

Module 10

Compression Members

Lesson

26

Short Compression Members under Axial Load with Biaxial Bending

Instructional Objectives:

At the end of this lesson, the student should be able to:

- understand the behaviour of short columns under axial load and biaxial bending,
- understand the concept of interaction surface,
- identify the load contour and interaction curves of P_u-M_u in a interaction surface,
- mention the limitation of direct application of the interaction surface in solving the problems,
- explain the simplified method of design and analysis of short columns under axial load and biaxial bending,
- apply the IS code method in designing and analysing the reinforced concrete short columns under axial load and biaxial bending.

10.26.1 Introduction

Beams and girders transfer their end moments into the corner columns of a building frame in two perpendicular planes. Interior columns may also have biaxial moments if the layout of the columns is irregular. Accordingly, such columns are designed considering axial load with biaxial bending. This lesson presents a brief theoretical analysis of these columns and explains the difficulties to apply the theory for the design. Thereafter, simplified method, as recommended by IS 456, has been explained with the help of illustrative examples in this lesson.

10.26.2 Biaxial Bending

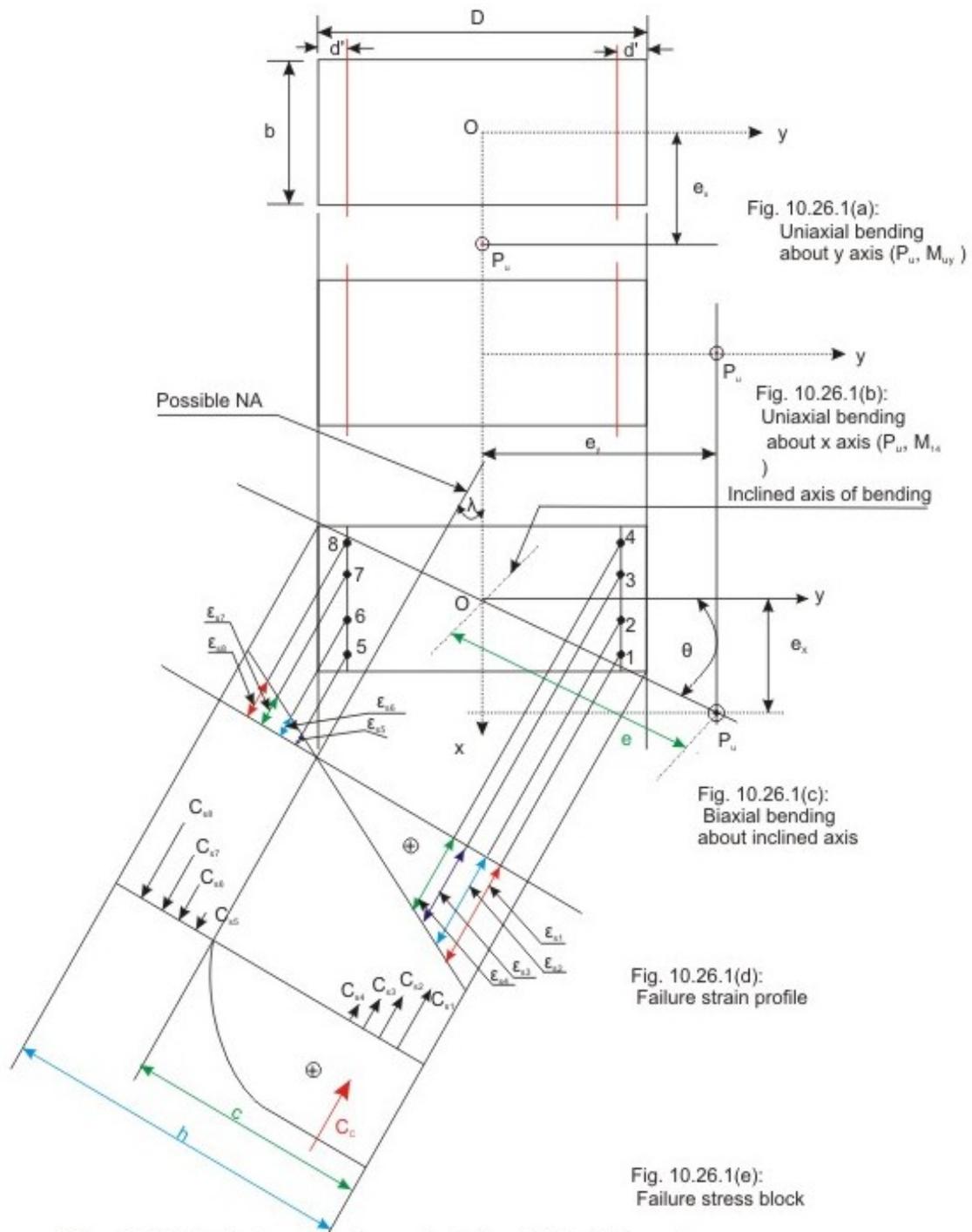


Fig. 10.26.1: Column under uniaxial and biaxial bending

Figures 10.26.1a and b present column section under axial load and uniaxial bending about the principal axes x and y , respectively. Figure 10.26.1c

presents the column section under axial load and biaxial bending. The eccentricities e_x and e_y of Fig.10.26.1c are the same as those of Fig.10.26.1a (for e_x) and Fig.10.26.1b (for e_y), respectively. Thus, the biaxial bending case (case c) is the resultant of two uniaxial bending cases a and b. The resultant eccentricity e , therefore, can be written as (see Fig.10.26.1c):

$$e = (e_x^2 + e_y^2)^{1/2} \quad (10.55)$$

Designating the moments of cases a, b and c by M_{ux} , M_{uy} and M_u , respectively, we can write:

$$M_u = (M_{ux}^2 + M_{uy}^2)^{1/2} \quad (10.56)$$

and the resultant M_u is acting about an inclined axis, so that

$$\tan \theta = e_x/e_y = M_{uy}/M_{ux} \quad (10.57)$$

the angle of inclination θ is measured from y axis.

This inclined resultant axis shall also be the principal axis if the column section including the reinforcing bars is axisymmetric. In such a situation, the biaxial bending can be simplified to a uniaxial bending with the neutral axis parallel to the resultant axis of bending.

The reinforced concrete column cross-sections are, in general, non-axisymmetric with reference to the longitudinal axis and, therefore, the neutral axis is not parallel to the resultant axis of bending (θ is not equal to λ in Fig.10.26.1c). Moreover, it is extremely laborious to find the location of the neutral axis with successive trials. However, failure strain profile and stress block can be drawn for a given location of the neutral axis. Figs.10.25.1d and e present the strain profile and stress block, respectively, of the section shown in Fig.10.25.1c.

10.26.3 Interaction Surface

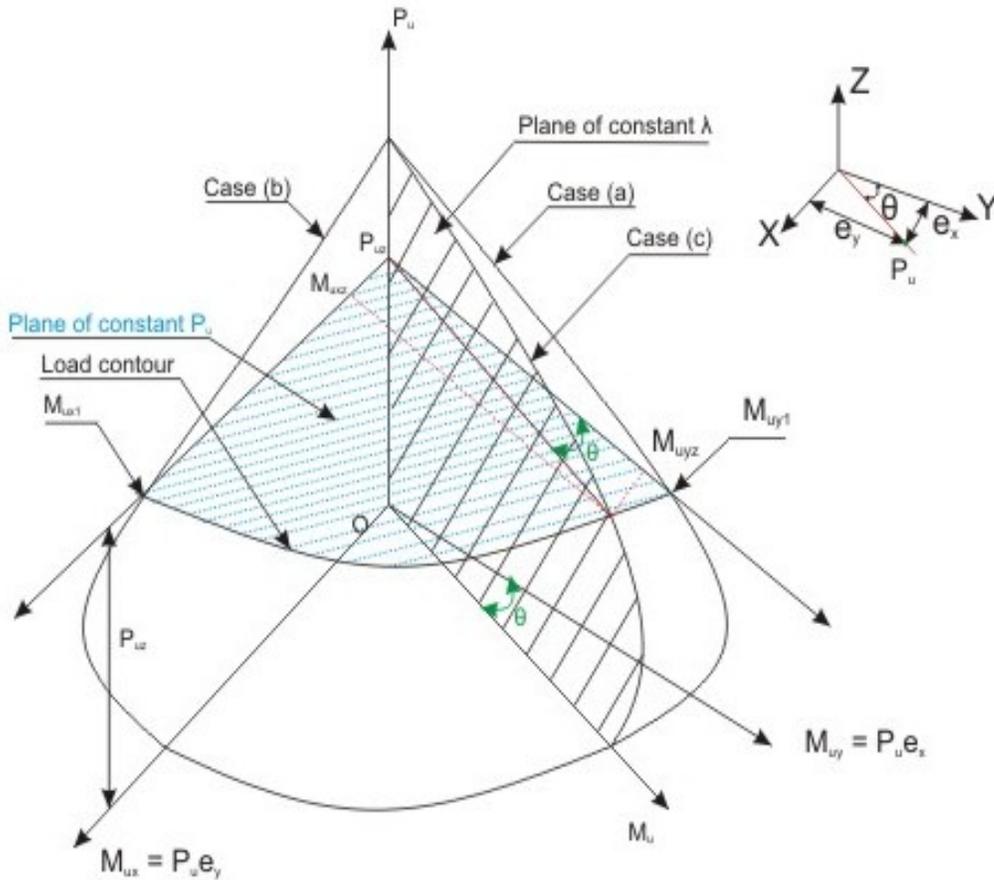


Fig. 10.26.2: Interaction diagram under axial load and biaxial bending

Figure 10.26.2 can be visualised as a three-dimensional plot of P_u - M_{ux} - M_{uz} , wherein two two-dimensional plots of P_u - M_{uz} and P_u - M_{ux} are marked as case (a) and case (b), respectively. These two plots are the interaction curves for the columns of Figs.10.26.1a and b, respectively. The envelope of several interaction curves for different axes will generate the surface, known as interaction surface.

The interaction curve marked as case (c) in Fig.10.26.2, is for the column under biaxial bending shown in Fig.10.26.1c. The corresponding axis of bending is making an angle θ with the y axis and satisfies Eq.10.57. It has been explained in Lesson 24 that a column subjected to a pair of P and M will be safe if their respective values are less than P_u and M_u , given by its interaction curve. Extending the same in the three-dimensional figure of interaction surface, it is also acceptable that a column subjected to a set of P_u , M_{uz} and M_{ux} is safe if the set of values lies within the surface. Since P_u is changing in the direction of z , let us designate the moments and axial loads as mentioned below:

M_{uxz} = design flexural strength with respect to major axis xx under biaxial loading, when $P_u = P_{uz}$,

M_{uyz} = design flexural strength with respect to minor axis yy under biaxial loading, when $P_u = P_{uz}$,

M_{ux1} = design flexural strength with respect to major axis xx under uniaxial loading, when $P_u = P_{uz}$, and

M_{uy1} = design flexural strength with respect to minor axis yy under uniaxial loading, when $P_u = P_{uz}$.

The above notations are also shown in Fig.10.26.2.

All the interaction curves, mentioned above, are in planes perpendicular to xy plane. However, the interaction surface has several curves parallel to xy plane, which are planes of constant P_u . These curves are known as load contour, one such load contour is shown in Fig.10.26.2, when $P_u = P_{uz}$. Needless to mention that the load is constant at all points of a load contour. These load contour curves are also interaction curves depicting the interaction between the biaxial bending capacities.

10.26.4 Limitation of Interaction Surface

The main difficulty in preparing an exact interaction surface is that the neutral axis for the case (c) of Fig.10.26.1c will not, in general, be perpendicular to the line joining the loading point P_u and the centre of the column (Fig.10.26.1c). This will require several trials with c and λ , where c is the distance of the neutral axis and λ angle made by the neutral axis with the x axis, as shown in Fig.10.26.1c. Each trial will give a set of P_u , M_{ux} and M_{uy} . Only for a particular case, the neutral axis will be perpendicular to the line joining the load point P_u to the centre of the column. This search makes the process laborious. Moreover, several trials with c and λ , giving different values of h (see Fig.10.26.1c), may result in a failure surface with wide deviations, particularly as the value of P_u will be increasing.

Accordingly, the design of columns under axial load with biaxial bending is done by making approximations of the interaction surface. Different countries adopted different approximate methods. Clause 39.6 of IS 456 recommends one method based on Bresler's formulation, also known as "Load Contour Method", which is taken up in the following section. (For more information, please refer to: "Design Criteria for Reinforced Columns under Axial Load and Biaxial Bending", by B. Bresler, J. ACI, Vol.32, No.5, 1960, pp.481-490).

10.26.5 IS Code Method for Design of Columns under Axial Load and Biaxial Bending

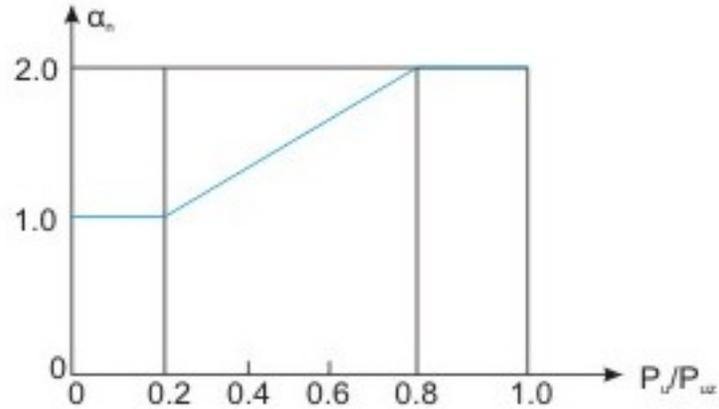


Fig. 10.26.3: Exponent α_n versus P_u/P_{uz}

IS 456 recommends the following simplified method, based on Bresler's formulation, for the design of biaxially loaded columns. The relationship between M_{uxz} and M_{uyz} for a particular value of $P_u = P_{uz}$, expressed in non-dimensional form is:

$$(M_{ux}/M_{ux1})^{\alpha_n} + (M_{uy}/M_{uy1})^{\alpha_n} \leq 1 \quad (10.58)$$

where M_{ux} and M_{uy} = moments about x and y axes due to design loads, and

α_n is related to P_u/P_{uz} , (Fig.10.26.3), where

$$\begin{aligned} P_{uz} &= 0.45 f_{ck} A_c + 0.75 f_y A_{sc} \\ &= 0.45 A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc} \end{aligned} \quad (10.59)$$

where A_g = gross area of the section, and

A_{sc} = total area of steel in the section

M_{uxz} , M_{uyz} , M_{ux1} and M_{uy1} are explained in sec.10.26.3 earlier.

It is worth mentioning that the quantities M_{ux} , M_{uy} and P_u are due to external loadings applied on the structure and are available from the analysis, whereas M_{ux1} , M_{uy1} and P_{uz} are the capacities of the column section to be considered for the design.

Equation 10.58 defines the shape of the load contour, as explained earlier (Fig.10.26.2). That is why the method is also known as "Load Contour Method". The exponent α_n of Eq.10.58 is a constant which defines the shape of the load

contour and depends on the value of P_u . For low value of the axial load, the load contour is approximated as a straight line and, in that case, $\alpha_n = 1$. On the other hand, for high values of axial load, the load contour is approximated as a quadrant of a circle, when $\alpha_n = 2$. For intermediate load values, the value of α_n lies between 1 and 2. Chart 64 of SP-16 presents the load contour and Fig.10.26.3 presents the relationship between α_n and P_u/P_{uz} . The mathematical relationship between α_n and P_u/P_{uz} is as follows:

$$\alpha_n = 1.0, \text{ when } P_u/P_{uz} \leq 0.2$$

$$\alpha_n = 0.67 + 1.67 P_u/P_{uz}, \text{ when } 0.2 < (P_u/P_{uz}) < 0.8$$

$$\alpha_n = 2.0, \text{ when } (P_u/P_{uz}) \geq 0.8$$

(10.60)

10.26.6 Solution of Problems using IS Code Method

The IS code method, as discussed in sec.10.26.5, can be employed to solve both the design and analysis types of problems. The only difference between the design and analysis type of problems is that a trial section has to be assumed including the percentage of longitudinal reinforcement in the design problems. However, these data are available in the analysis type of problems. Therefore, a guide line is given in this section for assuming the percentage of longitudinal reinforcement for the design problem. Further, for both types of problems, the eccentricities of loads are to be verified if they are more than the corresponding minimum eccentricities, as stipulated in cl.25.4 of IS 456. Thereafter, the relevant steps are given for the solution of the two types of problems.

(a) Selection of trial section for the design type of problems

As mentioned in sec.10.24.2(i) of Lesson 24, the preliminary dimensions are already assumed during the analysis of structure (mostly statically indeterminate). Thus, the percentage of longitudinal steel is the one parameter to be assumed from the given P_u , M_{ux} , M_{uy} , f_{ck} and f_y . Pillai and Menon (Ref. No. 4) suggested a simple way of considering a moment of approximately 15 per cent in excess (lower percentage up to 5 per cent if P_u/P_{uz} is relatively high) of the resultant moment

$$M_u = (1.15)(M_{ux}^2 + M_{uy}^2)^{1/2}$$

(10.61)

as the uniaxial moment for the trial section with respect to the major principal axis xx, if $M_{ux} \geq M_{uy}$; otherwise, it should be with respect to the minor principal axis.

The reinforcement should be assumed to be distributed equally on four sides of the section.

(b) Checking the eccentricities e_x and e_y for the minimum eccentricities

Clause 25.4 of IS 256 stipulates the amounts of the minimum eccentricities and are given in Eq.10.3 of sec.10.21.11 of Lesson 21. However, they are given below as a ready reference.

$$e_{xmin} \geq \text{greater of } (l/500 + b/30) \text{ or } 20 \text{ mm} \quad \dots (10.3)$$

$$e_{ymin} \geq \text{greater of } (l/500 + D/30) \text{ or } 20 \text{ mm}$$

where l , b and D are the unsupported length, least lateral dimension and larger lateral dimension, respectively. The clause further stipulates that for the biaxial bending, it is sufficient to ensure that the eccentricity exceeding the minimum value about one axis at a time.

(c) Steps for the solution of problems

The following are the steps for the solution of both analysis and design types of problems while employing the method recommended by IS 456.

(i) Verification of eccentricities

It is to be done determining $e_x = M_{ux}/P_u$ and $e_y = M_{uy}/P_u$ from the given data of P_u , M_{ux} and M_{uy} ; and e_{xmin} and e_{ymin} from Eq.10.3 from the assumed b and D and given l .

(ii) Assuming a trial section including longitudinal reinforcement

This step is needed only for the design type of problem, which is to be done as explained in (a) above.

(iii) Determination of M_{ux1} and M_{uy1}

Use of design charts should be made for this. M_{ux1} and M_{uy1} , corresponding to the given P_u , should be significantly greater than M_{ux} and M_{uy} , respectively. Redesign of the section should be done if the above are not satisfied for the design type of problem only.

(iv) Determination of P_{uz} and α_n

The values of P_{uz} and α_n can be determined from Eqs.10.59 and 10.60, respectively. Alternatively, P_{uz} can be obtained from Chart 63 of SP-16.

(v) Checking the adequacy of the section

This is done either using Eq.10.58 or using Chart 64 of SP-16.

10.26.7 Illustrative Example

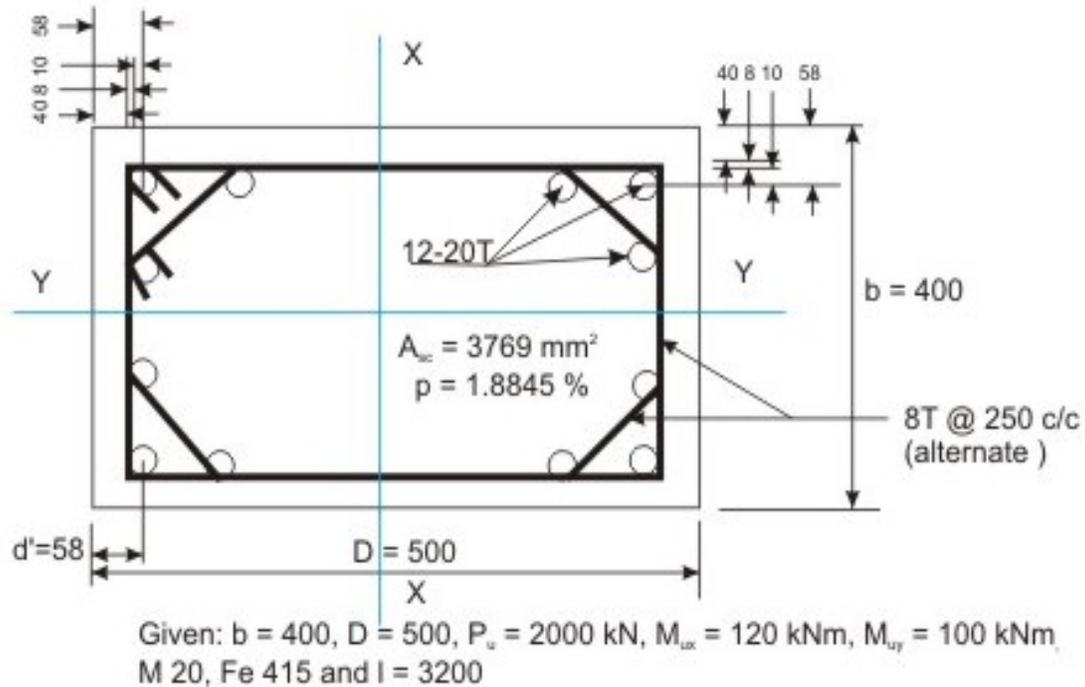


Fig. 10.26.4: Problem 1

Problem 1:

Design the reinforcement to be provided in the short column of Fig.10.26.4 is subjected to $P_u = 2000$ kN, $M_{uX} = 130$ kNm (about the major principal axis) and $M_{uY} = 120$ kNm (about the minor principal axis). The unsupported length of the column is 3.2 m, width $b = 400$ mm and depth $D = 500$ mm. Use M 25 and Fe 415 for the design.

Solution 1:

Step 1: Verification of the eccentricities

Given: $l = 3200$ mm, $b = 400$ mm and $D = 500$ mm, we have from Eq.10.3 of sec.10.26.6b, the minimum eccentricities are:

$$e_{x\min} = \text{greater of } (3200/500 + 400/30) \text{ and } 20 \text{ mm} = 19.73 \text{ mm or } 20 \text{ mm} = 20 \text{ mm}$$

$e_{ymin} = \text{greater of } (3200/500 + 500/30) \text{ and } 20 \text{ mm} = 23.07 \text{ mm or } 20 \text{ mm} = 23.07 \text{ mm}$

Again from $P_u = 2000 \text{ kN}$, $M_{ux} = 130 \text{ kNm}$ and $M_{uy} = 120 \text{ kNm}$, we have $e_x = M_{ux}/P_u = 130(10^6)/2000(10^3) = 65 \text{ mm}$ and $e_y = M_{uy}/P_u = 120(10^6)/2000(10^3) = 60 \text{ mm}$. Both e_x and e_y are greater than e_{xmin} and e_{ymin} , respectively.

Step 2: Assuming a trial section including the reinforcement

We have $b = 400 \text{ mm}$ and $D = 500 \text{ mm}$. For the reinforcement, $M_u = 1.15(M_{ux}^2 + M_{uy}^2)^{1/2}$, from Eq.10.61 becomes 203.456 kNm . Accordingly, we get

$$P_u/f_{ck}bD = 2000(10^3)/(25)(400)(500) = 0.4$$

$$M_u/f_{ck}bD^2 = 203.456(10^6)/(25)(400)(500)(500) = 0.0814$$

Assuming $d' = 60 \text{ mm}$, we have $d'/D = 0.12$. From Charts 44 and 45, the value of p/f_{ck} is interpolated as 0.06. Thus, $p = 0.06(25) = 1.5 \text{ per cent}$, giving $A_{sc} = 3000 \text{ mm}^2$. Provide 12-20 mm diameter bars of area 3769 mm^2 , actual p provided = 1.8845 per cent. So, $p/f_{ck} = 0.07538$.

Step 3: Determination of M_{ux1} and M_{uy1}

We have $P_u/f_{ck}bD = 0.4$ and $p/f_{ck} = 0.07538$ in step 2. Now, we get $M_{ux1}/f_{ck}bD^2$ from chart corresponding to $d' = 58 \text{ mm}$ (Fig.10.26.4) i.e., $d'/D = 0.116$. We interpolate the values of Charts 44 and 45, and get $M_{ux1}/f_{ck}bD^2 = 0.09044$. So, $M_{ux1} = 0.0944(25)(400)(500)(500)(10^{-6}) = 226.1 \text{ kNm}$.

For M_{ux1} , $d'/b = 58/400 = 0.145$. In a similar manner, we get $M_{uy1} = 0.0858(25)(400)(400)(500)(10^{-6}) = 171.6 \text{ kNm}$.

As M_{ux1} and M_{uy1} are significantly greater than M_{ux} and M_{uy} , respectively, redesign of the section is not needed.

Step 4: Determination of P_{uz} and α_n

From Eq.10.59, we have $P_{uz} = 0.45(25)(400)(500) + \{0.75(415) - 0.45(25)\}(3769) = 3380.7 \text{ kN}$.

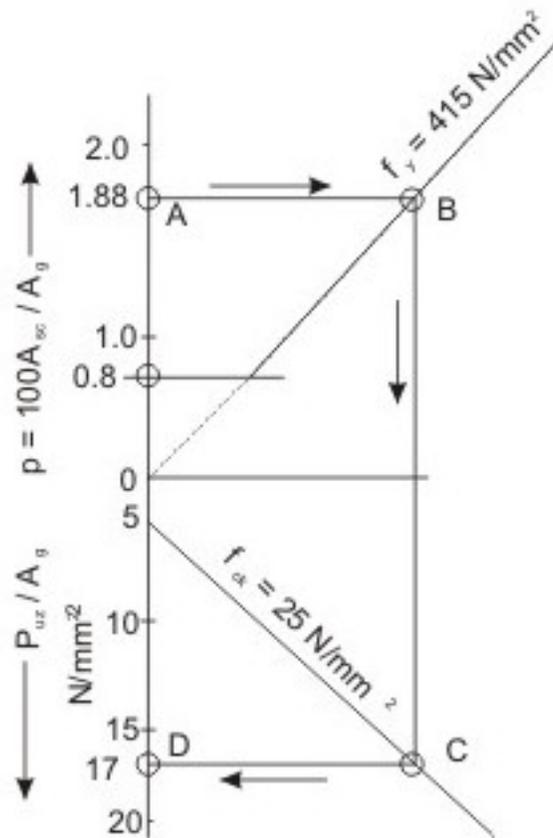


Fig. 10.26.5: Chart 63 of SP-16 in Problem 1
(not to scale)

Alternatively, Chart 63 may be used to find P_{uz} as explained. From the upper section of Chart 63, a horizontal line AB is drawn at $p = 1.8845$, to meet the Fe 415 line B (Fig.10.26.5). A vertical line BC is drawn from B to meet M 25 line at C. Finally, a horizontal line CD is drawn from C to meet P_{uz}/A_g at 17. This gives $P_{uz} = 17(400)(500) = 3400$ kN. The difference between the two values, 19.3 kN is hardly 0.57 per cent, which is due to the error in reading the value from the chart. However, any one of the two may be employed.

Now, the value of α_n is obtained from Eq.10.60 for $P_u/P_{uz} = 2000/3380.7 = 0.5916$, i.e., $0.2 < P_u/P_{uz} < 0.8$, which gives, $\alpha_n = 0.67 + 1.67 (P_u/P_{uz}) = 1.658$. Alternatively, α_n may be obtained from Fig.10.26.3, drawn to scale.

Step 5: Checking the adequacy of the section

Using the values of M_{ux} , M_{ux1} , M_{uy} , M_{uy1} and α_n in Eq.10.58, we have $(130/226.1)^{1.658} + (120/171.6)^{1.658} = 0.9521 < 1.0$. Hence, the design is safe.

Alternatively, Chart 64 may be used to determine the point $(M_{ux}/M_{ux1}), (M_{uy}/M_{uy1})$ is within the curve of $P_u/P_{uz} = 0.5916$ or not.

Here, $M_{ux}/M_{ux1} = 0.5749$ and $M_{uy}/M_{uy1} = 0.6993$. It may be seen that the point is within the curve of $P_u/P_{uz} = 0.5916$ of Chart 64 of SP-16.

Step 6: Design of transverse reinforcement

As per cl.26.5.3.2c of IS 456, the diameter of lateral tie should be $> (20/4)$ mm diameter. Provide 8 mm diameter bars following the arrangement shown in Fig.10.26.4. The spacing of lateral tie is the least of :

- (a) 400 mm = least lateral dimension of column,
- (b) 320 mm = sixteen times the diameter of longitudinal reinforcement (20 mm),
- (c) 300 mm

Accordingly, provide 8 mm lateral tie alternately @ 250 c/c (Fig.10.26.4).

10.26.8 Practice Questions and Problems with Answers

Q.1: Explain the behaviour of a short column under biaxial bending as the resultant of two uniaxial bending.

A.1: See sec. 10.26.2

Q.2: Draw one interaction surface for a short column under biaxial bending and show typical interaction curves and load contour curve. Explain the safety of a column with reference to the interaction surface when the column is under biaxial bending.

A.2: See sec.10.26.3 and Fig.10.26.2.

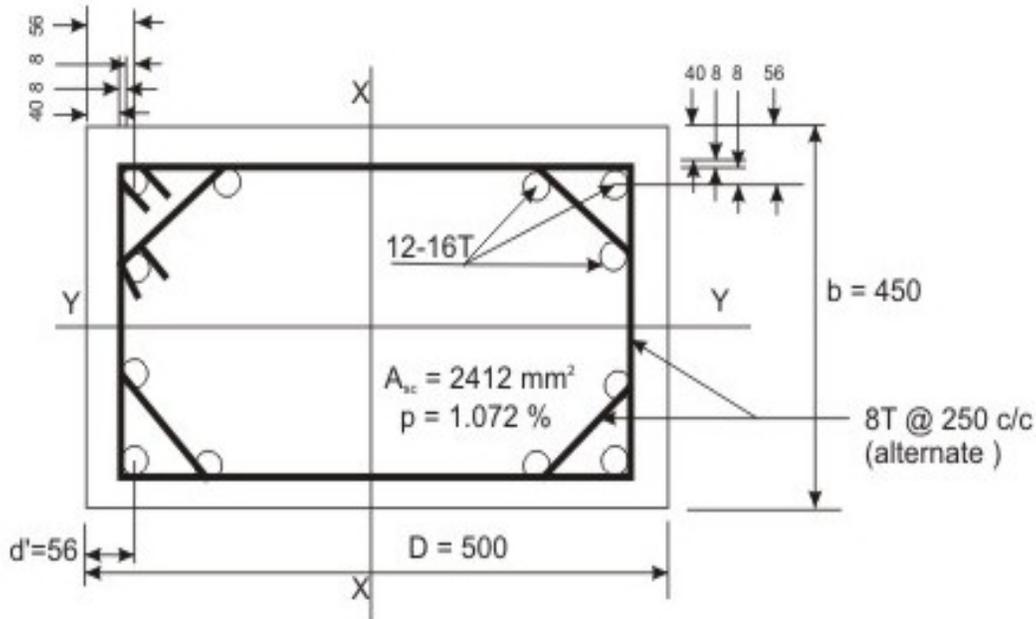
Q.3: Discuss the limitation of the interaction curve.

A.3: See sec.10.26.4.

Q.4: Illustrate the IS code method of design of columns under biaxial bending.

A.4: See sec.10.26.5.

Q.5:



Given: $b = 450$, $D = 500$, $P_u = 1600$ kN, $M_{ux} = 120$ kNm, $M_{uy} = 100$ kNm,
M 20, Fe 415, $l = 3200$, $A_{sc} = 2412$ mm², tie 8T @ 250 c/c (alternate)

Fig. 10.26.6: Q.5

Analyse the safety of the short column of unsupported length 3.2 m, $b = 450$ mm, $D = 500$ mm, as shown in Fig.10.26.6, having 12-16 mm diameter bars as longitudinal reinforcement and 8 mm diameter bars as lateral tie @ 250 mm c/c, when subjected to $P_u = 1600$ kN, $M_{ux} = 120$ kNm and $M_{uy} = 100$ kNm. Use M 25 and Fe 415.

A.5:

Step 1: Verification of the eccentricities

From the given data: $l = 3200$ mm, $b = 450$ mm and $D = 500$ mm,

$$e_{xmin} = 3200/500 + 450/30 = 21.4 > 20 \text{ mm, so, } 21.4 \text{ mm}$$

$$e_{ymin} = 3200/500 + 5000/30 = 23.06 > 20 \text{ mm, so, } 23.06 \text{ mm}$$

$$e_x = M_{ux}/P_u = 120(10^3)/1600 = 75 \text{ mm}$$

$$e_y = M_{uy}/P_u = 100(10^3)/1600 = 62.5 \text{ mm}$$

So, the eccentricities e_x and e_y are $\gg e_{xmin}$ and e_{ymin} .

Step 2: Determination of M_{ux1} and M_{uy1}

Given data are: $b = 450$ mm, $D = 500$ mm, $f_{ck} = 25$ N/mm², $f_y = 415$ N/mm², $P_u = 1600$ kN, $M_{ux} = 120$ kNm, $M_{uy} = 100$ kNm and $A_{sc} = 2412$ mm² (12-16 mm diameter bars).

We have $p = (100)(2412)/(450)(500) = 1.072$ per cent, and $d'/D = 56/500 = 0.112$, $d'/b = 56/450 = 0.124$, $P_u/f_{ck}bD = 1600/(25)(450)(500) = 0.2844$ and $p/f_{ck} = 1.072/25 = 0.043$. We get $M_{ux1}/f_{ck}bD^2$ from Charts 44 and 45 as 0.09 and 0.08, respectively. Linear interpolation gives $M_{ux1}/f_{ck}bD^2$ for $d'/D = 0.112$ as 0.0876. Thus,

$$M_{ux1} = (0.0876)(25)(450)(500)(500) = 246.376 \text{ kNm}$$

Similarly, interpolation of values (0.09 and 0.08) from Charts 44 and 45, we get $M_{uy1}/f_{ck}db^2 = 0.085$ for $d'/b = 0.124$. Thus

$$M_{uy1} = (0.085)(25)(500)(450)(450) = 215.156 \text{ kNm}$$

Step 3: Determination of P_{uz} and α_n

From Eq.10.59, $P_{uz} = 0.45(25)(450)(500) + \{0.75(415) - 0.45(25)\}(2412) = 3254.85$ kN. This gives $P_u/P_{uz} = 1600/3254.85 = 0.491574$.

From Eq.10.60, $\alpha_n = 0.67 + 1.67(P_u/P_{uz}) = 0.67 + 1.67(0.491574) = 1.4909$.

Step 4: Checking the adequacy of the section

From Eq.10.58, we have: $(120/246.376)^{1.4909} + (100/215.156)^{1.4909} = 0.6612 < 1$.

Hence, the section is safe to carry $P_u = 1600$ kN, $M_{ux} = 120$ kNm and $M_{uy} = 100$ kNm.

10.26.9 References

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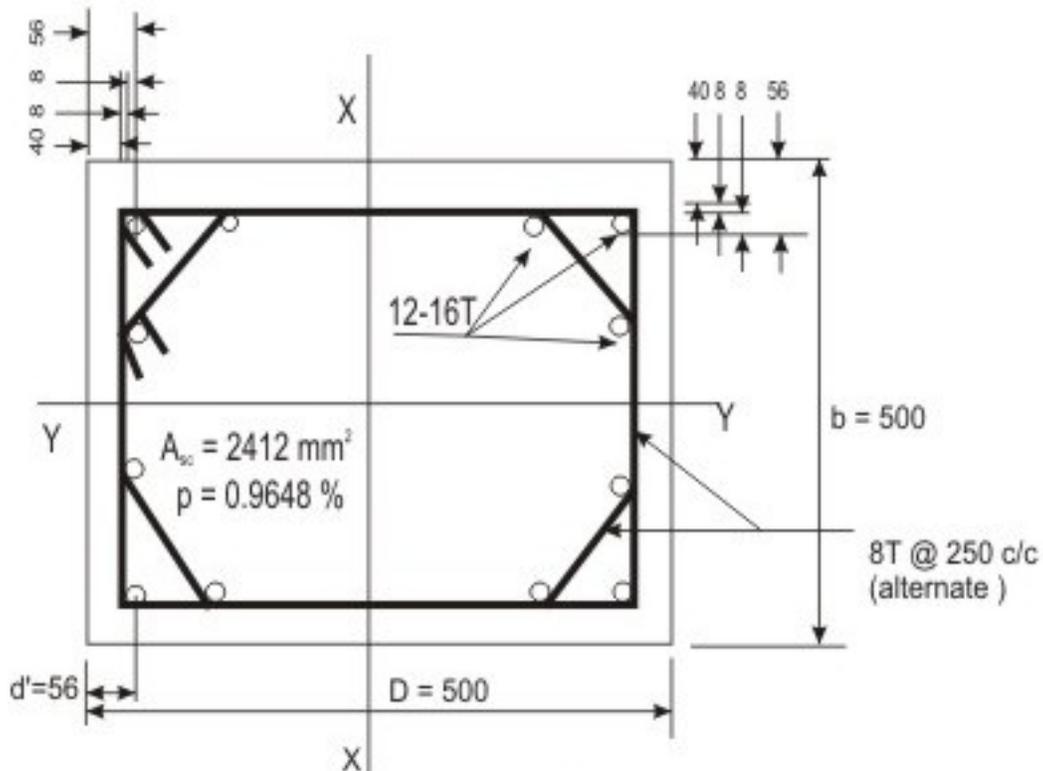
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10.26.10 Test 26 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1:



Given: $b = D = 500$, $P_u = 1800$ kN, $M_{ux} = 160$ kNm, $M_{uy} = 150$ kNm,
M 20, Fe 415, $l = 3500$, $A_{sc} = 2412 \text{ mm}^2$, tie 8T @ 250 c/c (alternate)

Fig. 10.26.7: TQ.1

Analyse the safety of the short square column of unsupported length = 3.5 m, $b = D = 500$ mm, as shown in Fig.10.26.7, with 12-16 mm diameter bars as longitudinal reinforcement and 8 mm diameter bars as lateral tie @ 250 mm c/c, when subjected to $P_u = 1800$ kN, $M_{ux} = 160$ kNm and $M_{uy} = 150$ kNm.

A.TQ.1:

Step 1: Verification of the eccentricities

From the given data: $l = 3500$ mm, $b = D = 500$ mm, we have

e_{min} in both directions (square column) = $(3500/500) + (500/30) = 23.67$ mm

$e_x = 160(10^3)/1800 = 88.88$ mm and $e_y = 150(10^3)/1800 = 83.34$ mm

Therefore, e_x and $e_y \gg e_{min}$.

Step 2: Determination of M_{ux1} and M_{uy1}

We have the given data: $b = D = 500$ mm, $f_{ck} = 25$ N/mm², $f_y = 415$ N/mm², $P_u = 1800$ kN, $M_{ux} = 160$ kNm, $M_{uy} = 150$ kNm and $A_{sc} = 2412$ mm² (12-16 mm diameter bars).

The percentage of longitudinal reinforcement $p = 241200/(500)(500) = 0.9648$ per cent, and $d'/D = 56/500 = 0.112$ and $p/f_{ck} = 0.9648/25 = 0.03859$. Linear interpolation of values of $M_{ux1}/f_{ck}bD^2$ from Charts 44 and 45 for $d'/D = 0.112$ is obtained as 0.08. Thus,

$$M_{ux1} = (0.08)(25)(500)(500)(500) = 250 \text{ kNm}$$

$$M_{uy1} = M_{ux1} = 250 \text{ kNm (square column)}$$

Step 3: Determination of P_{uz} and α_n

From Eq.10.59,

$$P_{uz} = 0.45(25)(500)(500) + \{0.75(415) - 0.45(25)\}(2415) = 3536.1 \text{ kN.}$$

$$P_u/P_{uz} = 1800/3536.1 = 0.509.$$

$$\text{From Eq.10.60, } \alpha_n = 0.67 + 1.67(0.509) = 1.52.$$

Step 4: Checking the adequacy of the section

$$\text{From Eq.10.58, we have: } (160/250)^{1.52} + (150/250)^{1.52} = 0.967 < 1.$$

Hence, the section can carry $P_u = 1800$ kN, $M_{ux} = 160$ kNm and $M_{uy} = 150$ kNm.

10.26.11 Summary of this Lesson

This lesson explains the behaviour of short columns under axial load and biaxial bending with the help of interaction surface, visualised as a three-dimensional plot of P_u - M_{ux} - M_{uy} . The interaction surface has a set of interaction curves of P_u - M_u and another set of interaction curves of M_{uxz} - M_{uyz} at constant P_{uz} , also known as load contour. The design and analysis of short columns are also explained with the help of derived equations and design charts of SP-16. Numerical examples in the illustrative example, practice problems and test will help in understanding the application of the theory in solving the analysis and design types of problems of short columns under axial load and biaxial bending.