# Module
# 10
# Compression Members

# Lesson 23

# Short Compression Members under Axial Load with Uniaxial Bending

## Instructional Objectives:

At the end of this lesson, the student should be able to:

- draw the strain profiles for different locations of the depth of the neutral axis,
- explain the behaviour of such columns for any one of the strain profiles,
- name and identify the three modes of failure of such columns,
- explain the interaction diagram and divide into the three regions indicating three modes of failure,
- identify the three modes of failure from the depth of neutral axis,
- identify the three modes of failure from the eccentricity of the axial load,
- determine the area of compressive stress block, distance of the centroid of the area of the compressive stress block from the highly compressed edge when the neutral axis is within and outside the cross-section of the column,
- determine the compressive stress of concrete and tensile/compressive stress of longitudinal steel for any location of the neutral axis within or outside the cross-section of the column,
- write the two equations of equilibrium,
- explain the need to recast the equations in non-dimensional form for their use in the design of such columns.

## 10.23.1 Introduction

Short reinforced concrete columns under axial load with uniaxial bending behave in a different manner than when it is subjected to axial load, though columns subjected to axial load can also carry some moment that may appear during construction or otherwise. The behaviour of such columns and the three modes of failure are illustrated in this lesson. It is explained that the moment $M$, equivalent to the load $P$ with eccentricity $e$ (= $M/P$), will act in an interactive manner. A particular column with specific amount of longitudinal steel, therefore, can carry either a purely axial load $P_o$ (when $M$ = 0), a purely moment $M_o$ (when $P$ = 0) or several pairs of $P$ and $M$ in an interactive manner. Hence, the needed interaction diagram of columns, which is a plot of $P$ versus $M$, is explained discussing different positions of neutral axis, either outside or within the cross-section of the column.

Depending on the position of the neutral axis, the column may or may not have tensile stress to be taken by longitudinal steel. In the compression region however, longitudinal steel will carry the compression load along with the concrete as in the case of axially loaded column.

## 10.23.2 Behaviour of Short Columns under Axial Load and Uniaxial Moment

Normally, the side columns of a grid of beams and columns are subjected to axial load $P$ and uniaxial moment $M_x$ causing bending about the major axis xx, hereafter will be written as $M$. The moment $M$ can be made equivalent to the axial load $P$ acting at an eccentricity of $e$ (= $M/P$). Let us consider a symmetrically reinforced short rectangular column subjected to axial load $P_u$ at an eccentricity of $e$ to have $M_u$ causing failure of the column.

Figure 10.21.11b of Lesson 21 presents two strain profiles IN and EF. For the strain profile IN, the depth of the neutral axis $kD$ is less than $D$, i.e., neutral axis is within the section resulting the maximum compressive strain of 0.0035 on the right edge and tensile strains on the left of the neutral axis forming cracks. This column is in a state of collapse for the axial force $P_u$ and moment $M_u$ for which IN is the strain profile. Reducing the eccentricity of the load $P_u$ to zero, we get the other strain profile EF resulting in the constant compressive strain of 0.002, which also is another collapse load. This axial load $P_u$ is different from the other one, i.e., a pair of $P_u$ and $M_u$, for which IN is the profile. For the strain profile EF, the neutral axis is at infinity ($k = \alpha$).

Figure 10.21.11c of Lesson 21 presents the strain profile EF with two more strain profiles IH and JK intersecting at the fulcrum point V. The strain profile IH has the neutral axis depth $kD = D$, while other strain profile JK has $kD > D$. The load and its eccentricity for the strain profile IH are such that the maximum compressive strain reaches 0.0035 at the right edge causing collapse of the column, though the strains throughout the depth is compressive and zero at the left edge. The strain profile JK has the maximum compressive strain at the right edge between 0.002 and 0.0035 and the minimum compressive strain at the left edge. This strain profile JK also causes collapse of the column since the maximum compressive strain at the right edge is a limiting strain satisfying assumption (ii) of sec. 10.21.10 of Lesson 21.

The four strain profiles, IN, EF, JK and IH of Figs.10.21.11b and c, separately cause collapse of the same column when subjected to four different pairs of $P_u$ and $M_u$. This shows that the column may collapse either due to a uniform constant strain throughout (= 0.002 by EF) or due to the maximum compressive strain at the right edge satisfying assumption (ii) of sec.10.21.10 of

Lesson 21 irrespective of the strain at the left edge (zero for IH and tensile for IN). The positions of the neutral axis and the eccentricities of the load are widely varying as follows:

(i) For the strain profile EF, *kD* is infinity and the eccentricity of the load is zero.

(ii) For the strain profile JK, *kD* is outside the section ($D < kD < \alpha$), with appropriate eccentricity having compressive strain in the section.

(iii) For the strain profile IH, *kD* is just at the left edge of the section ($kD = D$), with appropriate eccentricity having zero and 0.0035 compressive strains at the left and right edges, respectively.

(iv) For the strain profile IN, *kD* is within the section ($kD < D$), with appropriate eccentricity having tensile strains on the left of the neutral axis and 0.0035 compressive strain at the right edge.

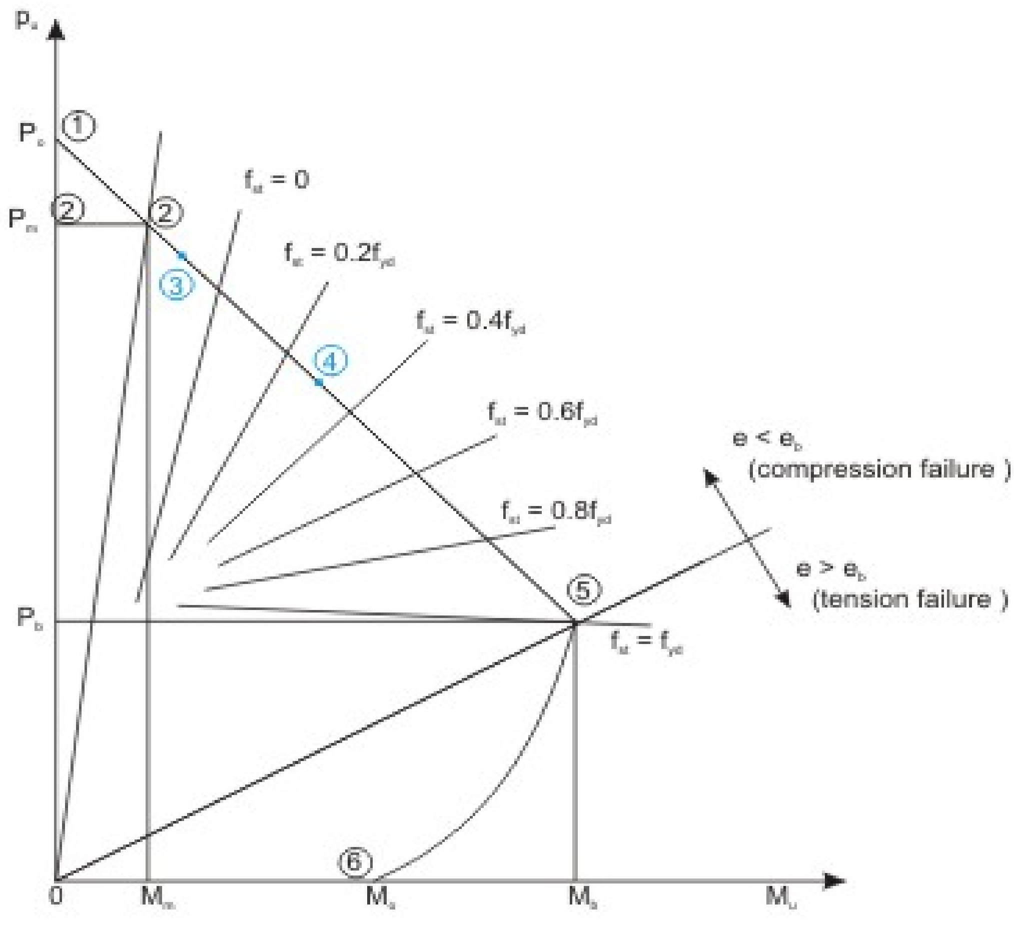


**Fig. 10.23.1**: Typical interaction diagram

It is evident that gradual increase of the eccentricity of the load $P_u$ from zero is changing the strain profiles from EF to JK, IH and then to IN. Therefore, we can accept that if we increase the eccentricity of the load to infinity, there will be only $M_u$ acting on the column. Designating by $P_o$ as the load that causes collapse of the column when acting alone and $M_o$ as the moment that also causes collapse when acting alone, we mark them in Fig.10.23.1 in the vertical and horizontal axes. These two points are the extreme points on the plot of $P_u$ versus $M_u$, any point on which is a pair of $P_u$ and $M_u$ (of different magnitudes) that will cause collapse of the same column having the neutral axis either outside or within the column.

The plot of $P_u$ versus $M_u$ of Fig.10.23.1 is designated as interaction diagram since any point on the diagram gives a pair of values of $P_u$ and $M_u$ causing collapse of the same column in an interactive manner. Following the same logic, several alternative column sections with appropriate longitudinal steel bars are also possible for a particular pair of $P_u$ and $M_u$. Accordingly for the purpose of designing the column, it is essential to understand the different modes of failure of columns, as given in the next section.

## 10.23.3 Modes of Failure of Columns

The two distinct categories of the location of neutral axis, mentioned in the last section, clearly indicate the two types of failure modes: (i) compression failure, when the neutral axis is outside the section, causing compression throughout the section, and (ii) tension failure, when the neutral axis is within the section developing tensile strain on the left of the neutral axis. Before taking up these two failure modes, let us discuss about the third mode of failure i.e., the balanced failure.

### (A) Balanced failure

Under this mode of failure, yielding of outer most row of longitudinal steel near the left edge occurs simultaneously with the attainment of maximum compressive strain of 0.0035 in concrete at the right edge of the column. As a result, yielding of longitudinal steel at the outermost row near the left edge and crushing of concrete at the right edge occur simultaneously. The different yielding strains of steel are determined from the following:

(i) For mild steel (Fe 250): $\varepsilon_y = 0.87 f_y / E_s$

(10.12)

(ii) For cold worked deformed bars: $\varepsilon_y = 0.87 f_y / E_s + 0.002$

(10.13)

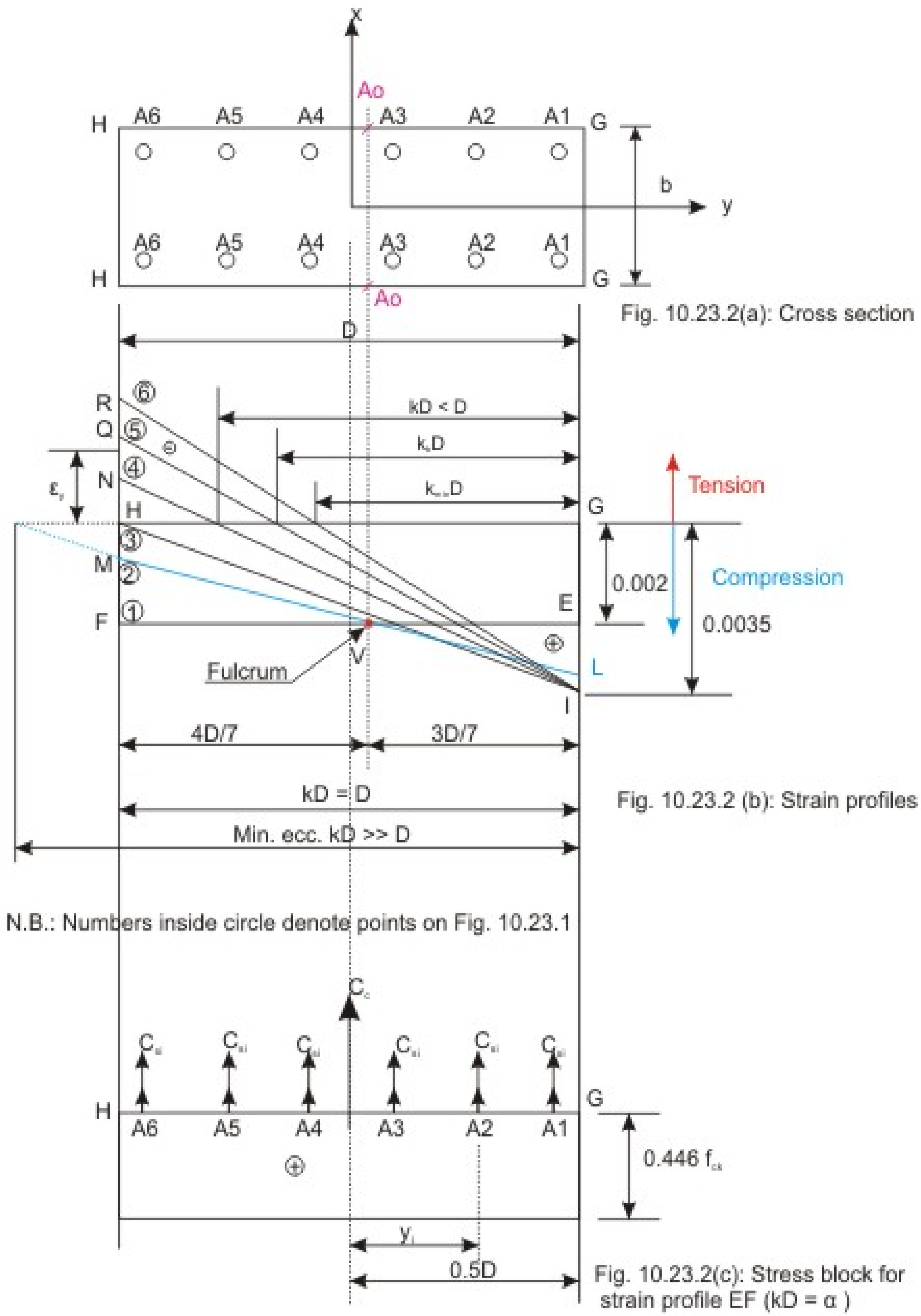


**Fig. 10.23.2**: Strain profiles and stress block for the strain profile EF

The corresponding numerical values are 0.00109, 0.0038 and 0.00417 for Fe 250, Fe 415 and Fe 500, respectively. Such a strain profile is known a balanced strain profile which is shown by the strain profile IQ in Fig.10.23.2b with a number 5. This number is shown in Fig.10.23.1 lying on the interaction diagram causing collapse of the column. The depth of the neutral axis is designated as $k_bD$ and shown in Fig.10.23.2b. The balanced strain profile IQ in Fig.10.23.2b also shows the strain $\varepsilon_y$ whose numerical value would change depending on the grade of steel as mentioned earlier. It is also important to observe that this balanced profile IQ does not pass trough the fulcrum point V in Fig.10.23.2b, while other profiles 1, 2 and 3 i.e., EF, LM and IN pass through the fulcrum point V as none of them produce tensile strain any where in the section of the column. The neutral axis depth for the balanced strain profile IQ is less than $D$, while the same for the other three are either equal to or more than $D$.

To have the balanced strain profile IQ causing balanced failure of the column, the required load and moment are designated as $P_b$ and $M_b$ , respectively and shown in Fig.10.23.1 as the coordinates of point 5. The corresponding eccentricity of the load $P_b$ is defined by the notation $e_b$ (= $M_b/P_b$). The four parameters of the balanced failure are, therefore, $P_b$, $M_b$, $e_b$ and $k_b$ (the coefficient of the neutral axis depth $k_bD$).

## (B) Compression failure

Compression failure of the column occurs when the eccentricity of the load $P_u$ is less than that of balanced eccentricity ($e < e_b$) and the depth of the neutral axis is more than that of balanced failure. It is evident from Fig.10.23.2b that these strain profiles may develop tensile strain on the left of the neutral axis till $kD = D$. All these strain profiles having $1 > k > k_b$ will not pass through the fulcrum point V. Neither the tensile strain of the outermost row of steel on the left of the neutral axis reaches $\varepsilon_y$.

On the other hand, all strain profiles having $kD$ greater than $D$ pass through the fulcrum point V and cause compression failure (Fig.10.23.2b). The loads causing compression failure are higher than the balanced load $P_b$ having the respective eccentricities less than that of the load of balanced failure. The extreme strain profile is EF marked by 1 in Fig.10.23.2b. Some of these points causing compression failure are shown in Fig.10.23.1 as 1, 2, 3 and 4 having $k > k_b$, either within or outside the section.

Three such strain profiles are of interest and need further elaboration. One of them is the strain profile IH (Fig.10.23.2b) marked by point 3 (Fig.10.23.1) for which $kD = D$. This strain profile develops compressive strain in the section with zero strain at the left edge and 0.0035 in the right edge as explained in sec. 10.23.2. Denoting the depth of the neutral axis by $D$ and eccentricity of the load for this profile by $e_D$, we observe that the other strain profiles LM and EF (Fig.10.23.2b), marked by 2 and 1 in Fig.10.23.1, have the respective $kD > D$ and $e < e_D$.

The second strain profile is EF (Fig.10.23.2b) marked by point 1 in Fig.10.23.1 is for the maximum capacity of the column to carry the axial load $P_o$ when eccentricity is zero and for which moment is zero and the neutral axis is at infinity. This strain profile has also been discussed earlier in sec.10.23.2.

The third important strain profile LM, shown in Fig.10.23.2b and by point 2 in Figs.10.23.1 and 2, is also due to another pair of collapse $P_m$ and $M_m$, having the capacity to accommodate the minimum eccentricity of the load, which hardly can be avoided in practical construction or for other reasons. The load $P_m$ , as seen from Fig.10.23.1, is less than $P_o$ and the column can carry $P_m$ and $M_m$ in an interactive mode to cause collapse. Hence, a column having the capacity to carry the truly concentric load $P_o$ (when $M$ = 0) shall not be allowed in the design. Instead, its maximum load shall be restricted up to $P_m$ (< $P_o$) along with $M_m$ (due to minimum eccentricity). Accordingly, the actual interaction diagram to be used for the purpose of the design shall terminate with a horizontal line 22' at point 2 of Fig.10.23.1. Point 2 on the interaction diagram has the capacity of $P_m$ with $M_m$ having eccentricity of $e_m$ (= $M_m/P_m$) and the depth of the neutral axis is >> $D$ (Fig.10.23.2b).

It is thus seen that from points 1 to 5 (i.e., from compression failure to balanced failure) of the interaction diagram of Fig.10.23.1, the loads are gradually decreasing and the moments are correspondingly increasing. The eccentricities of the successive loads are also increasing and the depths of neutral axis are decreasing from infinity to finite but outside and then within the section up to $k_bD$ at balanced failure (point 5). Moreover, this region of compression failure can be subdivided into two zones: (i) zone from point 1 to point 2, where the eccentricity of the load is less than the minimum eccentricity that should be considered in the actual design as specified in IS 456, and (ii) zone from point 2 to point 5, where the eccentricity of the load is equal to or more than the minimum that is specified in IS 456. It has been mentioned also that the first zone from point 1 to point 2 should be avoided in the design of column.

### (C) Tension failure

Tension failure occurs when the eccentricity of the load is greater than the balanced eccentricity $e_b$. The depth of the neutral axis is less than that of the balanced failure. The longitudinal steel in the outermost row on the left of the neutral axis yields first. Gradually, with the increase of tensile strain, longitudinal steel of inner rows, if provided, starts yielding till the compressive strain reaches 0.0035 at the right edge. The line IR of Fig.10.23.2b represents such a profile for which some of the inner rows of steel bars have yielded and compressive strain has reached 0.0035 at the right edge. The depth of the neutral axis is designated by ($k_{min}D$).

It is interesting to note that in this region of the interaction diagram (from 5 to 6 in Fig.10.23.1), both the load and the moment are found to decrease till point 6 when the column fails due to $M_o$ acting alone. This important behaviour is explained below starting from the failure of the column due to $M_o$ alone at point 6 of Fig.10.23.1.

At point 6, let us consider that the column is loaded in simple bending to the point (when $M = M_o$) at which yielding of the tension steel begins. Addition of some axial compressive load $P$ at this stage will reduce the previous tensile stress of steel to a value less than its yield strength. As a result, it can carry additional moment. This increase of moment carrying capacity with the increase of load shall continue till the combined stress in steel due to additional axial load and increased moment reaches the yield strength.

## 10.23.4 Interaction Diagram

It is now understood that a reinforced concrete column with specified amount of longitudinal steel has different carrying capacities of a pair of $P_u$ and $M_u$ before its collapse depending on the eccentricity of the load. Figure 10.23.1 represents one such interaction diagram giving the carrying capacities ranging from $P_o$ with zero eccentricity on the vertical axis to $M_o$ (pure bending) on the horizontal axis. The vertical axis corresponds to load with zero eccentricity while the horizontal axis represents infinite value of eccentricity. A radial line joining the origin O of Fig.10.23.1 to point 2 represents the load having the minimum eccentricity. In fact, any radial line represents a particular eccentricity of the load. Any point on the interaction diagram gives a unique pair of $P_u$ and $M_u$ that causes the state of incipient failure. The interaction diagram has three distinct zones of failure: (i) from point 1 to just before point 5 is the zone of compression failure, (ii) point 5 is the balanced failure and (iii) from point 5 to point 6 is the zone of tension failure. In the compression failure zone, small eccentricities produce failure of concrete in compression, while large eccentricities cause failure triggered by yielding of tension steel. In between, point 5 is the critical point at which both the failures of concrete in compression and steel in yielding occur simultaneously.

The interaction diagram further reveals that as the axial force $P_u$ becomes larger the section can carry smaller $M_u$ before failing in the compression zone. The reverse is the case in the tension zone, where the moment carrying capacity $M_u$ increases with the increase of axial load $P_u$. In the compression failure zone, the failure occurs due to over straining of concrete. The large axial force produces high compressive strain of concrete keeping smaller margin available for additional compressive strain line to bending. On the other hand, in the tension failure zone, yielding of steel initiates failure. This tensile yield stress reduces with the additional compressive stress due to additional axial load. As a result, further moment can be applied till the combined stress of steel due to axial force and increased moment reaches the yield strength.

Therefore, the design of a column with given $P_u$ and $M_u$ should be done following the three steps, as given below:

(i) Selection of a trial section with assumed longitudinal steel,

(ii) Construction of the interaction diagram of the selected trial column section by successive choices of the neutral axis depth from infinity (pure axial load) to a very small value (to be found by trial to get $P$ = 0 for pure bending),

(iii) Checking of the given $P_u$ and $M_u$, if they are within the diagram.

We will discuss later whether the above procedure should be followed or not. Let us first understand the corresponding compressive stress blocks of concrete for the two distinct cases of the depth of the neutral axis: (i) outside the cross-section and (ii) within the cross-section in the following sections.

## 10.23.5 Compressive Stress Block of Concrete when the Neutral Axis Lies Outside the Section

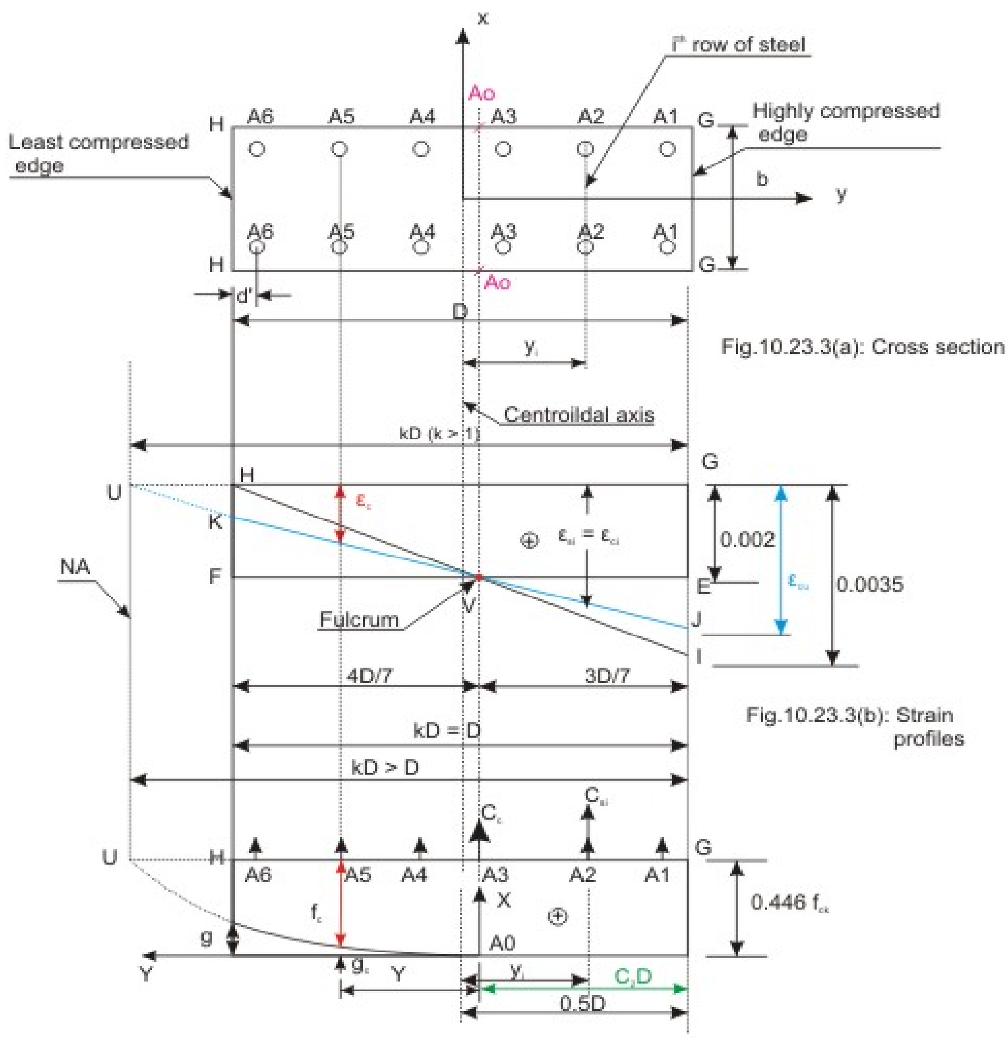


Fig.10.23.3(c): Stress block for the strain propife JK (kD > D)

**Fig.10.23.3**: Cross section of column, strain profiles and stress block for the strain profile JK (kD > D)

Figure 10.23.3c presents the stress block for a typical strain profile JK having neutral axis depth $kD$ outside the section ($k$ > 1). The strain profile JK in Fig.10.23.3b shows that up to a distance of 3$D$/7 from the right edge (point AO), the compressive strain is ≥ 0.002 and, therefore, the compressive stress shall remain constant at 0.446$f_{ck}$. The remaining part of the column section of length 4$D$/7, i.e., up to the left edge, has reducing compressive strains (but not zero). The stress block is, therefore, parabolic from AO to H which becomes zero at U (outside the section). The area of the compressive stress block shall be obtained subtracting the parabolic area between AO to H from the rectangular area between G and H. To establish the expression of this area, it is essential to know the equation of the parabola between AO and U, whose origin is at AO. The positive coordinates of $X$ and $Y$ are measured from the point AO upwards and to the left, respectively. Let us assume that the general equation of the parabola as

$X = aY^2 + bY + c$
(10.14)

The values of $a$, $b$ and $c$ are obtained as follows:

(i) At $Y = 0$, $X = 0$, at the origin: gives $c = 0$

(ii) At $Y = 0$, $dX/dY = 0$, at the origin: gives $b = 0$

(iii) At $Y = (kD - 3D/7)$, i.e., at point U, $X = 0.446f_{ck}$: gives $a = 0.446f_{ck}/D^2(k-3/7)^2$.

Therefore, the equation of the parabola is:

$X = \{0.446f_{ck}/D^2(k - 3/7)^2\}Y^2$
(10.15)

The value of $X$ at the point H (left edge of the column), $g$ is now determined from Eq.10.15 when $Y = 4D/7$, which gives

$g = 0.446\ f_{ck}\ \{4/(7k - 3)\}^2$
(10.16)

Hence, the area of the compressive stress block $= 0.446\ f_{ck}\ D\ [1 - (4/21)\{4/(7k - 3)\}^2]$

$= C_1\ f_{ck}\ D$
(10.17)

where $C_1 = 0.446[1 - (4/21)\{4/(7k - 3)\}^2]$
(10.18)

Equation 10.17 is useful to determine the area of the stress block for any value of $k > 1$ (neutral axis outside the section) by substituting the value of $C_1$ from Eq.10.18. The symbol $C_1$ is designated as the coefficient for the area of the stress block.

The position of the centroid of the compressive stress block is obtained by dividing the moment of the stress block about the right edge by the area of the stress block. The moment of the stress block is obtained by subtracting the moment of the parabolic part between AO and H about the right edge from the moment of the rectangular stress block of full depth $D$ about the right edge. The expression of the moment of the stress block about the right edge is:

$0.446\ f_{ck}\ D(D/2) - (1/3)(4D/7)\ 0.446\ f_{ck}\ \{4/(7k - 3)\}^2\ \{3D/7 + (3/4)(4D/7)\}$

$= 0.446\, f_{ck}\, D^2\, [(1/2) – (8/49)\{4/(7k – 3)\}^2]$
(10.19)

Dividing Eq.10.19 by Eq.10.17, we get the distance of the centroid from the right edge is:

$D[(1/2) – (8/49)\{4/(7k – 3)\}^2]/[1 – (4/21)\{4/(7k – 3)\}^2]$
(10.20)

$= C_2\, D$
(10.21)

where $C_2$ is the coefficient for the distance of the centroid of the compressive stress block of concrete measured from the right edge and is:

$C_2 = [(1/2) – (8/49)\{4/(7k – 3)\}^2]/[1 – (4/21)\{4/(7k – 3)\}^2]$
(10.22)

Table 10.4 presents the values of $C_1$ and $C_2$ for different values of $k$ greater than 1, as given in Table H of SP-16. For a specific depth of the neutral axis, $k$ is known. Using the corresponding values of $C_1$ and $C_2$ from Table 10.4, area of the stress block of concrete and the distance of centroid from the right edge are determined from Eqs.10.17 and 10.21, respectively.

Table 10.4 Stress block parameters $C_1$ and $C_2$ when the neutral axis is outside the section

| $K$ | $C_1$ | $C_2$ |
|---|---|---|
| 1.00 | 0.361 | 0.416 |
| 1.05 | 0.374 | 0.432 |
| 1.10 | 0.384 | 0.443 |
| 1.20 | 0.399 | 0.458 |
| 1.30 | 0.409 | 0.468 |
| 1.40 | 0.417 | 0.475 |
| 1.50 | 0.422 | 0.480 |
| 2.00 | 0.435 | 0.491 |
| 2.50 | 0.440 | 0.495 |
| 3.00 | 0.442 | 0.497 |
| 4.00 | 0.444 | 0.499 |

It is worth mentioning that the area of the stress block is $0.446 f_{ck}D$ and the distance of the centroid from the right edge is $0.5D$, when $k$ is infinite. Values of $C_1$ and $C_2$ at $k = 4$ are very close to those when $k = \infty$. In fact, for the practical interaction diagrams, it is generally adequate to consider values of $k$ up to about 1.2.

## 10.23.6 Determination of Compressive Stress Anywhere in the Section when the Neutral Axis Lies outside the Section

The compressive stress of concrete at any point between G and AO of Fig.10.23.3c is constant at $0.446f_{ck}$ as the strain in this zone is equal to or greater than 0.002. So, we can write

$$f_c = 0.446f_{ck} \text{ if } 0.002 \le \varepsilon_c \le 0.0035 \quad (10.23)$$

However, compressive stress of concrete between AO and H is to be determined using the equation of parabola. Let us determine the concrete stress $f_c$ at a distance of $Y$ from the origin AO. From Fig.10.23.3c, we have

$$f_c = 0.446 f_{ck} - g_c \quad (10.24)$$

where $g_c$ is as shown in Fig.10.23.3c and obtained from Eq.10.15. Thus, we get

$$f_c = 0.446 f_{ck} - \{0.446 f_{ck}/D^2(k - 3/7)^2\}Y^2$$

or

$$f_c = 0.446 f_{ck} \{1 - Y^2/(kD - 3D/7)^2\} \quad (10.25)$$

Designating the strain of concrete at this point by $\varepsilon_c$ (Fig.10.23.3b), we have from similar triangles

$$\varepsilon_c/0.002 = 1 - Y/(kD - 3D/7)$$

which gives

$$Y = \{1 - (\varepsilon/0.002)\}(kD - 3D/7) \quad (10.26)$$

Substituting the value of $Y$ from Eq.10.26 in Eq.10.25, we have

$$f_c = 0.446 f_{ck} [2(\varepsilon_c/0.002) - (\varepsilon_c/0.002)^2], \text{ if } 0 \le \varepsilon_c < 0.002 \quad (10.27)$$

## 10.23.7 Compressive Stress Block of Concrete when the Neutral Axis is within the Section

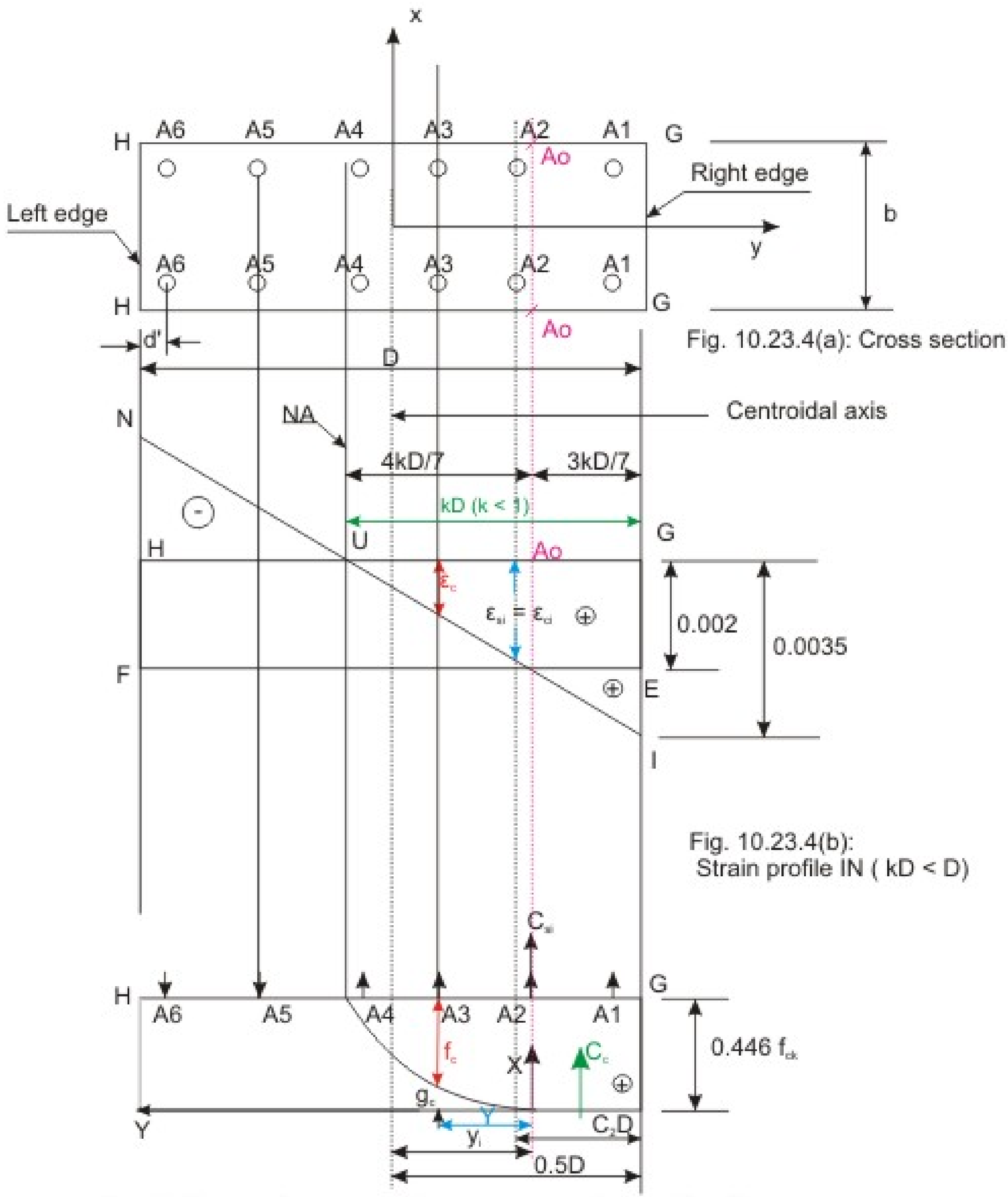


Fig. 10.23.4(a): Cross section

Fig. 10.23.4(b): Strain profile IN ( kD < D)

Fig. 10.23.4(c): Stress block for the strain profile IN (kD < D)

**Fig. 10.23.4**: Cross section, strain profile IN and stress block for strain profile IN

Figure 10.23.4c presents the stress block for a typical strain profile IN having neutral axis depth = *kD* within the section (*k* < 1). The strain profile IN in Fig.10.23.4b shows that from a to AO, i.e., up to a distance of 3*kD*/7 from the right edge, the compressive strain is ≥ 0.002 and, therefore, the compressive stress shall remain constant at $0.446f_{ck}$. From AO to U, i.e., for a distance of $4kD/7$, the strain is reducing from 0.002 to zero and the stress in this zone is parabolic as shown in Fig.10.23.4c. The area of the stress block shall be obtained subtracting the parabolic area between AO and U from the total rectangular area between G and U. As in the case when the neutral axis is outside the section (sec.10.23.5), we have to establish the equation of the parabola with AO as the origin and the positive coordinates $X$ and $Y$ are measured from the point AO upwards for $X$ and from the point AO to the left for $Y$, as shown in Fig.10.23.4c. Proceeding in the same manner as in sec.10.23.5 and assuming the same equation of the parabola as in Eq.10.14, the values of a, b and c are obtained as:

(i) At $Y = 0$, $X = 0$, at the origin: gives $c = 0$

(ii) At $Y = 0$, $dX/dY = 0$, at the origin: gives $b = 0$

(iii) At U, (i.e., at $Y = 4kD/7$), $X = 0.446\ f_{ck}$: gives $a = 0.446\ f_{ck}/(4kD/7)^2$.

Therefore, the equation of the parabola OR is:

$$X = \{0.446\ f_{ck}/(4kD/7)^2\}Y^2 \tag{10.28}$$

The area of the stress block = $0.446\ f_{ck}\ kD – (1/3)\ 0.446\ f_{ck}\ (4kD/7) = 0.36\ f_{ck}\ kD$, the same as obtained earlier in Eq.3.9 of Lesson 4 for flexural members. Similarly, the distance of the centroid can be obtained by dividing the moment of area of stress block about the right edge by the area of the stress block. The result is the same as in Eq.3.12 for the flexural members. Therefore, we have

Area of the stress block = $0.36\ f_{ck}\ kD$ (10.29)

The distance of the centroid of the stress block from the right edge = $0.42kD$ (10.30)

Thus, the values of $C_1$ and $C_2$ of Eqs.10.17 and 10.21, respectively, are 0.36 and 0.42 when the neutral axis is within the section. It is to be noted that the coefficients $C_1$ and $C_2$ are multiplied by $Df_{ck}$ and $D$, respectively when the neutral axis is outside the section. However, they are to be multiplied here, when the neutral axis is within the section, by $kDf_{ck}$ and $kD$, respectively.

It is further to note that though the expressions of the area of stress block and the distance of the centroid of the stress block from the right edge are the same as those for the flexural members, the important restriction of the maximum depth of the neutral axis $x_{umax}$ in the flexural members is not applicable in case of column. By this restriction, the compression failure of the flexural members is avoided. In case of columns, compression failure is one of the three modes of failure.

## 10.23.8 Determination of Compressive Stress Anywhere in the Compressive Zone when the Neutral Axis is within the Section

The compressive stress at any point between G and AO of Fig.10.23.4c is constant at $0.446f_{ck}$ as the strain in this zone is equal to or greater than 0.002. So, we can write

$f_c = 0.446\, f_{ck}$ if $0.002 \le \varepsilon_c \le 0.0035$ .... (10.23)

However, the compressive stress between AO and U is to be determined from the equation of the parabola. Let us determine the compressive stress $f_{ci}$ at a distance of $Y$ from the origin AO. From Fig.10.23.4c, we have

$$f_c = 0.446\, f_{ck} - g_c \quad (10.31)$$

where $g_c$ as shown in Fig.10.23.4c, is obtained from Eq.10.28. Thus, we get,

$$f_c = \{0.446\, f_{ck} - 0.446\, f_{ck}\, (4kD/7)^2\}Y^2 \quad (10.32)$$

Designating the strain of concrete at this point by $\varepsilon_c$ (Fig.10.23.4b), we have from similar triangles

$\varepsilon_c/0.002 = 1 - Y/(4kD/7)$, which gives

$$Y = \{1 - \varepsilon_c/0.002\}(4kD/7) \quad (10.33)$$

Substituting the value of $Y$ from Eq.10.33 in Eq.10.32, we get the same equation, Eq.10.27 of sec.10.23.6, when the neutral axis is outside the section. Therefore,

$$f_c = 0.446\, f_{ck}\, [2(\varepsilon_c/0.002) - (\varepsilon_c/0.002)^2] \ldots. \quad (10.26)$$

From the point U to the left edge H of the cross-section of the column, the compressive stress is zero. Thus, we have

$f_c = 0$ if $\varepsilon_c \le 0$

$f_c = 0.446\, f_{ck}$ if $\varepsilon_c \geq 0.002$

$f_c = 0.446\, f_{ck}\{2(\varepsilon_c/0.002) - (\varepsilon_c/0.002)^2\}$, if $0 \leq \varepsilon_c < 0.002$
(10.34)

## 10.23.9 Tensile and Compressive Stresses of Longitudinal Steel

Stresses are compressive in all the six rows (A1 to A6 of Figs.10.23.3a and c) of longitudinal steel provided in the column when the neutral axis depth $kD \geq D$. However, they are tensile on the left side of the neutral axis and compressive on the right side of the neutral axis (Figs.10.23.4a and c) when $kD < D$. These compressive or tensile stresses of longitudinal steel shall be calculated from the strain $\varepsilon_{si}$ at that position of the steel which is obtained from the strain profile considered for the purpose.

It should be remembered that the linear strain profiles are based on the assumption that plane sections remain plane. Moreover, at the location of steel in a particular row, the strain of steel $\varepsilon_{si}$ shall be the same as that in the adjacent concrete $\varepsilon_{ci}$. Thus, the strain of longitudinal steel can be calculated from the particular strain profile if the neutral axis is within or outside the cross-section of the column.

The corresponding stresses $f_{si}$ of longitudinal steel are determined from the strain $\varepsilon_{si}$ (which is the same as that of $\varepsilon_{ci}$ in the adjacent concrete) from the respective stress-strain diagrams of mild steel (Fig.1.2.3 of Lesson 2) and High Yield Strength Deformed bars (Fig.1.2.4 of Lesson2). The values are summarized in Table 10.5 below as presented in Table A of SP-16.

Table 10.5 Values of compressive or tensile $f_{si}$ from known values of $\varepsilon_{si}$ of longitudinal steel (Fe 250, Fe 415 and Fe 500)

| Fe 250 | | Fe 415 | | Fe 500 | |
|---|---|---|---|---|---|
| Strain $\varepsilon_{si}$ | Stress (N/mm$^2$) $f_{si}$ | Strain $\varepsilon_{si}$ | Stress (N/mm$^2$) $f_{si}$ | Strain $\varepsilon_{si}$ | Stress (N/mm$^2$) $f_{si}$ |
| < 0.00109 | $\varepsilon_{si}$ ($E_s$) | < 0.00144 | $\varepsilon_{si}$ ($E_s$) | < 0.00174 | $\varepsilon_{si}$ ($E_s$) |
| $\geq$ 0.00109 | 217.5 (= 0.87 $f_y$) | 0.00144 | 288.7 | 0.00174 | 347.8 |
| | | 0.00163 | 306.7 | 0.00195 | 369.6 |
| | | 0.00192 | 324.8 | 0.00226 | 391.3 |
| | | 0.00241 | 342.8 | 0.00277 | 413.0 |

| | | | | |
|---|---|---|---|---|
| | 0.00276 | 351.8 | 0.00312 | 423.9 |
| | 0.00380 | 360.9 | 0.00417 | 434.8 |

Notes: 1. Linear interpolation shall be done for intermediate values.
2. Strain at initial yield = $f_y/E_s$
3. Strain at final yield = $f_y/E_s + 0.002$

## 10.23.10 Governing Equations

A column subjected to $P_u$ and $M_u$ (= $P_u e$) shall satisfy the two equations of equilibrium, viz., $\sum V = 0$ and $\sum M = 0$, taking moment of vertical forces about the centroidal axis of the column. The two governing equation are, therefore,

$$P_u = C_c + C_s \tag{10.35}$$

$$M_u = C_c \text{ (appropriate lever arm)} + C_s \text{ (appropriate lever arm)} \tag{10.36}$$

where $C_c$ = Force due to concrete in compression

$C_s$ = Force due to steel either in compression when $kD \geq D$ or in tension and compression when $kD < D$

However, two points are to be remembered while expanding the equation $\sum V = 0$. The first is that while computing the force of steel in compression, the force of concrete that is not available at the location of longitudinal steel has to be subtracted. The second point is that the total force of steel shall consist of the summation of forces in every row of steel having different stresses depending on the respective distances from the centroidal axis. These two points are also to be considered while expanding the other equation $\sum M = 0$. Moreover, negative sign should be used for the tensile force of steel on the left of the neutral axis when $kD < D$.

It is now possible to draw the interaction diagram of a trial section for the given values of $P_u$ and $M_u$ following the three steps mentioned in sec.10.23.4. However, such an attempt should be avoided for the reason explained below.

It has been mentioned in sec.10.23.2 that any point on the interaction diagram gives a pair of values of $P_u$ and $M_u$ causing collapse. On the other hand, it is also true that for the given $P_u$ and $M_u$, several sections are possible. Drawing of interaction diagrams for all the trial sections is time consuming. Therefore, it is necessary to recast the interaction diagram selecting appropriate non-dimensional parameters instead of $P_u$ versus $M_u$ as has been explained in this lesson. Non-dimensional interaction diagram has the advantage of selecting alternative sections quickly for a given pair of $P_u$ and $M_u$. It is worth mentioning that all the aspects of the behaviour of column and the modes of failure shall remain valid in constructing the more versatile non-dimensional interaction diagram, which is taken up in Lesson 24.

## 10.23.11 Practice Questions and Problems with Answers

**Q.1:** Draw four typical strain profiles of a short, rectangular and symmetrically reinforced concrete column causing collapse subjected to different pairs of $P_u$ and $M_u$ when the depths of the neutral axis are (i) less than the depth of column $D$, (ii) equal to the depth of column $D$, (iii) $D < kD < \infty$ and (iv) $kD = \infty$. Explain the behaviour of column for each of the four strain profiles.

**A.1:** See sec. 10.23.2.

**Q.2:** Name and explain the three modes of failures of short, rectangular and symmetrically reinforced concrete columns subjected to axial load $P_u$ uniaxial moment $M_u$.

**A.2:** See sec.10.23.3.

**Q.3:** Draw a typical interaction diagram, and explain the three zones representing three modes of failure of a short, rectangular and symmetrically reinforced concrete column subjected to axial load $P_u$ and uniaxial moment $M_u$.

**A.3:** See sec.10.23.4.

**Q.4:** (a) Draw the compressive stress block of concrete of a short, rectangular and symmetrically reinforced concrete column subjected to axial load $P_u$ and uniaxial moment $M_u$, when the neutral axis lies outside the section.

(b) Derive expressions of determining the area of the compressive stress block of concrete and distance of the centroid of the compressive stress block from the highly compressed right edge for a column of Q4(a).

**A.4:** See sec.10.23.5.

**Q.5:** Derive expression of determining the stresses anywhere within the section of a column of Q4.

**A.5:** See sec.10.23.6.

**Q.6:** (a) Draw the compressive stress block of concrete of a short, rectangular and symmetrically reinforced concrete column subjected to axial load $P_u$ and uniaxial moment $M_u$, when the neutral axis is within the section.

(b) Derive expressions of determining the area of the compressive stress block of concrete and distance of the centroid of the compressive stress block from the compressed right edge for a column of Q6(a).

**A.6:** See sec.10.23.7.

**Q.7:** Derive expression of determining the compressive stress in the compression zone of a column of Q6.

**A.7:** See sec.10.23.8.

**Q.8:** Explain the principle of determining the stresses (both tensile and compressive) of longitudinal steel of a short, rectangular and symmetrically reinforced concrete column subjected to axial load $P_u$ and uniaxial moment $M_u$.

**A.8:** See sec.10.23.9.

**Q.9:** (a) Write the governing equations of equilibrium of a short, rectangular and symmetrically reinforced concrete column subjected to axial load $P_u$ and uniaxial moment $M_u$.

(b) Would you use the equations of equilibrium for the design of a short, rectangular and symmetrically reinforced concrete column for a given pair of $P_u$ and $M_u$? Justify your answer.

**A.9:** See sec.10.23.10.

## 10.23.12 References


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## 10.23.13 Test 23 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

**TQ.1:** Each of the following statements has four possible answers. Choose the correct answer (2 x 5 = 20 marks)

**(a)** The designed axial load of a short column has the theoretical carrying capacity before it collapses

(i) $P = P_o$ only as obtained from the interaction diagram on the vertical axis.

(ii) $P$ = Designed axial load with the code stipulated minimum eccentricity only.

(iii) A pair of $P_b$ and $M_b$ only.

(iv) All of the above.

**A.TQ.1a:** (iv)

**(b)** A short column in compression failure due to an axial load $P_u$ and uniaxial moment $M_u$ may have

(i) $kD = 0$ and $e = 0$

(ii) $kD = \infty$ and $e = 0$

(iii) $kD = 0$ and $e = \infty$

(iv) $kD = \infty$ and $e = \infty$

**A.TQ.1b:** (ii)

**(c)** The fulcrum of the strain profile of a short column is a point through which

(i) The strain profiles causing compression failure will pass.

(ii) The strain profile causing balanced failure will pass.

(iii) The strain profiles having no tension and causing compression failure will pass.

(iv) The strain profiles causing tension failure will pass.

**A.TQ.1c:** (iii)

**(d)** The maximum compressive strain of concrete in balanced failure of a short column subjected to $P_b$ and $M_b$ is

(i) 0.0035

(ii) 0.0035 minus 0.75 times the tensile strain of steel

(iii) 0.002

(iv) None of the above

**A.TQ.1d:** (i)

**TQ.2:** (a) Draw the compressive stress block of concrete of a short, rectangular and symmetrically reinforced concrete column subjected to axial load $P_u$ and uniaxial moment $M_u$, when the neutral axis is within the section.

(b) Derive expressions of determining the area of the compressive stress block of concrete and distance of the centroid of the compressive stress block from the compressed right edge for a column of TQ.2 (a). (10 + 20 = 30)

**A.TQ.2:** See sec.10.23.7.

## 10.23.14 Summary of this Lesson

Illustrating the behaviour of short, rectangular and symmetrically reinforced rectangular columns under axial load $P_u$ and uniaxial bending $M_u$, this lesson explains the three modes of failure and the interaction diagram of such columns. The different possible strain profiles, and the compressive stress blocks are drawn and explained when the neutral axis is within and outside the cross-section of the column. Determination of compressive stresses of concrete and tensile/compressive stresses of longitudinal steel are explained. The governing equations of equilibrium are introduced to illustrate the need for recasting them in non-dimensional form for the purpose of design of such columns.