Module
6

Shear, Bond, Anchorage, Development Length and Torsion

Lesson
13

Limit State of Collapse in Shear

Instructional Objectives:

At the end of this lesson, the student should be able to:

- name and explain the three different failure modes of reinforced concrete beams under the combined effects of bending moment and shear force,
- define nominal shear stress τ_v of rectangular and *T*-beams of uniform and varying depths under the combined effects of bending moment and shear force,
- name the two parameters on which the design shear strength of concrete depends,
- find out the maximum shear stress of concrete beams τ_{cmax} with shear reinforcement,
- locate the critical sections for shear in beams,
- explain when and why do we consider enhanced shear strength of concrete,
- explain why the minimum shear reinforcement is provided in any beam,
- determine the amount of minimum shear reinforcement to be provided in any beam,
- specify the three different ways of providing shear reinforcement in a beam,
- design the shear reinforcement in a beam for each of the three methods mentioned above,
- design the shear reinforcement closed to the support of a beam,
- specify the conditions to be satisfied for the curtailment of tension reinforcement when designing shear reinforcement,
- place the vertical stirrups in a beam.

6.13.1 Introduction

This lesson explains the three failure modes due to shear force in beams and defines different shear stresses needed to design the beams for shear. The critical sections for shear and the minimum shear reinforcement to be provided in beams are mentioned as per IS 456. The design of shear reinforcement has been illustrated in Lesson 14 through several numerical problems including the curtailment of tension reinforcement in flexural members.
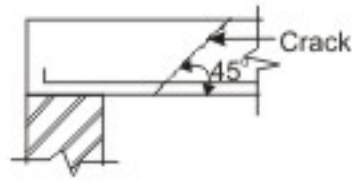
6.13.2 Failure Modes due to Shear

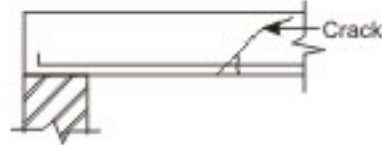Fig.6.13.1(a): Web shear progresses along dotted dotted lines

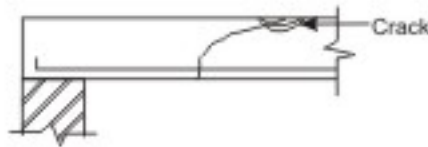Fig.6.13.1(b): Flexural tension (steel yields)

Fig.6.13.1(c): Flexural compression
(concrete crushes in compression)

Fig.6.13.1: Failure modes

Bending in reinforced concrete beams is usually accompanied by shear, the exact analysis of which is very complex. However, experimental studies confirmed the following three different modes of failure due to possible combinations of shear force and bending moment at a given section (Figs. 6.13.1a to c):

(i)   Web shear (Fig. 6.13.1a)
(ii) Flexural tension shear (Fig. 6.13.1b)
(iii) Flexural compression shear (Fig. 6.13.1c)

Web shear causes cracks which progress along the dotted line shown in Fig. 6.13.1a. Steel yields in flexural tension shear as shown in Fig. 6.13.1b, while concrete crushes in compression due to flexural compression shear as shown in Fig. 6.13.1c. An in-depth presentation of the three types of failure modes is beyond the scope here. Only the salient points needed for the routine design of beams in shear are presented here.

6.13.3 Shear Stress

The distribution of shear stress in reinforced concrete rectangular, *T* and *L*-beams of uniform and varying depths depends on the distribution of the normal stress. However, for the sake of simplicity the nominal shear stress τ_v is considered which is calculated as follows (IS 456, cls. 40.1 and 40.1.1):

NA

d

τ_v

Curve line
(i)

Straight line
(ii)

b

Note:

(i) Actual distribution
(ii) Average distribution

(a) Rectangular beam

NA

b_w

d

τ_v

(i)

(ii)

(b) T-beam

Note:

(i) Actual distribution
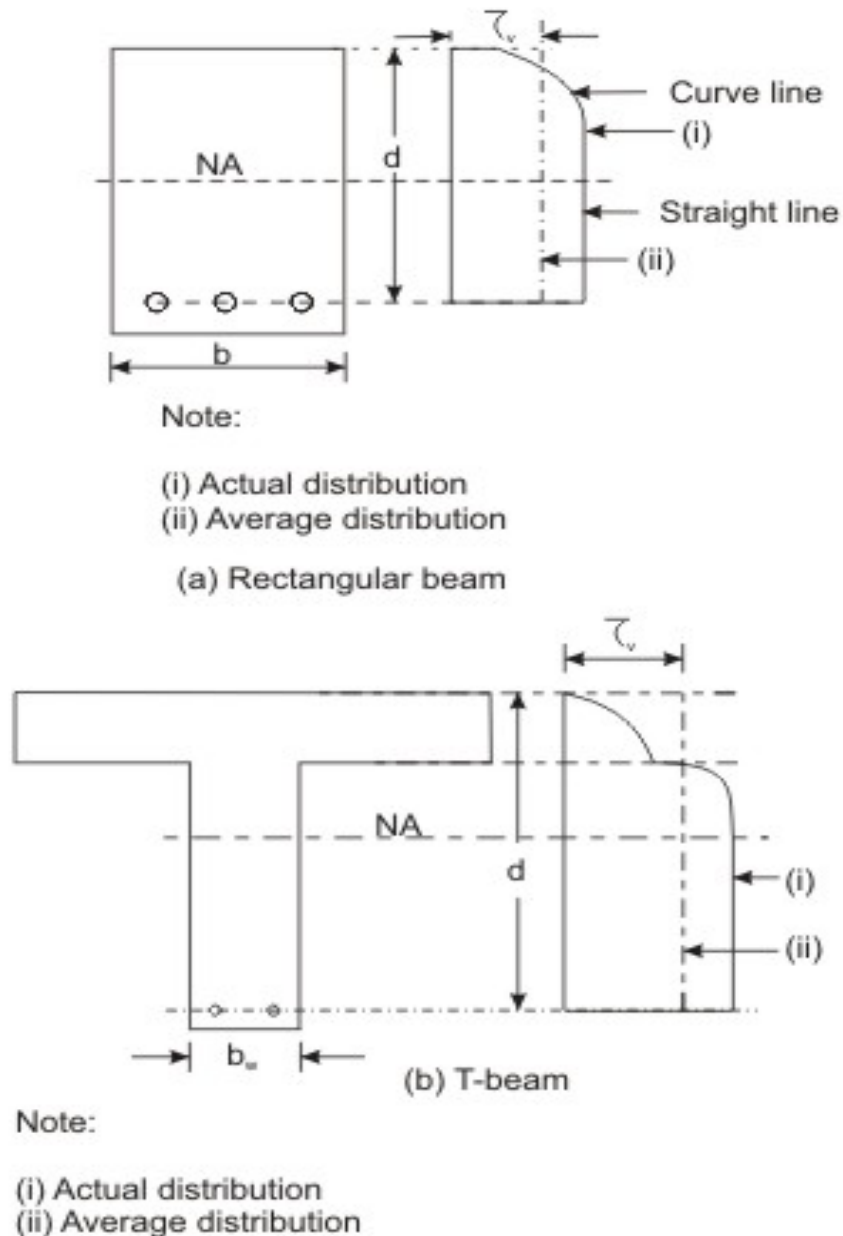(ii) Average distribution

Fig. 6.13.2: Distribution of shear stress and average shear stress

(i) In beams of uniform depth (Figs. 6.13.2a and b):

$$\tau_v = \frac{V_u}{b\,d}$$

(6.1)

where V_u = shear force due to design loads,

b = breadth of rectangular beams and breadth of the web b_w for flanged beams, and

d = effective depth.

(ii) In beams of varying depth:

$$\tau_v = \frac{V_u \pm \dfrac{M_u}{d}\tan\beta}{b\,d}$$

(6.2)

where τ_v, V_u, b or b_w and d are the same as in (i),

M_u = bending moment at the section, and

β = angle between the top and the bottom edges.

The positive sign is applicable when the bending moment M_u decreases numerically in the same direction as the effective depth increases, and the negative sign is applicable when the bending moment M_u increases numerically in the same direction as the effective depth increases.

6.13.4 Design Shear Strength of Reinforced Concrete

Recent laboratory experiments confirmed that reinforced concrete in beams has shear strength even without any shear reinforcement. This shear strength (τ_c) depends on the grade of concrete and the percentage of tension steel in beams. On the other hand, the shear strength of reinforced concrete with the reinforcement is restricted to some maximum value τ_{cmax} depending on the grade of concrete. These minimum and maximum shear strengths of reinforced concrete (IS 456, cls. 40.2.1 and 40.2.3, respectively) are given below:

6.13.4.1   Design shear strength without shear reinforcement (IS 456, cl. 40.2.1)

Table 19 of IS 456 stipulates the design shear strength of concrete τ_c for different grades of concrete with a wide range of percentages of positive tensile steel reinforcement. It is worth mentioning that the reinforced concrete beams must be provided with the minimum shear reinforcement as per cl. 40.3 even when τ_v is less than τ_c given in Table 6.1.

Table 6.1 Design shear strength of concrete, τ_c in N/mm^2

(100 A_s/b d)	Grade of concrete				
	M 20	M 25	M 30	M 35	M40 and above
≤ 0.15	0.28	0.29	0.29	0.29	0.30
0.25	0.36	0.36	0.37	0.37	0.38
0.50	0.48	0.49	0.50	0.50	0.51
0.75	0.56	0.57	0.59	0.59	0.60
1.00	0.62	0.64	0.66	0.67	0.68
1.25	0.67	0.70	0.71	0.73	0.74
1.50	0.72	0.74	0.76	0.78	0.79
1.75	0.75	0.78	0.80	0.82	0.84
2.00	0.79	0.82	0.84	0.86	0.88
2.25	0.81	0.85	0.88	0.90	0.92
2.50	0.82	0.88	0.91	0.93	0.95
2.75	0.82	0.90	0.94	0.96	0.98
≥ 3.00	0.82	0.92	0.96	0.99	1.01

In Table 6.1, A_s is the area of longitudinal tension reinforcement which continues at least one effective depth beyond the section considered except at support where the full area of tension reinforcement may be used provided the detailing is as per IS 456, cls. 26.2.2 and 26.2.3.

6.13.4.2 Maximum shear stress τ_{cmax} with shear reinforcement (cls. 40.2.3, 40.5.1 and 41.3.1)

Table 20 of IS 456 stipulates the maximum shear stress of reinforced concrete in beams τ_{cmax} as given below in Table 6.2. Under no circumstances, the nominal shear stress in beams τ_v shall exceed τ_{cmax} given in Table 6.2 for different grades of concrete.

Table 6.2 Maximum shear stress, τ_{cmax} in N/mm^2

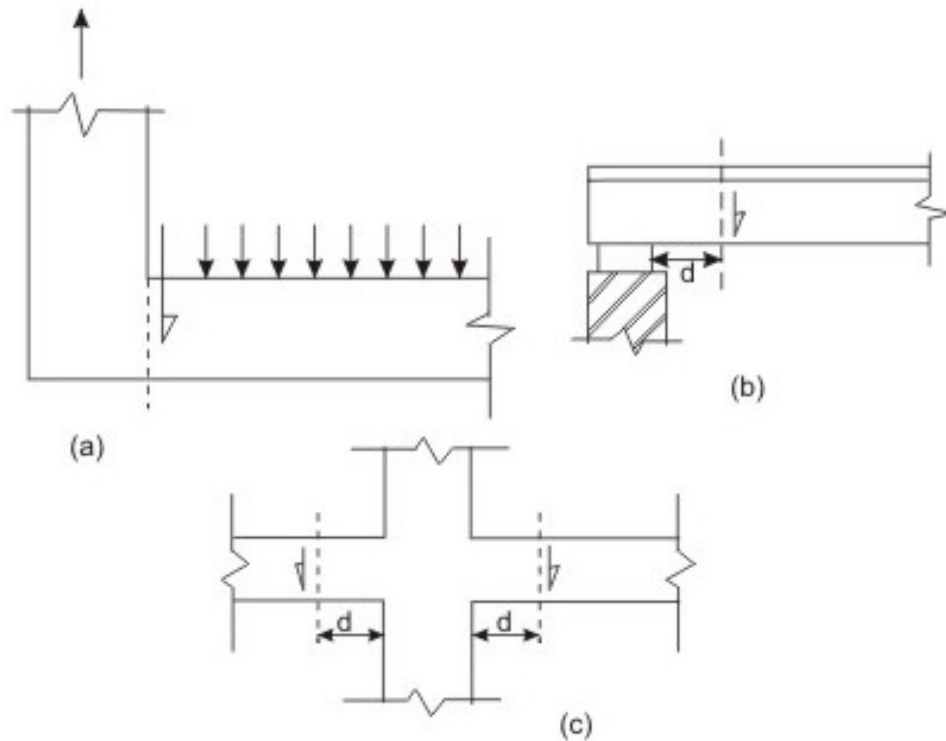| Grade of concrete	M 20	M 25	M 30	M 35	M 40 and above
τ_{cmax}, N/mm^2	2.8	3.1	3.5	3.7	4.0

6.13.5 Critical Section for Shear

Fig. 6.13.3: Support conditions for locating factored shear force

Clauses 22.6.2 and 22.6.2.1 stipulate the critical section for shear and are as follows:

For beams generally subjected to uniformly distributed loads or where the principal load is located further than $2d$ from the face of the support, where d is the effective depth of the beam, the critical sections depend on the conditions of supports as shown in Figs. 6.13.3 a, b and c and are mentioned below.

(i) When the reaction in the direction of the applied shear introduces tension (Fig. 6.13.3a) into the end region of the member, the shear force is to be computed at the face of the support of the member at that section.

(ii) When the reaction in the direction of the applied shear introduces compression into the end region of the member (Figs. 6.13.3b and c), the shear force computed at a distance d from the face of the support is to be used for the design of sections located at a distance less than d from the face of the support. The enhanced shear strength of sections close to supports, however, may be considered as discussed in the following section.

6.13.6 Enhanced Shear Strength of Sections Close to Supports (cl. 40.5 of IS 456)
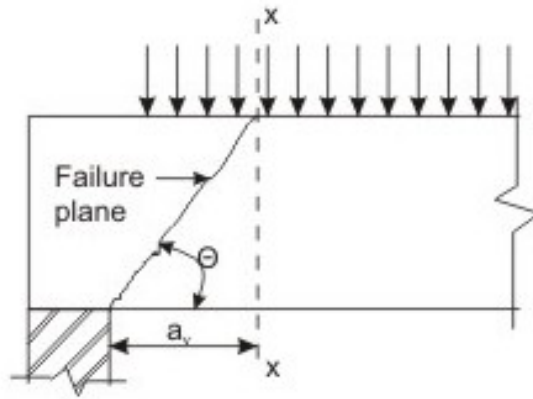

Fig. 6.13.4: Shear failure near support

Figure 6.13.4 shows the shear failure of simply supported and cantilever beams without shear reinforcement. The failure plane is normally inclined at an angle of 30° to the horizontal. However, in some situations the angle of failure is more steep either due to the location of the failure section closed to a support or for some other reasons. Under these situations, the shear force required to produce failure is increased.

Such enhancement of shear strength near a support is taken into account by increasing the design shear strength of concrete to $(2d\tau_c/a_v)$ provided that the design shear stress at the face of the support remains less than the value of τ_{cmax} given in Table 6.2 (Table 20 of IS 456). In the above expression of the enhanced shear strength

d = effective depth of the beam,

τ_c = design shear strength of concrete before the enhancement as given in Table 6.1 (Table 19 of IS 456),

a_v = horizontal distance of the section from the face of the support (Fig. 6.13.4).

Similar enhancement of shear strength is also to be considered for sections closed to point loads. It is evident from the expression $(2d\tau_c /a_v)$ that when a_v is equal to $2d$, the enhanced shear strength does not come into picture. Further, to increase the effectivity, the tension reinforcement is recommended to be extended on each side of the point where it is intersected by a possible failure plane for a distance at least equal to the effective depth, or to be provided with an equivalent anchorage.

6.13.7 Minimum Shear Reinforcement (cls. 40.3, 26.5.1.5 and 26.5.1.6 of IS 456)

Minimum shear reinforcement has to be provided even when τ_v is less than τ_c given in Table 6.1 as recommended in cl. 40.3 of IS 456. The amount of minimum shear reinforcement, as given in cl. 26.5.1.6, is given below.

The minimum shear reinforcement in the form of stirrups shall be provided such that:

$$\frac{A_{sv}}{b \, s_v} \geq \frac{0.4}{0.87 \, f_y}$$

(6.3)

where A_{sv} = total cross-sectional area of stirrup legs effective in shear,

s_v = stirrup spacing along the length of the member,

b = breadth of the beam or breadth of the web of the web of flanged beam b_w, and

f_y = characteristic strength of the stirrup reinforcement in N/mm^2 which shall not be taken greater than 415 N/mm^2.

The above provision is not applicable for members of minor structural importance such as lintels where the maximum shear stress calculated is less than half the permissible value.

The minimum shear reinforcement is provided for the following:

(i)   Any sudden failure of beams is prevented if concrete cover bursts and the bond to the tension steel is lost.

(ii)   Brittle shear failure is arrested which would have occurred without shear reinforcement.

(iii)   Tension failure is prevented which would have occurred due to shrinkage, thermal stresses and internal cracking in beams.

(iv)   To hold the reinforcement in place when concrete is poured.

(v)   Section becomes effective with the tie effect of the compression steel.

Further, cl. 26.5.1.5 of IS 456 stipulates that the maximum spacing of shear reinforcement measured along the axis of the member shall not be more than 0.75 d for vertical stirrups and d for inclined stirrups at 45°, where d is the effective depth of the section. However, the spacing shall not exceed 300 mm in any case.

6.13.8 Design of Shear Reinforcement (cl. 40.4 of IS 456)

When τ_v is more than τ_c given in Table 6.1, shear reinforcement shall be provided in any of the three following forms:

    (a) Vertical stirrups,
    (b) Bent-up bars along with stirrups, and
    (c) Inclined stirrups.

In the case of bent-up bars, it is to be seen that the contribution towards shear resistance of bent-up bars should not be more than fifty per cent of that of the total shear reinforcement.

The amount of shear reinforcement to be provided is determined to carry a shear force V_{us} equal to

$$V_{us} = V_u - \tau_c\, b\, d$$
(6.4)

where b is the breadth of rectangular beams or b_w in the case of flanged beams.

The strengths of shear reinforcement V_{us} for the three types of shear reinforcement are as follows:

(a) Vertical stirrups:

$$V_{us} = \frac{0.87\ f_y\ A_{sv}\ d}{s_v}$$
(6.5)

(b) For inclined stirrups or a series of bars bent-up at different cross-sections:

$$V_{us} = \frac{0.87\ f_y\ A_{sv}\ d}{s_v}(\sin\alpha + \cos\alpha)$$
(6.6)

(c) For single bar or single group of parallel bars, all bent-up at the same cross-section:

$$V_{us} = 0.87 \, f_y \, A_{sv} \, s_v \, \sin\alpha$$

(6.7)

where A_{sv} = total cross-sectional area of stirrup legs or bent-up bars within a distance s_v,

s_v = spacing of stirrups or bent-up bars along the length of the member,

τ_v = nominal shear stress,

τ_c = design shear strength of concrete,

b = breadth of the member which for the flanged beams shall be taken as the breadth of the web b_w,

f_y = characteristic strength of the stirrup or bent-up reinforcement which shall not be taken greater than 415 N/mm^2,

α = angle between the inclined stirrup or bent-up bar and the axis of the member, not less than 45°, and

d = effective depth.

The following two points are to be noted:

(i)   The total shear resistance shall be computed as the sum of the resistance for the various types separately where more than one type of shear reinforcement is used.

(ii)   The area of stirrups shall not be less than the minimum specified in cl. 26.5.1.6.

6.13.9 Shear Reinforcement for Sections Close to Supports

As stipulated in cl. 40.5.2 of IS 456, the total area of the required shear reinforcement A_s is obtained from:

$$A_s = a_v \, b \, (\tau_v - 2d \, \tau_c / a_v)/0.87 \, f_y$$

and $\geq 0.4 \, a_v \, b/0.87 \, f_y$

(6.8)

For flanged beams, b will be replaced by b_w, the breadth of the web of flanged beams.

This reinforcement should be provided within the middle three quarters of a_v, where a_v is less than d, horizontal shear reinforcement will be effective than vertical.

Alternatively, one simplified method has been recommended in cl. 40.5.3 of IS 456 and the same is given below.

The following method is for beams carrying generally uniform load or where the principal load is located further than $2d$ from the face of support. The shear stress is calculated at a section a distance d from the face of support. The value of τ_c is calculated in accordance with Table 6.1 and appropriate shear reinforcement is provided at sections closer to the support. No further check for shear at such sections is required.

6.13.10   Curtailment of Tension Reinforcement in Flexural Members (cl. 26.2.3.2 of IS 456)

Curtailment of tension reinforcement is done to provide the required reduced area of steel with the reduction of the bending moment. However, shear force increases with the reduction of bending moment. Therefore, it is necessary to satisfy any one of following three conditions while terminating the flexural reinforcement in tension zone:

(i) The shear stress τ_v at the cut-off point should not exceed two-thirds of the permitted value which includes the shear strength of the web reinforcement. Accordingly,

$$\tau_v \leq (2/3)\ (\tau_c + V_{us}/b\ d)$$

or $\quad V_{us} \geq (1.5\ \tau_v - \tau_c)\ b\ d$

(6.9)

(ii) For each of the terminated bars, additional stirrup area should be provided over a distance of three-fourth of effective depth from the cut-off point. The additional stirrup area shall not be less than $0.4\ b\ s/f_y$, where b is the breadth of rectangular beams and is replaced by b_w, the breadth of the web for flanged beams, s = spacing of additional stirrups and f_y is the characteristic strength of stirrup reinforcement in N/mm^2. The value of s shall not exceed $d/(8\ \beta_b)$, where β_b is the ratio of area of bars cut-off to the total area of bars at that section, and d is the effective depth.

(iii) For bars of diameters 36 mm and smaller, the continuing bars provide double the area required for flexure at the cut-off point. The shear stress should not exceed three-fourths that permitted. Accordingly,

$$\tau_v \leq (3/4)(\tau_c + V_{us}/b\,d)$$

or $\quad V_{us} \geq (1.33\,\tau_v - \tau_c)\,b\,d$
(6.10)

In the above expression b is the breadth of the rectangular beams which will be b_w in the case of flanged beams.

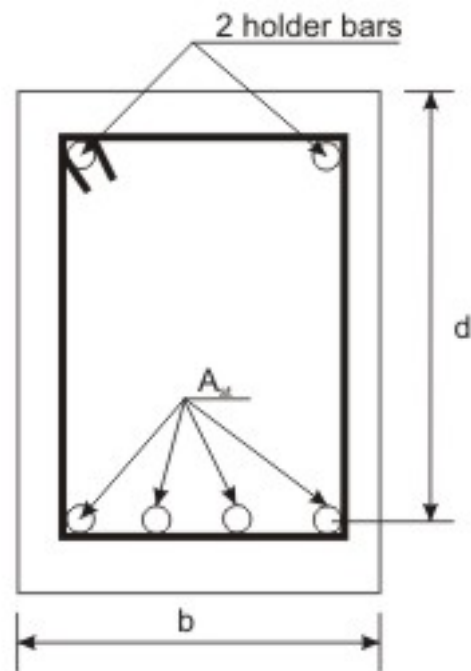## 6.13.11 Placement of Stirrups

Fig. 6.13.5: Placement of stirrups

The stirrups in beams shall be taken around the outer-most tension and compression bars. In T and L-beams, the stirrups will pass around longitudinal bars located close to the outer face of the flange. In the rectangular beams, two holder bars of diameter 10 or 12 mm are provided if there is no particular need for compression reinforcement (Fig. 6.13.5).

6.13.12 Practice Questions and Problems with Answers

Q.1: Name and explain the three different failure modes of reinforced concrete beams under the combined effects of bending moment and shear force.

A.1:   Sec. 6.13.2

Q.2:   Define nominal shear stress τ_v of rectangular and *T*-beams of (i) uniform depth and (ii) varying depth subjected to bending moment and shear force.

A.2:   Sec. 6.13.3

Q.3:   What is meant by "Design shear strength of concrete τ_c" ?

A.3:   Sec. 6.13.4

Q.4:   On what parameters τ_c of beams without shear reinforcement depends ? How do you get τ_c for different grades of concrete ?

A.4: τ_c depends on (i) grade of concrete and (ii) percentage of tensile steel in the beam.

Table 19 of cl. 40.2.1 of IS 456 gives the values of τ_c and the same table is presented in Table 6.1 of sec. 6.13.4.1 of this lesson.

Q.5:   How do you know the maximum shear stress of concrete beams τ_{cmax} with shear reinforcement ?

A.5:   Sec. 6.13.4.2

Q.6:   How do you determine the critical sections for shear in a beam ?

A.6:   Sec. 6.13.5

Q.7:   When and why do we consider enhanced shear strength of concrete ?

A.7:   Sec. 6.13.6

Q.8:   How do we determine the minimum shear reinforcement in rectangular and *T*-beams ? Why do we provide the minimum shear reinforcement ?

A.8:   Sec. 6.13.7

Q.9:   What are the three different ways to provide shear reinforcement ? Explain the method of design of each of them.

A.9:   Sec. 6.13.8

Q.10: How do we design the shear reinforcement close to the support of a beam ?

A.10: Sec. 6.13.9

Q.11: State the conditions to be satisfied for the curtailment of tension reinforcement when designing the shear reinforcement.

A.11: Sec. 6.13.10

Q.12: How do we place the vertical stirrups in a beam ?

A.12: Sec. 6.13.11

6.13.13 References:

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2. Limit State Design of Reinforced Concrete, 2nd Edition, by P.C.Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2002.
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5. Limit State Design of Reinforced Concrete Structures, by P.Dayaratnam, Oxford & I.B.H. Publishing Company Pvt. Ltd., New Delhi, 2004.
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14. Reinforced Concrete Designer's Handbook, 10th Edition, by C.E.Reynolds and J.C.Steedman, E & FN SPON, London, 1997.

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6.13.14 Test 13 with Solutions

Maximum Marks = 50,   Maximum Time = 30 minutes

Answer all questions.

TQ.1: Define nominal shear stress τ_v of rectangular and *T*-beams of (i) uniform depth and (ii) varying depth subjected to bending moment and shear force.

(5 marks)

A.TQ.1: Sec. 6.13.3

TQ.2: How do you determine the critical sections for shear in a beam ?   (5 marks)

A.TQ.2: Sec. 6.13.5

TQ.3: When and why do we consider enhanced shear strength of concrete?   (5 marks)

A.TQ.3: Sec. 6.13.6

TQ.4: How do we determine the minimum shear reinforcement in rectangular and *T*-beams? Why do we provide the minimum shear reinforcement ? (5 marks)

A.TQ.4: Sec. 6.13.7

TQ.5: What are the three different ways to provide shear reinforcement ? Explain the   method of design of each of them.   (5 marks)

A.TQ.5: Sec. 6.13.8

TQ.6: How do we design the shear reinforcement close to the support of a beam? (5 marks)

A.TQ.6: Sec. 6.13.9

TQ.7: State the conditions to be satisfied for the curtailment of tension reinforcement when designing the shear reinforcement.   (5 marks)

A.TQ.7: Sec. 6.13.10

6.13.15 Summary of this Lesson

Learning the different failure modes, the shear stresses and the design procedure of beams subjected to shear, this lesson explains the design procedure with special reference to curtailment of tension reinforcement in flexural members. Solutions of problems as illustrated in Lesson 14 and given in the practice problems and test, students will be thoroughly conversant with the design of rectangular and *T*-beams subjected to shear following the limit state of collapse as recommended by IS 456..