

Module

11

Foundations - Theory  
and Design

Lesson

29

Design of Foundations

## Instructional Objectives:

At the end of this lesson, the student should be able to:

- understand and apply the design considerations to satisfy the major and other requirements of the design of foundations,
- design the plain concrete footings, isolated footings for square and rectangular columns subjected to axial loads with or without the moments, wall footings and combined footings, as per the stipulations of IS code.

### 11.29.1 Introduction

The two major and some other requirements of foundation structures are explained in Lesson 28. Different types of shallow and deep foundations are illustrated in that lesson. The design considerations and different codal provisions of foundation structures are also explained. However, designs of all types of foundations are beyond the scope of this course. Only shallow footings are taken up for the design in this lesson. Several numerical problems are illustrated applying the theoretical considerations discussed in Lesson 28. Problems are solved explaining the different steps of the design.

### 11.29.2 Numerical Problems

#### Problem 1:

Design a plain concrete footing for a column of 400 mm x 400 mm carrying an axial load of 400 kN under service loads. Assume safe bearing capacity of soil as 300 kN/m<sup>2</sup> at a depth of 1 m below the ground level. Use M 20 and Fe 415 for the design.

#### Solution 1:

Plain concrete footing is given in secs.11.28.2(A)1 and 11.28.5(b).

#### Step 1: Transfer of axial force at the base of column

It is essential that the total factored loads must be transferred at the base of column without any reinforcement. For that the bearing resistance should be greater than the total factored load  $P_u$ .

Here, the factored load  $P_u = 400(1.5) = 600$  kN.

The bearing stress, as per cl.34.4 of IS 456 and given in Eqs.11.7 and 8 of sec.11.28.5(g) of Lesson 28, is

$$\sigma_{br} = 0.45 f_{ck} (A_1/A_2)^{1/2} \quad (11.7)$$

with a condition that

$$(A_1/A_2)^{1/2} \leq 2.0 \quad (11.8)$$

Since the bearing stress  $\sigma_{br}$  at the column-footing interface will be governed by the column face, we have  $A_1 = A_2 = 400(400) = 160000 \text{ mm}^2$ . Using  $A_1 = A_2$ , in Eq.11.7, we have

$$P_{br} = \text{Bearing force} = 0.45 f_{ck} A_1 = 0.45(20)(160000)(10^{-3}) = 1440 \text{ kN} > P_u (= 600 \text{ kN}).$$

Thus, the full transfer of load  $P_u$  is possible without any reinforcement.

### Step 2: Size of the footing

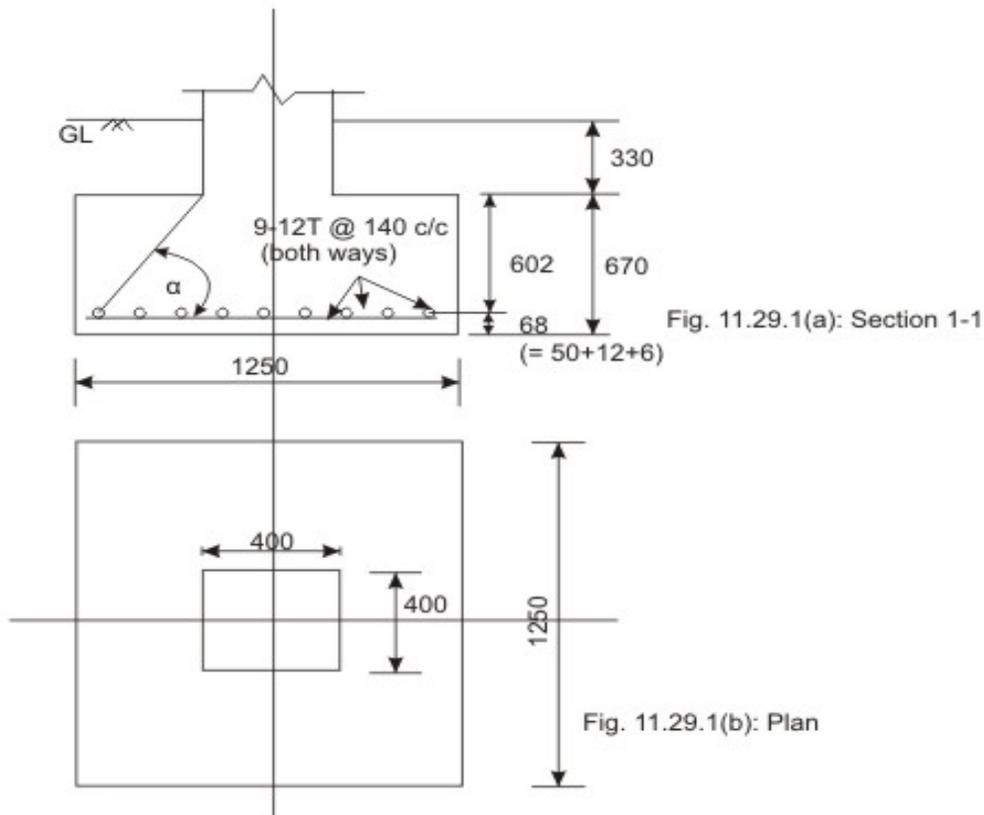


Fig. 11.29.1: Problem 1

Let us assume the weight of footing and back fill soil as 15 per cent of  $P_U$ . Then, the base area required =  $400(1.15)/300 = 1.533 \text{ m}^2$ . Provide  $1250 \times 1250 \text{ mm}$  ( $= 1.5625 \text{ m}^2$ ) as shown in Fig.11.29.1. The bearing pressure  $q_a = 400(1.15)/(1.25)(1.25) = 294.4 \text{ kN/m}^2$ .

### Step 3: Thickness of footing

The thickness of the footing  $h$  is governed by Eq.11.3 of sec.11.28.5 of Lesson 28. From Eq.11.3, we have

$$\begin{aligned} \tan \alpha &\leq 0.9\{(100q_a/f_{ck}) + 1\}^{1/2} \quad \dots (11.3) \\ &\leq 0.9\{[100(0.2944)/20] + 1\}^{1/2} \\ &\leq 1.415 \end{aligned}$$

We have from Fig.11.29.1a:

$$h = \{(1250 - 400)/2\}(\tan \alpha) = 601.375 \text{ mm}$$

Provide  $1250 \times 1250 \times 670 \text{ mm}$  block of plain concrete.

### Step 4: Minimum reinforcement

The plain concrete block  $1250 \times 1250 \times 670$  shall be provided with the minimum reinforcement 0.12 per cent for temperature, shrinkage and tie action.

$$\text{Minimum } A_{st} = 0.0012(1250)(670) = 1005.0 \text{ mm}^2.$$

Provide 9 bars of 12 mm diameter ( $= 1018 \text{ mm}^2$ ) both ways as shown in Fig.11.29.1b. The spacing of bars =  $(1250 - 50 - 12)/8 = 148.5 \text{ mm c/c}$ . Provide the bars @  $140 \text{ mm c/c}$ .

### Step 5: Check for the gross base pressure

Assuming unit weights of concrete and soil as  $24 \text{ kN/m}^3$  and  $20 \text{ kN/m}^3$

Service load =  $400.00 \text{ kN}$

Weight of footing =  $(0.67)(1.25)(1.25)(24) = 25.125 \text{ kN}$

Weight of soil =  $(0.33)(1.25)(1.25)(20) = 10.3125 \text{ kN}$

Total =  $435.4375 \text{ kN}$

$$q_a = 435.4375/(1.25)(1.25) = 278.68 \text{ kN/m}^2 < 300 \text{ kN/m}^2$$

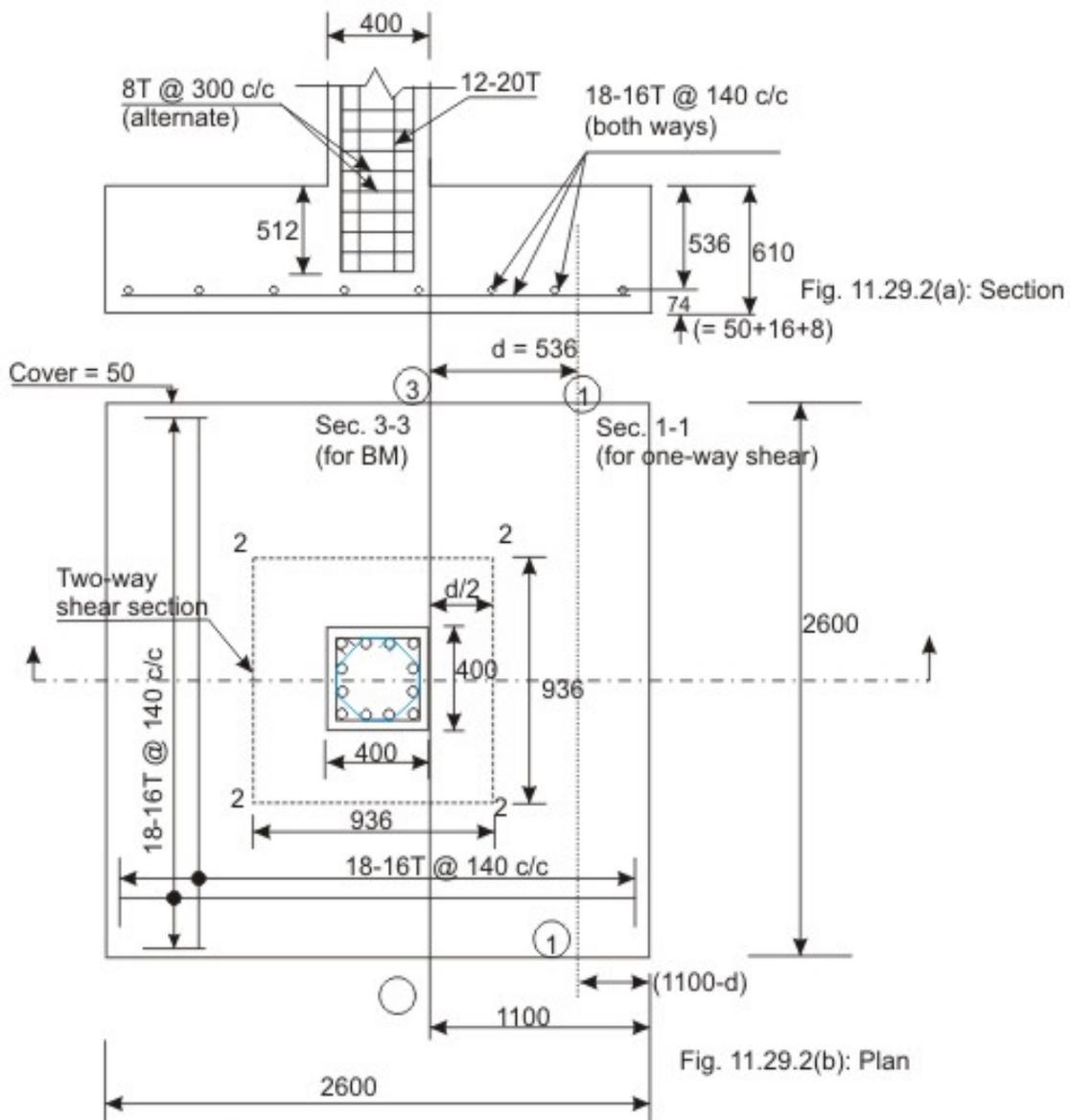
Hence, o.k.

**Problem 2:**

Design an isolated footing for a square column, 400 mm x 400 mm with 12-20 mm diameter longitudinal bars carrying service loads of 1500 kN with M 20 and Fe 415. The safe bearing capacity of soil is 250 kN/m<sup>2</sup> at a depth of 1 m below the ground level. Use M 20 and Fe 415.

**Solution 2:**

**Step 1: Size of the footing**



**Fig. 11.29.2: Problem 2**

Given  $P = 1500$  kN,  $q_c = 250$  kN/m<sup>2</sup> at a depth of 1 m below the ground level. Assuming the weight of the footing and backfill as 10 per cent of the load, the base area required =  $1500(1.1)/250 = 6.6$  m<sup>2</sup>. Provide 2.6 m x 2.6 m, area = 6.76 m<sup>2</sup> (Fig.11.29.2b).

### Step 2: Thickness of footing slab based on one-way shear

Factored soil pressure =  $1500(1.5)/(2.6)(2.6) = 0.3328$  N/mm<sup>2</sup>, say, 0.333 N/mm<sup>2</sup>.

Assuming  $p = 0.25\%$  in the footing slab, for M 20 concrete  $\tau_c = 0.36$  N/mm<sup>2</sup> (Table 19 of IS 456).  $V_u = 0.36(2600)d$  and  $V_u$  (actual) =  $0.333(2600)(1100 - d)$ . From the condition that  $V_u$  should be more than or equal to the actual  $V_u$ , we have (see Fig.11.29.2a, sec.11):

$$0.36(2600)d \geq 0.333(2600)(1100 - d)$$

So,  $d \geq 528.57$  mm.

Provide  $d = 536$  mm. The total depth becomes  $536 + 50 + 16 + 8$  (with 50 mm cover and diameter of reinforcing bars = 16 mm) = 610 mm.

### Step 3: Checking for two-way shear

The critical section is at a distance of  $d/2$  from the periphery of the column. The factored shear force (Fig.11.29.2b, sec. 2222) =  $0.333\{(2600)^2 - (400 + d)^2\}(10)^{-3} = 1959.34$  kN. Shear resistance is calculated with the shear strength =  $k_s \tau_c = k_s(0.25)(f_{ck})^{1/2}$ ; where  $k_s = 0.5 + \beta_c$  (cl. 31.6.3 of IS 456). Here  $\beta_c = 1.0$ ,  $k_s = 1.5 \not> 1$ ; so  $k_s = 1.0$ . This gives shear strength of concrete =  $0.25(f_{ck})^{1/2} = 1.118$  N/mm<sup>2</sup>. So, the shear resistance =  $(1.118) 4 (936)(536) = 2243.58$  kN > 1959.34 kN. Hence, ok.

Thus, the depth of the footing is governed by one-way shear.

### Step 4: Gross bearing capacity

Assuming unit weights of concrete and soil as 24 kN/m<sup>3</sup> and 20 kN/m<sup>3</sup>, respectively:

Given, the service load = 1500 kN

Weight of the footing =  $2.6(2.6)(0.61)(24) = 98.967$  kN

Weight of soil =  $2.6(2.6)(1.0-0.61)(20) = 52.728$  kN (Assuming the depth of the footing as 1.0 m).

Total = 1635.2 kN

Gross bearing pressure =  $1635.2/(2.6)(2.6) = 241.893 \text{ kN/m}^2 < 250 \text{ kN/m}^2$ .  
Hence, ok.

### Step 5: Bending moment

The critical section (Fig.11.29.2b, sec.3.3), is at the face of the column.

$$M_u = 0.333(2600)(1100)(550) \text{ Nmm} = 523.809 \text{ kNm}$$

Moment of resistance of the footing =  $Rbd^2$  where  $R = 2.76$  (Table 3.3 of Lesson 5).

$$\text{Moment of resistance} = 2.76(2600)(536)(536) = 2061.636 \text{ kNm} > 523.809 \text{ kNm}.$$

Area of steel shall be determined from Eq.3.23 of Lesson 5, which is:

$$M_u = 0.87 f_y A_{st} d \{1 - (A_{st} f_y / f_{ck} bd)\} \quad \dots (3.23)$$

Substituting  $M_u = 523.809 \text{ kNm}$ ,  $f_y = 415 \text{ N/mm}^2$ ,  $f_{ck} = 20 \text{ N/mm}^2$ ,  $d = 536 \text{ mm}$ ,  $b = 2600 \text{ mm}$ , we have:

$$A_{st}^2 - 67161.44578 A_{st} + 181.7861758 (10^5) = 6$$

Solving, we get  $A_{st} = 2825.5805 \text{ mm}^2$ .

Alternatively, we can use Table 2 of SP-16 to get the  $A_{st}$  as explained below:

$M_u/bd^2 = 523.809(10^6)/(2600)(536)(536) = 0.7013 \text{ N/mm}^2$ . Table 2 of SP-16 gives  $\rho = 0.2034$ .

$$A_{st} = 0.2034(2600)(536)/100 = 2834.58 \text{ mm}^2.$$

This area is close to the other value =  $2825.5805 \text{ mm}^2$ .

However, one-way shear has been checked assuming  $\rho = 0.25\%$ . So, use  $\rho = 0.25\%$ . Accordingly,  $A_{st} = 0.0025(2600)(536) = 3484 \text{ mm}^2$ .

Provide 18 bars of 16 mm diameter (=  $3619 \text{ mm}^2$ ) both ways. The spacing of bars =  $\{2600 - 2(50) - 16\}/17 = 146.117 \text{ mm}$ . The spacing is 140 mm c/c (Fig.11.29.2).

The bending moment in the other direction is also the same as it is a square footing. The effective depth, however, is 16 mm more than 536 mm. But, the area of steel is not needed to be determined with  $d = 552 \text{ mm}$  as we are providing 0.25 per cent reinforcement based on one-way shear checking.

### Step 6: Development length

$L_d = f_s \phi / 4(\tau_{bd}) = 0.87(415)(16) / 4(1.6)(1.2) = 47(16) = 752 \text{ mm}$  (cl.26.2.1 of IS 456).

Length available =  $1100 - 50 = 1050 \text{ mm} > 752 \text{ mm}$ .

### Step 7: Transfer of force at the base of the column

$$P_u = 1500(1.5) = 2250 \text{ kN}$$

Compressive bearing resistance =  $0.45 f_{ck}(A_1/A_2)^{1/2}$ . For the column face  $A_1/A_2 = 1$  and for the other face  $A_1/A_2 > 2$  but should be taken as 2. In any case, the column face governs.

Force transferred to the base through column at the interface =  $0.45(20)(400)(400) = 1440 \text{ kN} < 2250 \text{ kN}$ . The balance force  $2250 - 1440 = 810 \text{ kN}$  has to be transferred by the longitudinal reinforcements, dowels or mechanical connectors. As it is convenient, we propose to continue the longitudinal bars (12-20 mm diameter) into the footing. The required development length of 12-20 mm diameter bars, assuming a stress level of  $0.87 f_y(810/2250) = 129.978 \text{ N/mm}^2$ , is  $129.978(20) / 4(1.6)(1.2)(1.25) = 270.8 \text{ mm}$ . Here  $\tau_{bd}$  for M 20 =  $1.2 \text{ N/mm}^2$ , increased factor of 1.6 is due to deformed bars and increased factor of 1.25 is for the compression.

Length available =  $610 - 50 - 16 - 16 - 16 = 512 \text{ mm} > 270.8 \text{ mm}$  (Fig.11.29.2a). Hence, o.k. The arrangement is shown in Fig.11.29.2c.

### Alternatively: Design of dowels

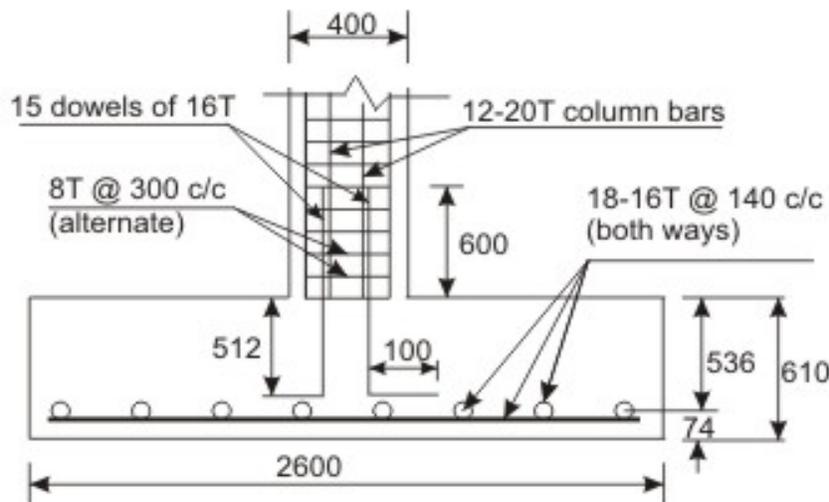


Fig. 11.29.2(c): Dowels

Fig. 11.29.2: Problem - 2

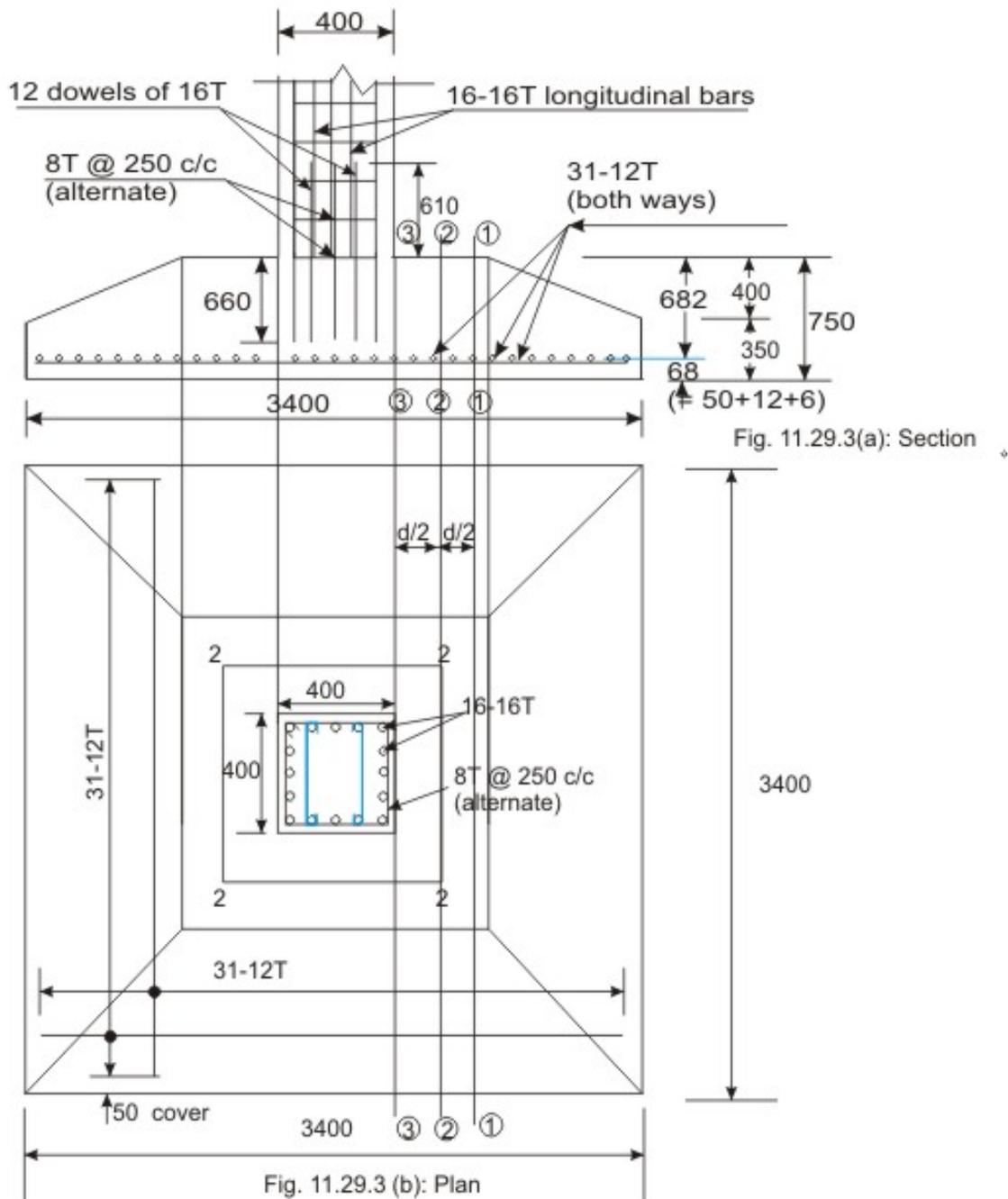
For the balance force 810 kN, the area of dowels =  $810000/0.67(415) = 2913.15 \text{ mm}^2$ . Minimum area =  $0.5(400)(400)/100 = 800 \text{ mm}^2 < 2913.15 \text{ mm}^2$  (cl.34.4.3 of IS 456). Therefore, number of 16 mm dowels =  $2913.15/201 = 15$ . The development length of 16 mm dowels in compression =  $0.87(415)(16)/4(1.6)(1.2)(1.25) = 601.76 \text{ mm}$ . Available vertical embedment length =  $610 - 50 - 16 - 16 - 16 = 512 \text{ mm}$ . So, the dowels will be extended by another 100 mm horizontally, as shown in Fig.11.29.2c.

**Problem 3:**

Design a sloped footing for a square column of 400 mm x 400 mm with 16 longitudinal bars of 16 mm diameter carrying a service load of 1400 kN. Use M 20 and Fe 415 both for column and footing slab. The safe bearing capacity of soil is  $150 \text{ kN/m}^2$ .

**Solution 3:**

**Step 1: Size of the footing**



**Fig. 11.29.3: Problem 3**

Given  $P = 1400$  kN and  $q_c = 150$  kN/m<sup>2</sup>. Assuming the weight of the footing and the back file as 10 per cent of the load, the required base area is:  $1400(1.1)/150 = 10.27$  m<sup>2</sup>. Provide 3400 x 3400 mm giving 11.56 m<sup>2</sup> (Fig.11.29.3b).

**Step 2: Thickness of footing slab based on one-way shear**

Factored bearing pressure =  $1400(1.5)/(3.4)(3.4) = 181.66 \text{ kN/m}^2 = 0.18166 \text{ N/mm}^2$ . Assuming 0.15 per cent reinforcement in the footing slab, Table 19 of IS 456 gives  $\tau_c$  for M 20 =  $0.28 \text{ N/mm}^2$ . From the condition that the one-way shear resistance  $\geq$  one-way shear force, we have at a distance  $d$  from the face of the column (sec.1-1 of Figs.11.29.3a and b).

$$0.28(3400)d \geq 0.18166(1500 - d)(3400)$$

or  $d \geq 590.24 \text{ mm}$ .

Provide total depth of footing as 670 mm, so that the effective depth =  $670 - 50 - 16 - 8 = 596 \text{ mm}$ . (The total depth is, however, increased to 750 mm in Step 7.)

### Step 3: Checking for two-way shear (See sec.11.28.5.d-2)

At the critical section 2222 (Figs.11.29.3a and b), the shear resistance =  $4(400 + 596)(596)(0.25)(f_{ck})^{1/2} = 2654.73 \text{ kN}$ .

The shear force =  $\{(3.4)(3.4) - (0.996)(0.996)\}0.18166 = 1919.78 \text{ kN} < 2654.73 \text{ kN}$ . Hence, o.k.

### Step 4: Gross bearing capacity

Assuming unit weights of concrete and soil as  $25 \text{ kN/m}^3$  and  $18 \text{ kN/m}^3$ , respectively, we have:

$$\text{Load on footing} = 1400.00 \text{ kN}$$

$$\text{Weight of footing} = (3.4)(3.4)(0.67)(25) = 193.63 \text{ kN}$$

Weight of soil =  $(3.4)(3.4)(1.25 - 0.67)(18) = 120.69 \text{ kN}$  (Assuming the depth of the footing as 1.25 m).

$$\text{Total} = 1714.32 \text{ kN}$$

Gross bearing capacity =  $1714.32 / (3.4)(3.4) = 148.30 \text{ kN/m}^2 < 150 \text{ kN/m}^2$ . Hence o.k.

### Step 5: Bending moment

We have to determine the area of steel in one direction as it is a square footing. So, we consider the lower effective depth which is 596 mm. The critical section is sec.33 (Figs.11.29.3a and b), where we have

$$M_u = 3400(1500)(0.18166)(1500)/2 = 694.8495 \text{ kNm}$$

$$M_u/bd^2 = 694.8495(10^6)/(3400)(596)(596) = 0.575 \text{ N/mm}^2$$

Table 2 of SP-16 gives,  $p = 0.165\%$ . Accordingly, area of steel =  $0.165(3400)(596)/100 = 3343.56 \text{ mm}^2$ . Provide 30 bars of 12 mm diameter (=  $3393 \text{ mm}^2$ ), both ways.

### Step 6: Development length

Development length of 12 mm diameter bars =  $0.87(415)(12)/4(1.6)(1.2) = 564.14 \text{ mm}$ . Hence, o.k.

### Step 7: Providing slope in the footing slab

Since the three critical sections (.i.e., of bending moment, two-way shear and one-way shear) are within a distance of 596 mm from the face of the column, the full depth of the footing slab is provided up to a distance of 700 mm from the face of the column. However, by providing slope the available section now is a truncated rectangle giving some less area for the one-way shear. Accordingly, the depth of the footing is increased from 670 mm to 750 mm. With a cover of 50 mm and bar diameter of 12 mm in both directions, the revised effective depth =  $750 - 50 - 12 - 6 = 682 \text{ mm}$ . Providing the minimum depth of 350 mm at the edge, as shown in Figs.11.29.3a and b, we check the one-way shear again, taking into account of the truncated rectangular cross-section at a distance of 682 mm from the face of the column.

$$\text{One-way shear force} = 0.18166(1500 - 682)(3400) = 505232.792 \text{ N}$$

$$\begin{aligned} \text{Area of truncated rectangle} &= 1800(682) + 1600(282) + 1600(682 - 282)/2 \\ &= 1998800 \text{ mm}^2 \end{aligned}$$

The shear stress =  $505232.792/1998800 = 0.2527 \text{ N/mm}^2 < 0.28 \text{ N/mm}^2$ . Hence, o.k.

### Step 8: Revised area of steel

The bending moment in step 5 is 694.8495 kNm at the face of the column. With  $d = 682 \text{ mm}$  now, we have

$$M_u/bd^2 = 694.8495(10^6)/(3400)(682)(682) = 0.4394 \text{ N/mm}^2$$

Table 2 of SP-16 gives,  $p$  is less than 0.15 per cent. Provide  $p = 0.15$  per cent due to the one-way shear. So,  $A_{st} = 0.15(3400)(682)/100 = 3478.2 \text{ mm}^2$ . Provide 31 bars of 12 mm ( $A_{st} = 3506 \text{ mm}^2$ ), both ways. Effectively, the number of bars has increased from 30 to 31 now.

### Step 9: Transfer of force at the base of the column

$P_u = 1400(1.5) = 2100$  kN. Compressive bearing resistance =  $0.45 f_{ck}(A_1/A_2)^{1/2} = 0.45(20)(1) = 9$  N/mm<sup>2</sup>.

Force transferred at the base through the column =  $9(400)(400)(10^{-3}) = 1440$  kN < 2100 kN.

Provide dowels for the excess  $(2100 - 1440) = 660$  kN. The area of dowels =  $660(10^3)/(0.67)(415) = 2373.67$  mm<sup>2</sup>. Minimum area of dowels =  $0.5(400)(400)/100 = 800$  mm<sup>2</sup>. Provide 12 dowels of 16 mm diameter (area = 2412 mm<sup>2</sup>).

The development length of 16 mm dowels =  $0.87(415)(16)/4(1.6)(1.2)(1.25) = 601.76$  mm.

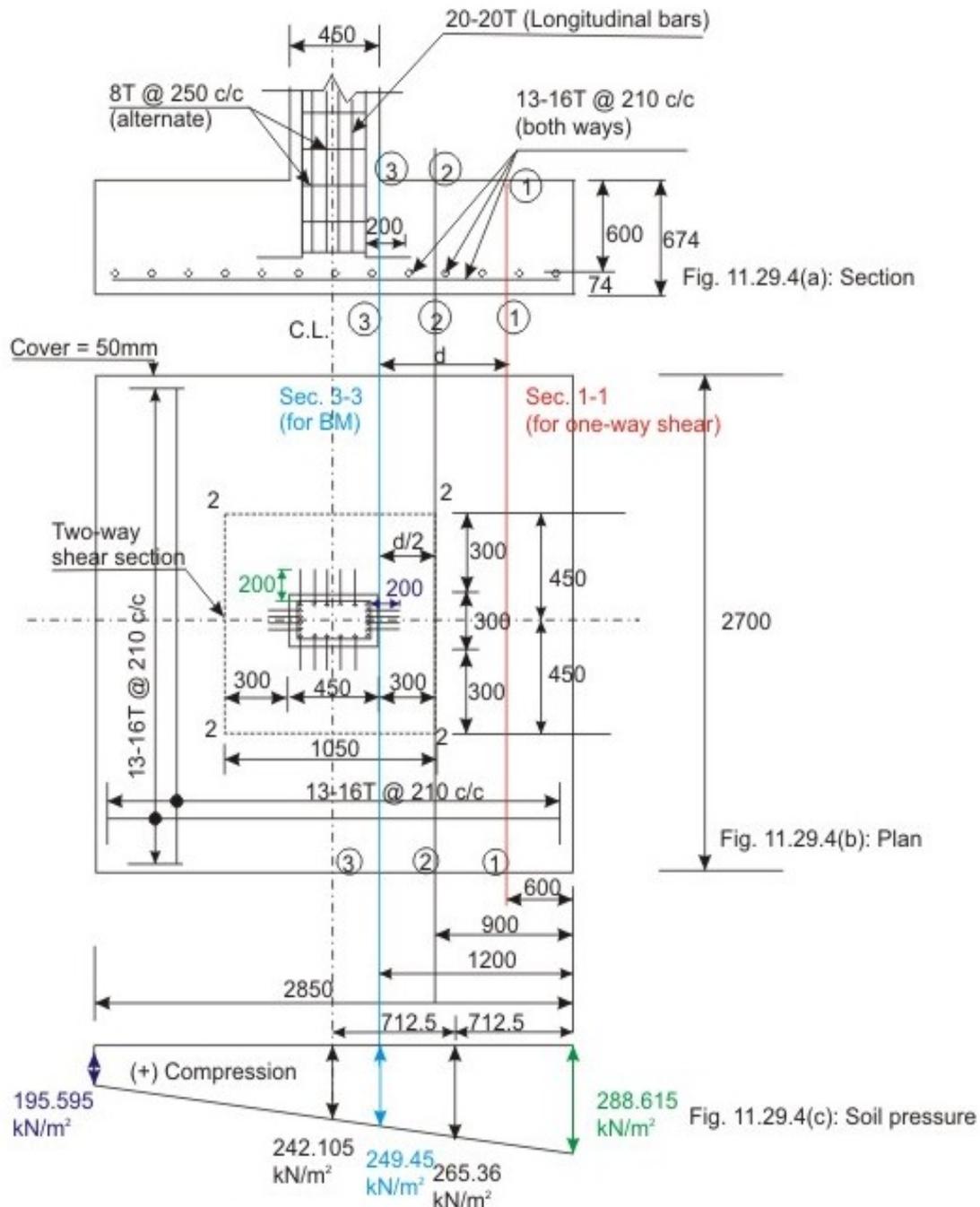
The vertical length available  $750 - 50 - 12 - 12 - 16 = 660$  mm > 601.76 mm. Hence, o.k. The arrangement of reinforcement and dowels is shown in Fig.11.29.3a and b.

### Problem 4:

Design one isolated footing for a column 300 mm x 450 mm, having 20 bars of 20 mm diameter ( $A_{st} = 4021$  mm<sup>2</sup>) of Problem 1 of sec.10.25.6 of Lesson 25 carrying  $P_u = 1620$  kN and  $M_u = 170$  kNm using M 25 and Fe 415. Assume that the moment is reversible. The safe bearing capacity of the soil is 200 kN/m<sup>2</sup> at a depth of 1 metre from ground level. Use M 25 and Fe 415 for the footing.

### Solution 4:

#### Step 1: Size of the footing



**Fig. 11.29.4: Problem 4**

Given  $P_u = 1620$  kN and  $M_u = 170$  kNm. The footing should be symmetric with respect to the column as the moment is reversible. Assuming the weights of footing and backfill as 15 per cent of  $P_u$ , the eccentricity of load  $P_u$  at the base is  $e = M_u/P(1.15) = 170(10^6)/1620(1.15)(10^3) = 91.25$  mm. This eccentricity may be taken as  $< L/6$  of the footing.

The factored bearing pressure is  $200(1.5) = 300 \text{ kN/m}^2$ . For the footing of length  $L$  and width  $B$ , we, therefore, have:

$$P_u/BL + 6M/BL^2 \leq 300$$

or,  $1620(1.15)/BL + 6(170)/BL^2 \leq 300$

or,  $BL^2 - 6.21L - 3.4 \leq 0$   
(1)

For the economic proportion, let us keep equal projection beyond the face of the column in the two directions. This gives

$$(L - 0.45)/2 = (B - 0.3)/2$$

or,  $B = L - 0.15$   
(2)

Using Eq.(2) in Eq.(1), we have

$$(L - 0.15) L^2 - 6.21L - 3.4 \leq 0$$

or  $L^3 - 0.15 L^2 - 6.21 L - 3.4 \leq 0$

We have  $L = 2.8 \text{ m}$  and  $B = 2.65 \text{ m}$ . Let us provide  $L = 2.85 \text{ m}$  and  $B = 2.70 \text{ m}$  (Fig.11.29.4b). We get the maximum and minimum pressures as

$1620(1.15)/(2.85)(2.70) \pm 170(6)/(2.7)(2.85)(2.85) = 242.105 \pm 46.51 = 288.615 \text{ kN/m}^2$  and  $195.595 \text{ kN/m}^2$ , respectively (Fig.11.29.4c). Both the values are less than  $300 \text{ kN/m}^2$ . Hence, o.k.

## Step 2: Thickness of footing slab based on one-way shear

The critical section (sec.11 of Figs.11.29.4a and b) is at a distance  $d$  from the face of the column. The average soil pressure at sec.11 is  $\{288.615 - (288.615 - 195.595)(1200 - d)/2850\} = 249.449 + 0.0326d$ .

The one-way shear force at sec.11 =  $(2.7)(1.2 - 0.001d)(249.449 + 0.0326d) \text{ kN}$ . Assuming 0.15 per cent reinforcement in the footing slab, the shear strength of M 25 concrete =  $0.29 \text{ N/mm}^2$ . Hence, the shear strength of the section =  $2700(d)(0.29)(10^3) \text{ kN}$ . From the condition that shear strength has to be  $\geq$  shear force, we have

$$2700(d)(0.29)(10^3) = (2.7)(1.2 - 0.001d)(249.449 + 0.0326d)$$

This gives,

$$d^2 + 15347.51534d - 9182171.779 = 0$$

Solving, we get  $d = 576.6198$ . Let us assume  $d = 600$  mm

### Step 3: Checking for two-way shear

At the critical section 2222 (Figs.11.29.4a and b), the shear resistance is obtained cl.31.6.31 of IS 456, which gives  $\tau_c = (0.5 + 450/300)(0.25)(25)^{1/2}$  but the multiplying factor  $(0.5 + 450/300) \not\geq 1.0$ . So, we have  $\tau_c = 0.25(25)^{1/2} = 1.25$  N/mm<sup>2</sup>. Hence, the shear resistance =  $(1.25)(2)\{(300 + 600) + (450 + 600)\}(600) = 2925$  kN.

Actual shear force is determined on the basis of average soil pressure at the centre line of the cross-section which is  $(195.595 + 288.615)/2 = 242.105$  kN/m<sup>2</sup> (Fig.11.29.4c). So, the actual shear force =  $V_u = (242.105)\{(2.7)(2.85) - (0.3 + 0.6)(0.45 + 0.6)\} = 1634.209$  kN < shear resistance (= 2925 kN). Hence, the depth of the footing is governed by one-way shear. With effective depth = 600 mm, the total depth of footing = 600 + 50 (cover) + 16 (bar dia) + 8 (half bar dia) = 674 mm.

### Step 4: Gross bearing capacity

Assuming the unit weights of concrete and soil as 25 kN/m<sup>3</sup> and 18 kN/m<sup>3</sup>, respectively, we have the bearing pressure for (i)  $P_u = 1620$  kN, (ii)  $M_u = 170$  kNm and (iii) self weight of footing and backfill soil.

(i) Due to  $P_u = 1620$  kN: pressure =  $1620/(2.7)(2.85) = 210.53$  kN/m<sup>2</sup>

(ii) Due to  $M_u = 170$  kNm: pressure =  $\pm 170(6)/(2.7)(2.85)(2.85) = \pm 46.51$  kN/m<sup>2</sup>

(iii) Self weight of footing of depth 674 mm and soil of  $(1000 - 674) = 326$  mm: pressure =  $0.674(25) + 0.326(18) = 22.718$  kN/m<sup>2</sup>

Thus, the maximum and minimum pressures are =  $210.53 + 22.718 \pm 46.51 = 279.758$  kN/m<sup>2</sup> and  $186.738$  kN/m<sup>2</sup> < 300 kN/m<sup>2</sup>. Hence, o.k.

### Step 5: Bending moment

(i) In the long direction (along the length = 2850 mm)

Bending moment at the face of column (sec.33 of Figs.11.29.4a and b) is determined where the soil pressure =  $288.615 - (288.615 - 195.595)(1200)/2850 = 249.45$  kN/m<sup>2</sup>. So, the bending moment =  $249.45(2.7)(1.2)(0.6) + (288.615 - 249.45)(2.7)(1.2)(2)/(2)(3) = 527.23$  kNm.

$M/Bd^2 = 527.23(10^6)/(2700)(616)(616) = 0.515 \text{ N/mm}^2 < 3.45 \text{ N/mm}^2$  for M 25 concrete.

Table 3 of SP-16 gives  $\rho = 0.1462 < 0.15$  per cent as required for one-way shear.

Thus,  $A_{st} = 0.15(2700)(616)/100 = 2494.8 \text{ mm}^2$ . Provide 13 bars of 16 mm diameter (area =  $2613 \text{ mm}^2$ ), spacing =  $(2700 - 100 - 16)/12 = 215.33 \text{ mm}$ , say 210 mm c/c.

**(ii) In the short direction ( $B = 2700 \text{ mm}$ )**

The average pressure on soil between the edge and centre of the footing =  $(288.615 + 242.105)/2 = 265.36 \text{ kN/m}^2$ . The bending moment is determined with this pressure as an approximation.

$$\text{Bending moment} = (265.36)(1.2)(0.6)(2.85) \text{ kNm} = 544.519 \text{ kNm}$$

$$M/Ld^2 = 544.519(10^6)/(2850)(600)(600) = 0.531$$

Table 3 of SP-16 gives  $\rho = 0.15068$ , which gives area of steel =  $0.15068(2850)(600)/100 = 2576.628 \text{ mm}^2$ . Provide 13 bars of 16 mm diameter (area =  $2613 \text{ mm}^2$ ) @ 210 mm c/c; i.e. the same arrangement in both directions.

**Step 6: Development length**

Development length of 16 mm diameter bars (M 25 concrete) =  $0.87(415)(16)/4(1.6)(1.4) = 644.73 \text{ mm}$ .

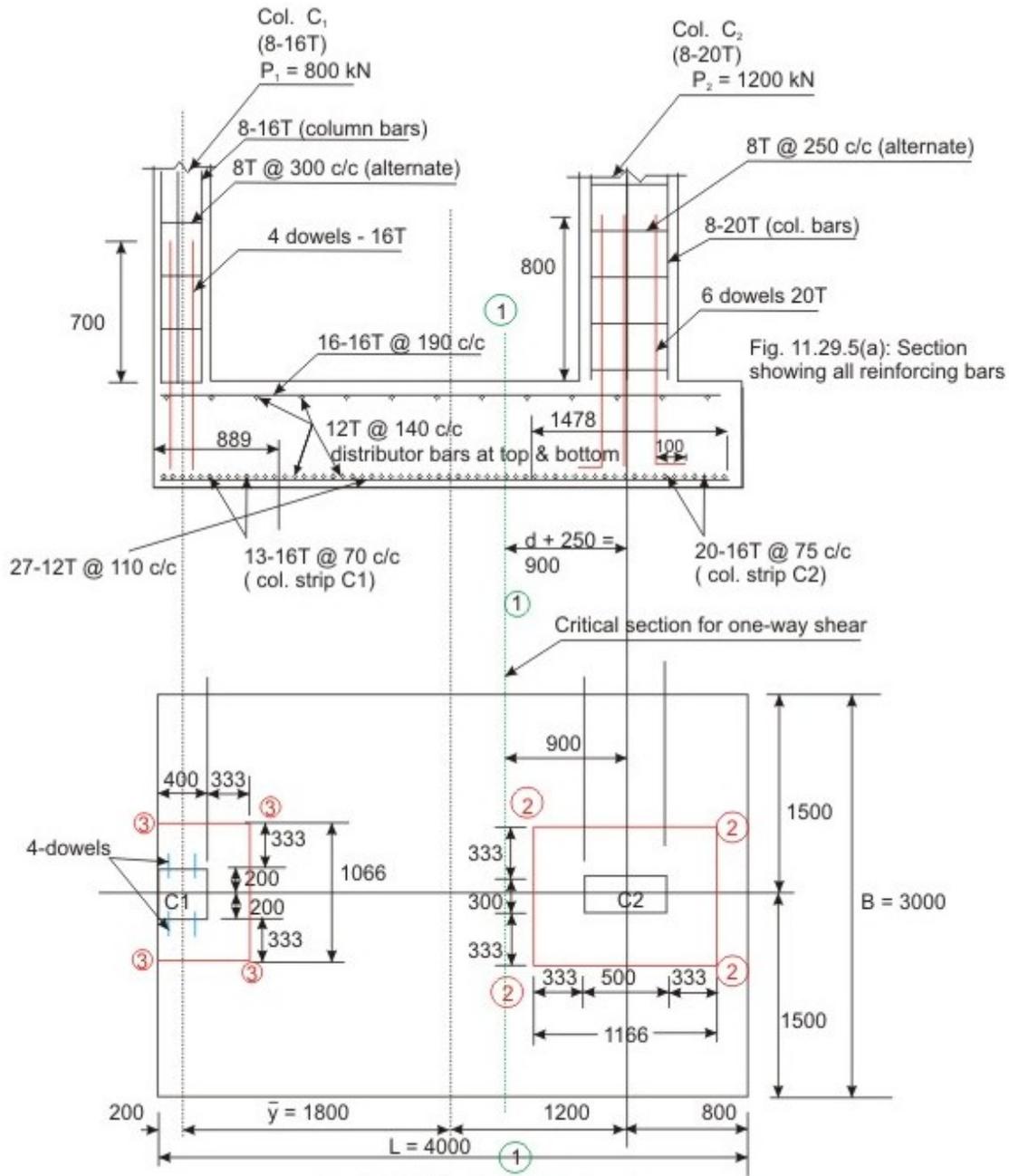
Length available =  $1200 - 50 - 8 = 1142 \text{ mm} > 644.73 \text{ mm}$ . Hence, o.k.

**Step 7: Transfer of force at the base of the column**

Since the column is having moment along with the axial force, some of the bars are in tension. The transfer of tensile force is not possible through the column-footing interface. So, the longitudinal bars of columns are to be extended to the footing. The required development length of 20 mm bars =  $0.87(415)/4(1.4)(1.6) = 805.92 \text{ mm}$ . Length available =  $600 \text{ mm} < 805.92 \text{ mm}$ . The bars shall be given  $90^\circ$  bend and then shall be extended by 200 mm horizontally to give a total length of  $600 + 8(20)$  (bend value) +  $200 = 960 \text{ mm} > 805.92 \text{ mm}$  (Figs.11.29.4 a and b).

The arrangement of reinforcement is shown in Figs.11.29.4a and b.

**Problem 5:**



**Fig. 11.29.5: Problem-5**

Design a combined footing for two columns  $C_1$ , 400 mm x 400 mm with 8 bars of 16 mm diameter carrying a service load of 800 kN and  $C_2$ , 300 mm x 500 mm with 8 bars of 20 mm diameter carrying a service load of 1200 kN (Figs.11.29.5a and b). The column  $C_1$  is flushed with the property line. The columns are at 3.0 m c/c distance. The safe bearing capacity of soil is 200 kN/m<sup>2</sup> at a depth of 1.5 m below the ground level. Use M 20 and Fe 415 for columns and footing.

**Solution :**

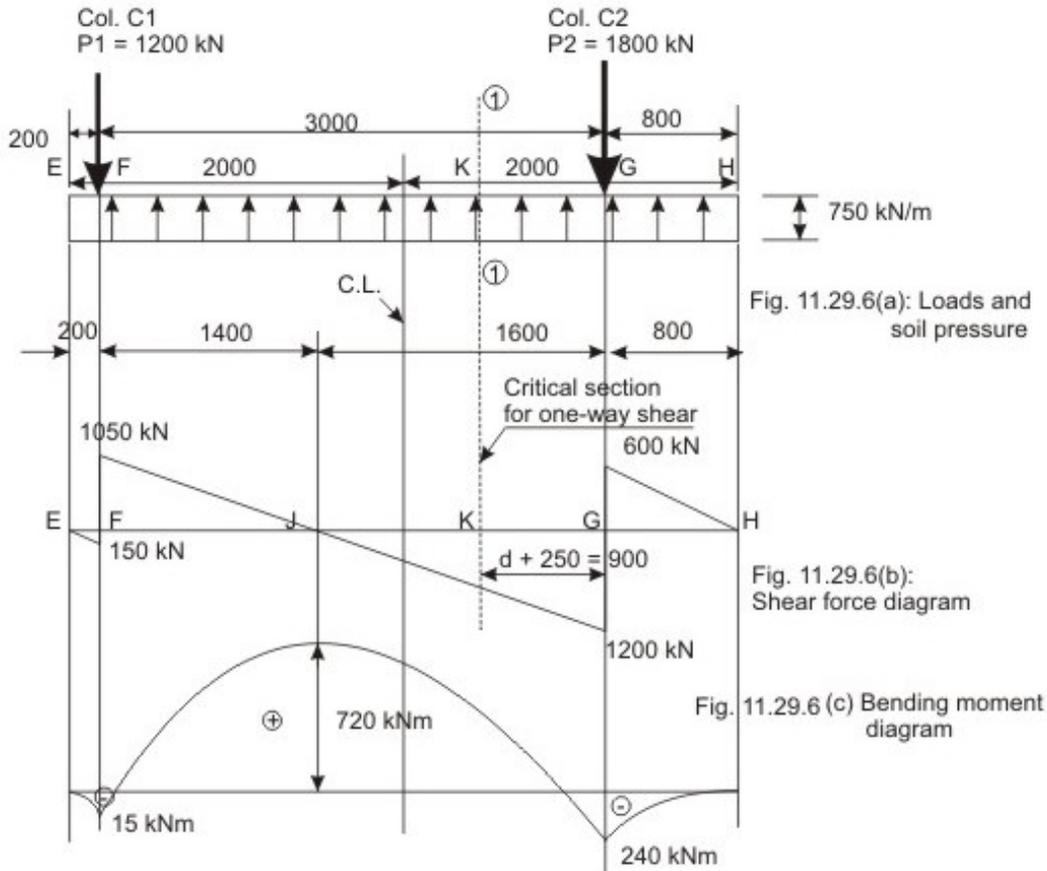
**Step 1: Size of the footing**

Assuming the weight of combined footing and backfill as 15 per cent of the total loads of the columns, we have the required base area, considering  $q_c = 200 \text{ kN/m}^2$ ,

$$\text{Area of the base} = (800 + 1200)(1.15)/200 = 11.5 \text{ m}^2.$$

It is necessary that the resultant of the loads of two columns and the centroid of the footing coincide so that a uniform distribution of soil pressure is obtained. Thus, the distance of the centroid of the footing  $\bar{y}$  from column  $C_1$  (Fig.11.29.5b) is:

$\bar{y} = 800(0) + 1200(3)/2000 = 1.8 \text{ m}$  (Fig.11.29.5b). Since  $\bar{y}$  is greater than half the c/c distance of column, a rectangular footing has to be designed. Let us provide 4 m x 3 m and the dimensions are shown in Fig.11.29.5b coinciding the centroid of the footing and the resultant line of action of the two loads, i.e. at a distance of 2 m from the left edge.



**Fig. 11.29.6:** Loads, soil pressure, shear force and bending moment diagrams of Problem 5

## Step 2: Thickness of footing slab based on one-way shear

Considering the footing as a wide beam of  $B = 3$  m in the longitudinal direction, the uniformly distributed factored load =  $(800 + 1200)(1.5)/4 = 750$  kN/m. Figures 11.29.6a, b and c present the column loads, soil pressure, shear force and bending moment diagrams.

The critical section of one-way shear is sec.11 (at point K) of Figs.11.29.5a and 11.29.6a, at a distance of  $d + 250$  mm from G (the location of column  $C_2$ ). The one-way shear force is

$$\text{Shear force} = (1600 - d - 250)1200/1600 = (1012.5 - 0.75d) \text{ kN}$$

Assuming  $p = 0.15$  per cent reinforcement in the footing slab, the shear strength of M 20 concrete =  $0.28 \text{ N/mm}^2$ . Hence, the shear strength of section 11 =  $(3000)d(0.28)(10^{-3})$  kN. From the condition that shear strength  $\geq$  shear force, we have

$(3000)d(0.28)(10^{-3}) \geq 1012.5 - 0.75d$ , which gives  $d \geq 636.79$  mm. Provide  $d = 650$  mm and the total depth =  $650 + 50 + 16 + 8 = 724$  mm (assuming cover = 50 mm and the diameter of bars = 16 mm).

## Step 3: Checking for two-way shear

### (i) Around column $C_2$

The effective depth along 4.0 m is  $650 + 16 = 666$  mm. The critical section for the two-way shear around column  $C_2$  is at a distance of  $666/2 = 333$  mm from the face of the column and marked by 2222 line in Fig.11.29.5b. The two-way punching shear force, considering the soil pressure =  $750/3 = 250 \text{ kN/m}^2$ , is

$$V_u = 1800 - (1.166)(0.966)(250) = 1518.411 \text{ kN}$$

As per cl.31.6.3.1 of IS 456, here  $k_s = 0.5 + (500/300)$  but  $\nless 1.0$ ; so,  $k_s = 1.0$ . Therefore, shear strength of concrete =  $0.25(20)^{1/2} (2)\{(300 + 666) + (500 + 666)\}(666) = 3174.92 \text{ kN} > 1518.411 \text{ kN}$ . Hence, o.k.

### (ii) Around column $C_1$

The effective depth of footing is 666 mm. The critical section is marked by 3333 in Fig.11.29.5b. The two-way punching shear =  $1200 - (1.066)(0.733)(250) = 1004.65 \text{ kN}$ . The resistance to two-way shear =  $0.25(20)^{1/2} (2)\{(1066 + 733)\}(666) = 2679.1 \text{ kN} > 1004.65 \text{ kN}$ . Hence, o.k.

Thus, the depth of the footing is governed by one-way shear.

#### Step 4: Gross bearing capacity

Assuming unit weights of concrete and soil as  $25 \text{ kN/m}^3$  and  $18 \text{ kN/m}^3$ , respectively, the gross bearing capacity under service load is determined below.

(i) Due to two loads:  $(800 + 1200)/(3)(4) = 166.67 \text{ kN/m}^2$

(ii) Due to weight of the footing: With a total depth of the footing = 724 mm, the pressure =  $0.724(25) = 18.1 \text{ kN/m}^2$ .

(iii) Due to backfill of  $1500 - 724 = 776 \text{ mm}$ , the pressure =  $0.776(18) = 13.968 \text{ kN/m}^2$ .

The total pressure =  $166.67 + 18.1 + 13.968 = 198.738 \text{ kN/m}^2 < 200 \text{ kN/m}^2$ .  
Hence, o.k.

#### Step 5: Bending moments (longitudinal direction)

##### (i) Maximum positive moment

Figure 11.29.6c shows the maximum positive bending moment = 720 kNm at a distance of 1.4 m from the column  $C_1$  (at point J). With effective depth  $d = 666 \text{ mm}$ , we have

$$M/Bd^2 = 720(10^6)/(3000)(666)(666) = 0.541 \text{ N/mm}^2$$

Table 2 of SP-16 gives  $\rho = 0.1553$  per cent.

$$A_{st} = 0.1553(3000)(666)/100 = 3102.894 \text{ mm}^2$$

Provide 16 bars of 16 mm diameter (area =  $3217 \text{ mm}^2$ ), spacing =  $(3000 - 50 - 16)/15 = 195.6 \text{ mm c/c}$ , say 190 mm c/c.

$$\text{Development length of 16 mm bars} = 47.01(16) = 752.16 \text{ mm}$$

$$\text{Length available} = 1600 - 50 - 16 = 1534 \text{ mm} > 752.16 \text{ mm}$$

Hence, o.k.

##### (ii) Maximum negative moment

Figure 11.29.6c shows the maximum negative moment = 240 kNm at a distance of 800 mm from the right edge. With the effective depth = 666 mm, we have

$$M/Bd^2 = 240(10^6)/(3000)(666)(666) = 0.018 \text{ N/mm}^2$$

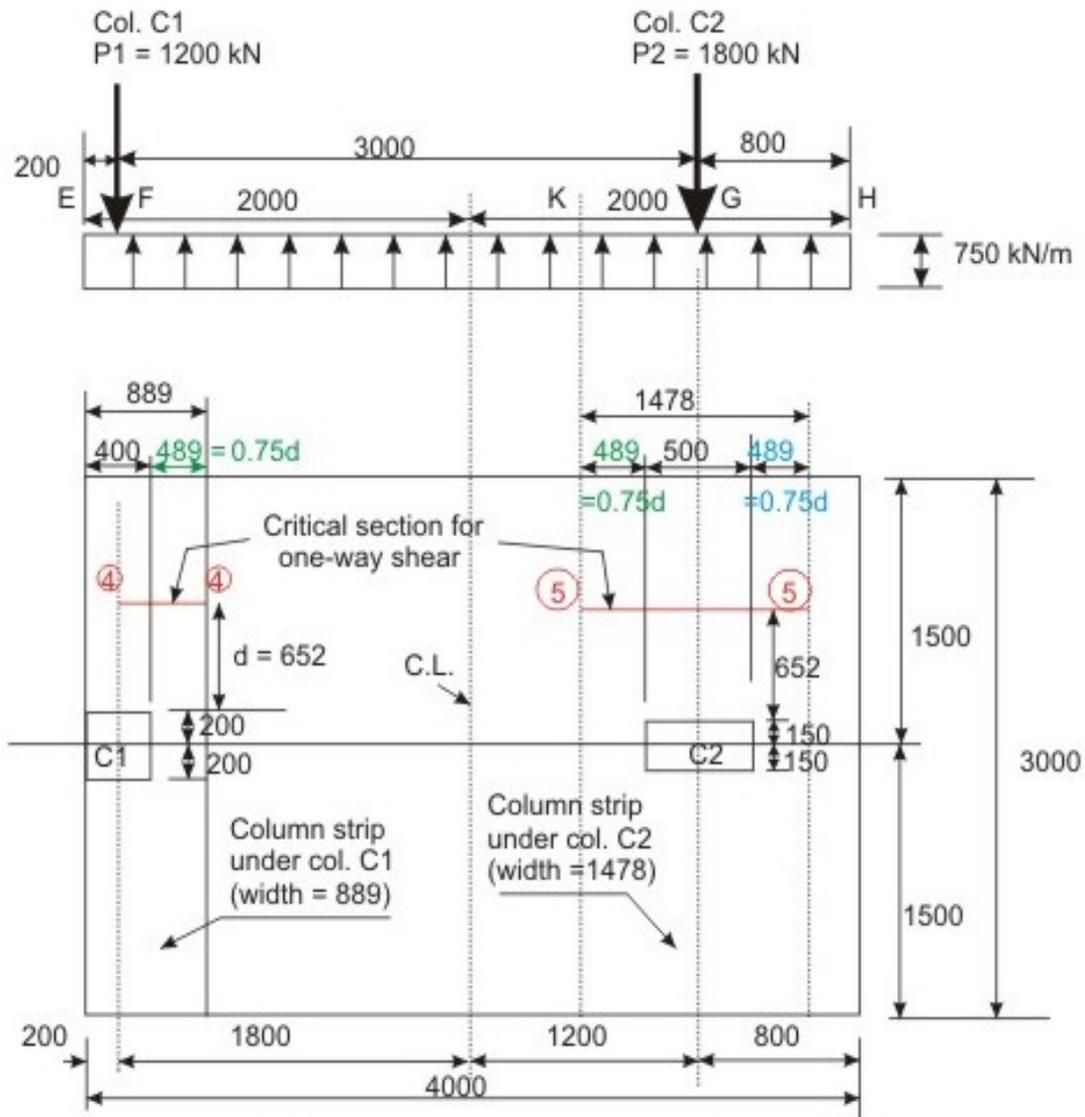
It is very nominal. So, provide 0.15 per cent steel, which gives  $A_{st} = 0.15(3000)(666)/100 = 2997 \text{ mm}^2$ . Provide 27 bars of 12 mm (area =  $3053 \text{ mm}^2$ ) at spacing =  $(3000 - 50 - 12)/26 = 113 \text{ mm c/c}$ ; say 110 mm c/c.

$$\text{Development length} = 47.01(12) = 564 \text{ mm}$$

$$\text{Length available} = 800 - 50 - 12 = 738 \text{ mm} > 564 \text{ mm}$$

Hence, o.k.

### Step 6: Design of column strip as transverse beam



**Fig. 11.29.7:** Column strips as transverse beams (Problem 5)

Figure 11.29.7 shows the two column strips under columns  $C_1$  and  $C_2$ .

### (i) Transverse beam under column $C_1$

The width of the transverse beam is  $0.75d$  from the face of column  $C_1$ . The effective depth is  $666 - 6 - 8 = 652$  mm, as the effective depth in the longitudinal direction = 666 mm, bottom bar diameter in longitudinal direction = 12 mm and assuming the bar diameter in the transverse direction as 16 mm. We have to check the depth and reinforcement in the transverse direction considering one-way shear and bending moment.

#### (A) One-way shear

The factored load for this transverse strip =  $1200/3 = 400$  kN/m. The section of the one-way shear in sec.44 (Fig.11.29.7) at a distance of  $d = 652$  mm from the face of column  $C_1$ . The width of the transverse strip =  $400 + 0.75(652) = 889$  mm.

One-way shear force in sec.44 =  $(1500 - 652 - 200)(400)(10^{-3}) = 259.2$  kN

$$\text{Shear stress developed} = 259.2(10^3)/(889)(652) = 0.447 \text{ N/mm}^2$$

To have the shear strength of concrete  $\geq 0.447 \text{ N/mm}^2$ , the percentage of reinforcement is determined by linear interpolation from Table 19 of IS 456, so that the depth of footing may remain unchanged. Table 19 of IS 456 gives  $p = 0.43125$ . Accordingly,  $A_{st} = 0.43125(889)(652)/100 = 2499.65 \text{ mm}^2$ . Provide 13 bars of 16 mm (area =  $2613 \text{ mm}^2$ ), spacing =  $889/12 = 74.08$  mm c/c, say 70 mm c/c. However, this area of steel shall be checked for bending moment consideration also.

#### (B) Bending moment at the face of column $C_1$ in the transverse strip under column

Bending moment =  $(1.3)(1.3)(400)/2 = 338$  kNm. We, therefore, have

$$M/(\text{width of strip})d^2 = 338(10^6)/(889)(652)(652) = 0.89 \text{ N/mm}^2$$

Table 2 of SP-16 gives  $p = 0.2608 < 0.43125$ . Hence, the area of steel as determine for one-way shear consideration is to be provided. Provide 13 bars of 16 mm @ 70 mm c/c in the column strip of width 889 mm under the column  $C_1$ .

Development length of 16 mm bars =  $47.01(16) = 752.16$  mm

Length available =  $1300 - 50 - 16 = 1234$  mm  $> 752.16$  mm

Hence, o.k.

### (ii) Transverse beam under column $C_2$

Figure 11.29.7 shows the strip of width =  $500 + 0.75d + 0.75d = 500 + 1.5(652) = 1478$  mm, considering the effective depth of footing = 652 mm.

#### (A) One-way shear

The factored load for this transverse strip =  $1800/3 = 600$  kN/m. The one-way shear section is marked by sec.5.5 in Fig.11.29.7 at a distance of  $d = 652$  mm from the face of the column  $C_2$ .

One-way shear in sec.5.5 (of width = 1478 mm) =  $(1500 - 652 - 150)(600)(10^{-3})$

$$= 418.8 \text{ kN}$$

The shear stress developed =  $418.8(10^3)/(1478)(652) = 0.434 \text{ N/mm}^2$

The corresponding percentage of area of steel, as obtained from Table 19 of IS 456 is,  $p = 0.404$  per cent. Accordingly,  $A_{st} = 0.404(1478)(652)/100 = 3893.17 \text{ mm}^2$ . Provide 20 bars of 16 mm (area =  $4021 \text{ mm}^2$ ), spacing =  $1478/19 = 77.78$  mm c/c, say 75 mm c/c. However, this area of steel shall be checked for bending moment consideration also.

#### (B) Bending moment at the face of column $C_2$ in the transverse strip under column $C_2$

The bending moment =  $(1.35)(1.35)(600)/2 = 546.75$  kNm. We, therefore, have

$$M/(\text{width of strip})d^2 = 546.75(10^6)/(1478)(652)(652) = 0.87 \text{ N/mm}^2$$

Table 2 of SP-16 gives:  $p = 0.2544 < 0.404$  per cent as required for one-way shear. So, provide 20 bars of 16 mm diameter @ 75 mm c/c in the column strip of width 1478 under column  $C_2$ .

Development length as calculated in Step (B) of col strip  $C_1 = 752.16$  mm

The length available =  $1350 - 50 - 16 = 1284$  mm  $> 752.16$  mm

Hence, o.k.

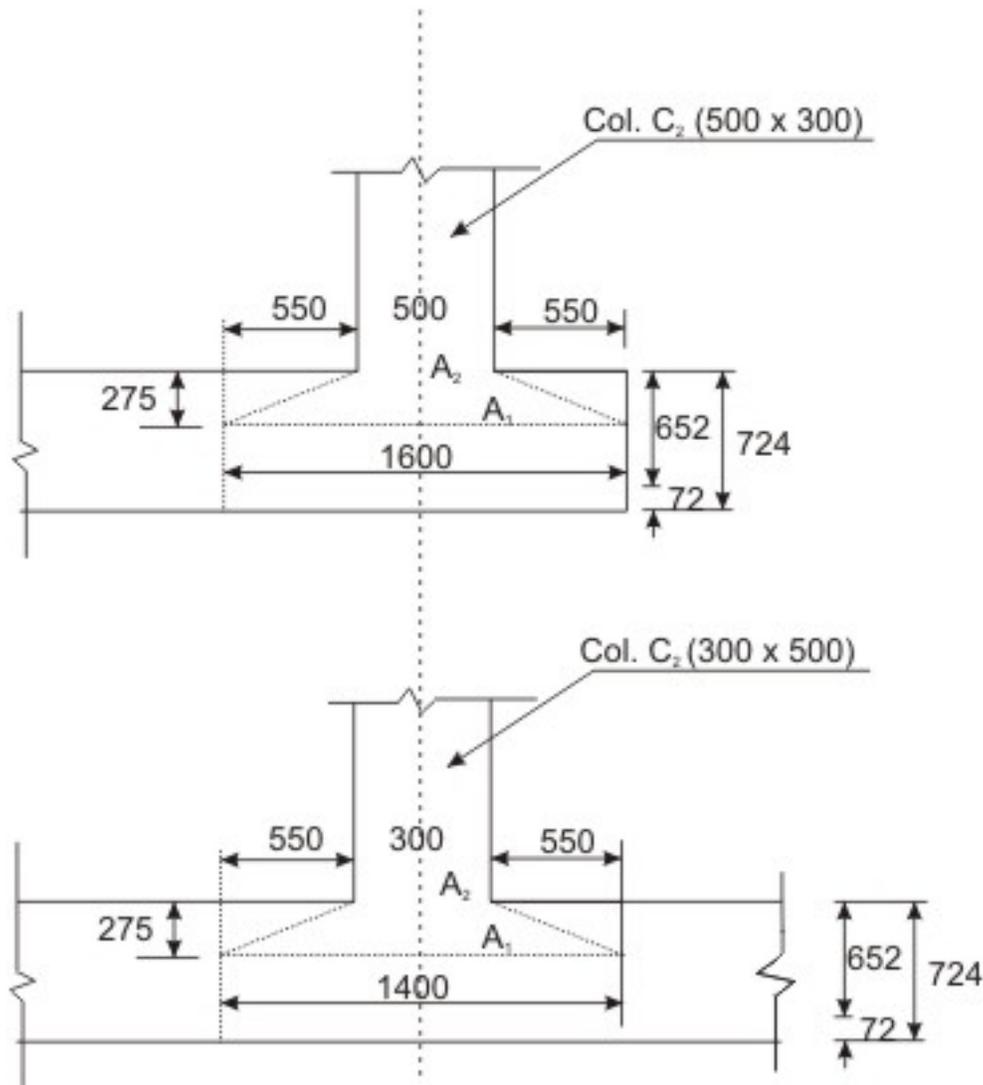
### Step 7: Transfer of forces at the base of the columns

#### (A) For column $C_1$

The limiting bearing stress at the column face governs where the bearing stress =  $0.45f_{ck} = 9 \text{ N/mm}^2$ , since the column  $C_1$  is at the edge of the footing.

The force that can be transferred =  $9(400)(400)(10^{-3}) = 1440 \text{ kN} > 1200 \text{ kN}$ , load of column  $C_1$ . Hence full transfer of force is possible without the need of any dowels. However, four 16 mm nominal dowels are provided as shown in Figs.11.29.5a and b.

(B) For column  $C_2$



**Fig. 11.29.8:** Dimensions of  $A_1$  and  $A_2$  of Problem 5

Figure 11.29.8 shows the dimensions to determine  $A_1$  and  $A_2$  for the column  $C_2$  and the footing. Accordingly,

$$A_1 = (1600)(1400) \text{ mm}^2, \text{ and}$$

$$A_2 = (300)(500) \text{ mm}^2$$

$$(A_1/A_2)^{1/2} = \{(16)(14)/(3)(5)\}^{1/2} = 3.86 \text{ but limited to } 2$$

The bearing stress at the column face =  $0.45f_{ck} = 9 \text{ N/mm}^2$ , and the bearing stress at the footing face =  $0.45f_{ck}(2) = 18 \text{ N/mm}^2$ . However, the bearing stress of  $9 \text{ N/mm}^2$  governs.

The force that can be transferred through the column  $C_2 = 9(300)(500) = 1350 \text{ kN} < 1800 \text{ kN}$ . For the excess force  $(1800 - 1350) = 450 \text{ kN}$ , dowels shall be provided. The area of dowels =  $450(10^3)/0.67(415) = 1618.414 \text{ mm}^2$ . The minimum area of dowels =  $0.5(300)(500)/100 = 750 \text{ mm}^2$ . Provide 6 dowels of 20 mm diameter (area =  $1885 \text{ mm}^2$ ).

The development length required in compression

$$= 0.97(415)(20)/4(1.6)(1.2)(1.25) = 752.2 \text{ mm.}$$

The length available =  $652 - 20 = 632 \text{ mm}$

Therefore, the dowels shall be given a  $90^\circ$  bend and shall be extended horizontally by 100 mm to have a total length of  $632 + 8(20) + 100 = 892 \text{ mm} > 752.2 \text{ mm}$  (Figs.11.29.5a and b).

### **Step 8: Distribution reinforcement**

Nominal distribution reinforcement shall be provided at top and bottom where the main reinforcement bars are not provided. The amount @ 0.12 per cent comes to  $0.12(1000)(652)/100 = 782.4 \text{ mm}^2/\text{metre}$ . Provide 12 mm diameter bars @ 140 mm c/c (area =  $808 \text{ mm}^2/\text{m}$ ).

All the reinforcing bars and dowels are shown in Figs.11.29.5a and b.

### 11.29.3 Practice Questions and Problems with Answers

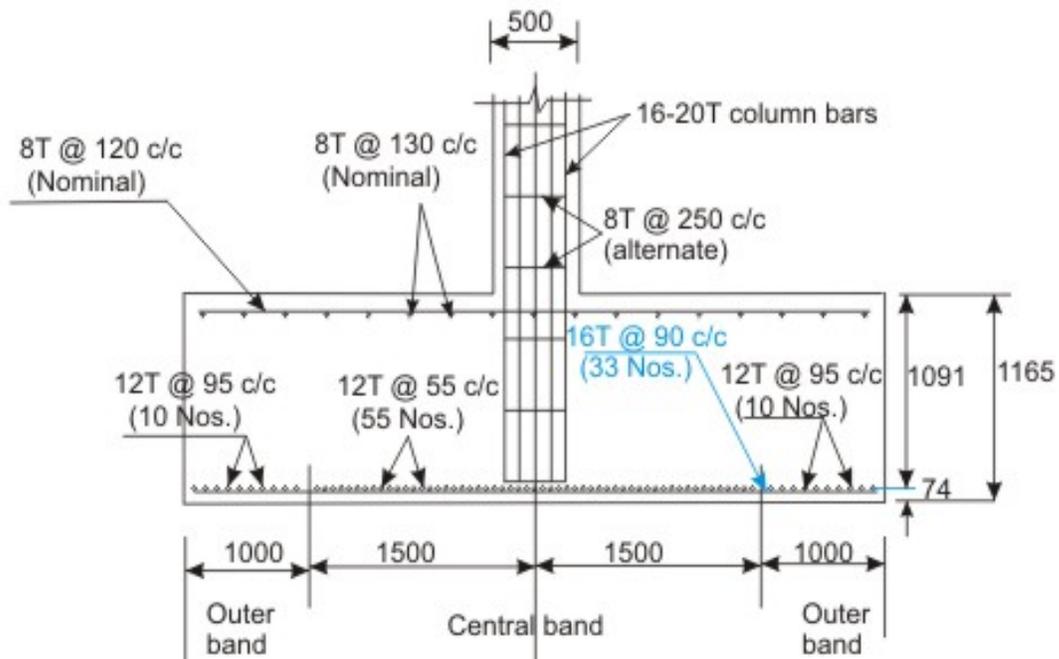


Fig. 11.29.9 (a) Section

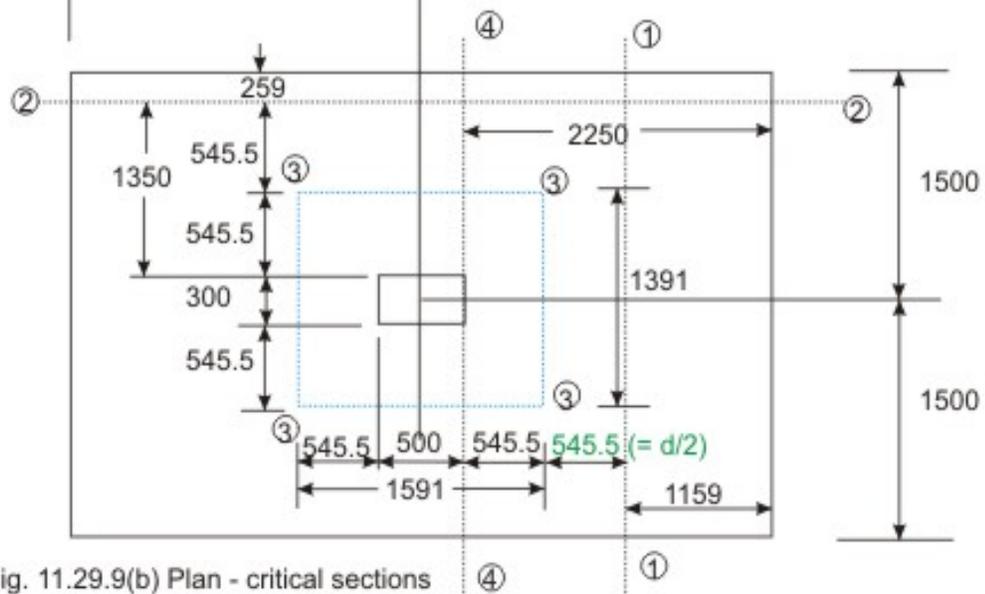
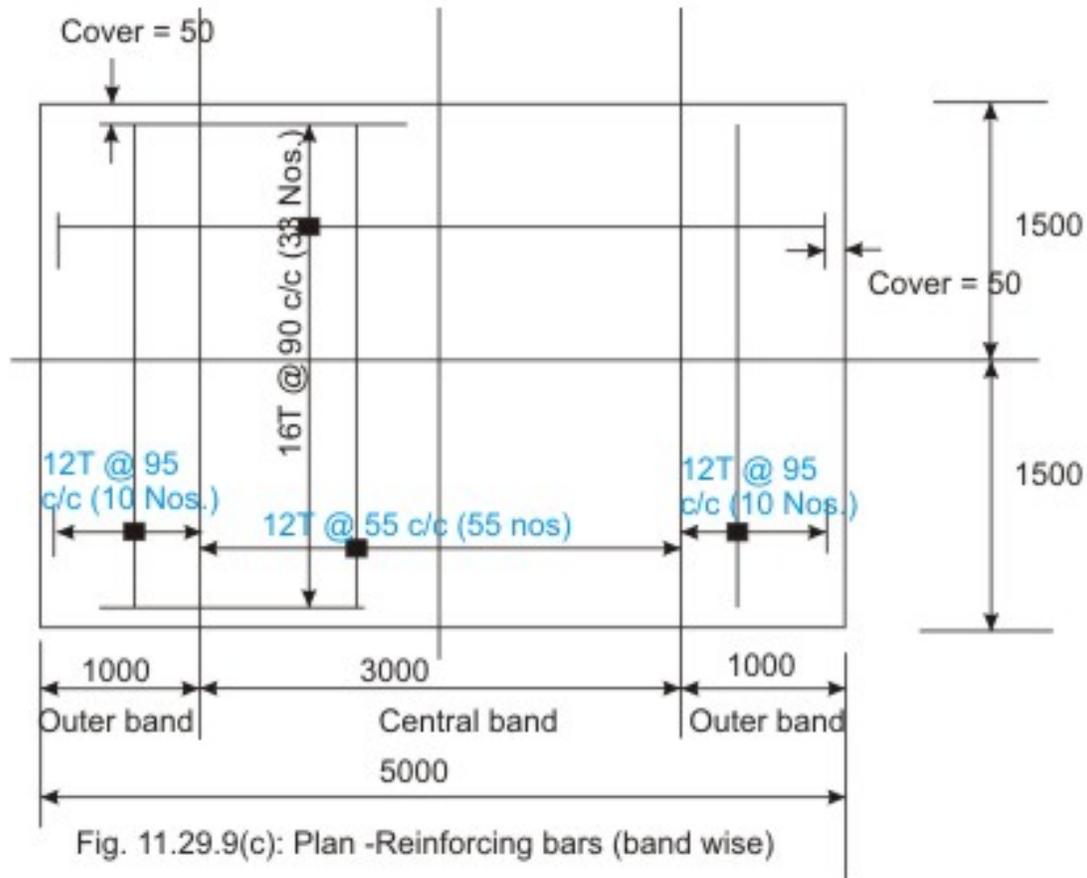


Fig. 11.29.9(b) Plan - critical sections

Fig. 11.29.9: Problem Q.1



**Fig. 11.29.9: Problem - Q.1**

**Q.1:** Design an isolated footing for a rectangular column, 300 mm x 500 mm with 16-20 mm diameter longitudinal bars carrying a service load of 3000 kN (Figs.11.29.9a and b). Assume safe bearing capacity of soil as 260 kN/m<sup>2</sup> at a depth of 1 m below ground level. Use M 20 and Fe 415.

**A.1:**

**Step 1: Size of the footing**

Given  $P = 3000$  kN and  $q_c = 260$  kN/m<sup>2</sup>. Assuming the weight of the footing and backfill as 25 per cent of the load, the base area required =  $3000(1.25)/260 = 14.42$  m<sup>2</sup>. Provide 3 m x 5 m giving 15 m<sup>2</sup> area.

**Step 2: Thickness of footing slab based on one-way shear**

(i) Along 5 m direction

The critical section of one-way shear is marked by sec.11 in Fig.11.29.9b.

$$\text{Factored soil pressure} = 3000(1.5)/15 = 300 \text{ kN/m}^2 = 0.3 \text{ N/mm}^2$$

Assuming 0.2 per cent reinforcement in the footing slab, Table 19 of IS 456 gives  $\tau_c$  for M 20 =  $0.32 \text{ N/mm}^2$ . From the condition that the shear resistance  $\geq$  shear force, we have:

$$0.32(3000)d \geq 0.3(2250 - d)(3000)$$

or  $d \geq 1088.71 \text{ mm}$

Using 16 mm diameter bars with 50 mm cover the total depth =  $1088.71 + 50 + 16 + 8 = 1162.71 \text{ mm}$ . Provide total depth of 1165 mm, so that  $d = 1091 \text{ mm}$ .

**(ii) Along 3 m direction**

The critical section of one-way shear is marked by sec.2-2 in Fig.11.29.9b.

$$\text{Resistance shear force} = 0.32(5000)(1091) = 1745.6 \text{ kN}$$

$$\text{Actual shear force} = 0.3(259)(5000) = 388.5 \text{ kN} < 1745.6 \text{ kN}$$

Hence, o.k.

**Step 3: Checking for two-way shear**

The critical section is marked by 3-3-3-3 in Fig. 11.29.9b.

The permissible shear strength of concrete M 20

$$= (0.5 + \beta) 0.25(f_{ck})^{1/2} = 0.25(20)^{1/2} = 1.118 \text{ N/mm}^2$$

Total perimeter of section 3-3-3-3 =  $2(500 + 1091 + 300 + 1091) = 5964 \text{ mm}$  giving area =  $5964(1091) = 6506724 \text{ mm}^2$ .

$$\text{Actual shear force} = \{15 - (1.591)(1.391)\}(300) = 3836.0757 \text{ kN}$$

$$\text{Shear stress} = 3836.0757(10^3)/6506724 = 0.5896 \text{ N/mm}^2 < 1.118 \text{ N/mm}^2$$

Hence, o.k.

The depth of the footing slab is governed by one-way shear here.

#### Step 4: Gross bearing capacity

Assuming unit weights of concrete and soil as  $25 \text{ kN/m}^3$  and  $20 \text{ kN/m}^3$ , respectively, we have

$$\text{Service load} = 3000 \text{ kN}$$

$$\text{Weight of footing slab} = 3(5)(1.165)(25) = 436.875 \text{ kN}$$

$$\text{Weight of backfill} = 3(5)(1.165)(20) = 349.5$$

$$\text{Total loads} = 3786.375 \text{ kN}$$

$$\text{Gross bearing capacity} = 3786.375/15 = 252.425 \text{ kN/m}^2 < 260 \text{ kN/m}^2$$

Hence, o.k.

#### Step 5: Bending moments

##### (i) In the long direction

The critical section of the bending moment in the long direction is marked by sec.4-4, (at the face of column) in Fig.11.29.9b. With factored soil pressure  $q = 0.3 \text{ N/mm}^2$ , the bending moment is  $0.3(3000)(2250)(2250)/2 = 2278.125 \text{ kNm}$ .

Moment of resistance of the section =  $RBd^2 = 2.76(3000)(1091)(1091)(10^{-6}) = 9855.53 \text{ kNm} > 2278.125 \text{ kNm}$ . Hence, o.k.

The area of steel is determined from Eq.3.23 of Lesson 5, which is,

$$M_u = 0.87 f_y A_{st} d \{1 - A_{st} f_y / f_{ck} b d\}$$

$$\text{or } 2278.125(10^6) = 0.87(415)(A_{st})(1091) - 0.87(A_{st}^2)(415)^2/(20)(3000)$$

$$\text{or } A_{st}^2 - 157734.9398 A_{st} + 912.249313(10^6) = 0,$$

which gives  $A_{st} = 6012.6245$ , i.e.,  $p = 0.1837$  per cent. However, we have to provide 0.2 per cent, as required for the one-way shear. So,  $A_{st} = 0.2(3000)(1091)/100 = 6546 \text{ mm}^2$ . Provide 33 bars of 16 mm diameter (area =  $6634 \text{ mm}^2$ , shown in Fig. 11.29.9c). The uniform spacing =  $(3000 - 50 - 16)/32 = 91.69 \text{ mm c/c}$ , say @ 90 mm c/c. Alternatively,

$$M_u/Bd^2 = 2278.125(10^6)/(3000)(1091)(1091) = 0.638$$

Table 2 of SP-16 gives  $p = 0.1834$  per cent, which is very close to the computed  $p = 0.1837$  per cent. So, provide 33 bars of 16 mm diameter @ 90 mm c/c in the long direction.

The development length of 16 mm bar is  $47.01(16) = 752.16$  mm

Length available =  $2250 - 50 - 16 = 2184$  mm  $> 752.16$  mm

Hence, o.k.

**(ii) In the short direction**

The critical section is marked by sec.5-5 in Fig.11.29.9b, (at the face of column) where the bending moment is:

$$0.3(5000)(1350)(1350)/2 = 1366.875 \text{ kNm}$$

We get

$$M_u/Ld^2 = 1366.875(10^6)/(5000)(1091)(1091) = 0.2296$$

This is a low value, which is not included in SP-16. So, provide 0.15 per cent to have

$$A_{st} = 0.15(5000)(1091)/100 = 8182.5 \text{ mm}^2$$

However, we have to check for one-way shear as the  $p$  provided 0.15 per cent is less than 0.2 per cent, required for the one-way shear and for which  $\tau_c = 0.28 \text{ N/mm}^2$  (Table 19 of IS 456). The one-way shear force, as calculated in step 2(ii) is 388.5 kN. The resistance force of shear at this section =  $0.28(5000)(1091) = 1527.4 \text{ kN} > 388.5 \text{ kN}$ . Hence, o.k.

Further, the total area of steel  $8182.5 \text{ mm}^2$  has to be distributed in the central band of width 3 m and two outer bands of width 0.75 m each as stipulated in cl.34.3.1c of IS 456. The reinforcement in the central band is obtained as given below:

$$A_{st} \text{ for central band} = (2/\beta + 1) \text{ (total area of steel)}$$

where  $\beta = 5/3 = 1.67$ . Thus,

$$\text{Area of steel in the central band} = (2/2.67)(8182.5) = 6129.21 \text{ mm}^2$$

Provide 55 bars of 12 mm (area =  $6220 \text{ mm}^2$ ) at the spacing of  $3000/54 = 55.55$  mm c/c, say, @ 55 mm c/c. Each of two outer band needs  $(8182.5 - 6129.21)/2 = 1026.65 \text{ mm}^2$ . Provide 10 bars of 12 mm diameter (area =  $1131 \text{ mm}^2$ ). The

spacing =  $(1000 - 50)/10 = 95$  mm c/c, say @ 90 mm c/c. Thus, the total area of steel provided =  $6220 + 1131 + 1131 = 8482$  mm<sup>2</sup> > 8182 mm<sup>2</sup> (Fig. 11.29.9c).

The development length required for 12 mm bars =  $(47.01)(12) = 564.12$  mm

Length available =  $1350 - 50 - 16 = 1284$  mm > 564.12 mm

Hence, o.k.

### Step 6: Transfer of force at the base of the column

Factored load =  $3000(1.5) = 4500$  kN. From the limiting bearing stress at the column-footing interface  $0.45 f_{ck}(A_1/A_2)^{1/2}$ , we have

(i) At the column face, where  $A_1 = A_2$ ; bearing stress =  $0.45 f_{ck} = 9.0$  N/mm<sup>2</sup>

(ii) At the footing face,  $A_1 = 15$  m<sup>2</sup> and  $A_2 = (0.3)(0.5) = 0.15$  m<sup>2</sup>. But,  $(A_1/A_2)^{1/2} \not\geq 2$ . So, the bearing stress =  $2(9) = 18$  N/mm<sup>2</sup>.

Therefore, the column face governs. The force that can be transferred through bearing =  $9(500)(300) = 1350$  kN. The excess force to be transferred by dowels =  $4500 - 1350 = 3150$  kN.

Area of dowels =  $3150(10^3)/0.67(415) = 11328.9$  mm<sup>2</sup>. Minimum area of dowels =  $0.5(300)(500)/100 = 750$  mm<sup>2</sup>. So, the number of 20 mm dowels = 36. Instead of providing 36 dowels, it is convenient to extend the sixteen column bars of 20 mm diameter to the footing.

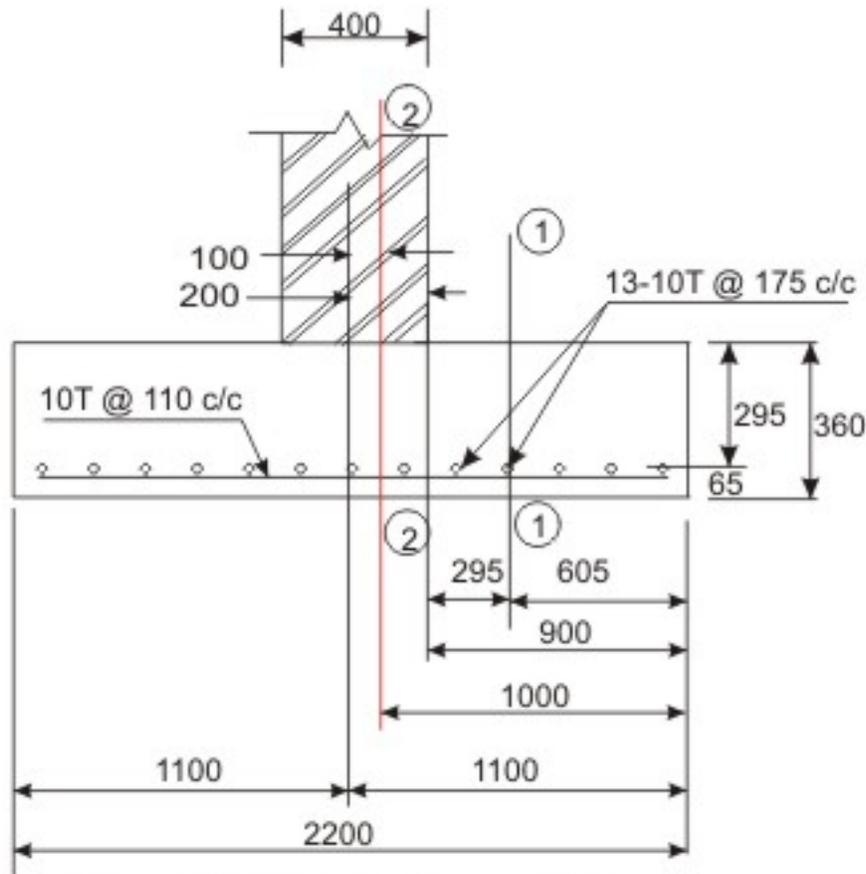
The development length required =  $0.87(415)(20)/4(1.6)(1.25)(1.2) = 752.2$  mm

Length available =  $1091 - 74 = 1017$  mm > 752.2 mm

### Step 7: Nominal reinforcement

Clause 34.5.2 stipulates to provide @ 360 mm<sup>2</sup> per metre length in each direction on each force, when the thickness of footing slab is greater than one metre. So, the minimum reinforcement is 8 mm bars @ 130 mm c/c (area = 387 mm<sup>2</sup>/m), wherever there is no other reinforcement.

The reinforcement bars are shown in Figs.11.29.9a and c.



**Fig. 11.29.10: Problem - Q. 2**

**Q.2:** Design a reinforced concrete footing for a wall of 400 mm thickness transmitting a load of 200 kN/m (Fig.11.29.10) under service condition. Assume the safe bearing capacity of soil as 100 kN/m<sup>2</sup> at a depth of 1 m below the ground level. Use M 20 and Fe 415.

**A.2:**

**Step 1: Size of the footing**

Given axial load = 200 kN/m and safe bearing capacity of soil  $q_c = 100$  kN/m<sup>2</sup> at a depth of 1 m below the ground level. Assuming the self weight of footing and backfill as 10 per cent, the area of the base required =  $200(1.1)/100 = 2.2$  m<sup>2</sup>. Provide width of 2.2 m for every one metre to get the required area of 2.2 m<sup>2</sup>.

**Step 2: Thickness of footing slab based on shear**

The critical section of shear is marked as sec.1-1 in Fig.11.29.10, at a distance of effective depth  $d$  of the footing.

$$\text{Factored soil pressure} = 200(1.5)/2.2 = 136.36 \text{ kN/m}^2 = 0.13636 \text{ N/mm}^2.$$

Assuming 0.15 per cent reinforcement in the footing slab, the shear strength of M 20  $\tau_c = 0.28 \text{ N/mm}^2$ . From the condition that the shear strength of the section  $\geq$  actual shear force in sec.11, we have:

$$0.28(1000)d \geq (0.13636)(1000)(900 - d)$$

This gives  $d \geq 294.755 \text{ mm}$ . Using cover of 50 mm and diameter of reinforcing bar as 10 mm, the total depth of footing = 295 + 50 + 10 + 5 = 360 mm. This gives effective depth = 295 mm.

### Step 3: Checking for the moment

The critical section for the bending moment is marked as sec.2-2 in Fig.11.29.10, where the bending moment (factored) is,

$$M_u = (1)(0.13636)(1000)(1000)/2 = 68.18 \text{ kNm/m}$$

The capacity of the section =  $(2.76)(1000)(295)(295) = 240.189 \text{ kNm/m}$  > 68.18 kNm/m. Hence, o.k.

$$M/bd^2 = 68.18(10^6)/1000(295)(295) = 0.784 \text{ N/mm}^2$$

Table 2 of SP-16 gives  $p = 0.2282$  per cent. Accordingly,  $A_{st} = 0.2282(1000)(295)/100 = 673.19 \text{ mm}^2/\text{m}$ . Provide 10 mm diameter bars @ 110 mm c/c which gives  $714 \text{ mm}^2/\text{m} > 673.19 \text{ mm}^2/\text{m}$ .

### Step 4: Development length

$$\text{The development length of 10 mm bars} = 47.01(10) = 470.1 \text{ mm}$$

$$\text{Length available} = 900 - 50 = 850 > 470.1$$

### Step 5: Distribution reinforcement

Minimum reinforcement @ 0.12 per cent should be provided longitudinally.

$$A_{st} = 0.12(2200)(360)/100 = 950.4 \text{ mm}^2$$

Provide 13 bars of 10 mm diameter (area = 1021 mm<sup>2</sup>). The spacing =  $(2200 - 50 - 10)/12 = 178.33 \text{ mm c/c}$ , say @ 175 mm c/c.

### Step 6: Transfer of loads at wall-footing base

$$\text{Bearing stress} = 0.45 f_{ck} (A_1/A_2)^{1/2} = 0.45(20)(1) = 9 \text{ N/mm}^2$$

The forces that can be transferred =  $9(1000)(2200)(10^{-3}) \text{ kN} = 19800 \text{ kN}$   
>> factored load of  $200(1.5) = 300 \text{ kN}$ . Hence, o.k.

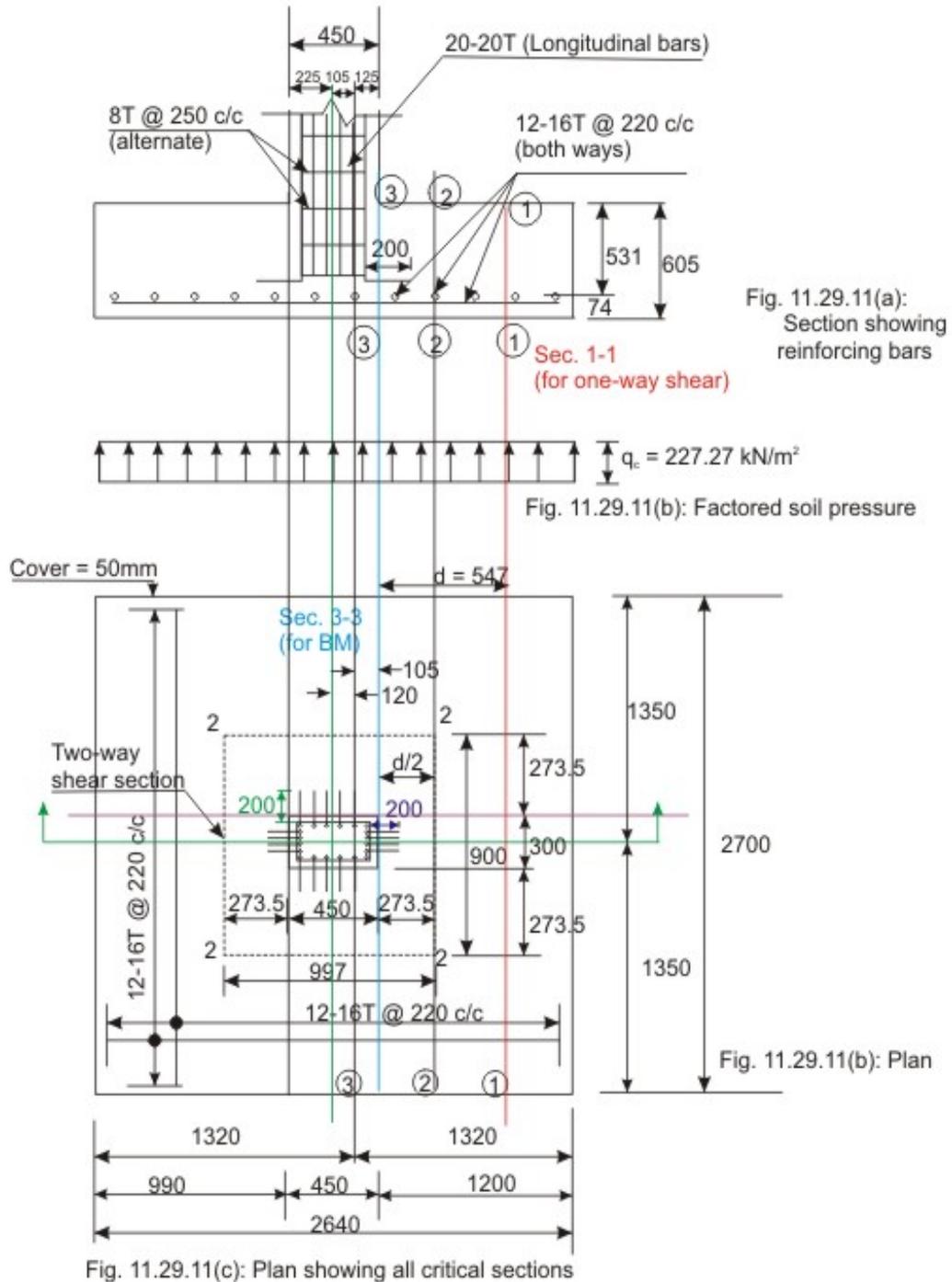
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## 11.29.5 Test 29 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.



**Fig. 11.29.11: TQ.1**

**TQ.1:** Redesign the footing of Problem 4 of section 11.29.2 (Fig.11.29.4) to have uniform base pressure considering that moments are due to dead load and hence irreversible (Fig.11.29.11). Other data of Problem 4 are: column size = 300 mm x 450 mm, longitudinal bars of column = 20 bars of 20 mm ( $A_{st} = 4021 \text{ mm}^2$ ),  $P_u = 1620 \text{ kN}$ ,  $M_u = 170 \text{ kNm}$ , safe bearing capacity of soil =  $200 \text{ kN/m}^2$  at a depth of 1 metre below the ground level, grade of concrete = M 25 and grade of steel = Fe 415.

(50  
marks)

**A.TQ.1:**

**Step 1: Size of the footing**

The required eccentricity between the centroids of column and footing =  $M_u/P_u = 170(10^6)/1620(10^3) = 104.938 \text{ mm}$ , say 105 mm.

Assuming the weight of the footing and backfill as 15 per cent, the required base area =  $1620(1.15)/200(1.5) = 6.21 \text{ m}^2$ .

It is desirable that the cantilever projections in the two directions from the respective column face should be equal. Accordingly, the footing is selected as 2.7 m x 2.64 m where the centre line of column is located at a distance of 105 mm left from the centre line of footing, as shown in Fig.11.29.11c. The arrangement shows that the cantilever projections are equal (1200 mm).

**Step 2: Thickness of footing based on one-way shear**

Factored soil pressure =  $1620/(2.7)(2.64) = 227.27 \text{ kN/m}^2$  (Fig.11.29.11b). The critical section of one-way shear is marked by sec.1-1 in Figs.11.29.11a and c, at a distance of  $d$  from the face of the column, where the factored shear  $V_u$  is,

$$V_u = 0.22727(2700)(1200 - d) = (736354.8 - 613.629 d) \text{ N}$$

Assuming 0.15 per cent reinforcement in the footing slab, Table 19 of IS 456 gives  $\tau_c = 0.29$  for M 25 concrete. Accordingly, the resistance shear of the section is  $0.29(2700)d = 783d$ . From the condition that the resistance shear  $\geq$  actual shear, we have

$783 d \geq 736354.8 - 613.629 d$ , which gives  $d \geq 527.237$ . Let us use full depth =  $527.237 + 50 + 16 + 8 = 601.237 \text{ mm}$ , say 605 mm, so that  $d = 605 - 50 - 8 = 547 \text{ mm}$  in the long direction and  $605 - 50 - 16 - 8 = 531 \text{ mm}$  in the short direction.

**Step 3: Checking for two-way shear**

The shear strength of concrete for two-way shear =  $0.25(f_{ck})^{1/2}$  (cl.31.6.3 of IS 456) =  $1.25 \text{ N/mm}^2$ . The resistance shear of the section is (sec.2222 of Fig.11.29.11c):

$$(1.25)(2)\{(450 + 547) + (300 + 547)\}(547) = 2521.67 \text{ kN}$$

The actual shear force in sec.2222 (Fig. 11.29.11c) is

$$\{(2.7)(2.64) - (0.997)(0.847)\}(227.27) = 1428.06 \text{ kN} < 2521.67 \text{ kN}$$

Hence, o.k.

#### Step 4: Gross bearing capacity

Assuming unit weights of concrete and soil as  $25 \text{ kN/m}^3$  and  $18 \text{ kN/m}^3$ , we have the factored gross bearing capacity as

$$\begin{aligned} q &= (1620)/(2.7)(2.64) + \{0.605(25) + 0.395(18)\} \\ &= 249.509 \text{ kN/m}^2 < 300 \text{ kN/m}^2 \end{aligned}$$

#### Step 5: Bending moment

##### (i) In the short direction

The critical section is marked by sec.33 in Fig.11.29.11c, at the face of column, where the bending moment is:

$$M_u = (2.7)\{(1.2)(1.2)/2\}227.27 = 441.82 \text{ kNm}$$

$M_u/Ld^2 = 441.82(10^6)/(2.7)(10^3)(547)(547) = 0.546 \text{ N/mm}^2 < 3.45 \text{ N/mm}^2$   
for M 25 and Fe 415.

Hence, the section has the capacity to resist  $M_u = 441.82 \text{ kNm}$ .

Table 3 of SP-16 gives  $p = 0.15488$  per cent. Accordingly,

$$A_{st} = 0.15488(2700)(547)/100 = 2287.42 \text{ mm}^2$$

Provide 12 bars of 16 mm diameter (area =  $2412 \text{ mm}^2$ ). The spacing =  $(2700 - 50 - 16)/11 = 239.45 \text{ mm c/c}$ , say  $220 \text{ mm c/c}$ . Provide 12 bars of 16 mm diameter @  $220 \text{ mm c/c}$ .

Development length of 16 mm diameter bars

$$= 0.87(415)(16)/4(1.4)(1.6) = 644.73 \text{ mm}$$

$$\text{Length available} = 1200 - 50 - 16 = 1134 \text{ mm} > 644.73 \text{ mm}$$

Hence, o.k.

**(ii) In the long direction**

The critical section is marked by sec.4-4 in Fig.11.29.11c, at the face of column, where the moment  $M_u$  is,

$$M_u = (2.64)\{(1.2)(1.2)/2\}(227.27) = 431.994 \text{ kNm}$$

$$M_u/Bd^2 = 431.994(10^6)/(2640)(531)(531) = 0.5803 \text{ N/mm}^2 < 3.45 \text{ N/mm}^2$$

Hence, the section can carry this moment. Table 3 of SP-16 gives  $\rho = 0.16509$  per cent. Accordingly,

$$A_{st} = 0.16509(2640)(531)/100 = 2314.2976 \text{ mm}^2$$

Provide 12 bars of 16 mm diameter (area = 2412 mm<sup>2</sup>). The spacing =  $(2640 - 50 - 16)/11 = 234 \text{ mm c/c}$ , say 220 mm c/c. Provide 12 bars of 16 mm diameter @ 220 mm c/c.

Development length as calculated in (i) for 16 mm diameter = 644.73 mm

$$\text{Length available} = 1200 - 50 - 16 = 1134 \text{ mm} > 644.73 \text{ mm}$$

Hence, o.k.

**Step 6: Transfer of force at the base of column**

Since the column is having moment along with the axial force, some of the bars are in tension. The transfer of tensile force is not possible through the column-footing interface. So, the longitudinal bars of column are to extended to the footing. The required development length of 20 mm bars =  $0.87(415)/4(1.4)(1.6) = 805.92 \text{ mm}$ . Length available = 531 mm < 805.92 mm. The bars shall be given 90° bend and then shall be extended by 200 mm horizontally to give a total length of  $531 + 8(20)$  (bend value) + 200 = 891 mm > 805.92 mm.

The arrangement of reinforcing bars is shown in Fig.11.29.11a.

## 11.29.6 Summary of this Lesson

This lesson illustrates the applications of the requirements for the design of foundations, explained in Lesson 28. Five illustrative examples, two practice problems and one test problem include plain concrete footings, isolated footings of square and rectangular columns transferring axial loads with or without moments to the footings, wall footings and combined footings. The solutions of illustrative problems and practice of the practice and test problems will give a clear understanding of the design of footings under different circumstances and soil conditions. All the designs are done following the stipulations of IS codes. It is, however, worth mentioning that all the footings explained in this lesson are shallow foundations. The design of deep foundations is beyond the scope of this course.