec. 13.34.9

Q.9: Establish the equations for determining the depth of neutral axis, moment of resistance and area of tension steel of an under-reinforced rectangular beam.

A.9: Sec. 13.34.10

Q.10: Write down the steps for solving the analysis type of problems of singly-reinforced rectangular beams.

A.10: Sec. 13.34.11

Q.11: Write down the steps for solving the design type of problems of singly-reinforced rectangular beams.

A.11: Sec. 13.34.12

Q.12: When do we go for doubly-reinforced beams? Establish the equation for determining areas of steel for the doubly reinforced rectangular beams.

A.12: Sec. 13.34.13

13.34.15 References

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13.34.16 Test 34 with Solutions

Maximum Marks = 50,   Maximum Time = 30 minutes

Answer all questions.

TQ.1: State the assumption for the design of members by working stress method. (15 marks)

A.TQ.1: Sec. 13.34.6

TQ.2: Establish the equations for determining the depth of neutral axis, moment of resistance and area of tension steel of an under-reinforced rectangular beam.

(15 marks)

A.TQ.2: Sec. 13.34.10

TQ.3: When do we go for doubly-reinforced beams? Establish the equation for determining areas of steel for the doubly-reinforced rectangular beams.

(20 marks)

A.TQ.3: Sec. 13.34.13

13.34.17 Summary of this Lesson

This lesson explains the philosophy of the analysis and design of singly-reinforced rectangular flexural members employing working stress method. The concepts of permissible stresses of concrete and reinforcement and the factors of safety are explained. The basic assumptions of the working stress method are mentioned. Balanced and under-reinforced sections are explained establishing the governing equations for the analysis and design of such members. The solution procedures for the two types of problems, viz., analysis and design types, are given in steps. Due to the architect's restrictions or for the functional requirements, the doubly-reinforced rectangular beams are needed to be designed. The governing equations of doubly-reinforced rectangular beams are established.