Module 10

Compression Members

Lesson 22

Short Axially Loaded Compression Members

Instructional Objectives:

At the end of this lesson, the student should be able to:

- state additional assumptions regarding the strengths of concrete and steel for the design of short axially loaded columns,
- specify the values of design strengths of concrete and steel,
- derive the governing equation for the design of short and axially loaded tied columns,
- derive the governing equation for the design of short and axially loaded spiral columns,
- derive the equation to determine the pitch of helix in spiral columns,
- apply the respective equations to design the two types of columns by direct computation,
- use the charts of SP-16 to design these two types of columns subjected to axial loads as per IS code.

10.22.1 Introduction

Tied and helically bound are the two types of columns mentioned in sec.10.21.3 of Lesson 21. These two types of columns are taken up in this lesson when they are short and subjected to axially loads. Out of several types of plan forms, only rectangular and square cross-sections are covered in this lesson for the tied columns and circular cross-section for the helically bound columns. Axially loaded columns also need to be designed keeping the provision of resisting some moments which normally is the situation in most of the practical columns. This is ensured by checking the minimum eccentricity of loads applied on these columns as stipulated in IS 456. Moreover, the design strengths of concrete and steel are further reduced in the design of such columns. The governing equations of the two types of columns and the equation for determining the pitch of the helix in continuously tied column are derived and explained. The design can be done by employing the derived equation i.e., by direct computation or by using the charts of SP-16. Several numerical examples are solved to explain the design of the two types of columns by direct computation and using the charts of SP-16.

10.22.2 Further Assumptions Regarding the Strengths of Concrete and Steel

All the assumptions required for the derivation of the governing equations are given in sec.10.21.11 of Lesson 21. The stress-strain diagrams of mild steel (Fe 250) and cold worked deformed bars (Fe 415 and Fe 500) are given in Figs.1.2.3 and 4, respectively of Lesson 2. The stress block of compressive part of concrete is given in Fig.3.4.1.9 of Lesson 4, which is used in the design of beam by limit state of collapse. The maximum design strength of concrete is shown as constant at 0.446 f_{ck} when the strain ranges from 0.002 to 0.0035. The maximum design stress of steel is 0.87 f_y.

Sections 10.21.4 and 12 of Lesson 21 explain that all columns including the short axially loaded columns shall be designed with a minimum eccentricity (cls. 25.4 and 39.2 of IS 456). Moreover, the design strengths of concrete and steel are further reduced to 0.4 f_{ck} and 0.67 f_y, respectively, to take care of the minimum eccentricity of 0.05 times the lateral dimension, as stipulated in cl.39.3 of IS 456. It is noticed that there is not attempt at strain compatibility. Also the phenomenon of creep has not been directly considered.

$$e_{x\,min} \geq \text{ greater of } (l/500 + D/30) \text{ or } 20 \text{ mm}$$

(10.3)

$$e_{y\,min} \geq \text{ greater of } (l/500 + b/30) \text{ or } 20 \text{ mm}$$

The maximum values of l_{ex}/D and l_{ey}/b should not exceed 12 in a short column as per cl.25.1.2 of IS 456. For a short column, when the unsupported length l = l_{ex} (for the purpose of illustration), we can assume l = 12 D (or 12b when b is considered). Thus, we can write the minimum eccentricity = 12D/500 + D/30 = 0.057D, which has been taken as 0.05D or 0.05b as the maximum amount of eccentricity of a short column.

It is, therefore, necessary to keep provision so that the short columns can resist the accidental moments due to the allowable minimum eccentricity by lowering the design strength of concrete by ten per cent from the value of 0.446f_{ck}, used for the design of flexural members. Thus, we have the design strength of concrete in the design of short column as (0.9)(0.446f_{ck}) = 0.4014f_{ck}, say 0.40 f_{ck}. The reduction of the design strength of steel is explained below.

For mild steel (Fe 250), the design strength at which the strain is 0.002 is f_y/1.15 = 0.87f_y. However, the design strengths of cold worked deformed bars (Fe 415 and Fe 500) are obtained from Fig.1.2.4 of Lesson 2 or Fig.23A of IS 456. Table A of SP-16 presents the stresses and corresponding strains of Fe 415 and Fe 500. Use of Table A of SP-16 is desirable as it avoids error while reading from figures (Fig.1.2.4 or Fig.23A, as mentioned above). From Table A of SP-16, the corresponding design strengths are obtained by making linear interpolation. These values of design strengths for which the strain is 0.002 are as follows:

(i) Fe 415: $\{0.9f_{yd} + 0.05f_{yd}(0.002 – 0.00192)/(0.00241 – 0.00192)\} = 0.908f_{yd} = 0.789f_y$

(ii) Fe 500: $\{0.85f_{yd} + 0.05f_{yd}(0.002 – 0.00195)/(0.00226 – 0.00195)\} = 0.859f_{yd} = 0.746f_y$

A further reduction in each of three values is made to take care of the minimum eccentricity as explained for the design strength of concrete. Thus, the acceptable design strength of steel for the three grades after reducing 10 per cent from the above mentioned values are $0.783f_y$, $0.710f_y$ and $0.671f_y$ for Fe 250, Fe 415 and Fe 500, respectively. Accordingly, cl. 39.3 of IS 456 stipulates $0.67f_y$ as the design strength for all grades of steel while designing the short columns. Therefore, the assumed design strengths of concrete and steel are $0.4f_{ck}$ and $0.67f_y$, respectively, for the design of short axially loaded columns.

10.22.3 Governing Equation for Short Axially Loaded Tied Columns

Factored concentric load applied on short tied columns is resisted by concrete of area A_c and longitudinal steel of areas A_{sc} effectively held by lateral ties at intervals (Fig.10.21.2a of Lesson 21). Assuming the design strengths of concrete and steel are $0.4f_{ck}$ and $0.67f_y$, respectively, as explained in sec. 10.22.2, we can write

$$P_u = 0.4f_{ck} A_c + 0.67f_y A_{sc} \tag{10.4}$$

where P_u = factored axial load on the member,

f_{ck} = characteristic compressive strength of the concrete,

A_c = area of concrete,

f_y = characteristic strength of the compression reinforcement, and

A_{sc} = area of longitudinal reinforcement for columns.

The above equation, given in cl. 39.3 of IS 456, has two unknowns Ac and A_{sc} to be determined from one equation. The equation is recast in terms of A_g, the gross area of concrete and p, the percentage of compression reinforcement employing

$$A_{sc} = pA_g/100 \tag{10.5}$$

$$A_c = A_g(1 - p/100) \tag{10.6}$$

Accordingly, we can write

$$P_u/A_g = 0.4f_{ck} + (p/100)\,(0.67f_y - 0.4f_{ck}) \tag{10.7}$$

Equation 10.7 can be used for direct computation of A_g when P_u, f_{ck} and f_y are known by assuming p ranging from 0.8 to 4 as the minimum and maximum percentages of longitudinal reinforcement. Equation 10.4 also can be employed to determine A_g and p in a similar manner by assuming p. This method has been illustrated with numerical examples and is designated as Direct Computation Method.

On the other hand, SP-16 presents design charts based on Eq.10.7. Each chart of charts 24 to 26 of SP-16 has lower and upper sections. In the lower section, P_u/A_g is plotted against the reinforcement percentage p(= $100A_s/A_g$) for different grades of concrete and for a particular grade of steel. Thus, charts 24 to 26 cover the three grades of steel with a wide range of grades of concrete. When the areas of cross-section of the columns are known from the computed value of P_u/A_g, the percentage of reinforcement can be obtained directly from the lower section of the chart. The upper section of the chart is a plot of P_u/A_g versus P_u for different values of A_g. For a known value of P_u, a horizontal line can be drawn in the upper section to have several possible A_g values and the corresponding P_u/A_g values. Proceeding vertically down for any of the selected P_u/A_g value, the corresponding percentage of reinforcement can be obtained. Thus, the combined use of upper and lower sections of the chart would give several possible sizes of the member and the corresponding A_{sc} without performing any calculation. It is worth mentioning that there may be some parallax error while using the charts. However, use of chart is very helpful while deciding the sizes of columns at the preliminary design stage with several possible alternatives.

Another advantage of the chart is that, the amount of compression reinforcement obtained from the chart are always within the minimum and maximum percentages i.e., from 0.8 to 4 per cent. Hence, it is not needed to examine if the computed area of steel reinforcement is within the allowable range as is needed while using Direct Computation Method. This method is termed as SP-16 method while illustrating numerical examples.

10.22.4 Governing Equation of Short Axially Loaded Columns with Helical Ties

Columns with helical reinforcement take more load than that of tied columns due to additional strength of spirals in contributing to the strength of columns. Accordingly, cl. 39.4 recommends a multiplying factor of 1.05 regarding the strength of such columns. The code further recommends that the ratio of volume of helical reinforcement to the volume of core shall not be less than 0.36 (A_g/A_c – 1) (f_{ck}/f_y), in order to apply the additional strength factor of 1.05 (cl. 39.4.1). Accordingly, the governing equation of the spiral columns may be written as

$$P_u = 1.05(0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}) \tag{10.8}$$

All the terms have been explained in sec.10.22.3.

Earlier observations of several investigators reveal that the effect of containing holds good in the elastic stage only and it gets lost when spirals reach the yield point. Again, spirals become fully effective after spalling off the concrete cover over the spirals due to excessive deformation. Accordingly, the above two points should be considered in the design of such columns. The first point is regarding the enhanced load carrying capacity taken into account by the multiplying factor of 1.05. The second point is maintaining specified ratio of volume of helical reinforcement to the volume of core, as specified in cl.39.4.1 and mentioned earlier.

The second point, in fact, determines the pitch p of the helical reinforcement, as explained below with reference to Fig.10.21.2b of Lesson 21.

$$\text{Volume of helical reinforcement in one loop} = \pi(D_c - \phi_{sp})\, a_{sp} \tag{10.9}$$

$$\text{Volume of core} = (\pi/4)\, D_c^2\, p \tag{10.10}$$

where D_c = diameter of the core (Fig.10.21.2b)

ϕ_{sp} = diameter of the spiral reinforcement (Fig.10.21.2b)

a_{sp} = area of cross-section of spiral reinforcement

p = pitch of spiral reinforcement (Fig.10.21.2b)

To satisfy the condition of cl.39.4.1 of IS 456, we have

$$\{\pi(D_c - \phi_{sp})\, a_{sp}\}/\{(\pi/4)\, D_c^2\ p\} \geq 0.36(A_g / A_c - 1)\,(f_{ck} / f_y)$$

which finally gives

$$p \leq 11.1(D_c - \phi_{sp})\, a_{sp}\ f_y /(D^2 - D_c^2)\ f_{ck}$$

(10.11)

Thus, Eqs.10.8 and 11 are the governing equations to determine the diameter of column, pitch of spiral and area of longitudinal reinforcement. It is worth mentioning that the pitch *p* of the spiral reinforcement, if determined from Eq.10.11, automatically satisfies the stipulation of cl.39.4.1 of IS 456. However, the pitch and diameter of the spiral reinforcement should also satisfy cl. 26.5.3.2 of IS 456:2000.

10.22.5 Illustrative Examples

Problem 1:

Design the reinforcement in a column of size 400 mm x 600 mm subjected to an axial load of 2000 kN under service dead load and live load. The column has an unsupported length of 4.0 m and effectively held in position and restrained against rotation in both ends. Use M 25 concrete and Fe 415 steel.

Solution 1:

Step 1: To check if the column is short or slender

Given l = 4000 mm, b = 400 mm and D = 600 mm. Table 28 of IS 456 = $l_{ex} = l_{ey}$ = 0.65(l) = 2600 mm. So, we have

l_{ex}/D = 2600/600 = 4.33 < 12

l_{ey}/b = 2600/400 = 6.5 < 12

Hence, it is a short column.

Step 2: Minimum eccentricity

$e_{x\,min}$ = Greater of (l_{ex}/500 + D/30) and 20 mm = 25.2 mm

$e_{y\,min}$ = Greater of (l_{ey}/500 + b/30) and 20 mm = 20 mm

0.05 D = 0.05(600) = 30 mm > 25.2 mm (= $e_{x\,min}$)

$0.05\, b = 0.05(400) = 20$ mm $= 20$ mm $(= e_{y\,min})$

Hence, the equation given in cl.39.3 of IS 456 (Eq.10.4) is applicable for the design here.

Step 3: Area of steel

Fro Eq.10.4, we have

$$P_u = 0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc} \quad \ldots. \quad (10.4)$$

$$3000(10^3) = 0.4(25)\{(400)(600) - A_{sc}\} + 0.67(415)\, A_{sc}$$

which gives,

$$A_{sc} = 2238.39 \text{ mm}^2$$

Provide 6-20 mm diameter and 2-16 mm diameter rods giving 2287 mm^2 (> 2238.39 mm^2) and p = 0.953 per cent, which is more than minimum percentage of 0.8 and less than maximum percentage of 4.0. Hence, o.k.

Step 4: Lateral ties

The diameter of transverse reinforcement (lateral ties) is determined from cl.26.5.3.2 C-2 of IS 456 as not less than (i) $\phi/4$ and (ii) 6 mm. Here, ϕ = largest bar diameter used as longitudinal reinforcement = 20 mm. So, the diameter of bars used as lateral ties = 6 mm.

The pitch of lateral ties, as per cl.26.5.3.2 C-1 of IS 456, should be not more than the least of

(i) the least lateral dimension of the column = 400 mm

(ii) sixteen times the smallest diameter of longitudinal reinforcement bar to be tied = 16(16) = 256 mm

(iii) 300 mm

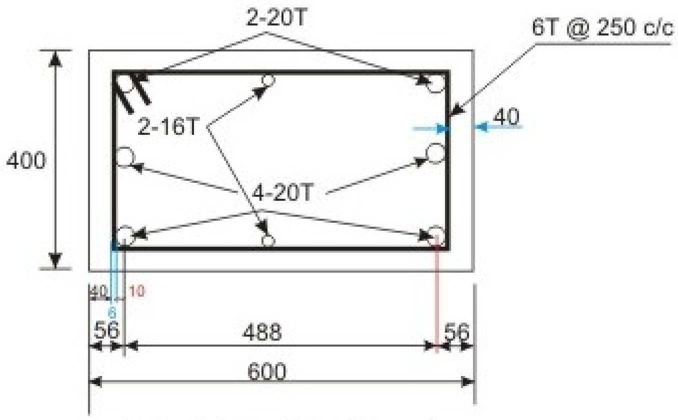

Fig. 10.22.1: Problem 1

Let us use p = pitch of lateral ties = 250 mm. The arrangement of longitudinal and transverse reinforcement of the column is shown in Fig. 10.22.1.

Problem 2:

Design the column of Problem 1 employing the chart of SP-16.

Solution 2:

Steps 1 and 2 are the same as those of Problem 1.

Step 3: Area of steel

$P_u/A_g = 3000(10^3)/(600)(400) = 12.5 \text{ N/mm}^2$

From the lower section of Chart 25 of SP-16, we get p = 0.95% when P_u/A_g = 12.5 N/mm^2 and concrete grade is M 25. This gives A_{sc} = 0.95(400)(600)/100 = 2288 mm^2. The results of both the problems are in good agreement. Marginally higher value of A_{sc} while using the chart is due to parallax error while reading the value from the chart. Here also, 6-20 mm diameter bars + 2-16 mm diameter bars (A_{sc} provided = 2287 mm^2) is o.k., though it is 1 mm^2 less.

Step 4 is the same as that of Problem 1. Figure 10.22.1, thus, is also the figure showing the reinforcing bars (longitudinal and transverse reinforcement) of this problem (same column as that of Problem 1).

Problem 3:

Design a circular column of 400 mm diameter with helical reinforcement subjected to an axial load of 1500 kN under service load and live load. The column has an unsupported length of 3 m effectively held in position at both ends but not restrained against rotation. Use M 25 concrete and Fe 415 steel.

Solution 3:

Step 1: To check the slenderness ratio

Given data are: unsupported length l = 3000 mm, D = 400 mm. Table 28 of Annex E of IS 456 gives effective length $l_e = l$ = 3000 mm. Therefore, l_e/D = 7.5 < 12 confirms that it is a short column.

Step 2: Minimum eccentricity

e_{min} = Greater of (l/500 + D/30) or 20 mm = 20 mm

0.05 D = 0.05(400) = 20 mm

As per cl.39.3 of IS 456, e_{min} should not exceed 0.05D to employ the equation given in that clause for the design. Here, both the eccentricities are the same. So, we can use the equation given in that clause of IS 456 i.e., Eq.10.8 for the design.

Step 3: Area of steel

From Eq.10.8, we have

$$P_u = 1.05(0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}) \qquad \ldots \;(10.8)$$

$$A_c = A_g - A_{sc} = 125714.29 - A_{sc}$$

Substituting the values of P_u, f_{ck}, A_g and f_y in Eq.10.8,

$$1.5(1500)(10^3) = 1.05\{0.4(25)(125714.29 - A_{sc}) + 0.67(415)\, A_{sc}\}$$

we get the value of A_{sc} = 3304.29 mm^2. Provide 11 nos. of 20 mm diameter bars (= 3455 mm^2) as longitudinal reinforcement giving p = 2.75%. This p is between 0.8 (minimum) and 4 (maximum) per cents. Hence o.k.

Step 4: Lateral ties

It has been mentioned in sec.10.22.4 that the pitch p of the helix determined from Eq.10.11 automatically takes care of the cl.39.4.1 of IS 456. Therefore, the pitch is calculated from Eq.10.11 selecting the diameter of helical reinforcement from cl.26.5.3.2 d-2 of IS 456. However, automatic satisfaction of cl.39.4.1 of IS 456 is also checked here for confirmation.

Diameter of helical reinforcement (cl.26.5.3.2 d-2) shall be not less than greater of (i) one-fourth of the diameter of largest longitudinal bar, and (ii) 6 mm.

Therefore, with 20 mm diameter bars as longitudinal reinforcement, the diameter of helical reinforcement = 6 mm.

From Eq.10.11, we have

Pitch of helix $p \le 11.1(D_c - \phi_{sp})\, a_{sp}\, f_y/(D^2 - D_c^2)\, f_{ck}$ … (10.11)

where $D_c = 400 - 40 - 40 = 320$ mm, $\phi_{sp} = 6$ mm, $a_{sp} = 28$ mm^2, $f_y = 415$ N/mm^2, $D = 400$ mm and $f_{ck} = 25$ N/mm^2.

So, $p \le 11.1(320 - 6)(28)(415)/(400^2 - 320^2)(25) \le 28.125$ mm

As per cl.26.5.3.2 d-1, the maximum pitch is the lesser of 75 mm and 320/6 = 53.34 mm and the minimum pitch is lesser of 25 mm and 3(6) = 18 mm. We adopt pitch = 25 mm which is within the range of 18 mm and 53.34 mm. So, provide 6 mm bars @ 25 mm pitch forming the helix.

Checking of cl. 39.4.1 of IS 456

The values of helical reinforcement and core in one loop are obtained from Eqs.10.8 and 9, respectively. Substituting the values of D_c, ϕ_{sp}, a_{sp} and pitch p in the above two equations, we have

Volume of helical reinforcement in one loop = 27632 mm^3 and

Volume of core in one loop = 2011428.571 mm^3.

Their ratio = 27632/2011428.571 = 0.0137375

$0.36(A_g/A_c - 1)(f_{ck}/f_y) = 0.012198795$

It is, thus, seen that the above ratio (0.0137375) is not less than $0.36(A_g/A_c - 1)(f_{ck}/f_y)$.

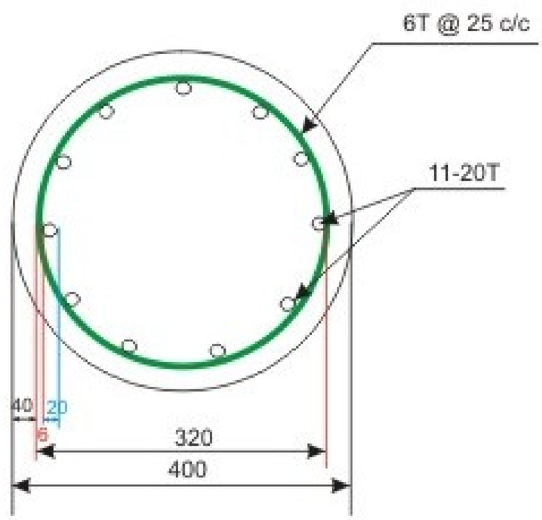

Fig. 10.22.2: Problem 3

Hence, the circular column of diameter 400 mm has eleven longitudinal bars of 20 mm diameter and 6 mm diameter helix with pitch p = 25 mm. The reinforcing bars are shown in Fig.10.22.2.

10.22.6 Practice Questions and Problems with Answers

Q.1: State and explain the values of design strengths of concrete and steel to be considered in the design of axially loaded short columns.

A.1: See sec. 10.22.2.

Q.2: Derive the governing equation for determining the dimensions of the column and areas of longitudinal bars of an axially loaded short tied column.

A.2: See sec. 10.22.2.

Q.3: Derive the governing equation for determining the diameter and areas of longitudinal bars of an axially loaded circular spiral short column.

A.3: First and second paragraph of sec. 10.22.4.

Q.4: Derive the expression of determining the pitch of helix in a short axially loaded spiral column which satisfies the requirement of IS 456.

A.4: See third paragraph onwards up to the end of sec. 10.22.4.

Q.5: Design a short rectangular tied column of b = 300 mm having the maximum amount of longitudinal reinforcement employing the equation given in cl.39.3 of IS 456, to carry an axial load of 1200 kN under service dead load and live load using M 25 and Fe 415. The column is effectively held in position at both ends and restrained against rotation at one end. Determine the unsupported length of the column.

A.5:

Step 1: Dimension *D* and area of steel A_{sc}

Substituting the values of P_u = 1.5(1200) = 1800 kN and A_{sc} = 0.04(300)D in Eq.10.4, we have

$$1800(10^3) = 0.4(25)(300D)(1 - 0.04) + 0.67(415)(0.04)(300D)$$

we get D = 496.60 mm. Use 300 mm x 500 mm column.

A_{sc} = 0.04(300)(500) = 6000 mm^2, provide 4-36 mm diameter + 4-25 mm diameter bars to give 4071 + 1963 = 6034 mm^2 > 6000 mm^2.

Step 2: Lateral ties

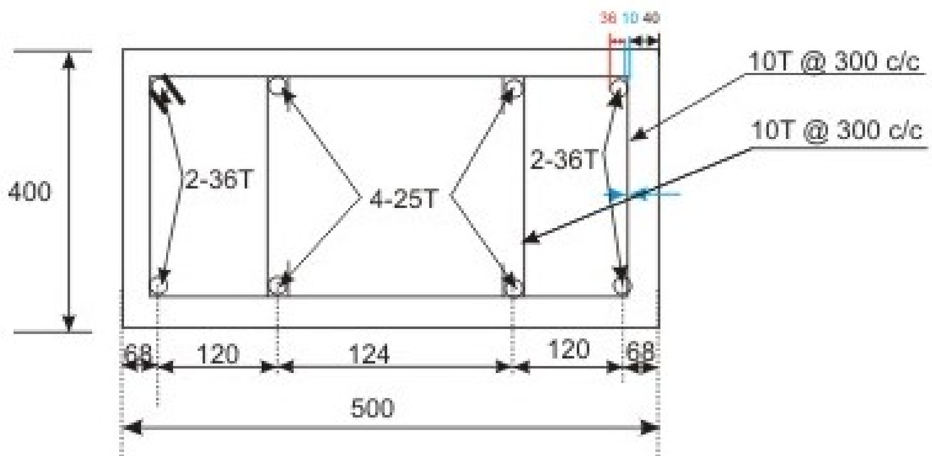

Fig. 10.22.3: Q.5

Diameter of lateral ties shall not be less than the larger of (i) 36/4 = 9 mm and (ii) 6 mm. Use 10 mm diameter bars as lateral ties.

Pitch of the lateral ties p shall not be more than the least of (i) 300 mm, (ii) 16(25) = 400 mm and (iii) 300 mm.

So, provide 10 mm diameter bars @ 300 mm c/c. The reinforcement bars are shown in Fig.10.22.3.

The centre to centre distance between two corner longitudinal bas along 500 mm direction is 500 – 2(4) + 10 + 18) = 364 mm which is less than 48 (diameter of lateral tie). Hence, the arrangement is satisfying Fig.9 of cl. 26.5.3.2 b-2 of IS 456.

Step 3: Unsupported length

As per the stipulation in cl. 25.1.2 of IS 456, the column shall be considered as short if l_{ex} = 12(D) = 6000 mm and l_{ey} = 12(300) = 3600 mm. For the type of support conditions given in the problem, Table 28 of IS 456 gives unsupported length is the least of (i) $l = l_{ex}/0.8$ = 6000/0.8 = 7500 mm and (ii) $l_{ey}/0.8$ = 3600/0.8 = 4500 mm. Hence, the unsupported length of the column is 4.5 m if the minimum eccentricity clause (cl. 39.3) is satisfied, which is checked in the next step.

Step 4: Check for minimum eccentricity

According to cl. 39.4 of IS 456, the minimum eccentricity of 0.05b or 0.05D shall not exceed as given in cl. 25.4 of IS 456. Thus, we have

(i) 0.05(500) = l/500 + 500/30 giving l = 4165 mm

(ii) 0.05(300) = l/500 + 300/10 giving l = 2500 mm

Therefore, the column shall have the unsupported length of 2.5 m.

Q.6: (a) Suggest five alternative dimensions of square short column with the minimum longitudinal reinforcement to carry a total factored axial load of 3000 kN using concrete of grades 20, 25, 30, 35 and 40 and Fe 415. Determine the respective maximum unsupported length of the column if it is effectively held in position at both ends but not restrained against rotation. Compare the given factored load of the column with that obtained by direct computation for all five alternative columns.

(b) For each of the five alternative sets of dimensions obtained in (a), determine the maximum factored axial load if the column is having maximum longitudinal reinforcement (i) employing SP-16 and (ii) by direct computation.

A.6:

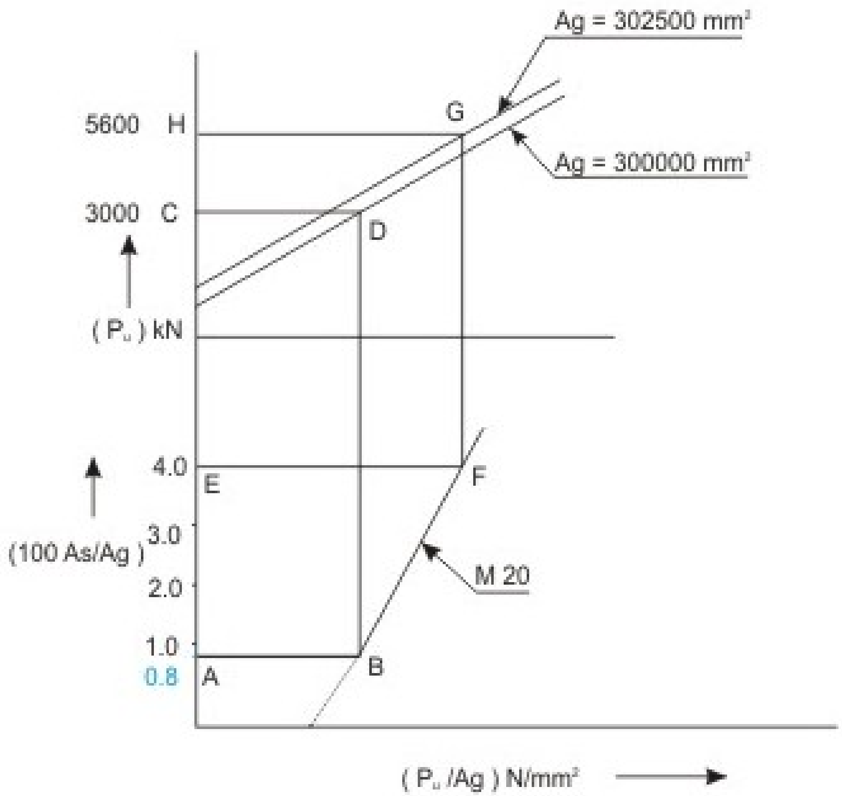

Fig. 10.22.4: Q.6, Chart 25 of SP-16 (not to scale)

Solution of Part (a):

Step 1: Determination of A_g and column dimensions b (= D)

Chart 25 of SP-16 gives all the dimensions of five cases. The two input data are P_u = 3000 kN and 100 A_s/A_g = 0.8. In the lower section of Chart 25, one horizontal line AB is drawn starting from A where p = 0.8 (Fig.10.22.4) to meet the lines for M 20, 25, 30, 35 and 40 respectively. In Fig.10.22.4, B is the meeting point for M 20 concrete. Separate vertical lines are drawn from these points of intersection to meet another horizontal line CD from the point C where P_u = 3000 kN in the upper section of the figure. The point D is the intersecting point. D happens to be on line when A_g = 3000 cm^2. Otherwise, it may be in between two liens with different values of A_g. For M 20, A_g = 3000 cm^2. However, in case the point is in between two lines with different values of A_g, the particular A_g has to be computed by linear interpolation. Thus, all five values of A_g are obtained.

The dimension b = D = $\sqrt{300000}$ = 550 mm. Other four values are obtained similarly. Table 10.1 presents the values of A_g and D along with other results as explained below.

Step 2: Unsupported length of each column

The unsupported length l is determined from two considerations:

(i) Clause 25.1.2 of IS 456 mentions that the maximum effective length l_{ex} is 12 times b or D (as $b = D$ here for a square column). The unsupported length is related to the effective length depending on the type of support. In this problem Table 28 of IS 456 stipulates $l = l_{ex}$. Therefore, maximum value of $l = 12\ D$.

(ii) The minimum eccentricity of cl. 39.3 should be more than the same as given in cl. 25.4. Assuming them to be equal, we get $l/500 + D/30 = D/20$, which gives $l = 8.33D$. For the column using M 20 and Fe 415, the unsupported length = 8.33(550) = 4581 mm. All unsupported lengths are presented in Table 10.1 using the equation

$$l = 8.33\ D \quad (1)$$

Step 3: Area of longitudinal steel

Step 1 shows that the area provided for the first case is 550 mm x 550 mm = 302500 mm^2, slightly higher than the required area of 300000 mm^2 for the practical aspects of construction. However, the minimum percentage of the longitudinal steel is to the calculated as 0.8 per cent of area required and not area provided (vide cl. 26.5.3.1 b of IS 456). Hence, for this case A_{sc} = 0.8(300000)/100 = 2400 mm^2. Provide 4-25 mm diameter + 4-12 mm diameter bars (area = 1963 + 452 = 2415 mm^2). Table 10.1 presents this and other areas of longitudinal steel obtained in a similar manner.

Step 4: Factored load by direct computation

Equation 10.4 is employed to calculate the factored load by determining A_c from A_g and A_{st}. With a view to comparing the factored loads, we will use the values of A_g as obtained from the chart and not the A_g actually provided. From Eq.10.4, we have

$$P_u \text{ from direct computation} = 0.4(f_{ck})(0.992\ A_g) + 0.67(f_y)(0.008)A_g$$

or $$P_u = A_g(0.3968\ f_{ck} + 0.00536\ f_y) \quad (2)$$

For the first case when A_g = 300000 mm^2, f_{ck} = 20 N/mm^2, and f_y = 415 N/mm^2, Eq.(2) gives P_u = 3048.12 kN. This value and other values of factored loads obtained from the direct computation are presented in Table 10.1.

Table 10.1 Results of Q.6a (Minimum Longitudinal Steel), given factored P_u = 3000 kN

Concrete grade	Gross area of concrete (A_g)		$b = D$ (cm)	Area of steel (A_{sc})			P_u by direct computation	l (m)
	Required (cm^2)	Provided (cm^2)		Required (cm^2)	Provided (cm^2)	Bars		
M 20	3000	3025	55	24	24.15	4-25 + 4-12	3048.12	4.581
M 25	2500	2500	50	20	20.60	4-20 + 4-16	3036.10	4.165
M 30	2200	2209	47	17.60	17.85	2.25 + 4-16	3108.25	3.915
M 35	1800	1806	42.5	14.40	14.57	2-28 + 2-12	2900.23	3.540
M 40	1600	1600	40	12.80	13.06	2-20 + 6-12	2895.42	3.332

Solution of Part (b):

Step 1: Determination of P_u

Due to the known dimensions of the column section, the A_g is now known. With known A_g and reinforcement percentage $100A_s/A_g$ as 4 per cent, the factored P_u shall be determined. For the first case, when $b = D =$ 550 mm, A_g = 302500 mm^2. In Chart 25, we draw a horizontal line EF from E, where $100A_s/A_g$ = 4 in the lower section of the chart (see Fig.10.22.4) to meet the M 20 line at F. Proceeding vertically upward, the line FG intersects the line A_g = 302500 at G. A horizontal line towards left from G, say GH, meets the load axis at H where P_u = 5600 kN. Similarly, P_u for other sets are determined and these are presented in Table 10.2, except for the last case when M 40 is used, as this chart has ended at p = 3.8 per cent.

Step 2: Area of longitudinal steel

The maximum area of steel, 4 per cent of gross area of column = 0.04(550)(550) = 12100 mm^2. Provide 12-36 mm diameter bars to have the actual area of steel = 12214 mm^2 > 12100 mm^2, as presented in Table 10.2.

Step 3: Factored P_u from direct computation

From Eq,10.4, as in Step 4 of the solution of Part (a) of this question, we have

$$P_u = 0.4 f_{ck} (A_g - A_{sc}) + 0.67 f_y A_{sc} \qquad (3)$$

Substituting the values of A_g and A_{sc} actually provided, we get the maximum P_u of the same column when the longitudinal steel is the maximum. For the first case when A_g = 302500 mm^2, A_{sc} = 12214 mm^2, f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2, we get P_u = 5718.4 kN. This value along with other four values are presented in Table 10.2.

Remarks:

Tables 10.1 and 10.2 reveal that two sets of results obtained from charts of SP-16 and by direct computation methods are in good agreement. However, values obtained from the chart are marginally different from those obtained by direct computation both on the higher and lower sides. These differences are mainly due to personal error (parallax error) while reading the values with eye estimation from the chart.

Table 10.2 Results of Q.6(b) (Maximum Longitudinal Steel) given the respective A_g

Concrete grade	$b = D$ (cm)	Gross concrete area (A_s) (cm^2)	Area of steel (A_{sc})			P_u = Factored load	
			Required (cm^2)	Provided (cm^2)	Bars (No.)	SP-chart (kN)	Direct Computation (kN)
M 20	55	3025	121	122.14	12-36	5600	5718.4
M 25	50	2500	100	101.06	8-36 + 4-25	5200	5208.9
M 30	47	2209	88.36	88.97	8-32 + 4-28	5000	5017.8
M 35	42.5	1806.25	72.25	73.69	12-28	4500	4474.5
M 40	40	1600	64	64.46	8-28 + 4-32	Not available	4249.2

Q.7: Design a short, helically reinforced circular column with minimum amount of longitudinal steel to carry a total factored axial load of 3000 kN with the same support condition as that of Q.6, using M 25 and Fe 415. Determine its unsupported length. Compare the results of the dimension and area of longitudinal steel with those of Q.6(a) when M 25 and Fe 415 are used.

A.7:

Step 1: Diameter of helically reinforced circular column

As per cl. 39.4 of IS 456, applicable for short spiral column, we get from Eq.10.8

$$P_u = 1.05(0.4 f_{ck} A_c + 0.67 f_y A_{sc}) \qquad .... \; (10.8)$$

Given data are: P_u = 3000 kN, $A_c = \pi/4 \, (D^2)(0.992)$, $A_{sc} = 0.008(\pi/4) \, D^2$, f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2. So, we have

$$3000(10^3) = 1.05(12.1444)(\pi/4)D^2$$

giving D = 547.2 mm and A_g = 235264.2252 mm^2. Provide diameter of 550 mm.

Step 2: Area of longitudinal steel

Providing 550 mm diameter, the required A_g has been exceeded. Clause 26.5.3.1b stipulates that the minimum amount of longitudinal bar shall be determined on the basis of area required and not area provided for any column. Accordingly, the area of longitudinal steel = 0.008 A_g = 0.008(235264.2252) = 1882.12 mm^2. Provide 6-20 mm diameter bars (area = 1885 mm^2) as longitudinal steel, satisfying the minimum number of six bar (cl. 26.5.3.1c of IS 456).

Step 3: Helical reinforcement

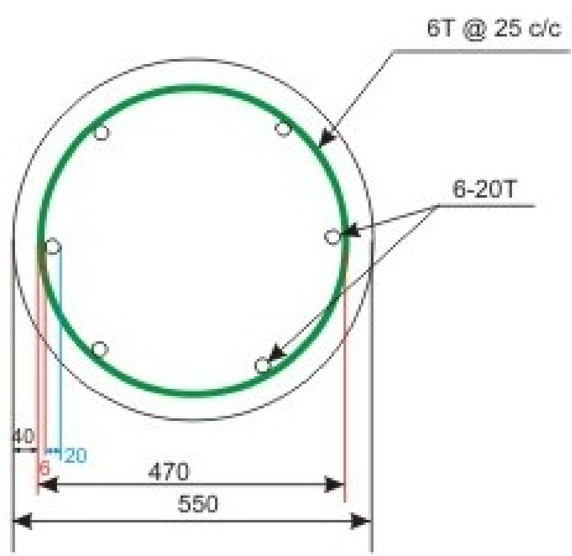

Fig. 10.22.5: Q.7

Minimum diameter of helical reinforcement is greater of (i) 20/4 or (ii) 6 mm. So, provide 6 mm diameter bars for the helical reinforcement (cl. 26.5.3.2d-2 of IS 456). The pitch of the helix p is determined from Eq.10.11 as follows:

$$p \le 11.1(D_c - \phi_{sp}) \, a_{sp} \, f_y/(D^2 - D_c^2) \, f_{ck} \qquad .... \; (10.11)$$

Using D_c = 550 – 40 – 40 = 470 mm, ϕ_{sp} = 6 mm, a_{sp} = 28 mm^2, D = 550 mm, f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2, we get

$$p \le 11.1(470 - 6)(28)(415)/(550^2 - 470^2)(25) \le 29.34 \text{ mm}$$

Provide 6 mm diameter bar @ 25 mm c/c as helix. The reinforcement bars are shown in Fig. 10.22.5. Though use of Eq.10.11 automatically checks the stipulation of cl. 39.4.1 of IS 456, the same is checked as a ready reference in Step 4 below.

Step 4: Checking of cl. 39.4.1 of IS 456

Volume of helix in one loop = $\pi(D_c - \phi_{sp})\, a_{sp}$ …. (10.9)

Volume of core = $(\pi/4)\, D_c^2\, (p)$ …. (10.10)

The ratio of Eq.10.9 and Eq.10.10 = $4(D_c - \phi_{sp})\, a_{sp}/D_c^2\, p$

= 4(470 – 6)(28)/(470)(470)(25) = 0.009410230874

This ratio should not be less than $0.36(A_g/A_c - 1)(f_{ck}/f_y)$

= $0.36\{(D^2/D_c^2) - 1)\}\,(f_{ck}/f_y)$ = 0.008011039177

Hence, the stipulation of cl. 39.4.1 is satisfied.

Step 5: Unsupported length

The unsupported length shall be the minimum of the two obtained from (i) short column requirement given in cl. 25.1.2 of IS 456 and (ii) minimum eccentricity requirement given in cls. 25.4 and 39.3 of IS 456. The two values are calculated separately:

(i) $l = l_e = 12D = 12(550) = 6600$ mm

(ii) $l/500 + D/30 = 0.05\, D$ gives l = 4583.3 mm

So, the unsupported length of this column = 4.58 m.

Step 6: Comparison of results

Table 10.3 presents the results of required and actual gross areas of concrete and area of steel bars, dimensions of column and number and diameter of longitudinal reinforcement of the helically reinforced circular and the square columns of Q.6(a) when M 20 and Fe 415 are used for the purpose of comparison.

Table 10.3 Comparison of results of circular and square columns with minimum longitudinal steel (P_u = 3000 kN, M 25, Fe 415)

Column shape and Q.No.	Gross concrete area			Area of steel		
	Required (cm^2)	Provided (cm^2)	Dimension *D* (cm)	Required (cm^2)	Provided (cm^2)	Bar dia. and No. (mm,No.)
Circular (Q.7)	2352.64	2376.78	55	18.82	18.85	6-20
Square (Q.6(a)	2500	2500	50	20	20.6	4-20 + 4-16

10.22.7 References

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14. Reinforced Concrete Designer's Handbook, 10th Edition, by C.E.Reynolds and J.C.Steedman, E & FN SPON, London, 1997.
15. Indian Standard Plain and Reinforced Concrete – Code of Practice (4th Revision), IS 456: 2000, BIS, New Delhi.
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10.22.8 Test 22 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions carrying equal marks.

TQ.1: Derive the expression of determining the pitch of helix in a short axially loaded spiral column which satisfies the requirement of IS 456. (20 marks)

A.TQ.1: See third paragraph onwards up to the end of sec. 10.22.4.

TQ.2: Design a square, short tied column of $b = D = 500$ mm to carry a total factored load of 4000 kN using M 20 and Fe 415. Draw the reinforcement diagram.

(30 marks)

A.TQ.2:

Step 1: Area of longitudinal steel

Assuming p as the percentage of longitudinal steel, we have A_c = (500)(500)(1 – 0.01p), A_{sc} = (500)(500)(0.01p), f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2. Using these values in Eq.10.4

$$P_u = 0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{st} \qquad \text{…. (10.4)}$$

or $$4000000 = 0.4(20)(250000)(1 - 0.01p) + 0.67(415)(250000)(0.01p)$$

we get p = 2.9624, which gives A_{sc} = (2.9624)(500)(500)/100 = 7406 mm^2. Use 4-36 + 4-25 + 4-22 mm diameter bars (4071 + 1963 + 1520) = 7554 mm^2 > 7406 mm^2 as longitudinal reinforcement.

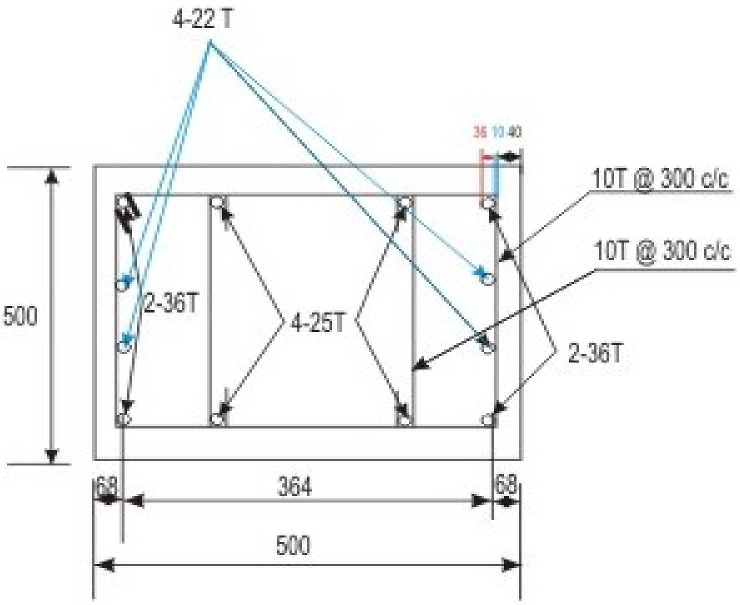

Fig. 10.22.6: TQ.2

Step 2: Lateral ties

Diameter of tie is the greater of (i) 36/4 and (ii) 6 mm. Provide 10 mm diameter lateral ties.

The pitch of the lateral ties is the least of (i) 500 mm, (ii) 16(22) = 352 mm, and (iii) 300 mm. Provide 10 mm diameter @ 300 mm c/c. The reinforcement bars are shown in Fig.10.22.6. It is evident that the centre to centre distance between two corner bars = 364 mm is less than 48 times the diameter of lateral ties = 480 mm (Fig.9 of cl. 26.5.3.2b-2 of IS 456).

10.22.9 Summary of this Lesson

This lesson illustrates the additional assumptions made regarding the strengths of concrete and steel for the design of short tied and helically reinforced columns subjected to axial loads as per IS 456. The governing equations for determining the areas of cross sections of concrete and longitudinal steel are derived and explained. The equation for determining the pitch of the helix for circular columns is derived. Several numerical examples are solved to illustrate the applications of the derived equations and use of charts of SP-16 for the design of both tied and helically reinforced columns. The results of the same problem by direct computation are compared with those obtained by employing the charts of SP-16 are compared. The differences of results, if any, are discussed. Understanding the illustrative examples and solving the practice and test problems will explain the applications of the equation and use of charts of SP-16 for designing these two types of columns.