Module 5

Flanged Beams – Theory and Numerical Problems

Lesson
12
Flanged Beams – Numerical Problems (Continued)

Instructional Objectives:

At the end of this lesson, the student should be able to:

- identify the two types of problems – analysis and design types,
- apply the formulations to design the flanged beams.

5.12.1 Introduction

Lesson 10 illustrates the governing equations of flanged beams and Lesson 11 explains their applications for the solution of analysis type of numerical problems. It is now necessary to apply them for the solution of design type, the second type of the numerical problems. This lesson mentions the different steps of the solution and solves several numerical examples to explain their step-by-step solutions.

5.12.2 Design Type of Problems

We need to assume some preliminary dimensions of width and depth of flanged beams, spacing of the beams and span for performing the structural analysis before the design. Thus, the assumed data known for the design are: D_f, b_w, D, effective span, effective depth, grades of concrete and steel and imposed loads.

There are four equations: (i) expressions of compressive force C, (ii) expression of the tension force T, (iii) $C = T$ and (iv) expression of M_u in terms of C or T and the lever arm {M = (C or T) (lever arm)}. However, the relative dimensions of D_f, D and x_u and the amount of steel (under-reinforced, balanced or over-reinforced) influence the expressions. Accordingly, the respective equations are to be employed assuming a particular situation and, if necessary, they need to be changed if the assumed parameters are found to be not satisfactory. The steps of the design problems are as given below.

Step 1: To determine the factored bending moment M_u

Step 2: To determine the $M_{u,lim}$ of the given or the assumed section

The beam shall be designed as under-reinforced, balanced or doubly reinforced if the value of M_u is less than, equal to or more than $M_{u,lim}$. The design of over-reinforced beam is to be avoided as it does not increase the bending moment carrying capacity beyond $M_{u,lim}$ either by increasing the depth or designing a doubly reinforced beam.

Step 3: To determine x_u, the distance of the neutral axis, from the expression of M_u

Here, it is necessary to assume first that x_u is in the flange. Later on, it may be necessary to calculate x_u if the value is found to be more than D_f . This is to be done assuming first that $D_f/x_u < 0.43$ and then $D_f/x_u > 0.43$ separately.

Step 4: To determine the area(s) of steel

For doubly reinforced beams $A_{st} = A_{st,lim} + A_{st2}$ and A_{sc} are to be obtained, while only A_{st} is required to be computed for under-reinforced and balanced beams. These are calculated employing $C = T$ (for A_{st} and $A_{st,lim}$) and the expression of M_{u2} to calculate A_{st2} and A_{sc}.

Step 5: It may be necessary to check the x_u and A_{st} once again after Step 4

It is difficult to prescribe all the relevant steps of design problems. Decisions are to be taken judiciously depending on the type of problem. For the design of a balanced beam, it is necessary to determine the effective depth in Step 3 employing the expression of bending moment M_u. For such beams and for under-reinforced beams, it may be necessary to estimate the A_{st} approximately immediately after Step 2. This value of A_{st} will facilitate to determine x_u.

5.12.3 Numerical Problems

Four numerical examples are solved below explaining the steps involved in the design problems.

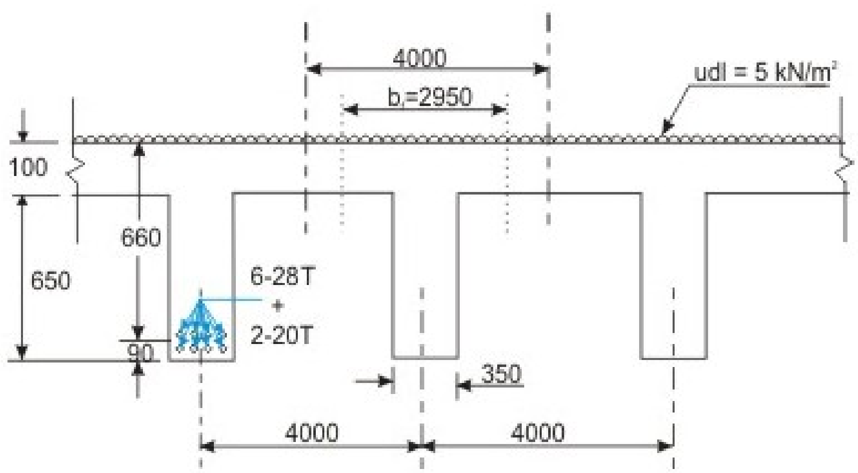

Fig. 5.12.1: Example 5

Ex.5: Design the simply supported flanged beam of Fig. 5.12.1, given the following: D_f = 100 mm, D = 750 mm, b_w = 350 mm, spacing of beams = 4000 mm c/c, effective span = 12 m, cover = 90 mm, d = 660 mm and imposed loads = 5 kN/m^2. Fe 415 and M 20 are used.

Solution:

Step 1: Computation of factored bending moment

Weight of slab per m^2 = (0.1) (1) (1) (25) = 2.5 kN/m^2

So, Weight of slab per m = (4) (2.5) = 10.00 kN/m

Dead loads of web part of the beam = (0.35) (0.65) (1) (25) = 5.6875 kN/m

Imposed loads = (4) (5) = 20 kN/m

Total loads = 30 + 5.6875 = 35.6875 kN/m

Factored Bending moment = (1.5) $\dfrac{(35.6875)\,(12)\,(12)}{8} = 963.5625$ kNm

Step 2: Computation of $x_{u,lim}$

Effective width of flange =(l_o/6) + b_w+ 6 D_f = (12000/6) + 350 + 600 = 2,950 mm.

$x_{u,max}$ = 0.48 d = 0.48 (660) = 316.80 mm. This shows that the neutral axis is in the web of this beam.

D_f/d = 100/660 = 0.1515 < 0.2, and

D_f/x_u = 100/316.8 = 0.316 < 0.43

The expression of $M_{u,lim}$ is obtained from Eq. 5.7 of Lesson 10 (case ii a of sec. 5.10.4.2) and is as follows:

$M_{u,lim} = 0.36(x_{u,max}/d)\{1 - 0.42\,(x_{u,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\,(b_f - b_w)\, D_f\,(d - D_f/2)$

= 0.36(0.48) {1 - 0.42(0.48)} (20) (350) (650) (650)

+ 0.45 (20) (2950 - 350) (100) (660 - 50) = 1,835.43 kNm

The design moment M_u = 963.5625 kNm is less than $M_{u,lim}$. Hence, one under-reinforced beam can be designed.

Step 3: Determination of x_u

Since the design moment M_u is almost 50% of $M_{u,lim}$, let us assume the neutral axis to be in the flange. The area of steel is to be calculated from the moment equation (Eq. 3.23 of Lesson 5), when steel is ensured to reach the design stress f_d = 0.87 (415) = 361.05 N/mm^2. It is worth mentioning that the term b of Eq. 3.23 of Lesson 5 is here b_f as the T- beam is treated as a rectangular beam when the neutral axis is in the flange.

$$M_u = 0.87\ f_y\ A_{st}\ d\left\{1 - \frac{A_{st}\ f_y}{f_{ck}\ b\ d}\right\}$$

(3.23)

Here, all but A_{st} are known. However, this will give a quadratic equation of A_{st} and the lower one of the two values will be provided in the beam. The above equation gives:

$$A_{st}^2 - 93831.3253\ A_{st} + 379416711.3 = 0$$

which gives the lower value of A_{st} as:

A_{st} = 4,234.722097 mm^2. The reason of selecting the lower value of A_{st} is explained in sec 3.6.4.8 of Lesson 6 in the solution of Design Problem 3.1.

Then, employing Eq. 3.16 of Lesson 5, we get

$$x_u = \frac{0.87\ f_y\ A_{st}}{0.36\ b\ f_{ck}}$$

(3.16)

or $\quad x_u$ = 71.98 mm.

Again, employing Eq. 3.24 of Lesson 5, we can determine x_u first and then A_{st} from Eq. 3.16 or 17 of Lesson 5, as explained in the next step.

Eq. 3.24 of Lesson 5 gives:

$$M_u = 0.36(x_u/d)\ \{1 - 0.42(x_u/d)\}\ f_{ck}\ b_f\ d^2$$

$$= 0.36\ (x_u)\ \{1 - 0.42\ (x_u/d)\}\ f_{ck}\ b_f\ d$$

$$963.5625\ (10^6) = 0.36\ (x_u)\ \{1 - 0.42\ (x_u/660)\}\ (20)\ (2950)\ (660)$$

or $\quad x_u$ = 72.03 mm.

The two values of x_u are the same. It is thus seen that, the value of x_u can be determined either first finding the value of A_{st}, from Eq. 3.23 of Lesson 5 or directly from Eq. 3.24 of Lesson 5 first and then the value of A_{st} can be determined.

Step 4: Determination of A_{st}

Equating $C = T$, we have from Eq. 3.17 of Lesson 5:

$$\frac{x_u}{d} = \frac{0.87\ f_y\ A_{st}}{0.36\ f_{ck}\ b_f\ d}$$

$$A_{st} = \frac{0.36\, f_{ck}\ b_f\ x_u}{0.87\ f_y} = \frac{0.36\ (20)\ (2950)\ (72.03)}{0.87\ (415)} = 4{,}237.41\ \text{mm}^2$$

Minimum A_{st} = $(0.85/f_y)\ b_w\ d$ = (0.85/415) (350) (660) = 473.13 mm^2

Maximum A_{st} = $0.04\ b_w\ D$ = (0.04) (350) (660) = 9,240 mm^2

Hence, A_{st} = 4,237.41 mm^2 is o.k.

Provide 6 - 28 T (= 3694 mm^2) + 2-20 T (= 628 mm^2) to have total A_{st} = 4,322 mm^2.

Ex.6: Design a beam in place of the beam of Ex.5 (Fig. 5.12.1) if the imposed loads are increased to 12 kN/m^2. Other data are: D_f = 100 mm, b_w = 350 mm, spacing of beams = 4000 mm c/c, effective span = 12 m simply supported and cover = 90 mm. Use Fe 415 and M 20.

Solution: As in Ex.5, b_f = 2,950 mm.

Step 1: Computation of factored bending moment

Weight of slab/m^2 = 2.5 kN/m^2 (as in Ex.1)

Imposed loads = 12.0 kN/m^2 (given)

Total loads = 14.5 kN/m^2

Total weight of slab and imposed loads = 14.5 (4) = 58.0 kN/m

Dead loads of the beam = 0.65 (0.35) (25) = 5.6875 kN/m

Total loads = 63.6875 kN/m

$$(M_u)_{\text{factored}} = \frac{1.5\,(63.6875)\,(12)\,(12)}{8} = 1{,}719.5625 \text{ kNm}$$

Step 2: Determination of $M_{u,lim}$

$M_{u,lim}$ of the beam of Ex.5 = 1,835.43 kNm. The factored moment of this problem (1,719.5625 kNm) is close to the value of $M_{u,lim}$ of the section.

Step 3: Determination of d

Assuming $D_f/d < 0.2$, we have from Eq. 5.7 of Lesson 10,

$M_u = 0.36(x_{u,max}/d)\{1 - 0.42\,(x_{u,max}/d)\}\, f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\,(b_f - b_w)\, D_f\,(d - D_f/2)$

$1719.5625\,(10^6) = 0.36(0.48)\,\{1 - 0.42(0.48)\}\,(20)\,(350)\,d^2$

$+ 0.45\,(2600)\,(20)\,(100)\,(d - 50)$

Solving the above equation, we get d = 624.09 mm, giving total depth = 624.09 + 90 = 715 mm (say).

Since the dead load of the beam is reduced due to decreasing the depth of the beam, the revised loads are calculated below:

Loads from the slab = 58.0 kN/m

Dead loads (revised) = 0.615 (0.35) (25) = 5.38125 kN/m

Total loads = 63.38125 kN/m

$$(M_u)_{\text{factored}} = \frac{1.5\,(63.38125)\,(12)\,(12)}{8} = 1{,}711.29 \text{ kNm}$$

Approximate value of A_{st}:

$$A_{st} = \frac{M_u}{0.87\, f_y\, (d - \frac{D_f}{2})} = \frac{1711.29\,(10^6)}{0.87\,(415)\,(625 - 50)} = 8{,}243.06 \text{ mm}^2$$

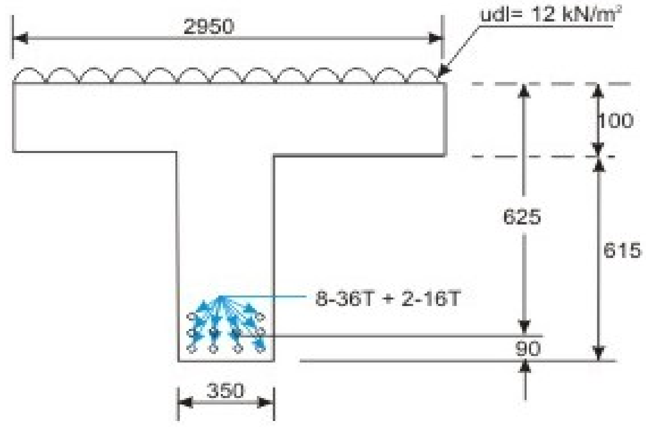

Fig. 5.12.2: Example 6

Step 4: Determination of A_{st} (Fig. 5.12.2)

$$x_u = x_{u,max} = 0.48\ (625) = 300 \text{ mm}$$

Equating T and C (Eq. 5.5 of Lesson 10), we have:

$$0.87\ f_y\ A_{st} = 0.36\ x_{u,max}\ b_w\ f_{ck} + 0.45\ f_{ck}\ (b_f - b_w)\ D_f$$

or $$A_{st} = \frac{0.36\ (300)\ (350)\ (20)\ +\ 0.45\ (20)\ (2600)\ (100)}{0.87\ (415)} = 8{,}574.98 \text{ mm}^2$$

Maximum $A_{st} = 0.04\ b\ D = 0.04\ (350)\ (715) = 10{,}010.00 \text{ mm}^2$

Minimum $A_{st} = (0.85/f_y)\ b_w\ d = (0.85/415)\ (350)\ (625) = 448.05 \text{ mm}^2$

Hence, $A_{st} = 8{,}574.98 \text{ mm}^2$ is o.k.

So, provide 8-36 T + 2-18 T = 8143 + 508 = 8,651 mm^2

Step 5: Determination of x_u

Using $A_{st} = 8{,}651 \text{ mm}^2$ in the expression of $T = C$ (Eq. 5.5 of Lesson 10), we have:

$$0.87\ f_y\ A_{st} = 0.36\ x_u\ b_w\ f_{ck} + 0.45\ f_{ck}\ (b_f - b_w)\ D_f$$

or $$x_u = \frac{0.87\ f_y\ A_{st} - 0.45\ f_{ck}\ (b_f - b_w)}{0.36\ b_w\ f_{ck}}$$

$$= \frac{0.87\,(415)\,(8651) - 0.45\,(20)\,(2600)\,(100)}{0.36\,(350)\,(20)} = 310.89 > x_{u,\max} \ \ (= 300 \ \text{mm})$$

So, A_{st} provided is reduced to 8-36 + 2-16 = 8143 + 402 = 8,545 mm^2. Accordingly,

$$x_u = \frac{0.87\,(415)\,(8545) - 0.45\,(20)\,(2600)\,(100)}{0.36\,(350)\,(20)} = 295.703\,\text{mm} < x_{u,\max} \ \ (= 300 \ \text{mm})$$

Step 6: Checking of M_u

D_f /d = 100/625 = 0.16 < 0.2

D_f /x_u = 100/215.7 = 0.33 < 0.43. Hence, it is a problem of case (iii a) and M_u can be obtained from Eq. 5.14 of Lesson 10.

So, $M_u = 0.36(x_u /d) \{1 - 0.42(x_u /d)\} f_{ck}\, b_f\, d^2 + 0.45\, f_{ck}\, (b_f - b_w)\, (D_f)\, (d - D_f /2)$

= 0.36 (295.703/625) {1 - 0.42 (295.703/625)} (20) (350) (625) (625)

+ 0.45 (20) (2600) (100) (625 - 50)

= 1,718.68 kNm > $(M_u)_{\text{design}}$ (= 1,711.29 kNm)

Hence, the design is o.k.

Ex.7: Determine the tensile reinforcement A_{st} of the flanged beam of Ex.5 (Fig. 5.12.1) when the imposed loads = 12 kN/m^2. All other parameters are the same as those of Ex.5: D_f = 100 mm, D = 750 mm, b_w = 350 mm, spacing of beams = 4000 mm c/c, effective span = 12 m, simply supported, cover = 90 mm and d = 660 mm. Use Fe 415 and M 20.

Solution:

Step 1: Computation of factored bending moment M_u

Dead loads of the slab (see Ex.5) = 2.5 kN/m^2

Imposed loads = 12.0 kN/m^2

Total loads = 14.5 kN/m^2

Loads/m = 14.5 (4) = 58.0 kN/m

Dead loads of beam = 0.65 (0.35) (25) = 5.6875 kN/m

Total loads = 63.6875 kN/m

Factored M_u = (1.5) (63.6875) (12) (12)/8 = 1,719.5625 kNm.

Step 2: Determination of $M_{u,lim}$

From Ex.5, the $M_{u,lim}$ of this beam = 1,835.43 kNm. Hence, this beam shall be designed as under-reinforced.

Step 3: Determination of x_u

Assuming x_u to be in the flange, we have from Eq. 3.24 of Lesson 5 and considering $b = b_f$,

$$M_u = 0.36x_u \{1 - 0.42(x_u/d)\} f_{ck}\, b_f\, d$$

$$1719.5625 (10^6) = 0.36\, x_u \{1 - 0.42\, (x_u/660)\} (20)\, (2950)\, (550)$$

Solving, we get x_u = 134.1 > 100 mm

So, let us assume that the neutral axis is in the web and $D_f/x_u < 0.43$, from Eq. 5.14 of Lesson 10 (case iii a of sec. 5.10.4.3), we have:

$$M_u = 0.36(x_u/d) \{1 - 0.42(x_u/d)\} f_{ck}\, b_w\, d^2 + 0.45\, f_{ck}\, (b_f - b_w)\, (D_f)\, (d - D_f/2)$$

$$= 0.36\, x_u \{1 - 0.42\, (x_u/660)\} (20)\, (350)\, (660)$$

$$+ 0.45\, (20)\, (2600)\, (100)\, (660 - 50)$$

Substituting the value of M_u = 1,719.5625 kNm in the above equation and simplifying,

$$x_u^2 - 1571.43\, x_u + 276042 = 0$$

Solving, we have x_u = 201.5 mm

$$D_f/x_u = 100/201.5 = 0.496 > 0.43.$$

So, we have to use Eq. 5.15 and 5.18 of Lesson 10 for y_f and M_u (case iii b of sec. 5.10.4.3). Thus, we have:

$$M_u = 0.36\, x_u \{1 - 0.42(\, x_u/d)\} f_{ck}\, b_w\, d + 0.45\, f_{ck}\, (b_f - b_w)\, y_f\, (d - y_f/2)$$

where, $y_f = (0.15\, x_u + 0.65\, D_f)$

So, $M_u = 0.36\, x_u\, \{1 - 0.42\, (x_u/660)\} (20)\, (350)\, (660)$

$+ 0.45 (20) (2600) (0.15 x_u + 65) (660 - 0.075 x_u - 32.5)$

or $\quad 1719.5625 (10^6) = 3.75165 (10^6) x_u - 795.15 x_u^2 + 954.4275 (10^6)$

Solving, we get $x_u = 213.63$ mm.

$D_f / x_u = 100/213.63 = 0.468 > 0.43$.
Hence, o.k.

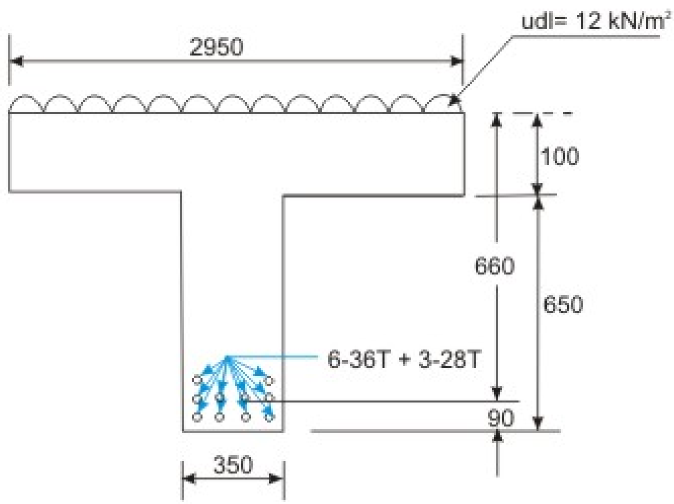

Fig. 5.12.3: Example 7

Step 4: Determination of A_{st}

Equating $C = T$ from Eqs. 5.16 and 5.17 of Lesson 10 (case iii b of sec. 5.10.4.3), we have:

$$0.87 f_y A_{st} = 0.36 f_{ck} b_w x_u + 0.45 f_{ck} (b_f - b_w) y_f$$

where, $\quad y_f = 0.15 x_u + 0.65 D_f$

Here, using $x_u = 213.63$ mm, $D_f = 100$ mm, we get

$$y_f = 0.15 (213.63) + 0.65 (100) = 97.04 \text{ mm}$$

So, $\quad A_{st} = \dfrac{0.36 (20) (350) (213.63) + 0.45 (20) (2600) (97.04)}{0.87 (415)} = 7{,}780.32 \text{ mm}^2$

Minimum $A_{st} = (0.85/f_y) (b_w) (d) = 0.85 (350) (660)/(415) = 473.13 \text{ mm}^2$

Maximum $A_{st} = 0.04 b_w D = 0.04 (350) (750) = 10{,}500 \text{ mm}^2$

Hence, $\quad A_{st} = 7{,}780.32 \text{ mm}^2$ is o.k.

Provide 6-36 T + 3-28 T (6107 + 1847 = 7,954 mm^2). Please refer to Fig. 5.12.3.

Step 5: Checking of x_u and M_u using A_{st} = 7,954 mm^2

From $T = C$ (Eqs. 5.16 and 5.17 of Lesson 10), we have

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, (b_f - b_w)\, y_f$$

where, $y_f = 0.15\, x_u + 0.65\, D_f$

or $0.87\,(415)\,(7954) = 0.36\,(20)\,(350)\,x_u + 0.45\,(20)\,(2600)\,(0.15\,x_u + 0.65\,D_f)$

or $x_u = 224.01$ mm

$D_f / x_u = 100/224.01 = 0.446 > 0.43$. Accordingly, employing Eq. 5.18 of Lesson 10 (case iii b of sec. 5.10.4.3), we have:

So, $M_u = 0.36\, x_u\, \{1 - 0.42(x_u/d)\}\, f_{ck}\, b_w\, d + 0.45\, f_{ck}\, (b_f - b_w)\, y_f\, (d - y_f/2)$

$= 0.36\,(224.01)\{1 - 0.42\,(224.01/660)\}\,(20)\,(350)\,(660)$

$+ 0.45\,(20)\,(2600)\,\{(0.15)\,224.01 + 65\}\,\{(660) - 0.15\,(112) - 32.5\}$

$= 1{,}779.439$ kNm $> 1{,}719.5625$ kNm

Hence, o.k.

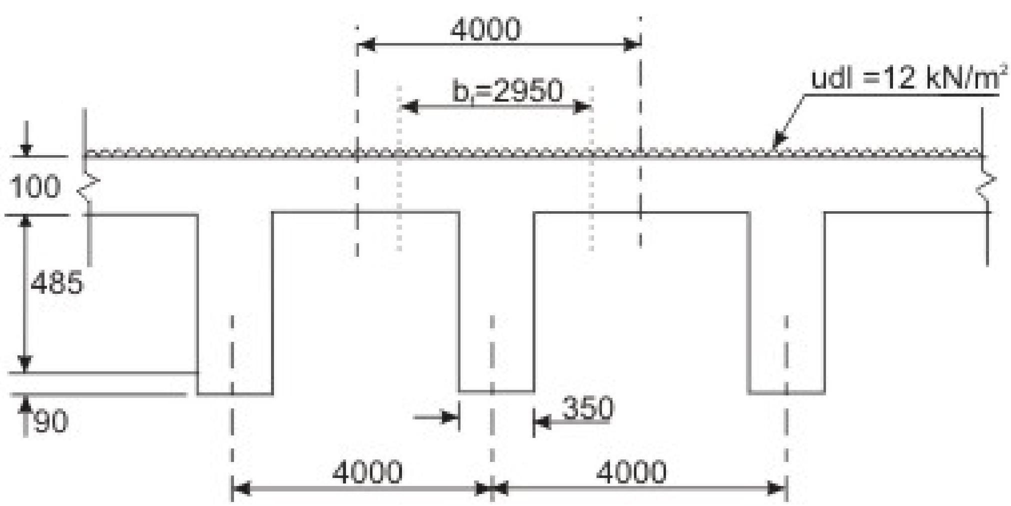

Fig. 5.12.4: Example 8

Ex.8: Design the flanged beam of Fig. 5.12.4, given in following: D_f = 100 mm, D = 675 mm, b_w = 350 mm, spacing of beams = 4000 mm c/c, effective span = 12 m simply supported, cover = 90 mm, d = 585 mm and imposed loads = 12 kN/m^2. Use Fe 415 and M 20.

Step 1: Computation of factored bending moment, M_u

Weight of slab/m^2 = (0.1) (25) = 2.5 kN/m^2

Imposed loads = 12.0 kN/m^2

Total loads = 14.5 kN/m^2

Total weight of slab + imposed loads/m = 14.5 (4) = 58 kN/m

Dead loads of beam = 0.575 (0.35) (25) = 5.032 kN/m

Total loads = 63.032 kN/m

Factored M_u = (1.5) (63.032) (12) (12)/8 = 1,701.87 kNm

Step 2: Determination of $M_{u,lim}$

Assuming the neutral axis to be in the web, $D_f /x_u <$ 0.43 and D_f /d = 100 / 585 = 0.17 < 0.2, we consider the case (ii a) of sec. 5.10.4.2 of Lesson 10 to get the following:

$$M_{u,lim} = 0.36\,(x_{u,max}/d)\,\{1 - 0.42\,(x_{u,max}/d)\}\,f_{ck}\,b_w\,d^2$$

$$+ 0.45\,f_{ck}\,(b_f - b_w)\,D_f\,(d - D_f/2)$$

$$= 0.36(0.48)\,\{1 - 0.42\,(0.48)\}\,(20)\,(350)\,(585)\,(585)$$

$$+ 0.45(20)\,(2600)\,(100)\,(585 - 50) = 1{,}582.4 \text{ kNm}$$

Since, factored $M_u > M_{u,lim}$, the beam is designed as doubly reinforced.

$$M_{u2} = M_u - M_{u,lim} = 1701.87 - 1582.4 = 119.47 \text{ kNm}$$

Step 3: Determination of area of steel

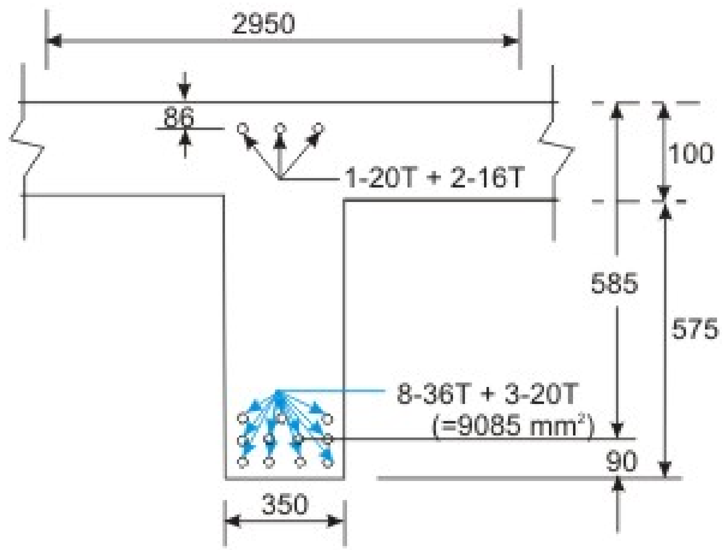

Fig. 5.12.5: Example 8

$A_{st,lim}$ is obtained equating $T = C$ (Eqs. 5.5 and 6 of Lesson 10).

$0.87\, f_y\, (A_{st,lim}) = 0.36\, b_w\, (x_{u,max}/d)\, d\, f_{ck} + 0.45\, f_{ck}\, (b_f - b_w)\, D_f$

or $$A_{st,lim} = \frac{0.36\, b_w\, (x_{u,\max}/d)\, d\, f_{ck} + 0.45\, f_{ck}\, (b_f - b_w)\, D_f}{0.87\, f_y}$$

$$= \frac{0.36\,(350)\,(0.48)\,(585)\,(20) + 0.45\,(20)\,(2600)\,(100)}{0.87\,(415)} = 8{,}440.98$$

mm^2

$$A_{sc} = \frac{M_{u2}}{(f_{sc} - f_{cc})\,(d - d')}$$ (Eq. 4.4 of Lesson 8).

where f_{sc} = 353 N/mm^2 for d'/d = 0.1

f_{cc} = 0.446 f_{ck} = 0.446 (20) = 8.92 N/mm^2

M_{u2} = 119.47 (10^6) Nmm

d' = 58.5 mm

d = 585 mm

Using the above values in the expression of A_{sc} (Eq. 4.4 of Lesson 8), we get

$$A_{sc} = 659.63 \text{ mm}^2$$

$$A_{st2} = \frac{A_{sc}\,(f_{sc} - f_{cc})}{0.87\, f_y}$$ (Eqs. 4.4 and 4.5 of Lesson 8).

Substituting the values of A_{sc}, f_{sc}, f_{cc} and f_y we get

$$A_{st2} = 628.48 \text{ mm}^2$$

Total $A_{st} = A_{st,lim} + A_{st2} = 8{,}440.98 + 628.48 = 9{,}069.46 \text{ mm}^2$

Maximum $A_{st} = 0.04\, b_w\, D = 0.04\,(350)\,(675) = 9{,}450 \text{ mm}^2$

and minimum $A_{st} = (0.85/f_y)\, b_w\, d = (0.85/415)\,(350)\,(585) = 419.37 \text{ mm}^2$

Hence, $A_{st} = 9{,}069.46 \text{ mm}^2$ is o.k.

Provide 8-36 T + 3-20 T = 8143 + 942 = 9,085 mm^2 for A_{st} and 1-20 + 2-16 = 314 + 402 = 716 mm^2 for A_{sc} (Fig. 5.12.5).

Step 4: To check for x_u and M_u (Fig. 5.12.5)

Assuming x_u in the web and $D_f/x_u < 0.43$ and using $T = C$ (case ii a of sec. 5.10.4.2 of Lesson 10 with additional compression force due to compression steel), we have:

$$0.87\, f_y\, A_{st} = 0.36\, b_w\, x_u\, f_{ck} + 0.45\,(b_f - b_w)\, f_{ck}\, D_f + A_{sc}\,(f_{sc} - f_{cc})$$

or $$0.87\,(415)\,(9085) = 0.36\,(350)\, x_u\,(20) + 0.45\,(2600)\,(20)\,(100)$$

$$+ 716\,\{353 - 0.45\,(20)\}$$

This gives $x_u = 275.33$ mm.

$$x_{u,max} = 0.48\,(d) = 0.48\,(585) = 280.8 \text{ mm.}$$

So, $x_u < x_{u,max}$, $D_f/x_u = 100/275.33 = 0.363 < 0.43$

and $D_f/d = 100/585 = 0.17 < 0.2$.

The assumptions, therefore, are correct. So, M_u can be obtained from Eq. 5.14 of sec. 5.10.4.3 of Lesson 10 with additional moment due to compression steel, as given below:

So, $M_u = 0.36\, b_w\, x_u\, f_{ck}\, (d - 0.42\, x_u) + 0.45\, (b_f - b_w)\, f_{ck}\, D_f\, (d - D_f/2)$

$+ A_{sc}\, (f_{sc} - f_{cc})\, (d - d')$

$= 0.36\, (350)\, (275.33)\, (20)\, \{585 - 0.42\, (275.33)\}$

$+ 0.45\, (2600)\, (20)\, (100)\, (585 - 50) + 716\, (344)\, (585 - 58.5)$

$= 325.66 + 1251.9 + 129.67 = 1{,}707.23$ kNm

Factored moment = 1,701.87 kNm < 1,707.23 kNm. Hence, o.k.

5.12.4 Practice Questions and Problems with Answers

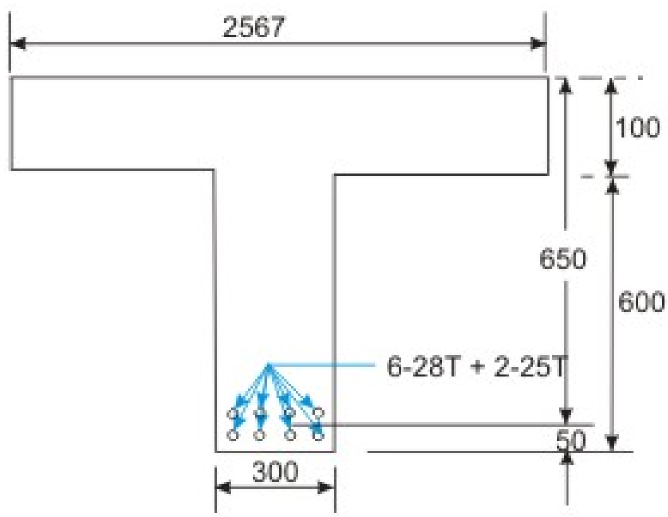

Fig. 5.12.6: Q. 1

Q.1: Determine the steel reinforcement of a simply supported flanged beam (Fig. 5.12.6) of D_f = 100 mm, D = 700 mm, cover = 50 mm, d = 650 mm, b_w = 300 mm, spacing of the beams = 4,000 mm c/c, effective span = 10 m and imposed loads = 10 kN/m^2. Use M 20 and Fe 415.

A.1: Solution:

Step 1: Computation of $(M_u)_{factored}$

Weight of slab = (0.1) (25) = 2.5 kN/m^2
Imposed loads = 10.0 kN/m^2

12.5 kN/m^2

Total loads per m = (12.5) (4) = 50 kN/m

Dead loads of beam = (0.3) (0.6) (25) = 4.50 kN/m

Total loads = 54.50 kN/m

Factored M_u = (1.5) (54.50) (10) (10)/8 = 1,021.87 kNm

Step 2: Determination of $M_{u,lim}$

Effective width of the flange $b_f = l_o/6 + b_w + 6\ D_f$ = (10,000/6) + 300 + 600 = 2,567 mm.

$x_{u,max}$ = 0.48 d = 0.48 (650) = 312 mm

Hence, the balanced neutral axis is in the web of the beam.

D_f/d = 100/650 = 0.154 < 0.2

D_f/x_u = 100/312 = 0.32 < 0.43

So, the full depth of flange is having a stress of 0.446 f_{ck}. From Eq. 5.7 of Lesson 10 (case ii a of sec. 5.10.4.2), we have,

$$M_{u,lim} = 0.36\,(x_{u,max}/d)\,\{1 - 0.42\,(x_{u,max}/d)\}\, f_{ck}\, b_w\, d^2$$

$$+ 0.45\, f_{ck}\,(b_f - b_w)\, D_f\,(d - D_f/2)$$

$$= 0.36(0.48)\,\{1 - 0.42(0.48)\}\,(20)\,(300)\,(650)\,(650)$$

$$+\ 0.45(20)\,(2267)\,(100)\,(650 - 50)$$

$$= 1573.92 \text{ kNm} > M_u\ (= 1021.87 \text{ kNm})$$

So, the beam will be under-reinforced one.

Step 3: Determination of x_u

Assuming x_u is in the flange, we have from Eq. 3.24 of Lesson 5 (rectangular beam when $b = b_f$).

$$M_u = 0.36\,(x_u/d)\,\{1 - 0.42\,(x_u/d)\}\, f_{ck}\, b_f\, d^2$$

$$= 0.36\, x_u\,\{1 - 0.42\,(x_u/d)\}\, f_{ck}\, b_f\, d$$

$$1021.87\,(10^6) = 0.36\, x_u\,\{1 - 0.42(x_u/650)\}\,(20)\,(2567)\,(650)$$

x_u = 89.55 mm^2 < 100 mm (Hence, the neutral axis is in the flange.)

Step 4: Determination of A_{st}

Equating $C = T$, we have from Eq. 3.17 of Lesson 5:

$$\frac{x_u}{d} = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck} b_f\, d}$$

or $$A_{st} = \frac{0.36\, f_{ck}\, b_f\, x_u}{0.87\, f_y} = \frac{0.36\,(20)\,(2567)\,(89.55)}{0.87\,(415)} = 4{,}584.12\ \text{mm}^2$$

Minimum $A_{st} = (0.85/f_y)\,(b_w)\,d = \dfrac{0.85\,(300)\,(650)}{415} = 399.39\ \text{mm}^2$

Maximum A_{st} = 0.04 $b_w D$ = (0.04) (300) (700) = 8,400 mm^2

So, A_{st} = 4,584.12 mm^2 is o.k.

Provide 6-28 T + 2-25 T = 3694 + 981 = 4,675 mm^2 (Fig. 5.12.6).

5.12.5 References

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13. Properties of Concrete, 4th Edition, 1st Indian reprint, by A.M.Neville, Longman, 2000.
14. Reinforced Concrete Designer's Handbook, 10th Edition, by C.E.Reynolds and J.C.Steedman, E & FN SPON, London, 1997.
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16. Design Aids for Reinforced Concrete to IS: 456 – 1978, BIS, New Delhi.

5.12.6 Test 12 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

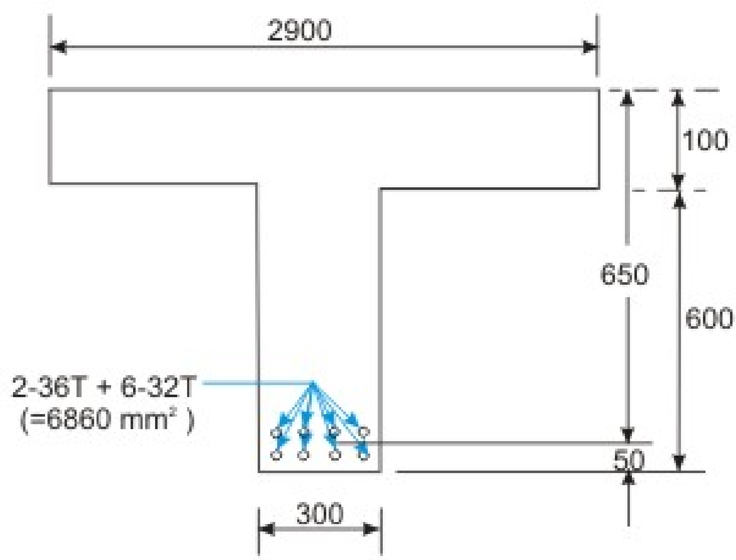

Fig. 5.12.7: TQ. 1

TQ.1: Determine the steel reinforcement A_{st} of the simply supported flanged beam of Q.1 (Fig. 5.12.6) having D_f = 100 mm, D = 700 mm, cover = 50 mm, d = 650 mm, b_w = 300 mm, spacing of the beams = 4,000 mm c/c, effective span = 12 m and imposed loads = 10 kN/m^2. Use M 20 and Fe 415.

A.TQ.1: Solution:

Step 1: Computation of (M_u)$_{factored}$

Total loads from Q.1 of sec. 5.12.4 = 54.50 kN/m

Factored M_u = (1.5) (54.50) (12) (12)/8 = 1,471.5 kNm

Step 2: Determination of $M_{u,lim}$

Effective width of flange = $l_o/6 + b_w + 6\, D_f$

= (12000/6) + 300 + 600 = 2,900 mm (Fig. 5.12.7)

$x_{u,max}$ = 0.48 d = 0.48 (650) = 312 mm

Hence, the balanced neutral axis is in the web.

D_f/d = 100/650 = 0.154 < 0.2

D_f/x_u = 100/312 = 0.32 < 0.43

So, the full depth of flange is having constant stress of 0.446 f_{ck}. From Eq. 5.7 of Lesson 10 (case ii a of sec. 5.10.4.2), we have

$$M_{u,lim} = 0.36\,(x_{u,max}/d)\,\{1 - 0.42\,(x_{u,max}/d)\}\, f_{ck}\, b_w\, d^2$$

$$+\, 0.45\, f_{ck}\,(b_f - b_w)\, D_f\,(d - D_f/2)$$

$$= 0.36(0.48)\,\{1 - 0.42(0.48)\}\,(20)\,(300)\,(650)\,(650)$$

$$+\ 0.45(20)\,(2600)\,(100)\,(650 - 50) = 1{,}753.74 \text{ kNm} > 1{,}471.5 \text{ kNm}$$

So, the beam will be under-reinforced.

Step 3: Determination of x_u

Assuming x_u to be in the flange, we have from Eq. 3.24 of Lesson 5 (singly reinforced rectangular beam when $b = b_f$):

$$M_u = 0.36\, x_u\,\{1 - 0.42\,(x_u/d)\}\, f_{ck}\, b_f\, d$$

or $$1471.5\,(10^6) = 0.36\,(x_u)\,\{1 - 0.42\,(x_u/650)\}\,(20)\,(2900)\,(650)$$

or $$x_u^2 - 1547.49\, x_u + 167.81\,(10^3) = 0$$

Solving, we have x_u = 117.34 mm > 100 mm

So, neutral axis is in the web.

Assuming $D_f / x_u < 0.43$, we have from Eq. 5.14 of Lesson 10 (case iii a of sec. 5.10.4.3),

$$M_u = 0.36\, x_u \{1 - 0.42\, (x_u / d)\}\, f_{ck}\, b_w\, d + 0.45\, f_{ck}\, (b_f - b_w)\, D_f\, (d - D_f / 2)$$

$$= 0.36\, x_u \{1 - 0.42\, (x_u / 650)\}\, (20)\, (300)\, (650)$$

$$+ 0.45(20)\, (2600)\, (100)\, (650 - 50)$$

or $\quad x_u^2 - 1547.62\, x_u + 74404.7 = 0$

Solving, we have $x_u = 49.67 < 100$ mm

However, in the above when it is assumed that the neutral axis is in the flange x_u is found to be 117.34 mm and in the second trial when x_u is assumed in the web x_u is seen to be 49.67 mm. This indicates that the full depth of the flange will not have the strain of 0.002, neutral axis is in the web and D_f / x_u is more than 0.43. So, we have to use Eq. 5.18 of Lesson 10, with the introduction of y_f from Eq. 5.15 of Lesson 10.

Assuming $D_f / x_u > 0.43$, from Eqs. 5.15 and 5.18 of Lesson 10 (case iii b of sec. 5.10.4.3), we have:

$$M_u = 0.36\, x_u \{1 - 0.42\, (x_u / d)\}\, f_{ck}\, b_w\, d + 0.45\, f_{ck}\, (b_f - b_w)\, y_f\, (d - y_f / 2)$$

where, $\quad y_f = (0.15\, x_u + 0.65\, D_f)$

So, $\quad M_u = 0.36\, x_u \{1 - 0.42\, (x_u / 650)\}\, (20)\, (300)\, (650)$

$$+ 0.45(20)\, (2600)\, (0.15\, x_u + 0.65)\, (650 - 0.075\, x_u - 0.325\, x_u)$$

or, $\quad 1471.5\, (10^6) = -\,1170.45\, x_u^2 + 3.45735\, x_u + 939.2175\, (10^6)$

Solving, we get $x_u = 162.9454$ mm. This shows that the assumption of $D_f / x_u > 0.43$ is correct as $D_f / x_u = 100 / 162.9454 = 0.614$.

Step 4: Determination of A_{st}

Equating $C = T$ from Eqs. 5.16 and 5.17 of Lesson 10 (case iii b of sec. 5.10.4.3), we have

$$0.87\, f_y\, A_{st} = 0.36\, f_{ck}\, b_w\, x_u + 0.45\, f_{ck}\, (b_f - b_w)\, y_f$$

or $$A_{st} = \frac{0.36\,(20)\,(30)\,(162.9454) + 0.45\,(20)\,(2600)\,\{0.15\,(162.9454) + 65\}}{0.87\,(415)}$$

$$= 974.829 + 5{,}796.81 = 6{,}771.639 \text{ mm}^2$$

Minimum $A_{st} = (0.85/f_y)\,(b_w)\,(d) = 0.85\,(300)\,(650)/415 = 399.39 \text{ mm}^2$

Maximum $A_{st} = 0.04\,(b_w)\,(D) = 0.04\,(300)\,(700) = 8{,}400 \text{ mm}^2$

So, $A_{st} = 6{,}771.639$ is o.k.

Provide 2-36 T + 6-32 T = 2035 + 4825 = 6,860 mm^2 > 6,771.639 mm^2 (Fig. 5.12.7).

5.12.7 Summary of this Lesson

This lesson explains the steps involved in solving the design type of numerical problems. Further, several examples of design type of numerical problems are illustrated explaining the steps of their solutions. Solutions of practice problems and test problems will give the readers confidence in applying the theory explained in Lesson 10 in solving the numerical problems.