

# Module 10

## Compression Members

Lesson

24

Preparation of Design  
Charts

Version 2 CE IIT, Kharagpur

## Instructional Objectives:

At the end of this lesson, the student should be able to:

- identify a design chart and understand the differences between a design chart and interaction diagram of  $P$  and  $M$ ,
- name the major design parameters of short columns subjected to axial loads and uniaxial bending,
- state the design parameters assumed before the design,
- state the design parameter actually designed for the column,
- explain the roles of each of the design parameters in increasing the strength capacities of column,
- name the two non-dimensional design parameters to prepare the design charts,
- derive the governing equations in four separate cases while preparing the design charts,
- mention the various points at which the values of the two non-dimensional parameters are determined to prepare the design charts,
- prepare the design chart of any short and rectangular column subjected to axial loads and uniaxial moment.

### 10.24.1 Introduction

Lesson 23 illustrates the different steps of determining the capacities of a short, rectangular, reinforced with steel bars, concrete column. Several pairs of collapse strengths  $P_u$  and  $M_u$  are to be determined for a column with specific percentage of longitudinal steel bars assuming different positions of the neutral axis. A designer has to satisfy that each of the several pairs of  $P_u$  and  $M_u$ , obtained from the structural analysis, is less than or equal to the respective strengths in form of pairs of  $P_u$  and  $M_u$  obtained from determining the capacities for several locations of the neutral axis. Thus, the design shall involve several trials of a particular cross-section of a column for its selection.

On the other hand, it is also possible to prepare non-dimensional interaction diagram selecting appropriate non-dimensional parameters. This would help to get several possible cross-sections with the respective longitudinal

steel bars. This lesson explains the preparation of such non-dimensional interaction diagrams which are also known as design charts.

Similar design charts of circular and other types of cross-sections can be prepared following the same procedure as that of rectangular cross-section. However, the stress block parameters, explained in Lesson 23, are to be established separately by summing up the forces and moment of several strips by dividing the cross-section of columns into the strips. This lesson is restricted to columns of rectangular cross-section which are symmetrically reinforced.

## 10.24.2 Design Parameters

The following are the four major design parameters to be determined for any column so that it has sufficient pairs of strengths ( $P_u$  and  $M_u$ ) to resist all critical pairs obtained from the analysis:

- (i) dimensions  $b$  and  $D$  of the rectangular cross-section,
- (ii) longitudinal steel reinforcing bars - percentage  $\rho$ , nature of distribution (equally on two or four sides) and  $d'/D$ ,
- (iii) grades of concrete and steel, and
- (iv) transverse reinforcement.

The roles and importance of each of the above four parameters are elaborated below:

### (i) Dimensions $b$ and $D$ of the rectangular cross-section

The strength of column depends on the two dimensions  $b$  and  $D$ . However, preliminary dimensions of  $b$  and  $D$  are already assumed for the analysis of structure, which are usually indeterminate statically. In the subsequent redesign, these dimensions may be revised, if needed, inviting re-analysis with the revised dimensions.

### (ii) Longitudinal steel reinforcing bars

It is a very important consideration to utilise the total area of steel bars effectively. The total area of steel, expressed in percentage  $\rho$  ranges from the minimum 0.8 to the maximum 4 per cent of the gross area of the cross-section. The bars may be distributed either equally on two sides or on all four sides judiciously having two or multiple rows of steel bars. The strain profiles of Fig.10.23.2 reveals that the rows of bars may be all in compression or both compression and tension depending on the location of the neutral axis. Accordingly, the total strength of the longitudinal bars is determined by adding all

the individual strengths of bars of different rows. The effective cover  $d'$ , though depends on the nominal cover, has to be determined from practical considerations of housing all the steel bars.

### **(iii) Grades of concrete and steel**

The dimensions  $b$  and  $D$  of the cross-section and the amount of longitudinal steel bars depend on the grades of concrete and steel.

### **(iv) Transverse reinforcement**

The transverse reinforcement, provided in form of lateral ties or spirals, are important for the following advantages in

- (a) preventing premature / local buckling of the longitudinal bars,
- (b) improving ductility and strength by the effect of confinement of the core concrete,
- (c) holding the longitudinal bars in position during construction, and
- (d) providing resistance against shear and torsion, if present.

However, the transverse reinforcement does not have a major contribution in influencing the capacities of the column. Moreover, the design of transverse reinforcement involves selection of bar diameter and spacing following the stipulations in the design code. The bar diameter of the transverse reinforcement also depends on the bar diameter of longitudinal steel. Accordingly, the transverse reinforcement is designed after finalizing other parameters mentioned above.

It is, therefore, clear that the design of columns mainly involves the determination of percentage of longitudinal reinforcement  $p$ , either assuming or knowing the dimensions  $b$  and  $D$ , grades of concrete and steel, distribution of longitudinal bars in two or multiple rows and  $d'/D$  ratio from the analysis or elsewhere. Needless to mention that any designed column should be able to resist several critical pairs of  $P_u$  and  $M_u$  obtained from the analysis of the structure. It is also a fact that several trials may be needed to arrive at the final selection revising any or all the assumed parameters. Accordingly, the design charts are prepared to give the results for the unknown parameter quickly avoiding lengthy calculations after selecting appropriate non-dimensional parameters.

Based on the above considerations and making the design simple, quick and fairly accurate, the following are the two non-dimensional parameters:

For axial load:  $P_u/f_{ck}bD$

For moment:  $M_u/f_{ck}bD^2$

The characteristic strength of concrete  $f_{ck}$  has been associated with the non-dimensional parameters as the grade of concrete does not improve the strength of the column significantly. The design charts prepared by SP-16 are assuming the constant value of  $f_{ck}$  for M 20 to avoid different sets of design charts for different grades of concrete. However, separate design charts are presented in SP-16 for three grades of steel (Fe 250, Fe 415 and Fe 500), four values of  $d'/D$  (0.05, 0.1, 0.15 and 0.2) and two types of distribution of longitudinal steel (distributed equally on two and four sides). Accordingly there are twenty-four design charts for the design of rectangular columns. Twelve separate design charts are also presented in SP-16 for circular sections covering the above mentioned three grades of steel and for values of  $d'/D$  ratio.

However, the unknown parameter  $p$ , the percentage of longitudinal reinforcement has been modified to  $p/f_{ck}$  in all the design charts of SP-16, so that for grades other than M 20, the more accurate value of  $p$  can be obtained by multiplying the  $p/f_{ck}$  with the actual grade of concrete used in the design of that column.

However, this lesson explains that it is also possible to prepare design chart taking into consideration the actual grade of concrete. As mentioned earlier, the design charts are prepared getting the pairs of values of  $P_u$  and  $M_u$  in non-dimensional form from the equations of equilibrium for different locations of the neutral axis. We now take up the respective non-dimensional equations for four different cases as follows:

- (a) When the neutral axis is at infinity, i.e.,  $kD = \infty$ , pure axial load is applied on the column.
- (b) When the neutral axis is outside the cross-section of the column, i.e.,  $\infty > kD \geq D$ .
- (c) When the neutral axis is within the cross-section of the column, i.e.,  $kD < D$ .
- (d) When the column behaves like a steel beam.

### 10.24.3 Non-dimensional Equation of Equilibrium when $k = \infty$ , (Pure Axial Load)

Figures 10.23.2b and c of Lesson 23 present the strain profile EF and the corresponding stress block for this case. As the load is purely axial, we need to

express the terms  $C_c$  and  $C_s$  of Eq.10.35 of sec.10.23.10 of Lesson 23. The total compressive force due to concrete of constant stress of  $0.446 f_{ck}$  is:

$$C_c = 0.446 f_{ck} b D \quad (10.37)$$

However, proper deduction shall be made for the compressive force of concrete not available due to the replacement by steel bars while computing  $C_s$ .

The force of longitudinal steel bars in compression is now calculated. The steel bars of area  $pbD/100$  are subjected to the constant stress of  $f_{sc}$  when the strain is 0.002. Subtracting the compressive force of concrete of the same area  $pbD/100$ , we have,

$$C_s = (pbD/100) (f_{sc} - 0.446 f_{ck}) \quad (10.38)$$

Thus, we have from Eq.10.35 of sec.10.23.10 of Lesson 23 after substituting the expressions of  $C_c$  and  $C_s$  from Eqs.10.37 and 10.38,

$$P_u = 0.446 f_{ck} b D + (pbD/100) (f_{sc} - 0.446 f_{ck}) \quad (10.39)$$

Dividing both sides of Eq.10.39 by  $f_{ck} bD$ , we have

$$(P_u/f_{ck} bD) = 0.446 + (p/100 f_{ck}) (f_{sc} - 0.446 f_{ck}) \quad (10.40)$$

Thus, Eq.10.40 is the only governing equation for this case to be considered.

#### 10.24.4 Non-dimensional Equations of Equilibrium when Neutral Axis is Outside the Section ( $\infty > kD \geq D$ )

Figures 10.23.3b and c of Lesson 23 present the strain profile JK and the corresponding stress block for this case. The expressions of  $C_c$ ,  $C_s$  and appropriate lever arms are determined to write the two equations of equilibrium (Eqs.10.35 and 36) of Lesson 23. While computing  $C_c$ , the area of parabolic stress block is determined employing the coefficient  $C_1$  from Table 10.4 of Lesson 23. Similarly, the coefficient  $C_2$ , needed to write the moment equation, is obtained from Table 10.4 of Lesson 23. The forces and the corresponding lever arms of longitudinal steel bars are to be considered separately and added for each of the  $n$  rows of the longitudinal bars. Thus, we have the first equation as,

$$P_u = C_1 f_{ck} bD + \sum_{i=1}^n (p_i bD/100)(f_{si} - f_{ci})$$

(10.41)

where  $C_1$  = coefficient for the area of stress block to be taken from Table 10.4 of Lesson 23,

$p_i = A_{si}/bD$  where  $A_{si}$  is the area of reinforcement in the  $i^{\text{th}}$  row,

$f_{si}$  = stress in the  $i^{\text{th}}$  row of reinforcement, taken positive for compression and negative for tension,

$f_{ci}$  = stress in concrete at the level of the  $i^{\text{th}}$  row of reinforcement, and

$n$  = number of rows of reinforcement.

Here also, the deduction of the compressive force of concrete has been made for the concrete replaced by the longitudinal steel bars.

Dividing both sides of Eq.10.41 by  $f_{ck}bD$ , we have

$$(P_u/f_{ck}bD) = C_1 + \sum_{i=1}^n (p_i /100 f_{ck})(f_{si} - f_{ci})$$

(10.42)

Similarly, the moment equation (Eq.10.36) becomes,

$$M_u = C_1 f_{ck}bD (D/2 - C_2D) + \sum_{i=1}^n (p_i bD/100)(f_{si} - f_{ci}) y_i$$

(10.43)

where  $C_2$  = coefficient for the distance of the centroid of the compressive stress block of concrete measured from the highly compressed right edge and is taken from Table 10.4 of Lesson 23, and

$y_i$  = the distance from the centroid of the section to the  $i^{\text{th}}$  row of reinforcement, positive towards the highly compressed right edge and negative towards the least compressed left edge.

Dividing both sides of Eq.10.43 by  $f_{ck}bD^2$ , we have

$$(M_u/f_{ck}bD^2) = C_1(0.5 - C_2) + \sum_{i=1}^n (p_i /100 f_{ck})(f_{si} - f_{ci})(y_i/D)$$

(10.44)

Equations 10.42 and 10.44 are the two non-dimensional equations of equilibrium in this case when  $\infty < kD \leq D$ .

### 10.24.5 Non-dimensional Equations of Equilibrium when the Neutral Axis is within the Section ( $kD < D$ )

The strain profile IN and the corresponding stress block of concrete are presented in Figs.10.23.4b and c for this case. Following the same procedure of computing  $C_c$ ,  $C_s$  and the respective lever arms, we have the first equation as

$$P_u = 0.36 f_{ck} kbD + \sum_{i=1}^n (p_i bD / 100) (f_{si} - f_{ci}) \quad (10.45)$$

Dividing both sides of Eq.10.45 by  $f_{ck}bD$ , we have

$$P_u / f_{ck}bD = 0.36 k + \sum_{i=1}^n (p_i / 100 f_{ck}) (f_{si} - f_{ci}) \quad (10.46)$$

and the moment equation (Eq.10.36) as

$$M_u = 0.36 f_{ck} kbD(0.5 - 0.42 k) D + \sum_{i=1}^n (p_i bD / 100) (f_{si} - f_{ci}) (y_i / D) \quad (10.47)$$

Dividing both sides of Eq.10.47 by  $f_{ck}bD^2$ , we have

$$(M_u / f_{ck}bD^2) = 0.36 k(0.5 - 0.42 k) + \sum_{i=1}^n (p_i / 100 f_{ck}) (f_{si} - f_{ci}) (y_i / D) \quad (10.48)$$

where  $k =$  Depth of the neutral axis/Depth of column, mentioned earlier in sec.10.21.10 and Fig.10.21.11 of Lesson 21.

Equations 10.46 and 10.48 are the two non-dimensional equations of equilibrium in this case.

### 10.24.6 Non-dimensional Equation of Equilibrium when the Column Behaves as a Steel Beam

This is a specific situation when the column is subjected to pure moment  $M_u = M_o$  only (Point 6 of the interaction diagram in Fig.10.23.1 of Lesson 23).

Since the column has symmetrical longitudinal steel on both sides of the centroidal axis of the column, the column will resist the pure moment by yielding of both tensile and compressive steel bars (i.e.,  $f_{si} = 0.87 f_y = f_{yd}$ ). Thus, we have only one equation (Eq.10.36 of Lesson 23), which becomes

$$M_u = \sum_{i=1}^n (p_i bD / 100) (0.87 f_y) (y_i/D) \quad (10.49)$$

Dividing both sides of Eq.10.49 by  $f_{ck} bD^2$ , we have

$$(M_u/f_{ck}bD^2) = \sum_{i=1}^n (p_i / 100 f_{ck}) (0.87 f_y) (y_i/D) \quad (10.50)$$

Equation 10.50 is the equation of equilibrium in this case.

### 10.24.7 Preparation of Design Charts

Design charts are prepared employing the equations of four different cases as given in secs.10.24.3 to 6. The advantage of employing the equations is that the actual grade of concrete can be taken into account, though it may not be worthwhile to follow this accurately. However, preparation of interaction diagram will help in understanding the behaviour of column with the change of neutral axis depth for the four cases mentioned in sec.10.24.2. The step by step procedure of preparing the design charts is explained below. It is worth mentioning that the values of  $(P_u/f_{ck}bD)$  and  $(M_u/f_{ck}bD^2)$  are determined considering different locations of the neutral axis for the four cases mentioned in sec.10.24.2.

#### Step 1: When the neutral axis is at infinity

The governing equation is Eq.10.40. The strain profile EF and the corresponding stress block are in Fig.10.23.2b and c of Lesson 23, respectively.

#### Step 2: When the column is subjected to axial load considering minimum eccentricity

Lesson 22 presents the design of short columns subjected to axial load only considering minimum eccentricity as stipulated in cl.29.3 of IS 456, employing Eq.10.4, which is as follows:

$$P_u = 0.4 f_{ck} b D + (pbD/100) (0.67 f_y - 0.4 f_{ck}) \dots \quad (10.4)$$

Dividing both sides of Eq.10.4 by  $f_{ck} bD$ , we have

$$(P_u/f_{ck} bD) = 0.4 + (p/100 f_{ck}) (0.67 f_y - 0.4 f_{ck}) \quad (10.51)$$

The  $P_u$  obtained from Eq.10.51 can also resist  $M_u$  as per cl.39.3 of IS 456. From the stipulation of cl. 39.3 of IS 456 and considering the maximum value of the minimum eccentricity as  $0.05D$ , we have

$$M_u = (P_u) (0.05)D = 0.02 f_{ck} bD^2 + (0.05 pbD^2/100) (0.67 f_y - 0.4 f_{ck})$$

Dividing both sides of the above equation by  $f_{ck}bD^2$ , we have

$$(M_u/f_{ck} bD^2) = 0.02 + (0.05p/100 f_{ck}) (0.67 f_y - 0.4 f_{ck}) \quad (10.52)$$

Equations 10.51 and 10.52 are the two equations to be considered in this case.

### Step 3: When the neutral axis is outside the section

Figures 10.23.3b and c of Lesson 23 present one strain profile JK and the corresponding stress block, respectively, out of a large number of values of  $k$  from 1 to infinity, only values up to about 1.2 are good enough to consider, as explained in sec.10.23.5 of Lesson 23. Accordingly, we shall consider only one point, where  $k = 1.1$ , in this case. With the help of Eqs.10.42 and 10.44, Table 10.4 for the values of  $C_1$  and  $C_2$ , Table 10.5 for the values of  $f_{si}$  and Eq.10.23 or Eq.10.27 for the values of  $f_{ci}$ , the non-dimensional parameters  $P_u/f_{ck} bD$  and  $M_u/f_{ck} bD^2$  are determined.

### Step 4: When the neutral axis is within the section

One representative strain profile IU and the corresponding stress block are presented in Fig.10.23.4b and c, respectively, of Lesson 23. The following six points of the interaction diagram are considered satisfactory for preparing the design charts:

- (a) Where the tensile stress of longitudinal steel is zero i.e.,  $kD = D - d'$ ,
- (b) Where the tensile stress of longitudinal steel is  $0.4f_y d = 0.4(0.87 f_y)$ ,
- (c) Where the tensile stress of longitudinal steel is  $0.8f_y d = 0.8(0.87 f_y)$ ,
- (d) Where the tensile stress of longitudinal steel is  $f_y d = 0.87f_y$  and strain  $= 0.87f_y/E_s$ , i.e., the initial yield point,
- (e) Where the tensile stress of longitudinal steel is  $f_y d = 0.87f_y$  and strain  $= 0.87f_y/E_s + 0.002$ , i.e., the final yield point,

(f) When the depth of the neutral axis is  $0.25D$ .

For all six points, the respective strain profile and the corresponding stress blocks can be drawn. Therefore, values of  $(P_u/f_{ck} bD)$  and  $(M_u/f_{ck} bD^2)$  are determined from Eqs.10.46 and 10.48, using Table 10.5 for  $f_{sc}$  and Eq.10.34 for  $f_{ci}$ .

### Step 5: When the column behaves like a steel beam

As explained in sec.10.24.6, Eq.10.50 is used to compute  $M_u/f_{ck} bD^2$  in this case.

### Step 6: Preparation of design chart

The ten pairs of  $(P_u/f_{ck} bD)$  and  $(M_u/f_{ck} bD^2)$  (one set each in steps 1, 2, 3 and 5 and six sets in step 4) can be plotted to prepare the desired design chart.

One illustrative example is taken up in the next section.

## 10.24.8 Illustrative Example

### Problem 1:

Prepare a design chart for a rectangular column with 3 per cent longitudinal steel distributed equally on two faces using M 25 and Fe 415, and considering  $d'/D = 0.15$ .

### Solution 1:

The solution of this problem is explained in six steps of the earlier section.

### Step 1: When the neutral axis is at infinity

Figures 10.23.2b and c present the strain profile EF and the corresponding stress block, respectively. Using the values of  $p = 3$  per cent,  $f_{ck} = 25 \text{ N/mm}^2$  and determining the value of  $f_{sc} = 327.7388 \text{ N/mm}^2$  (using linear interpolation from the values of Table 10.5 of Lesson 23), we get the value of  $(P_u/f_{ck} bD)$  from Eq.10.40 as

$$(P_u/f_{ck} bD) = 0.8259.$$

### Step 2: When the column is subjected to axial load considering minimum eccentricity

Using the value of  $p = 3$  per cent,  $f_{ck} = 25 \text{ N/mm}^2$  and  $f_y = 415 \text{ N/mm}^2$  in Eqs.10.51 and 10.52 of sec.10.24.6, we have

$$(P_u/f_{ck} bD) = 0.7217$$

$$(M_u/f_{ck} bD^2) = 0.0361$$

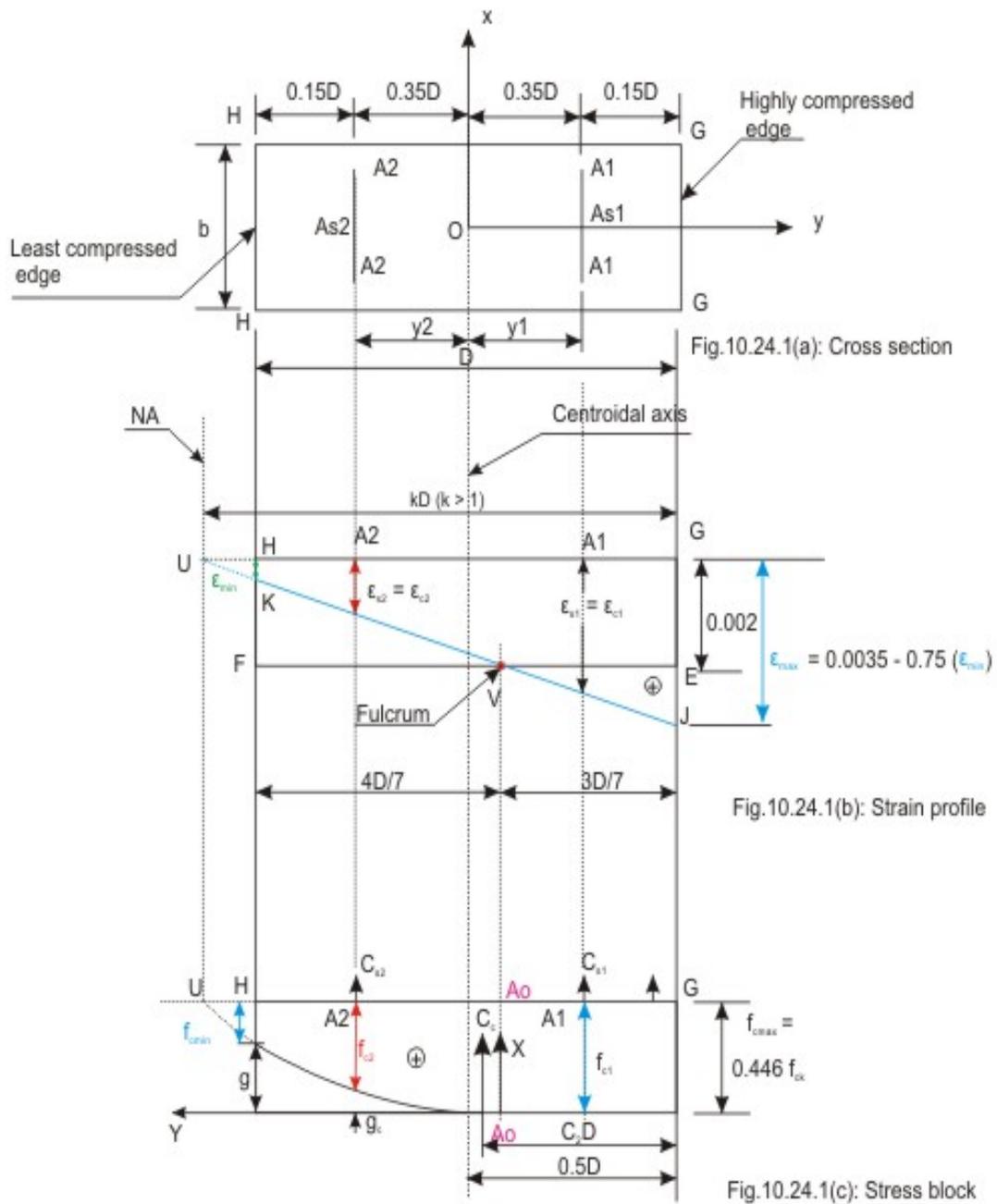


Fig.10.24.1: Problem 1 and Q. 3 (step 3, k=1.1)

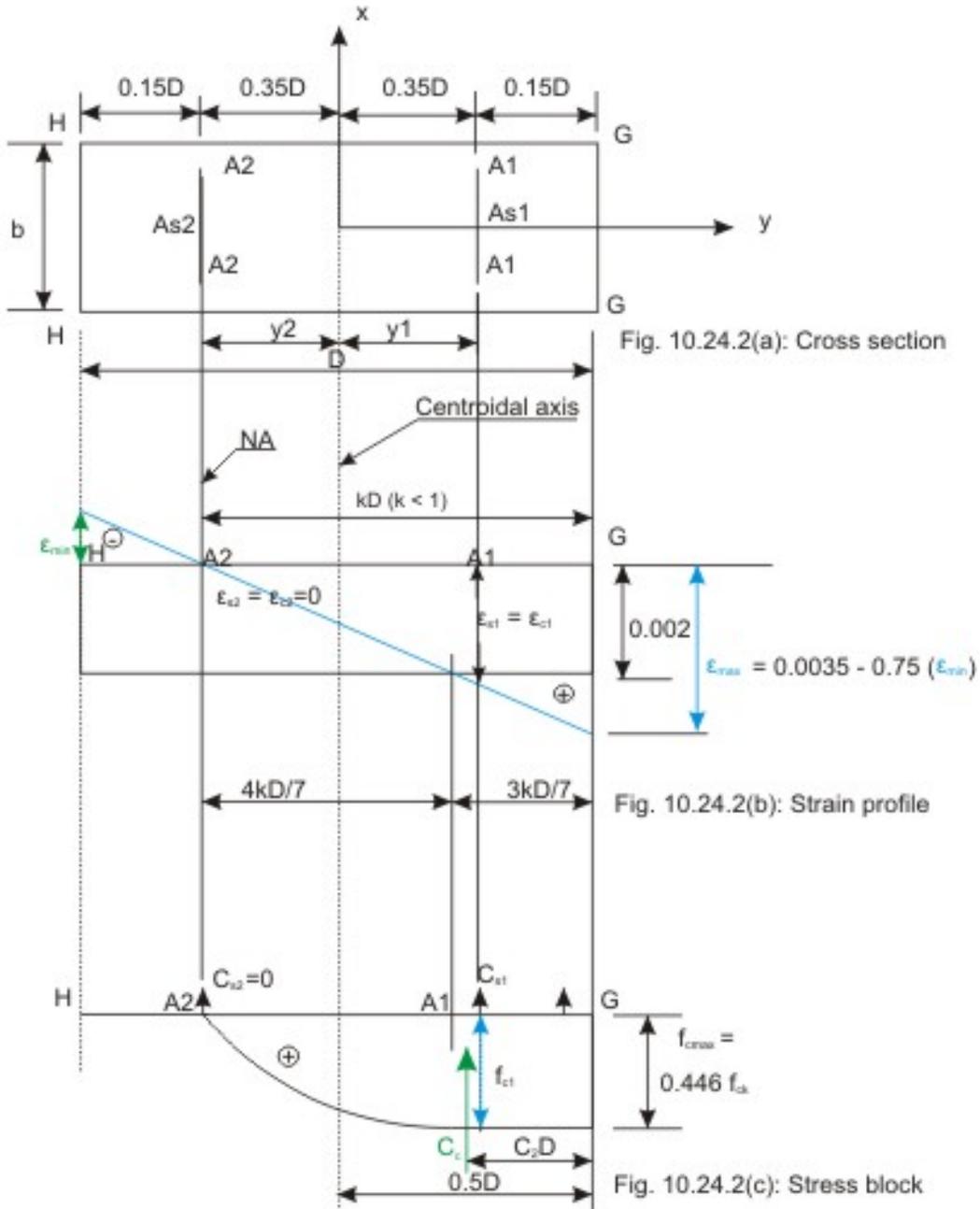
**Step 3: When the neutral axis depth = 1.1 D**

Figures 10.24.1a, b and c show the section of the column, strain profile JK and the corresponding stress block, respectively, for this case. We use Eqs.10.42 and 10.44 for determining the value of  $(P_u/f_{ck} bD)$  and  $(M_u/f_{ck} bD^2)$  for

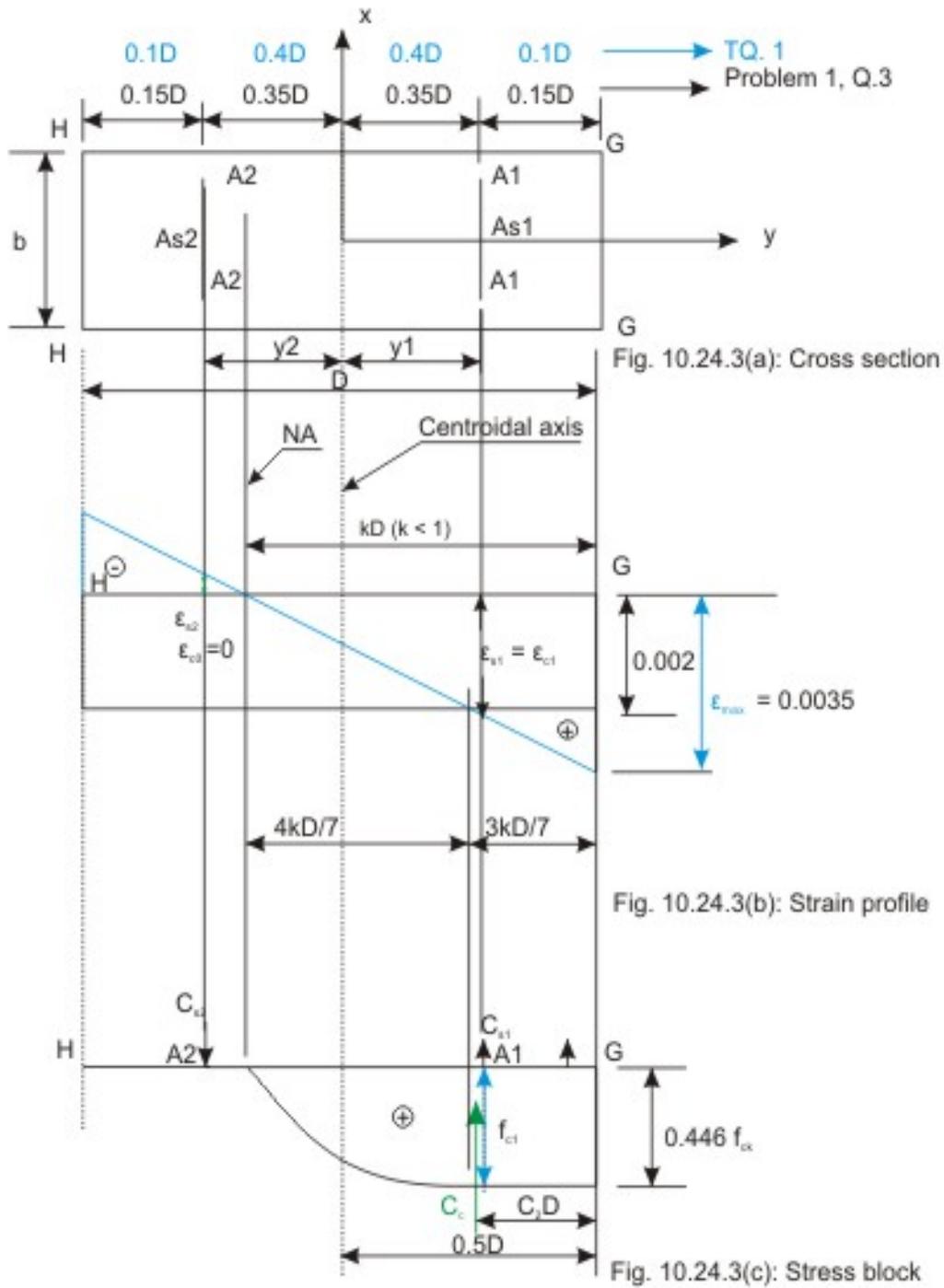
this case using  $k = 1.1$ ,  $f_{ck} = 25 \text{ N/mm}^2$ ,  $p_1 = p_2 = 1.5$ ,  $y_1/D = 0.35$  and  $y_2/D = -0.35$ . Values of  $C_1$ ,  $C_2$ ,  $f_{s1}$  and  $f_{s2}$ ,  $f_{c1}$  and  $f_{c2}$  are obtained from equations mentioned in Step 3 of sec.10.24.6. The values of all the quantities are presented in Table 10.6A, mentioning the source equation no., table no. etc. to get the two non-dimensional parameters as given below:

$$(P_u/f_{ck} bD) = 0.67405$$

$$(M_u/f_{ck} bD^2) = 0.06370$$



**Fig. 10.24.2:** Problem 1 and Q. 3 (step 4,  $f_{s2} = 0$ )



**Fig. 10.24.3:** Problem 1, Q.3 and TQ. 1 (step 4,  $f_{s2} = -0.4f_{yd}$ )

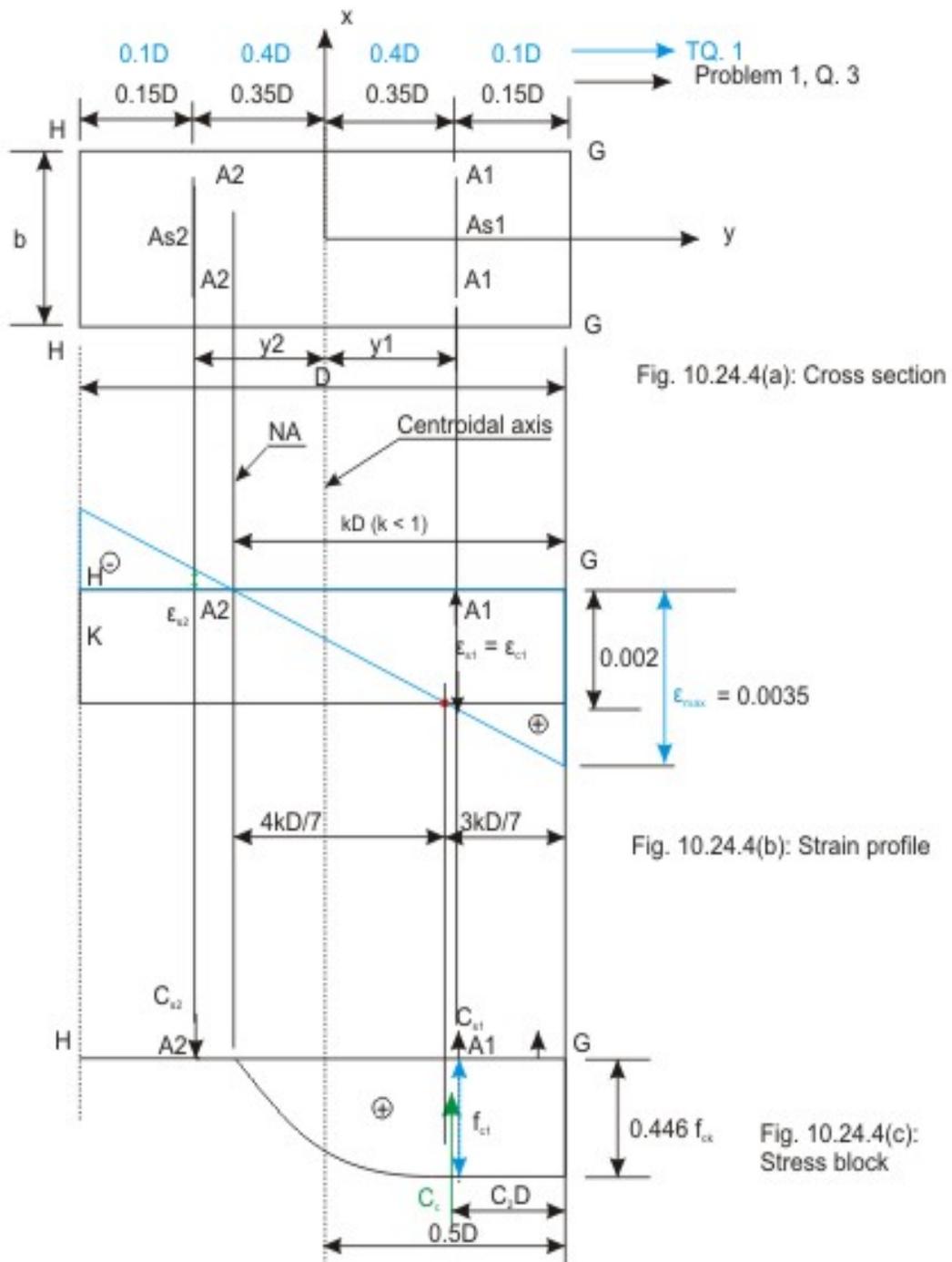
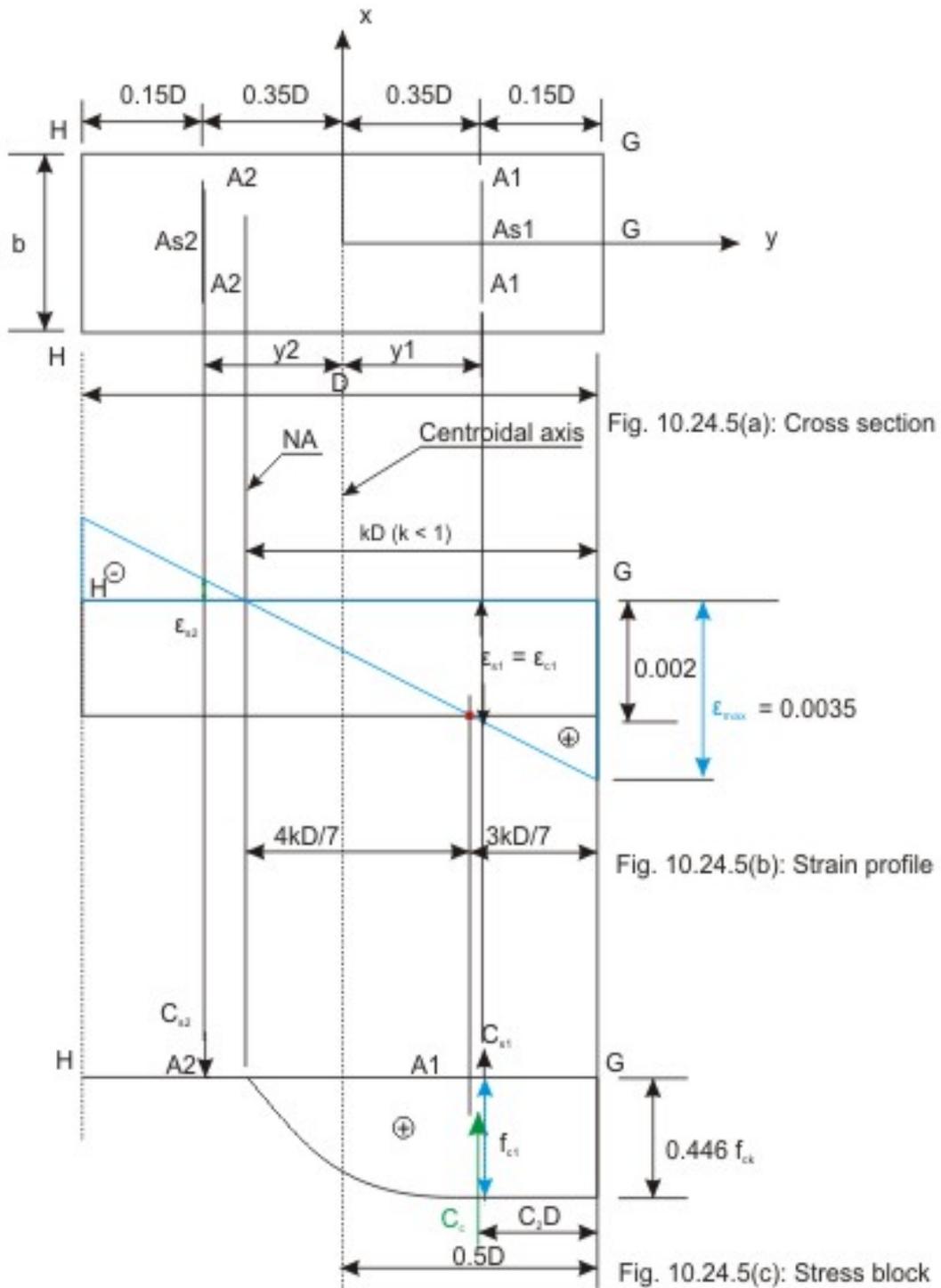
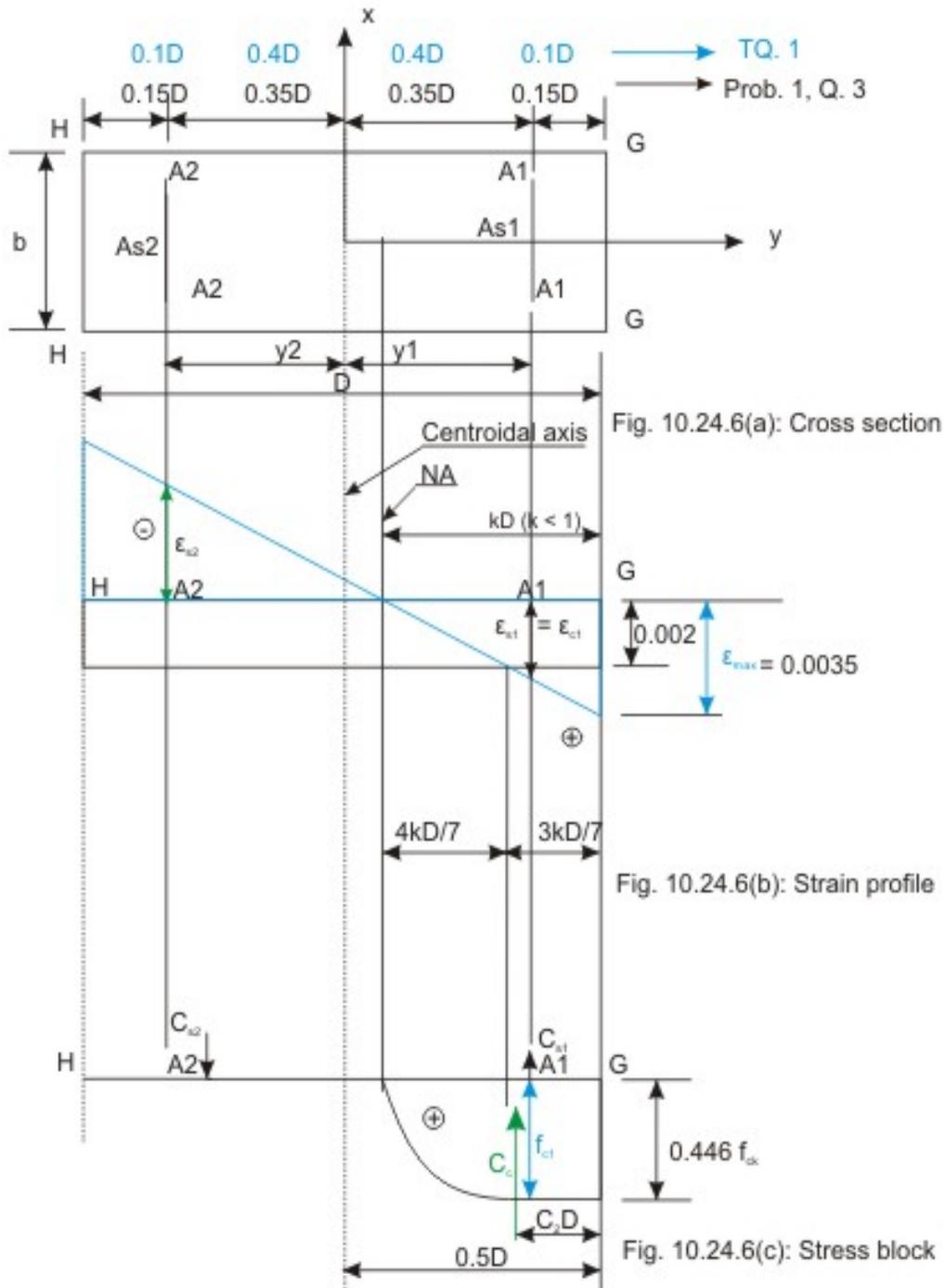


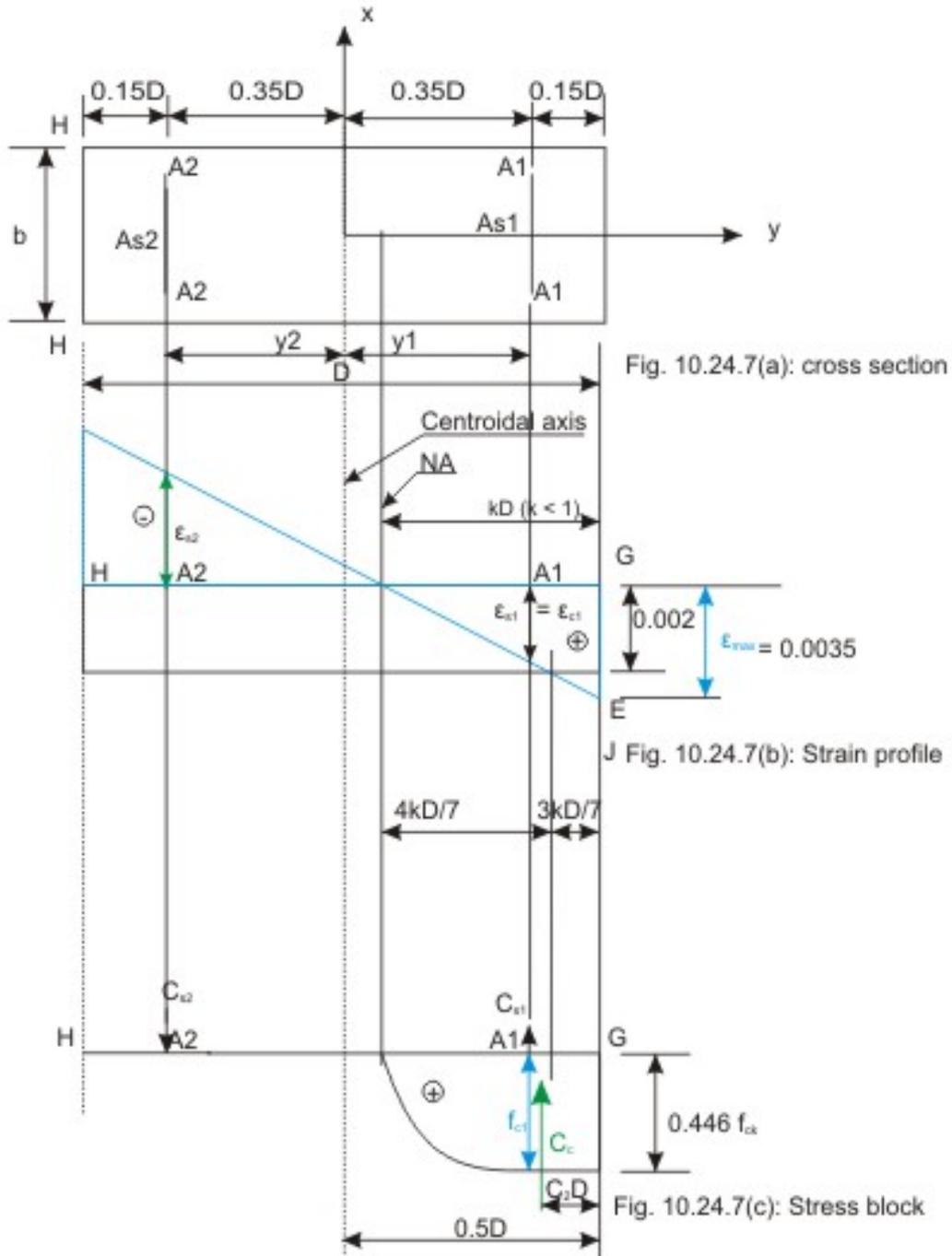
Fig. 10.24.4.: Problem 1, Q.3 and TQ. 1 (step 4,  $f_{s2} = -0.8f_{yd}$ )



**Fig. 10.24.5:** Problem 1 and Q.3 (step 4,  $f_{s2} = -f_{yd}$ , initial yield)



**Fig. 10.24.6:** Problem 1, Q.3 and TQ. 1 (step 4,  $f_{s2} = f_{yd}$ ), final yield



**Fig. 10.24.7: Problem 1 and Q.3 (step 4,  $k = 0.25$ )**

**Step 4: When the neutral axis is within the section**

In Step 4 of section 10.24.6, six different locations of neutral axis are mentioned; five of them (a to e) are specified by the magnitude of  $f_{s2}$  (tensile) of longitudinal steel and one of them is specified by the value of  $k = 0.25$ . The values of all the quantities are presented in Tables 10.6A and B, mentioning the source equation no., table no. etc.

Figures 10.24.2 to 10.24.7 present the respective strain profiles and the corresponding stress block separately for all six different locations of the neutral axis.

**Table 10.6A Parameters and results of Problem 1 of Section 10.24.8**

Given data:  $f_{ck} = 25 \text{ N/mm}^2$ ,  $f_y = 415 \text{ N/mm}^2$ ,  $p = 3 \text{ per cent}$ ,  $p_1 = p_2 = 1.5 \text{ per cent}$ ,

$$d'/D = 0.15$$

Note: Units of  $f_{si}$ ,  $f_{sc}$  and  $f_{ci}$  are in  $\text{N/mm}^2$ , (-) minus sign indicates tensile strain or stress.

Sl.No.	Given	$k = 1.1$	$f_{s2} = 0$	$f_{s2} = -0.4 f_{yd}$	$f_{s2} = 0.8 f_{yd}$
	Description				
1	Sec. No.	10.24.7	10.24.7	10.24.7	10.24.7
2	Step No.	3	4	4	4
3	Fig. No.	10.24.1	10.24.2	10.24.3	10.24.4
4	$\varepsilon_{s1} = \varepsilon_{c1}$	0.002829	0.00288	0.00275	0.00263
5	$\varepsilon_{s2} = \varepsilon_{c2}$	0.000744	0.0	-0.00072	-0.00144
6	Table No. of $f_{si}$ and $f_{sc}$	10.5	10.5	10.5	10.5
7	$f_{s1}$	352.407	352.871	351.669	348.392
8	$f_{s2}$	148.914	0.0	-144.42	-288.84
9	$f_{sc}$	NA	NA	NA	NA
10	Eq.Nos. of $f_{ci}$	10.23 and 10.27	10.34	10.34	10.34
11	$f_{c1}$	11.15	11.15	11.15	11.15
12	$f_{c2}$	6.757	0.0	0.0	0.0
13	Table No. of $C_1$ and $C_2$	10.4	NA	NA	NA
14	$C_1$	0.384	NA	NA	NA
15	$C_2$	0.443	NA	NA	NA
16	$y_1/D$	+0.35	+0.35	+0.35	+0.35
17	$y_2/D$	-0.35	-0.35	-0.35	-0.35
18	$k$	1.1	0.85	0.7046	0.6017
19	Eq.No. of $P_u/f_{ck} bD$	10.42	10.46	10.46	10.46
20	$P_u/f_{ck} bD$	0.6740	0.5110	0.3713	0.2457
21	Eq.No. of $M_u/f_{ck} bD^2$	10.44	10.48	10.48	10.48
22	$M_u/f_{ck} bD^2$	0.0643	0.1155	0.1536	0.1850

**Table 10.6B Parameters and results of Problem 1 of Section 10.24.8**

Given data:  $f_{ck} = 25 \text{ N/mm}^2$ ,  $f_y = 415 \text{ N/mm}^2$ ,  $\rho = 3 \text{ per cent}$ ,  $\rho_1 = \rho_2 = 1.5 \text{ per cent}$ ,

$$d/D = 0.15$$

Note: Units of  $f_{si}$ ,  $f_{sc}$  and  $f_{ci}$  are in  $\text{N/mm}^2$ , (-) minus sign indicates tensile strain or stress.

Sl.No.	Given	$f_{s2} = - f_{yd}$ (Initial yield)	$f_{s2} = - f_{yd}$ (Final yield)	$k = 0.25$
	Description			
1	Sec. No.	10.24.7	10.24.7	10.24.7
2	Step No.	4	4	4
3	Fig. No.	10.24.5	10.24.6	10.24.7
4	$\epsilon_{s1} = \epsilon_{c1}$	0.00256	0.00221	0.0014
5	$\epsilon_{s2} = \epsilon_{c2}$	-0.00180	-0.00380	-0.0084
6	Table No. of $f_{si}$ and $f_{sc}$	10.5	10.5	10.5
7	$f_{s1}$	346.754	335.484	281.0
8	$f_{s2}$	-361.05	-361.05	-361.05
9	$f_{sc}$	NA	NA	NA
10	Eq.Nos. of $f_{ci}$	10.34	10.34	10.34
11	$f_{c1}$	11.15	11.15	10.146
12	$f_{c2}$	0.0	0.0	0.0
13	Table No. of $C_1$ and $C_2$	NA	NA	NA
14	$C_1$	NA	NA	NA
15	$C_2$	NA	NA	NA
16	$y_1/D$	+0.35	+0.35	+0.35
17	$y_2/D$	-0.35	-0.35	-0.35
18	$k$	0.5607	0.4072	0.25
19	Eq.No. of $P_u/f_{ck} bD$	10.46	10.46	10.46
20	$P_u/f_{ck} bD$	0.1866	0.1246	0.0353
21	Eq.No. of $M_u/f_{ck} bD^2$	10.48	10.48	10.48
22	$M_u/f_{ck} bD^2$	0.1997	0.1921	0.1680

**Step 5: When the column behaves like a steel beam**

For this case, the parameter ( $M_u/f_{ck} bD^2$ ) is determined from Eq.10.50 using  $\rho_1 = \rho_2 = 1.5 \text{ per cent}$ ,  $f_{ck} = 25 \text{ N/mm}^2$ ,  $f_y = 415 \text{ N/mm}^2$ ,  $y_1/D = 0.35$  and  $y_2/D = -0.35$ . Thus, we get

$$(M_u/f_{ck} bD^2) = 0.15164$$

### Step 6: Final results of design chart

The values of ten pairs of  $(P_u/f_{ck} bD)$  and  $(M_u/f_{ck} bD^2)$  as obtained in steps 1 to 5 are presented in Sl. Nos. 1 to 10 of Table 10.6C. The design chart can be prepared by plotting these values.

**Table 10.6C** Final values of  $P_u/f_{ck} bD$  and  $M_u/f_{ck} bD^2$  of Problem 1 of Section 10.24.8

Sl. No.	Particulars about the point	$P_u/f_{ck} bD$	$M_u/f_{ck} bD^2$
1	$k = \alpha$	0.8259	0.0
2	Minimum eccentricity	0.7217	0.0361
3	$k = 1.1$	0.6740	0.0643
4	$f_{s2} = 0$	0.5110	0.1155
5	$f_{s2} = (-)0.4 f_{yd}$	0.3713	0.1536
6	$f_{s2} = (-)0.8 f_{yd}$	0.2457	0.1850
7	$f_{s2} = (-) f_{yd}$ (Initial yield)	0.1866	0.1997
8	$f_{s2} = (-) f_{yd}$ (Final yield)	0.1246	0.1921
9	$k = 0.25$	0.0353	0.1680
10	Steel Beam	0.0	0.1516

### 10.24.9 Practice Questions and Problems with Answers

**Q.1:** Why do we need to have non-dimensional design chart?

**A.1:** See sec. 10.24.1

**Q.2:** Name the different design parameters while designing a column. Mention which one is the most important parameter.

**A.2:** See sec. 10.24.2.

**Q.3:** Prepare a design chart for a rectangular column within three per cent longitudinal steel, equally distributed on two faces, using M 25 and Fe 250 and considering  $d'/D = 0.15$ .

**A.3:** The solution of this problem is obtained following the same six steps of Problem 1 of sec.10.24.8, except that the grade of steel here is Fe 250. Therefore, the final results and all the parameters are presented in Table 10.7 avoiding explaining step by step again.

Table 10.7 Final values of  $P_U/f_{ck} bD$  and  $M_U/f_{ck} bD^2$  of Q.3 of Section 10.24.9

Sl. No.	Particulars about the point	$P_U/f_{ck} bD$	$M_U/f_{ck} bD^2$
1	$k = \alpha$	0.6936	0.0
2	Minimum eccentricity	0.5890	0.0295
3	$k = 1.1$	0.5931	0.0354
4	$f_{s2} = 0$	0.4298	0.0871
5	$f_{s2} = (-)0.4 f_{yd}$	0.3438	0.1113
6	$f_{s2} = (-)0.8 f_{yd}$	0.2645	0.1323
7	$f_{s2} = (-) f_{yd}$ (Initial yield)	0.2268	0.1421
8	$f_{s2} = (-) f_{yd}$ (Final yield)	0.1559	0.1395
9	$k = 0.25$	0.0839	0.1248
10	Steel Beam	0.0	0.0914

### 10.24.10 References

1. Reinforced Concrete Limit State Design, 6<sup>th</sup> Edition, by Ashok K. Jain, Nem Chand & Bros, Roorkee, 2002.
2. Limit State Design of Reinforced Concrete, 2<sup>nd</sup> Edition, by P.C.Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2002.
3. Advanced Reinforced Concrete Design, by P.C.Varghese, Prentice-Hall of India Pvt. Ltd., New Delhi, 2001.
4. Reinforced Concrete Design, 2<sup>nd</sup> Edition, by S.Unnikrishna Pillai and Devdas Menon, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2003.
5. Limit State Design of Reinforced Concrete Structures, by P.Dayaratnam, Oxford & I.B.H. Publishing Company Pvt. Ltd., New Delhi, 2004.
6. Reinforced Concrete Design, 1<sup>st</sup> Revised Edition, by S.N.Sinha, Tata McGraw-Hill Publishing Company. New Delhi, 1990.
7. Reinforced Concrete, 6<sup>th</sup> Edition, by S.K.Mallick and A.P.Gupta, Oxford & IBH Publishing Co. Pvt. Ltd. New Delhi, 1996.
8. Behaviour, Analysis & Design of Reinforced Concrete Structural Elements, by I.C.Syal and R.K.Ummat, A.H.Wheeler & Co. Ltd., Allahabad, 1989.
9. Reinforced Concrete Structures, 3<sup>rd</sup> Edition, by I.C.Syal and A.K.Goel, A.H.Wheeler & Co. Ltd., Allahabad, 1992.
10. Textbook of R.C.C, by G.S.Birdie and J.S.Birdie, Wiley Eastern Limited, New Delhi, 1993.
11. Design of Concrete Structures, 13<sup>th</sup> Edition, by Arthur H. Nilson, David Darwin and Charles W. Dolan, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2004.
12. Concrete Technology, by A.M.Neville and J.J.Brooks, ELBS with Longman, 1994.
13. Properties of Concrete, 4<sup>th</sup> Edition, 1<sup>st</sup> Indian reprint, by A.M.Neville, Longman, 2000.

14. Reinforced Concrete Designer's Handbook, 10<sup>th</sup> Edition, by C.E.Reynolds and J.C.Steedman, E & FN SPON, London, 1997.
15. Indian Standard Plain and Reinforced Concrete – Code of Practice (4<sup>th</sup> Revision), IS 456: 2000, BIS, New Delhi.
16. Design Aids for Reinforced Concrete to IS: 456 – 1978, BIS, New Delhi.

### 10.24.11 Test 24 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

**TQ.1:** Determine the parameters including the two non-dimensional parameters,  $P_u$  and  $M_u$  of a rectangular reinforced concrete short column of  $b = 370$  mm,  $D = 530$  mm,  $d'/D = 0.1$  and having 8-25 mm diameter bars as longitudinal steel distributed equally on two sides using M 20 and Fe 415 for each of the following three cases:

- (a) when  $f_{s2} = -0.4 f_{yd}$
- (b) when  $f_{s2} = -0.8 f_{yd}$
- (c) when  $f_{s2} = -f_{yd}$  (at final yield)

(16 + 17 + 17 = 50)

**A.TQ.1:** This problem can be solved following the same procedure of explained in Step 4b, c and d of sec.10.24.7. The step by step calculations are not shown here and the final results are presented in Table 10.8.

**Table 10.8 Parameters and results of TQ.1 of Section 10.24.11**

Given data:  $f_{ck} = 20$  N/mm<sup>2</sup>,  $f_y = 415$  N/mm<sup>2</sup>,  $b = 370$  mm,  $D = 530$  mm, Longitudinal steel = 8-25 mm diameter equally distributed on two sides,  $d'/D = 0.15$

Sl.No.	Given	$f_{s2} = -0.4 f_{yd}$	$f_{s2} = 0.8 f_{yd}$	$f_{s2} = -f_{yd}$ (Final yield)
	Description			
1	Sec. No.	10.24.7	10.24.7	10.24.7
2	Step No.	4	4	4
3	Fig. No.	10.24.3	10.24.4	10.24.6
4	$\epsilon_{s1} = \epsilon_{c1}$	0.0030	0.0029	0.0027
5	$\epsilon_{s2} = \epsilon_{c2}$	-0.00072	-0.00144	-0.0038
6	Table No. of $f_{si}$ and $f_{sc}$	10.5	10.5	10.5
7	$f_{s1}$	354.1702	353.468	349.956

8	$f_{s2}$	-144.42	-288.84	-361.05
9	$f_{sc}$	NA	NA	NA
10	Eq.Nos. of $f_{ci}$	10.34	10.34	10.34
11	$f_{c1}$	11.15	11.15	11.15
12	$f_{c2}$	0.0	0.0	0.0
13	Table No. of $C_1$ and $C_2$	NA	NA	NA
14	$C_1$	NA	NA	NA
15	$C_2$	NA	NA	NA
16	$y_1/D$	+0.4	+0.4	+0.4
17	$y_2/D$	-0.4	-0.4	-0.4
18	$k$	0.7461	0.6371	0.4311
19	Eq.No. of $P_u/f_{ck} bD$	10.46	10.46	10.46
20	$P_u/f_{ck} bD$	0.3690	0.2572	0.1452
21	$P_u$ (kN)	1447.225	1008.792	569.568
22	Eq.No. of $M_u/f_{ck} bD^2$	10.48	10.48	10.48
23	$M_u/f_{ck} bD^2$	0.1481	0.1799	0.1899
24	$M_u$ (kNm)	307.777	374.125	394.779

### 10.24.12 Summary of this Lesson

This lesson explains the procedure of the preparation of design charts of rectangular reinforced concrete short columns subjected to axial load and uniaxial moment. Different positions of the neutral axis due to different pairs of  $P_u$  and  $M_u$  give rise to different strain profiles and stress blocks. Accordingly, the column may collapse when subjected to any pair of axial load and moment exceeding the capacities of the column. Design charts are very much useful to design the column avoiding lengthy numerical computations. Illustrative example, practice and test problems will help in understanding each step of the procedure to prepare the design chart.