

# Module 2

## Philosophies of Design by Limit State Method

# Lesson 3

## Philosophies of Design by Limit State Method

## Instructional Objectives:

At the end of this lesson, the student should be able to:

- categorically state the four common steps to be followed in any method of design,
- define the limit states,
- identify and differentiate the different limit states,
- state if the structures are to be designed following all the limit states,
- explain the concept of separate partial safety factors for loads and material strengths depending on the limit state being considered,
- justify the "size effect" of concrete in its strength,
- name the theory for the analysis of structures to be designed by limit states,
- name the method of analysis of statically indeterminate beams and frames, and slabs spanning in two direction at right angles,
- justify the need to redistribute the moments in statically indeterminate beams and frames,
- identify four reasons to justify the design of structures or parts of the structure by limit state method.

### 2.3.1 Introduction

In any method of design, the following are the common steps to be followed:

- (i) To assess the dead loads and other external loads and forces likely to be applied on the structure,
- (ii) To determine the design loads from different combinations of loads,
- (iii) To estimate structural responses (bending moment, shear force, axial thrust etc.) due to the design loads,
- (iv) To determine the cross-sectional areas of concrete sections and amounts of reinforcement needed.

Many of the above steps have lot of uncertainties. Estimation of loads and evaluation of material properties are to name a few. Hence, some suitable factors of safety should be taken into consideration depending on the degrees of such uncertainties.

Limit state method is one of the three methods of design as per IS 456:2000. The code has put more emphasis on this method by presenting it in a full section (Section 5), while accommodating the working stress method in Annex B of the code (IS 456). Considering rapid development in concrete technology and simultaneous development in handling problems of uncertainties, the limit state method is a superior method where certain aspects of reality can be explained in a better manner.

## 2.3.2 Limit State Method

### 2.3.2.1 What are limit states?

Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs. The design of structures by this method will thus ensure that they will not reach limit states and will not become unfit for the use for which they are intended. It is worth mentioning that structures will not just fail or collapse by violating (exceeding) the limit states. Failure, therefore, implies that clearly defined limit states of structural usefulness has been exceeded.

Limit state of collapse was found / detailed in several countries in continent fifty years ago. In 1960 Soviet Code recognized three limit states: (i) deformation, (ii) cracking and (iii) collapse.

### 2.3.2.2 How many limit states are there?

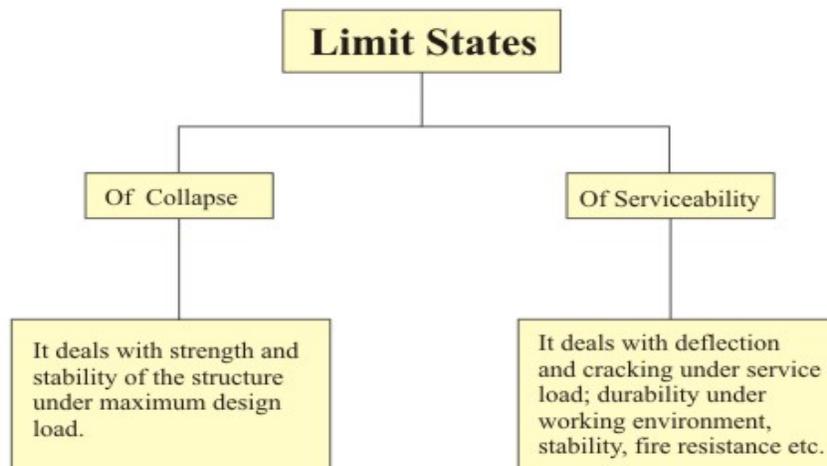
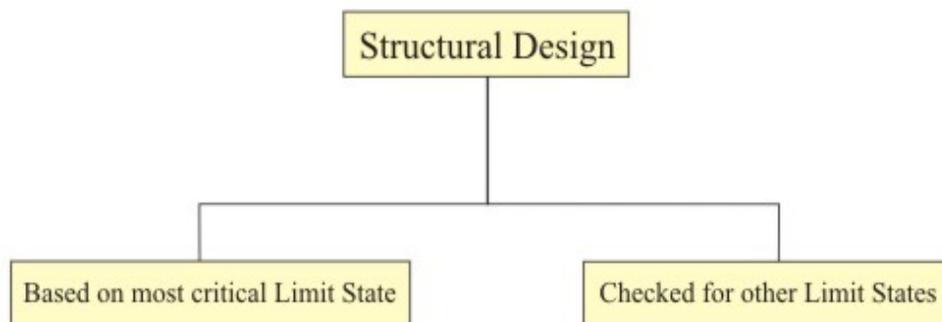


Fig. 2.3.1: Two main limit states

There are two main limit states: (i) limit state of collapse and (ii) limit state of serviceability (see Fig. 2.3.1).

(i) Limit state of collapse deals with the strength and stability of structures subjected to the maximum design loads out of the possible combinations of several types of loads. Therefore, this limit state ensures that neither any part nor the whole structure should collapse or become unstable under any combination of expected overloads.

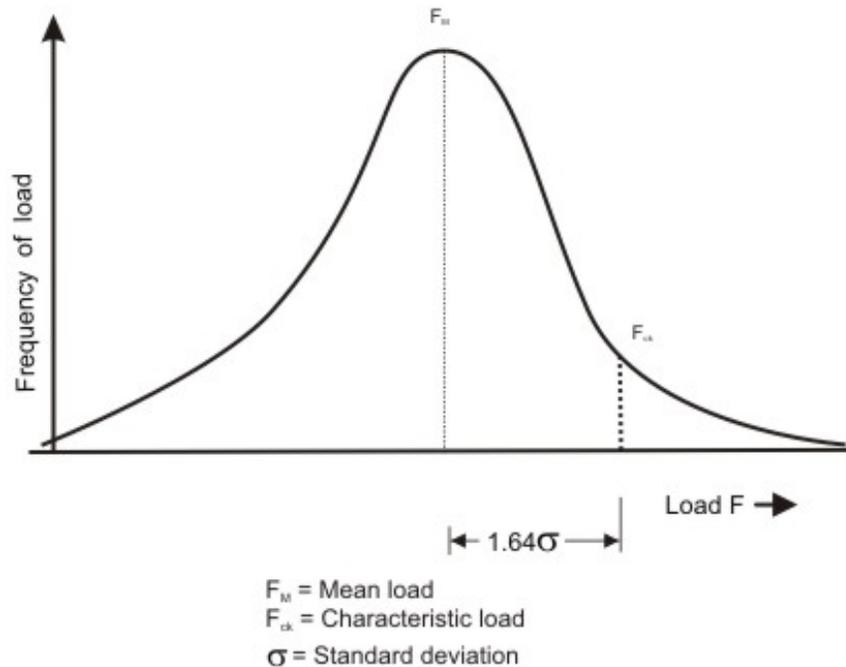
(ii) Limit state of serviceability deals with deflection and cracking of structures under service loads, durability under working environment during their anticipated exposure conditions during service, stability of structures as a whole, fire resistance etc.



**Fig. 2.3.2 : Structural Design**

All relevant limit states have to be considered in the design to ensure adequate degree of safety and serviceability. The structure shall be designed on the basis of the most critical limit state and shall be checked for other limit states (see Fig. 2.3.2).

### 2.3.2.3 Partial safety factors



**Fig. 2.3.3:** Characteristic loads  $F_{ck}$

The characteristic values of loads as discussed in sec. 1.1.5 are based on statistical data. It is assumed that in ninety-five per cent cases the characteristic loads will not be exceeded during the life of the structures (Fig. 2.3.3). However, structures are subjected to overloading also. Hence, structures should be designed with loads obtained by multiplying the characteristic loads with suitable factors of safety depending on the nature of loads or their combinations, and the limit state being considered. These factors of safety for loads are termed as partial safety factors ( $\gamma_f$ ) for loads. Thus, the design loads are calculated as

$$\text{(Design load } F_d) = \text{(Characteristic load } F) \text{ (Partial safety factor for load } \gamma_f) \quad (2.1)$$

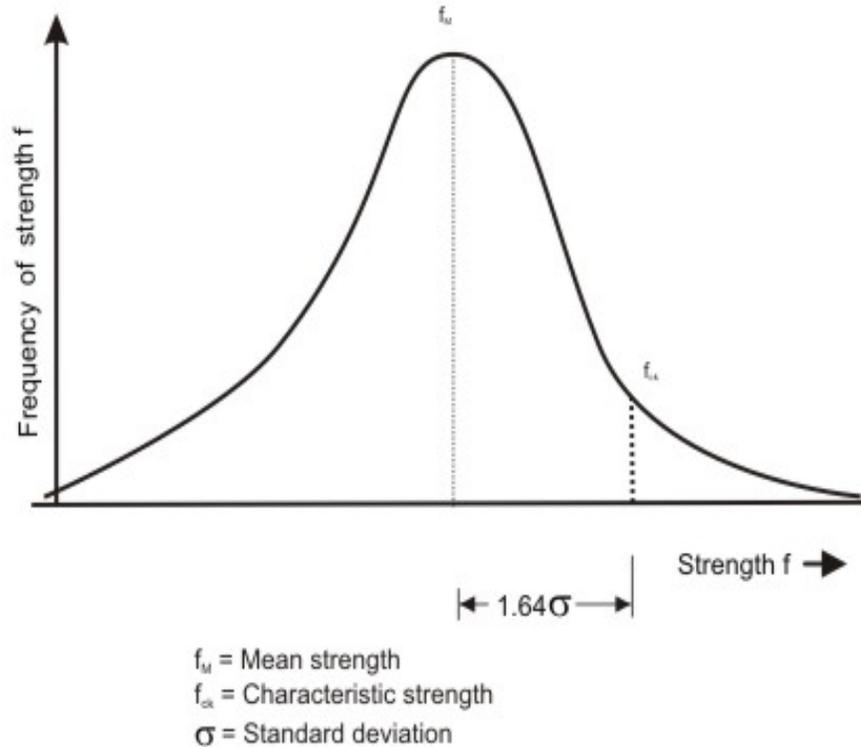
Respective values of  $\gamma_f$  for loads in the two limit states as given in Table 18 of IS 456 for different combinations of loads are furnished in Table 2.1.

**Table 2.1 Values of partial safety factor  $\gamma_f$  for loads**

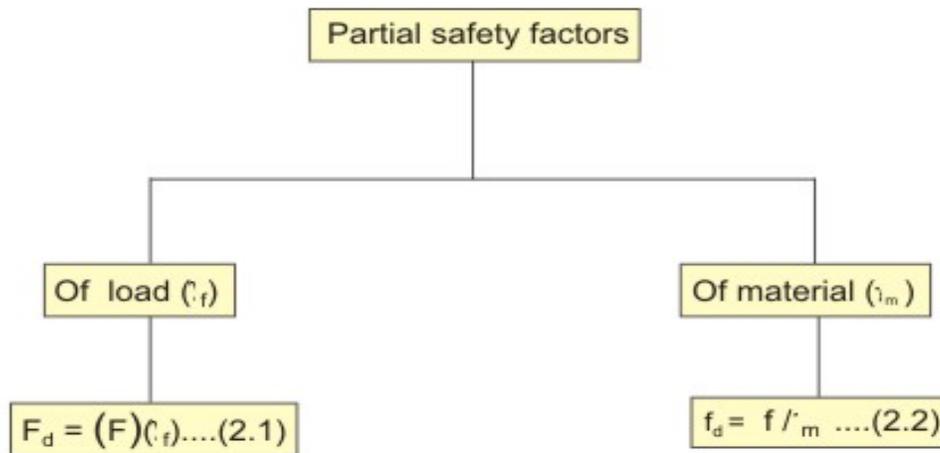
Load combinations	Limit state of collapse			Limit state of serviceability (for short term effects only)		
	DL	IL	WL	DL	IL	WL
DL + IL	1.5		1.0	1.0	1.0	-
DL + WL	1.5 or 0.9 <sup>1)</sup>	-	1.5	1.0	-	1.0
DL + IL + WL	1.2			1.0	0.8	0.8

NOTES:

- 1 While considering earthquake effects, substitute *EL* for *WL*.
  - 2 For the limit states of serviceability, the values of  $\gamma_f$  given in this table are applicable for short term effects. While assessing the long term effects due to creep the dead load and that part of the live load likely to be permanent may only be considered.
- <sup>1)</sup> This value is to be considered when stability against overturning or stress reversal is critical.



**Fig. 2.3.4: Characteristic strength  $f_{ck}$**



Where ,

$F_d$  = Design load

$\gamma_f$  = Partial safety factor of load

$f_d$  = Design strength of material

$\gamma_m$  = Partial safety factor of material

**Fig 2.3.5 : Partial safety factors**

Similarly, the characteristic strength of a material as obtained from the statistical approach is the strength of that material below which not more than five per cent of the test results are expected to fall (see Fig. 2.3.4). However, such characteristic strengths may differ from sample to sample also. Accordingly, the design strength is calculated dividing the characteristic strength further by the partial safety factor for the material ( $\gamma_m$ ), where  $\gamma_m$  depends on the material and the limit state being considered. Thus,

$$\text{Design strength of the material } f_d = \frac{\text{Characteristic strength of the material } f}{\text{Partial safety factor of the material } \gamma_m} \quad (2.2)$$

Both the partial safety factors are shown schematically in Fig. 2.3.5.

Clause 36.4.2 of IS 456 states that  $\gamma_m$  for concrete and steel should be taken as 1.5 and 1.15, respectively when assessing the strength of the structures or structural members employing limit state of collapse. However, when assessing the deflection, the material properties such as modulus of elasticity should be taken as those associated with the characteristic strength of the

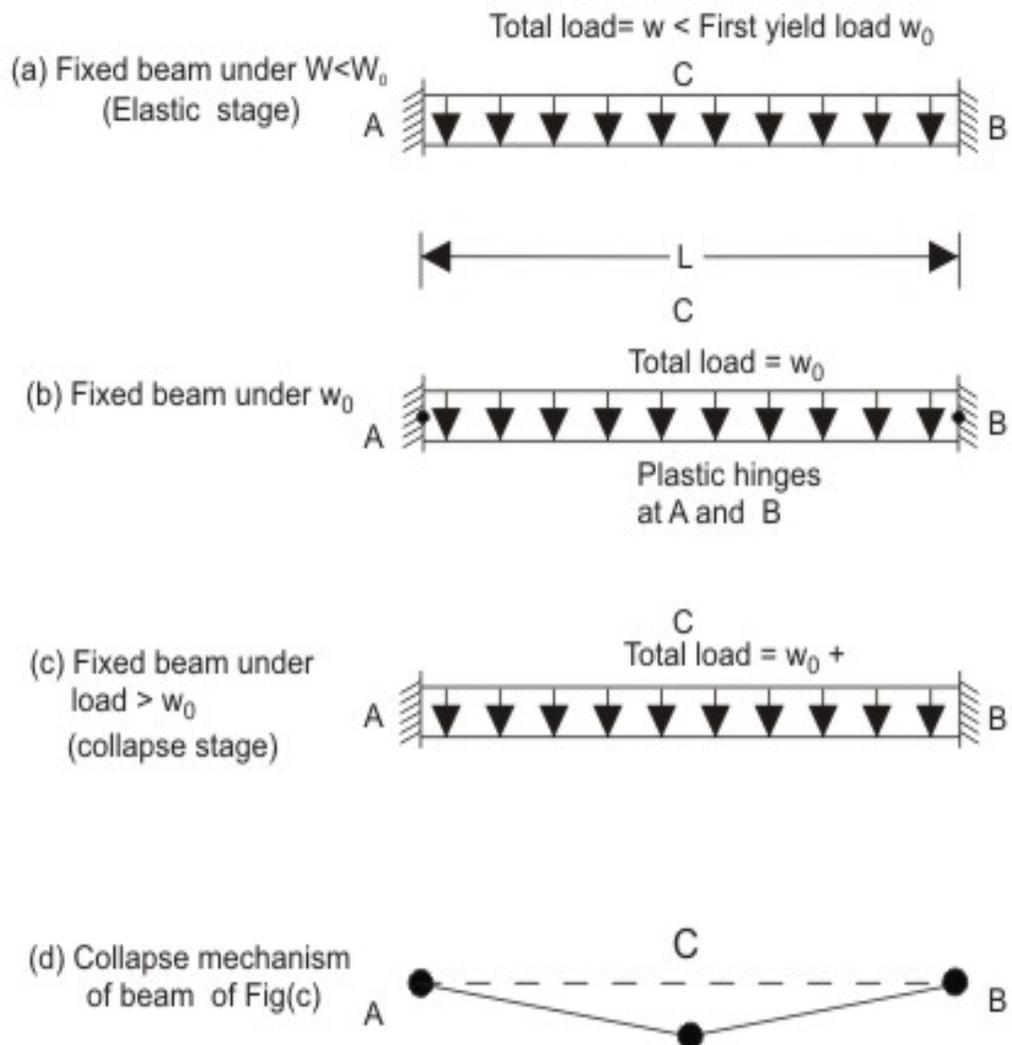
material. It is worth mentioning that partial safety factor for steel (1.15) is comparatively lower than that of concrete (1.5) because the steel for reinforcement is produced in steel plants and commercially available in specific diameters with expected better quality control than that of concrete.

Further, in case of concrete the characteristic strength is calculated on the basis of test results on 150 mm standard cubes. But the concrete in the structure has different sizes. To take the size effect into account, it is assumed that the concrete in the structure develops a strength of 0.67 times the characteristic strength of cubes. Accordingly, in the calculation of strength employing the limit state of collapse, the characteristic strength ( $f_{ck}$ ) is first multiplied with 0.67 (size effect) and then divided by 1.5 ( $\gamma_m$  for concrete) to have  $0.446 f_{ck}$  as the maximum strength of concrete in the stress block.

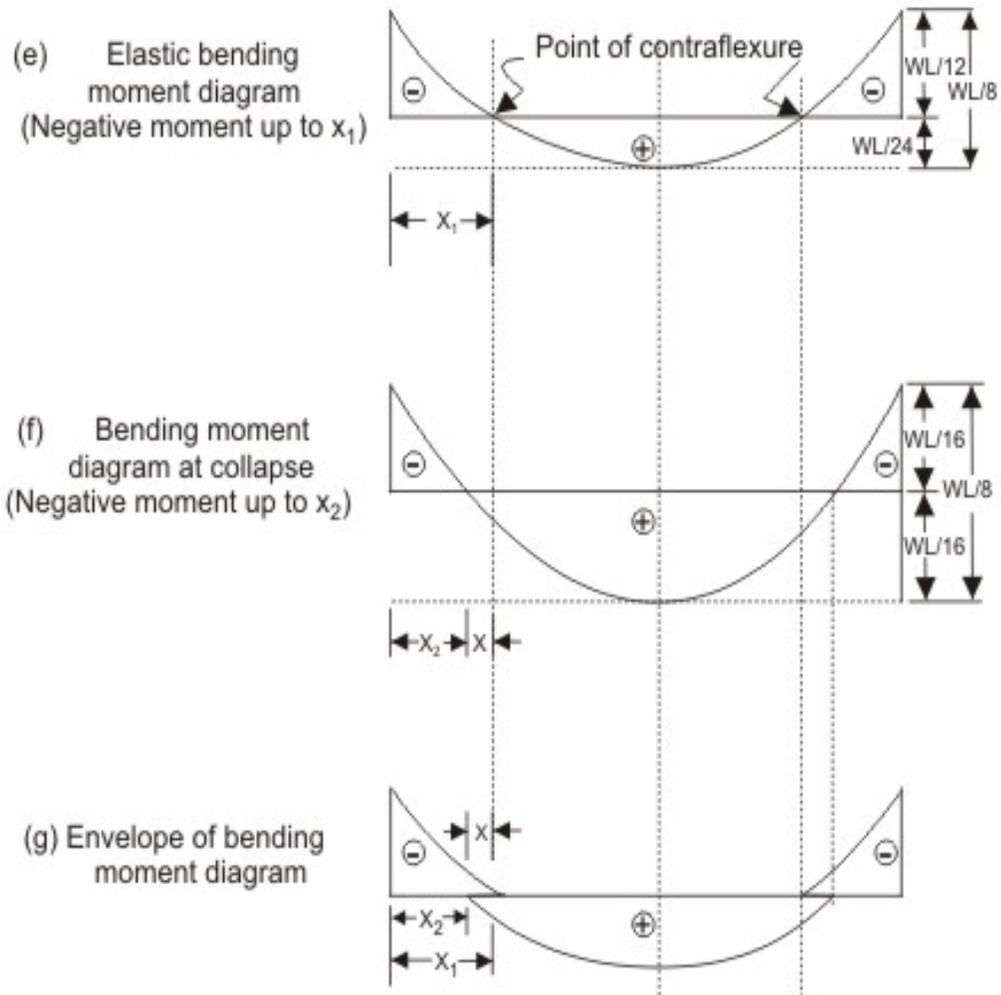
### 2.3.3 Analysis

Analysis of structure has been briefly mentioned in sec. 1.1.4 earlier. Herein, the analysis of structure, in the two limit states (of collapse and of serviceability), is taken up. In the limit state of collapse, the strength and stability of the structure or part of the structure are ensured. The resistances to bending moment, shear force, axial thrust, torsional moment at every section shall not be less than their appropriate values at that section due to the probable most unfavourable combination of the design loads on the structure. Further, the structure or part of the structure should be assessed for rupture of one or more critical sections and buckling due to elastic or plastic instability considering the effects of sway, if it occurs or overturning.

Linear elastic theory is recommended in cl. 22 of IS 456 to analyse the entire structural system subjected to design loads. The code further stipulates the adoption of simplified analyses for frames (cl. 22.4) and for continuous beams (cl. 22.5). For both the limit states the material strengths should be taken as the characteristic values in determining the elastic properties of members. It is worth mentioning that the statically indeterminate structures subjected to design loads will have plastic hinges at certain locations as the loads increase beyond the characteristic loads. On further increase of loads, bending moments do not increase in the locations of plastic hinges as they are already at the full capacities of bending moments. However, these plastic hinges undergo more rotations and the moments are now received by other sections which are less stressed. This phenomenon continues till the plastic hinges reach their full rotation capacities to form a mechanism of collapse of the structure. This is known as the redistribution of moments (Figs. 2.3.6 and 2.3.7). The theory and numerical problems of “Redistribution of moments” are presented elaborately in Lesson 38.



**Fig 2.3.6** : Redistribution of moments



**Fig 2.3.7 : Redistribution of moments**

The design of structure, therefore, should also ensure that the less stressed sections can absorb further moments with a view to enabling the structure to rotate till their full capacities. This will give sufficient warning to the users before the structures collapse. Accordingly, there is a need to redistribute moments in continuous beams and frames. Clause 37.1.1 stipulates this provision and the designer has to carry out the redistribution by satisfying the stipulated conditions there.

The analysis of slabs spanning in two directions at right angles should be performed by employing yield line theory or any other acceptable method. IS 456:2000 has illustrated alternative provisions for the simply supported and restrained slabs spanning in two directions in Annex D along with Tables 26 and 27 giving bending moment coefficients of these slabs for different possible boundary conditions. These provisions enable to determine the reinforcement

needed for bending moments in two directions and torsional reinforcement wherever needed.

### 2.3.4 Concluding Remarks

The limit state method is based on a stochastic process where the design parameters are determined from observations taken over a period of time. The concept of separate partial safety factors for loads and material strengths are based on statistical and probabilistic grounds. These partial safety factors for the material strengths are determined on the basis of reliability of preparations of concrete and reinforcement. The overloading of structure has been kept in mind while specifying the partial safety factors of loads.

The stress block of structures or parts of structure designed on the basis of limit state method subjected to the designed loads or collapse loads represents the stress-strain diagram at the defined states of collapse and satisfy the requirements of strength and stability. Simultaneous checking of these structures or parts of them for the limit state of serviceability ensures the deflection and cracking to remain within their limits. Thus, this method is more rational and scientific.

### 2.3.5 Practice Questions and Problems with Answers

**Q.1:** List the common steps of design of structures by any method of design.

**A.1:** The four steps are listed in section 2.3.1 under (i) to (iv).

**Q.2:** What are limit states?

**A.2:** See sec. 2.3.2.1

**Q.3:** How many limit states are there? Should a structure be designed following all the limit states?

**A.3:** See sec. 2.3.2.2

**Q.4:** Define partial safety factors of load and material. Write the expressions to determine the design load and design strength of the material from their respective characteristic values employing the corresponding partial safety factors.

**A.4:** See sec. 2.3.2.3 and Eqs.2.1 and 2.2

**Q.5:** What is size effect of concrete? What is its role in determining the material strength of concrete?

**A.5:** The characteristic strength of concrete is determined from the results of tests conducted on cube specimens of 150 mm dimension. The dimensions of concrete structures or in members of structure are different and widely varying. This has an effect on the strength of concrete in the structure. This is known as size effect.

Due to the size effect, the characteristic strength of concrete is reduced to 2/3 of its value and then further divided by the partial safety factor of the concrete ( $\gamma_m = 1.5$ ) to get the design strength of concrete ( $f_d$ ). Thus,

$$f_d = (f_{ck}) \frac{(2/3)}{1.5} = \frac{0.67 f_{ck}}{1.5} = 0.446 f_{ck}$$

**Q.6:** Which theory should be employed for the analysis of structural system to be designed element wise, by limit state method?

**A.6:** Linear elastic theory should be employed for the analysis of structural system subjected to design loads.

**Q.7:** Justify the need to do the redistribution of moments in statically indeterminate structures.

**A.7:** Statically indeterminate structures will have plastic hinges formed when loads increase from the characteristic values. These locations where plastic hinges are formed will undergo rotations at constant moment when the sections of lower stresses will receive the additional moments due to further increase of loads. This process will continue till sufficient plastic hinges are formed to have a mechanism of collapse (see Figs. 2.3.6 and 2.3.7).

The structures should have such a provision to avoid sudden failure at the failure of one critical section. The comparatively lower stressed sections, therefore, should be designed taking the redistribution of moments into account.

**Q.8:** What are the analytical methods for the design of simply supported and restrained slabs?

**A.8:** Yield line theory or any other acceptable method of analysis can be employed for these slabs. Alternatively, the method illustrated in Annex D of IS 456:2000 can also be used for the slabs spanning in two perpendicular directions.

**Q.9:** Give four reasons to justify the design of structures by limit state method.

**A.9:** The four reasons are:

- (i) Concept of separate partial safety factors of loads of different combinations in the two limit state methods.
- (ii) Concept of separate partial safety factors of materials depending on their quality control during preparation. Thus,  $\gamma_m$  for concrete is 1.5 and the same for steel is 1.15. This is more logical than one arbitrary value in the name of safety factor.
- (iii) A structure designed by employing limit state method of collapse and checked for other limit states will ensure the strength and stability requirements at the collapse under the design loads and also deflection and cracking at the limit state of serviceability. This will help to achieve the structure with acceptable probabilities that the structure will not become unfit for the use for which it is intended.
- (iv) The stress block represents in a more realistic manner when the structure is at the collapsing stage (limit state of collapse) subjected to design loads.

### 2.3.6 References

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### 2.3.7 Test 3 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

**TQ.1:** List the common steps of design of structures by any method of design.  
(8 marks)

**A.TQ.1:** See Ans. 1 of sec. 2.3.5.

**TQ.2:** What are limit states?  
(6 marks)

**A.TQ.2:** See Ans. 2 of sec. 2.3.5.

**TQ.3:** Draw schematic figures to explain (i) the different limit states and (ii) use of the limit states to design a structure.  
(6 + 6 = 12 marks)

**A.TQ.3:** The schematic figures are Figs. 2.3.1 and 2.3.2 and answers are in sec. 2.3.2.2

**TQ.4:** What is size effect of concrete? What is its role in determining the material strength of concrete?  
(6 marks)

**A.TQ.4:** See Ans. 5 of sec. 2.3.5.

**TQ.5:** Justify the need to do the redistribution of moments in statically indeterminate structures.  
(10 marks)

**A.TQ.5:** See Ans. 7 of sec. 2.3.5.

**TQ.6:** Give four reasons to justify the design of structures by limit state method.  
(8 marks)

**A.TQ.6:** See Ans. 9 of sec. 2.3.5.

### 2.3.8 Summary of this Lesson

This lesson defines and identifies different limit states. It explains the concept and defines the partial safety factors for loads and material strengths depending on the limit states to be considered in the design. It explains the theory for the analysis of structures to be carried out by the designer. Further, the redistribution of moments in statically indeterminate beams and frames is explained. Finally, the use of limit state method for the design of reinforced concrete structures has been justified.