Module 4

Doubly Reinforced Beams – Theory and Problems

Lesson 9

Doubly Reinforced Beams – Theory

Instructional Objectives:

At the end of this lesson, the student should be able to:

- design the amounts of compression and tensile reinforcement if the b, d, d', f_{ck}, f_y and M_u are given, and
- determine the moment of resistance of a beam if b, d, d', f_{ck}, f_y, A_{sc} and A_{st} are given.

4.9.1 Introduction

This lesson illustrates the application of the theory of doubly reinforced beams in solving the two types of problems mentioned in Lesson 8. Both the design and analysis types of problems are solved by (i) direct computation method, and (ii) using tables of SP-16. The step by step solution of the problems will help in understanding the theory of Lesson 8 and its application.

4.9.2 Numerical problems

4.9.2.1 Problem 4.1

Design a simply supported beam of effective span 8 m subjected to imposed loads of 35 kN/m. The beam dimensions and other data are: b = 300 mm, D = 700 mm, M 20 concrete, Fe 415 steel (Fig. 4.9.1). Determine f_{sc} from d'/d as given in Table 4.2 of Lesson 8.

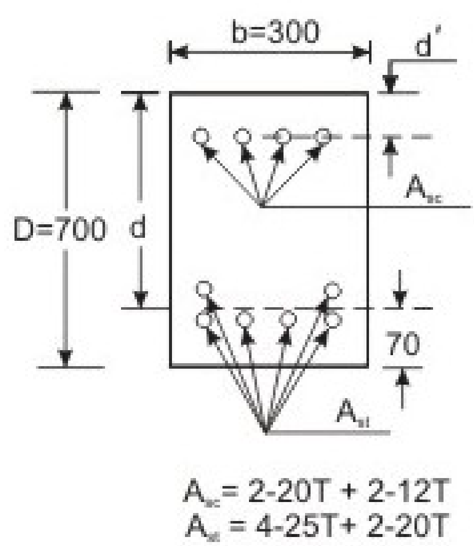


Fig. 4.9.1: Problem 4.1

(a) Solution by direct computation method

Dead load of the beam = 0.3 (0.7) (25) = 5.25 kN/m

Imposed loads (given) = 35.00 kN/m

Total loads = 5.25 + 35.00 = 40.25 kN/m

Factored bending moment = $(1.5)\,\dfrac{wl^2}{8} = \dfrac{(1.5)(40.25)(8)(8)}{8} = 482.96$ kNm

Assuming d' = 70 mm, d = 700 - 70 = 630 mm

$\dfrac{x_{u,\max}}{d} = 0.48$ gives $x_{u,\,max}$ = 0.48 (630) = 302.4 mm

Step 1: Determination of $M_{u,\,lim}$ and $A_{st,\,lim}$

$$M_{u,\,\lim} = 0.36\,(\frac{x_{u,\,\max}}{d})\,(1 - 0.42\,\frac{x_{u,\,\max}}{d})\,b\,d^2 f_{ck}$$

(4.2)

= 0.36(0.48) {1 – 0.42 (0.48)} (300) (630)^2 (20) (10^{-6}) kNm

= 328.55 kNm

$$A_{st,\,\lim} = \frac{M_{u,\,\lim}}{0.87\,f_y\,(d - 0.42\,x_{u,\,\max})} \qquad (6.8)$$

So, $$A_{st1} = \frac{328.55\,(10^6)\ \text{Nmm}}{0.87\,(415)\,\{\,630 - 0.42\,(0.48)\,630\,\}} = 1809.14\ \text{mm}^2$$

Step 2: Determination of M_{u2}, A_{sc}, A_{st2} and A_{st}

(Please refer to Eqs. 4.1, 4.4, 4.6 and 4.7 of Lesson 8.)

$M_{u2} = M_u - M_{u,lim} = 482.96 - 328.55 = 154.41$ kNm

Here, d'/d = 70/630 = 0.11

From Table 4.2 of Lesson 8, by linear interpolation, we get,

$$f_{sc} = 353 - \frac{353 - 342}{5} = 350.8 \text{ N/mm}^2$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})(d - d')} = \frac{154.41(10^6) \text{ Nmm}}{\{350.8 - 0.446(20)\}(630 - 70) \text{ N/mm}} = 806.517 \text{ mm}^2$$

$$A_{st2} = \frac{A_{sc}(f_{sc} - f_{cc})}{0.87 f_y} = \frac{806.517 (350.8 - 8.92)}{(0.87)(415)} = 763.694 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 1809.14 + 783.621 = 2572.834 \text{ mm}^2$$

Step 3: Check for minimum and maximum tension and compression steel.

(vide sec.4.8.5 of Lesson 8)

(i) In compression:

(a) Minimum $A_{sc} = \frac{0.2}{100}(300)(700) = 420 \text{ mm}^2$

(b) Maximum $A_{sc} = \frac{4}{100}(300)(700) = 8400 \text{ mm}^2$

Thus, 420 mm^2 < 806.517 mm^2 < 8400 mm^2 . Hence, o.k.

(ii) In tension:

(a) Minimum $A_{st} = \frac{0.85 \; b\, d}{f_y} \quad \frac{0.85(300)(630)}{415} = 387.1 \text{ mm}^2$

(b) Maximum $A_{st} = \frac{4}{100}(300)(700) = 8400 \text{ mm}^2$

Here, 387.1 mm^2 < 2572.834 mm^2 < 8400 mm^2 . Hence, o.k.

Step 4: Selection of bar diameter and numbers.

(i) for A_{sc}: Provide 2-20 T + 2-12 T (= 628 + 226 = 854 mm^2)

(ii) for A_{st}: Provide 4-25 T + 2-20 T (= 1963 + 628 = 2591 mm^2)

It may be noted that A_{st} is provided in two layers in order to provide adequate space for concreting around reinforcement. Also the centroid of the tensile bars is at 70 mm from bottom (Fig. 4.9.1).

(b) Solution by use of table of SP-16

For this problem, $\frac{M_u}{b\,d^2} = \frac{482.96\,(10^6)}{300\,(630)^2} = 4.056$ and $d'/d = \frac{70}{630} = 0.11.$

Table 50 of SP-16 gives p_t and p_c for $\frac{M_u}{b\,d^2} = 4$ and 4.1 and $d'/d = 0.1$ and $0.15.$ The required p_t and p_c are determined by linear interpolation. The values are presented in Table 4.3 to get the final p_t and p_c of this problem.

Table 4.3 Calculation of p_t and p_c

$\frac{M_u}{b\,d^2}$		$d'/d = 0.1$	$d'/d = 0.15$	$d'/d = 0.11$
4.0	p_t	1.337	1.360	$1.337 + \frac{0.023\,(0.01)}{0.05} = 1.342$
	p_c	0.401	0.437	$0.433 + \frac{0.036\,(0.01)}{0.05} = 0.408$
4.1	p_t	1.368	1.392	$1.368 + \frac{0.024\,(0.01)}{0.05} = 1.373$
	p_c	0.433	0.472	$0.433 + \frac{0.039\,(0.01)}{0.05} = 0.441$
4.056	p_t	Not Applicable (NA)	NA	$1.342 + \frac{0.031\,(0.056)}{0.1} = 1.3594$
	p_c	NA	NA	$0.408 + \frac{0.033\,(0.056)}{0.1} = 0.426$

So, $$A_{st} = \frac{1.3594\,(300)\,(630)}{100} = 2569.26\ \text{m}^2$$

and $$A_{sc} = \frac{0.426\,(300)\,(630)}{100} = 805.14\ \text{mm}^2$$

These values are close to those obtained by direct computation method where A_{st} = 2572.834 mm^2 and A_{sc} = 806.517 mm^2. Thus, by using table of SP-16 we get the reinforcement very close to that of direct computation method. Hence, provide

(i) for A_{sc}: 2-20 T + 2-12 T (= 628 + 226 = 854 mm^2)

(ii) for A_{st}: 4-25 T + 2-20 T (= 1963 + 628 = 2591 mm^2)

4.9.2.2 Problem 4.2

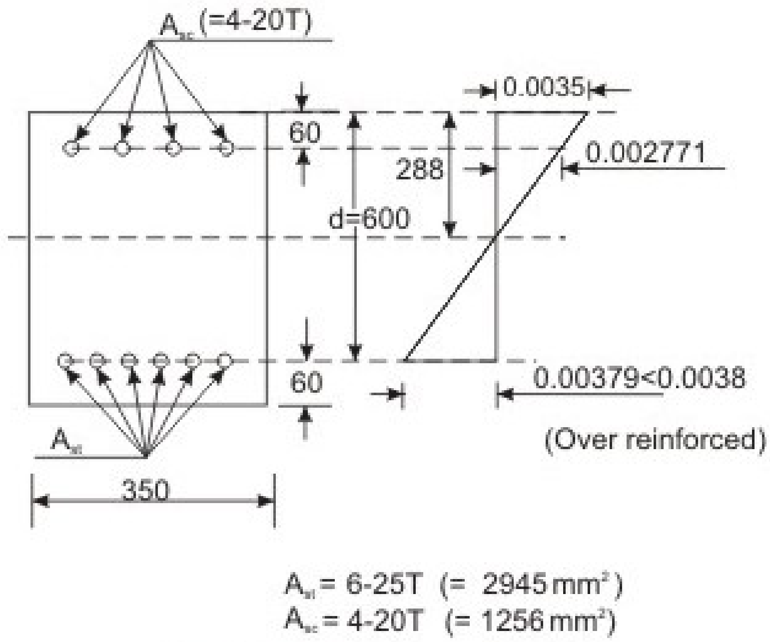


Fig. 4.9.2: Problem 4.2

Determine the ultimate moment capacity of the doubly reinforced beam of b = 350 mm, d' = 60 mm, d = 600 mm, A_{st} = 2945 mm^2 (6-25 T), A_{sc} = 1256 mm^2 (4-20 T), using M 20 and Fe 415 (Fig.4.9.2). Use direct computation method only.

Solution by direct computation method

Step 1: To check if the beam is under-reinforced or over-reinforced.

$$x_{u,\max} = 0.48\,(600) = 288 \text{ mm}$$

$$\varepsilon_{st} = \frac{\varepsilon_c\,(d - x_{u,\max})}{x_{u,\max}} = \frac{0.0035\,(600 - 288)}{288} = 0.00379$$

Yield strain of Fe 415 = $\dfrac{f_y}{1.15\,(E_s)} + 0.002 = \dfrac{415}{(1.15)\,(2)\,(10^5)} + 0.002$

= 0.0038 > 0.00379.

Hence, the beam is over-reinforced.

Step 2: To determine $M_{u,lim}$ and $A_{st,lim}$
(vide Eq. 4.2 of Lesson 8 and Table 3.1 of Lesson 5)

$$M_{u,\lim} = 0.36\,(\frac{x_{u,\,\max}}{d})\,(1 - 0.42\frac{x_{u,\,\max}}{d})\,b\,d^2 f_{ck}$$

$$= 0.36\ (0.48)\ \{1 - 0.42\ (0.48)\}\ (350)\ (600)^2\ (20)\ (10^{-6})\ \text{kNm}$$

$$= 347.67\ \text{kNm}$$

From Table 3.1 of Lesson 5, for f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2,

$$A_{st,\lim} = \frac{0.96\,(350)\,(600)}{100} = 2016\ \text{mm}^2$$

Step 3: To determine A_{st2} and A_{sc}
(vide Eqs.4.7 and 4.6 of Lesson 8)

$$A_{st2} = A_{st} - A_{st,\,lim} = 2945 - 2016 = 929\ \text{mm}^2$$

The required A_{sc} will have the compression force equal to the tensile force as given by 929 mm^2 of A_{st2}.

So, $$A_{sc} = \frac{A_{st2}\,(0.87\,f_y)}{(f_{sc} - f_{cc})}$$

For f_{sc} let us calculate ε_{sc}: (vide Eq. 4.9 of Lesson 8)

$$\varepsilon_{sc} = \frac{0.0035\,(x_{u,\,\max} - d')}{x_{u,\,\max}} = \frac{0.0035\,(288 - 60)}{288} = 0.002771$$

Table 4.1 of Lesson 8 gives:

$$f_{sc} = 351.8 + \frac{(360.9 - 351.8)\,(0.002771 - 0.002760)}{(0.00380 - 0.00276)} = 351.896\ \text{N/mm}^2$$

So, $A_{sc} = \dfrac{929\,(0.87)\,(415)}{\{351.89 - 0.446\,(20)\}} = 977.956\ \text{mm}^2$

Step 4: To determine M_{u2}, M_u and A_{st}

(Please refer to Eqs. 4.4 and 4.1 of Lesson 8)

$$M_{u2} = A_{sc}\,(f_{sc} - f_{cc})\,(d - d')$$

$$= 977.956\ \{351.896 - 0.446\ (20)\}\ (600 - 60)\ (10^{-6})\ \text{kNm}$$

$$= 181.12\ \text{kNm}$$

$$M_u = M_{u,\,lim} + M_{u2} = 347.67 + 181.12 = 528.79\ \text{kNm}$$

Therefore, with $A_{st} = A_{st,\,lim} + A_{st2} = 2016 + 929 = 2945\ \text{mm}^2$ the required A_{sc} = 977.956 mm^2 (much less than the provided 1256 mm^2). Hence, o.k.

4.9.3 Practice Questions and Problems with Answers

Q.1: Design a doubly reinforced beam (Fig. 4.9.3) to resist M_u = 375 kNm when b = 250 mm, d = 500 mm, d' = 75 mm, f_{ck} = 30 N/mm^2 and f_y = 500 N/mm^2, using (i) direct computation method and (ii) using table of SP-16.

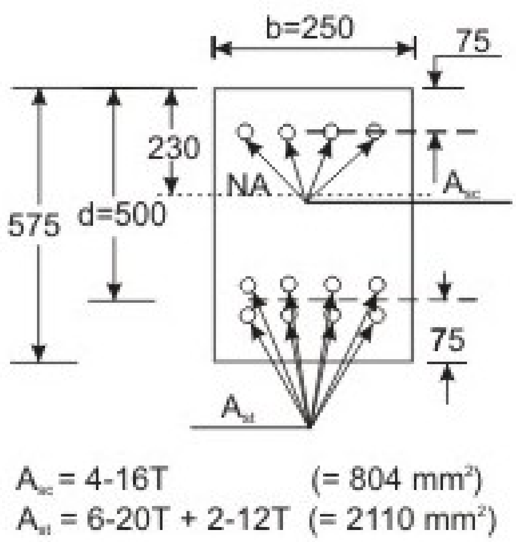


Fig. 4.9.3: Problem of Q 1

A.1: (A) Solution by direct computation method:

From the given data

$$M_{u,\,lim} = 0.36\,(\frac{x_{u,\max}}{d})\,(1-0.42\,\frac{x_{u,\max}}{d})\,b\,d^2\,f_{ck}$$

$$= 0.36\,(0.46)\,\{1 - 0.42\,(0.46)\}\,(250)\,(500)^2\,(30)\,(10^{-6})\text{ kNm}$$

$$= 250.51\text{ kNm}$$

Using the value of p_t = 1.13 from Table 3.1 of Lesson 5 for f_{ck} = 30 N/mm^2 and f_y = 500 N/mm^2,

$$A_{st,\ \mathrm{lim}} = \frac{1.13\,(250)\,(500)}{100} = 1412.5\ \mathrm{mm}^2$$

$$M_{u2} = 375 - 250.51 = 124.49\ \text{kNm}$$

From Table 4.2 of Lesson 8, for d'/d = 75/500 = 0.15 and f_y = 500 N/mm^2, we get f_{sc} = 395 N/mm^2

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})\,(d - d')} = \frac{124.49\ (10^6)}{\{395 - 0.446\,(30)\}\,(500 - 75)} = 767.56\ \mathrm{mm}^2$$

$$A_{st2} = \frac{A_{sc}\ (f_{sc} - f_{cc})}{0.87\,f_y} = \frac{767.56\ \{395 - 0.446\,(30)\}}{0.87\ (500)} = 673.37\ \mathrm{mm}^2$$

$$A_{st} = A_{st,\mathrm{lim}} + A_{st2} = 1412.5 + 673.37 = 2085.87\ \mathrm{mm}^2$$

Alternatively: (use of Table 4.1 of Lesson 8 to determine f_{sc} from ε_{sc})

$$x_{u,\ max} = 0.46\ (500) = 230\ \text{mm}$$

$$\varepsilon_{sc} = \frac{0.0035\,(230 - 75)}{230} = \frac{0.0035\,(155)}{230} = 0.002359$$

From Table 4.1

$$f_{sc} = 391.3 + \frac{(413.0 - 391.3)(0.002359 - 0.00226)}{(0.00277 - 0.00226)} = 395.512 \ \text{N/mm}^2$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})(d - d')} = \frac{124.49(10^6)}{\{395.512 - 0.446(30)\}(500 - 75)} = 766.53 \ \text{mm}^2$$

$$A_{st2} = \frac{A_{sc}(f_{sc} - f_{cc})}{0.87 f_y} = \frac{766.53 \ (382.132)}{0.87 \ (500)} = 673.369 \ \text{mm}^2$$

$$A_{st} = A_{st,\lim} + A_{st2} = 1412.5 + 673.369 = 2085.869 \ \text{mm}^2$$

Check for minimum and maximum A_{st} and A_{sc}

(i) Minimum $A_{st} = \dfrac{0.85 \ b \ d}{f_y} \ \dfrac{0.85(250) \ (500)}{500} = 212.5 \ \text{mm}^2$

(ii) Maximum $A_{st} = 0.04 \, b \, D = 0.04(250)(575) = 5750 \ \text{mm}^2$

(iii) Minimum $A_{st} = \dfrac{0.2 \ b \ D}{100} \ \dfrac{0.2(250) \ (575)}{100} = 287.5 \ \text{mm}^2$

(iv) Maximum $A_{st} = 0.04 \, b \, D = 0.04(250)(575) = 5750 \ \text{mm}^2$

Hence, the areas of reinforcement satisfy the requirements.

So, provide (i) 6-20 T + 2-12 T = 1885 + 226 = 2111 mm^2 for A_{st}

(ii) 4-16 T = 804 mm^2 for A_{sc}

(B) Solution by use of table of SP-16

From the given data, we have

$$\frac{M_u}{b \, d^2} = \frac{375(10^6)}{250(500)^2} = 6.0$$

$d'/d = 75/500 = 0.15$

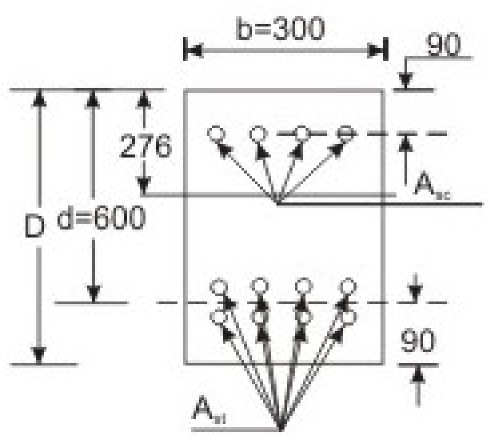
Table 56 of SP-16 gives: $p_t = 1.676$ and $p_c = 0.619$

So, $$A_{st} = \frac{(1.676)\,(250)\,(500)}{100} = 2095 \text{ mm}^2$$

and $$A_{sc} = \frac{(0.619\,(250)\,(500)}{100} = 773.75 \text{ mm}^2$$

These values are close to those of (A). Hence, provide 6-20 T + 2-12 T as A_{st} and 4-16 T as A_{sc}.

Q.2: Determine the moment of resistance of the doubly reinforced beam (Fig. 4.9.4) with b = 300 mm, d = 600 mm, d' = 90 mm, f_{ck} = 30 N/mm^2, f_y = 500 N/mm^2, A_{sc} = 2236 mm^2 (2-32 T + 2-20 T), and A_{st} = 4021 mm^2 (4-32 T + 4-16 T). Use (i) direct computation method and (ii) tables of SP-16.



A_{sc} = 2-32T + 2-20T (= 2236 mm^2)
A_{st} = 4-32T + 4-16T (= 4021 mm^2)

Fig. 4.9.4: Problem of Q 2

A.2: (i) Solution by direct computation method:

$x_{u,\,max} = 0.46\,(600) = 276$ mm

$$\varepsilon_{st} = \frac{0.0035\,(600 - 276)}{276} = 0.0041086$$

ε_{yield} = 0.00417. So ε_{st} < ε_{yield} i.e. the beam is over-reinforced.

For d'/d = 0.15 and f_y = 500 N/mm^2, Table 4.2 of Lesson 8 gives: f_{sc} = 395 N/mm^2 and with f_{ck} = 30 N/mm^2 , Table 3.1 of Lesson 5 gives $p_{t,\,lim}$ = 1.13.

$$A_{st,\mathrm{lim}} = \frac{1.13\,(300)\;(600)}{100} = 2034 \;\; \mathrm{mm}^2$$

$$M_{u,lim} = 0.36\,(\frac{x_{u,\max}}{d})\,(1 - 0.42\;\frac{x_{u,\max}}{d})\; b\; d^2\; f_{ck}$$

= 0.36 (0.46) {1 - 0.42 (0.46)} (300) (600)^2 (30) (10^{-6}) kNm

= 432.88 kNm

A_{st2} = 4021 - 2034 = 1987 mm^2

$$(A_{sc})_{required} = \frac{A_{st2}\;(0.87)\,f_y}{(f_{sc}\; - f_{cc}\;)} = \frac{1987\,(0.87)\,(500)}{\{395 - 0.446\,(30)\}} = 2264.94 \;\; \mathrm{mm}^2 \; > \; 2236 \;\; \mathrm{mm}^2$$

So, A_{st2} of 1987 mm^2 is not fully used. Let us determine A_{st2} required when A_{sc} = 2236 mm^2.

$$A_{st2} = \frac{A_{sc}\;(f_{sc}\; - f_{cc}\;)}{0.87\, f_y} = \frac{2236\;\{395\; - \;0.446\,(30)\}}{(0.87)\;(500)} = 1961.61 \;\; \mathrm{mm}^2$$

A_{st} = $A_{st,\,lim}$ + A_{st2} = 2034 + 1961.61 = 3995.61 mm^2 < 4021 mm^2.

Hence, o.k.

With A_{st2} = 1961.61 mm^2, M_{u2} = A_{st2} (0.87 f_y) (d - d')

= 1961.61 (0.87) (500) (600 - 75) (10^{-6}) kNm = 447.98268 kNm

Again, when A_{sc} = 2236 mm^2 (as provided)

$$M_{u2} = A_{sc}\,(f_{sc} - f_{cc})\,(d - d')$$

$= 2236\{395 - 0.446\,(30)\}\,(600 - 75)\,(10^{-6})$ kNm $= 447.9837$ kNm

$M_u = M_{u,\,lim} + M_{u2} = 432.88 + 447.98$ (M_{u2} is taken the lower of the two)

$= 880.86$ kNm

Hence, the moment of resistance of the beam is 880.86 kNm.

Alternatively f_{sc} can be determined from Table 4.1 of Lesson 8.

Using the following from the above:

$x_{u,\,max} = 276$ mm

$A_{st,lim} = 2034$ mm^2

$M_{u,lim} = 432.88$ kNm

$A_{st2} = 1987$ mm^2

To find $(A_{sc})_{required}$

$$\varepsilon_{st} = \frac{0.0035\,(276 - 90)}{276} = 0.00236$$

Table 4.1 of Lesson 8 gives:

$$f_{sc} = 391.3 + \frac{(413 - 391.3)\,(0.00236 - 0.00226)}{(0.00277 - 0.00226)} = 395.55 \text{ N/mm}^2$$

$$(A_{sc})_{required} = \frac{A_{st2}\,(0.87)\,f_y}{(f_{sc} - f_{cc})} = \frac{1987\,(0.87)\,(500)}{\{395.55 - 0.446\,(30)\}}$$

$$= 2261.68 \text{ mm}^2 > 2236 \text{ mm}^2$$

So, it is not o.k.

Let us determine A_{st2} required when $A_{sc} = 2236$ mm^2.

$$A_{st2} = \frac{A_{sc}\,(f_{sc} - f_{cc})}{0.87\, f_y} = \frac{2236\,\{395.55 - 0.446\,(30)\}}{(0.87)\,(500)} = 1964.44 \text{ mm}^2$$

$A_{st} = A_{st,\,lim} + A_{st2} = 2034 + 1964.44 = 3998.44 \text{ mm}^2 < 4021 \text{ mm}^2$.

So, o.k.

M_{u2} (when $A_{st2} = 1964.44 \text{ mm}^2$) $= A_{st2}\,(0.87\, f_y)\,(d - d')$

$= 1964.44\,(0.87)\,(500)\,(600 - 75)\,(10^{-6})$ kNm

$= 448.63$ kNm

For $A_{sc} = 2236 \text{ mm}^2$,

$$M_{u2} = A_{sc}\,(f_{sc} - f_{cc})\,(d - d')$$

$= 2236\,\{(395.55 - 0.446\,(30)\}\,(600 - 75)\,(10^{-6})$ kNm

$= 2236\,(382.17)\,(525)\,(10^{-6})$ kNm

$= 448.63$ kNm

Both the M_{u2} values are the same. So,

$M_u = M_{u,lim} + M_{u2} = 432.88 + 448.63$

$= 881.51$ kNm

Here, the $M_u = 881.51$ kNm.

(ii) Solution by using table of SP-16

From the given data:

$$p_t = \frac{4021\,(100)}{300\,(600)} = 2.234$$

$$p_c = \frac{2236\,(100)}{300\,(600)} = 1.242$$

$d'/d = 0.15$

Table 56 of SP-16 is used first considering d'/d = 0.15 and p_t = 2.234, and secondly, considering d'/d = 0.15 and p_c = 1.242. The calculated values of p_c and M_u/bd^2 for the first and p_t and M_u/bd^2 for the second cases are presented below separately. Linear interpolation has been done.

(i) When d'/d = 0.15 and p_t = 2.234

$$\frac{M_u}{b\,d^2} = 8.00 + \frac{(8.1 - 8.0)\,(2.234 - 2.218)}{(2.245 - 2.218)} = 8.06$$

$$p_c = 1.235 + \frac{(1.266 - 1.235)\,(0.016)}{(0.027)} = 1.253 > 1.242$$

So, this is not possible.

(ii) When d'/d = 0.15 and p_c = 1.242

$$\frac{M_u}{b\,d^2} = 8.00 + \frac{(8.1 - 8.0)\,(1.242 - 1.235)}{(1.266 - 1.235)} = 8.022$$

$$p_t = 2.218 + \frac{(2.245 - 2.218)\,(1.242 - 1.235)}{(1.266 - 1.235)} = 2.224 < 2.234$$

So, M_u = 8.022 (300) $(600)^2$ (10^{-6})= 866.376 kNm.

Hence, o.k.

4.9.4 References

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4.9.5 Test 9 with Solutions

Maximum Marks = 50,   Maximum Time = 30 minutes

Answer all questions.

TQ.1: Design a simply supported beam of effective span 8 m subjected to imposed loads of 35 kN/m. The beam dimensions and other data are: b = 300 mm, D = 700 mm, M 20 concrete, Fe 415 steel (Fig. 4.9.1). Determine f_{sc} from strain ε_{sc} as given in Table 4.1 of Lesson 8.

A.TQ.1: This problem is the same as Problem 4.1 in sec. 4.9.2.1 except that here the f_{sc} is to be calculated using Table 4.1 instead of Table 4.2.

Step 1: Here, the Step 1 will remain the same as that of Problem 4.1.

Step 2: Determination of M_{u2}, A_{sc}, A_{st2} and A_{st}

$$M_{u2} = M_u - M_{u,\ \lim} = 482.96 - 328.55 = 154.41 \text{ kNm}$$

From strain triangle: (Fig. 4.8.2 of Lesson 8)

$$\varepsilon_{sc} = \frac{0.0035\,(302.4 - 70)}{302.4} = 0.00269$$

$$f_{sc} \text{ (from Table 4.1 of Lesson 8)} = 342.8 + \frac{(351.8 - 342.8)}{(0.00276 - 0.00241)}(0.00269 - 0.00241)$$

$$= 350 \text{ N/mm}^2$$

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})\,(d - d')} = \frac{154.41\,(10^6)}{\{350 - 0.446\,(20)\}\,(630 - 70) \text{ N/mm}} = 808.41 \text{ mm}^2$$

$$A_{st2} = \frac{A_{sc}\,(f_{sc} - f_{cc})}{0.87\,f_y} = \frac{808.41\,(341.08)}{(0.87)\,(415)} = 763.696 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 1809.14 + 763.696 = 2572.836 \text{ mm}^2$$

$$A_{sc} = 808.41 \text{ mm}^2$$

Steps 3 & 4 will also remain the same as those of Problem 4.1.

Hence, provide 2-20 T + 2-12 T (854 mm^2) as A_{sc} and 4-25 T + 2-20 T (2591 mm^2) as A_{st} .

TQ.2: Determine the ultimate moment capacity of the doubly reinforced beam of b = 350 mm, d' = 60 mm, d = 600 mm, A_{st} = 2945 mm^2 (6-25 T), A_{sc} = 1256 mm^2 (4-20 T), using M 20 and Fe 415 (Fig. 4.9.2). Use table of SP-16 only.

A.TQ.2: Solution by using table of SP-16

This problem is the same as that of Problem 4.2 of sec. 4.9.2.2, which has been solved by direct computation method. Here, the same is to be solved by using SP-16.

The needed parameters are:

$d'/d = 60/600 = 0.1$

$$p_t = \frac{A_{st}\,(100)}{b\,d} = \frac{2945\,(100)}{350(600)} = 1.402$$

$$p_c = \frac{A_{sc}\,(100)}{b\,d} = \frac{1256\,(100)}{350\,(600)} = 0.5981$$

Here, we need to use Table 50 for f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2. The table gives values of M_u/bd^2 for (I) d'/d and p_t and (ii) d'/d and p_c. So, we will consider both the possibilities and determine M_u.

(i) Considering Table 50 of SP-16 when d'/d = 0.1 and p_t = 1.402:

Interpolating the values of M_u/bd^2 at p_t = 1.399 and 1.429, we get

$$\left(\frac{M_u}{b\,d^2}\right)_{p_t = 1.402} = 4.2 + \frac{(4.3 - 4.2)\,(1.402 - 1.399)}{(1.429 - 1.399)} = 4.21$$

the corresponding $\left(p_c\right)_{p_t = 1.402} = 0.466 + \dfrac{(0.498 - 0.466)\,(1.402 - 1.399)}{(1.429 - 1.399)} = 0.4692$

But, p_c provided is 0.5981 indicates that extra compression reinforcement has been used.

So, we get

$M_u = 4.21\ b\,d^2 = (4.21)\,(350)\,(600)^2\,(10^{-6}) = 530.46\ \text{kNm}$ when A_{st} = 2945 mm^2 and A_{sc} = 985.32 mm^2, i.e. 270.69 mm^2 (= 1256 - 985.32) of compression steel is extra.

(ii) Considering d'/d = 0.1 and p_c = 0.5981, we get by linear interpolation

$$\left(\frac{M_u}{b\,d^2}\right)_{p_c = 0.5981} = 4.6 + \frac{(4.7 - 4.6)\,(0.5981 - 0.595)}{(0.628 - 0.595)} = 4.61$$

the corresponding p_t is:

$$\left(p_t\right)_{p_c = 0.5981} = 1.522 + \frac{(1.533 - 1.522)\,(0.5981 - 0.595)}{(0.628 - 0.595)} = 1.5231$$

The provided p_t = 1.402 indicates that the tension steel is insufficient by 254.31 mm^2 as shown below:

Amount of additional A_{st} still required =

$$\frac{(1.5231 - 1.402)\,(350)\,(600)}{100} = 254.31 \text{ mm}^2$$

If this additional steel is provided, then the M_u of this beam becomes:

$$M_u = 4.61\ b\,d^2 = 4.61\,(350)\,(600)^2\,(10^{-6})\ \text{kNm} = 580.86\ \text{kNm}$$

The above two results show that the moment of resistance of this beam is the lower of the two. So, M_u = 530.46 kNm. By direct computation the M_u = 528.79 kNm. The two results are in good agreement.

4.9.6 Summary of this Lesson

This lesson presents solutions of four numerical problems covering both design and analysis types. These problems are solved by two methods: (i) direct computation method and (ii) using table of SP-16. Two problems are illustrated in the lesson and the other two are given in the practice problem and test of this lesson. The solutions will help in understanding the step by step application of the theory of doubly reinforced beams given in Lesson 8.