Module 4

Lecture 1

Measuring and modeling: Soil Hydraulic Characteristics

The experimental techniques for measuring various unsaturated soil parameters vary widely in terms of cost, complexity, and accuracy. The available techniques can be classified into laboratory-based or field-based methods. These techniques are presented separately in the following sections.

Moisture content measurement

Measurement of soil moisture content is important for establishing SWCC and qualitative understanding of hydraulic characteristics of the soil. Measurement of water content can be done gravimetrically in the laboratory on disturbed or undisturbed samples obtained by augers or rings, respectively. On the other hand, measurement of soil moisture in the field conditions requires instruments designed with the help of fundamental principles of physics and chemistry. Most of these sensors were developed by soil scientists for determining the crop growth. However, they have potential applications in the moisture measurement for field applications in geotechnical engineering. The moisture content in the field is estimated indirectly by determining the relationship to some property of the soil which can be readily measured by electrical resistance or neutron scattering or chemical reactivity. Some of the important techniques are described here.

<u>Time Domain Reflectrometry (TDR)</u>

Time domain reflectometry (TDR) is well known for the determination of soil water content and bulk electrical conductivity. A very fast electric pulse is injected into a waveguide, in TDR method, that carries the pulse to an isolated probe placed in the soil. The velocity of the pulse in the probe indirectly signifies the amount of moisture in the soil. The soils with high degree of saturation measures smaller velocities and vice-versa. The instrument directly gives the values in volumetric water content (%).The TDR method relies on graphical interpretation of the waveform reflected from that part of the waveguide, which is the probe.

The probes are connected to the instrument through a network of coaxial cables and multiplexers in field installation (Evett, 2003) as shown in Fig. 4.1. The mechanism of TDR instrumentations is illustrated in Fig. 4.2.

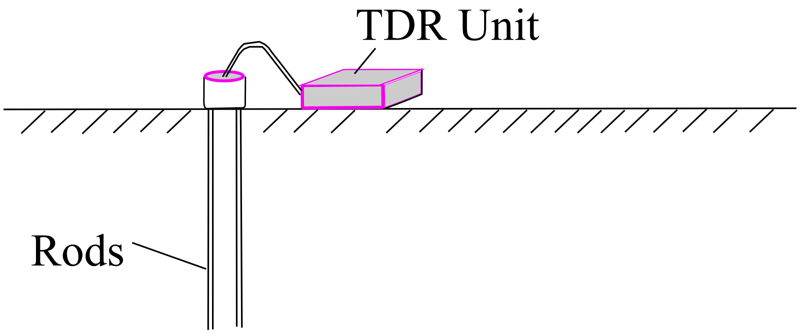


Fig. 4.1 Illustration showing the installation of TDR in the field

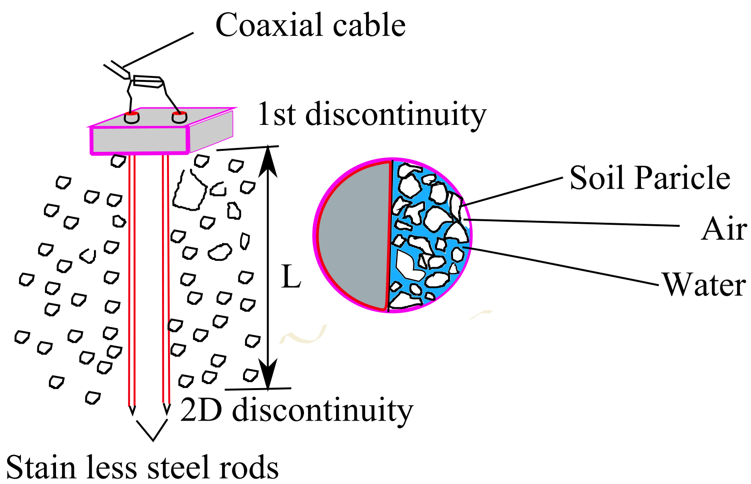


Fig. 4.2. Basic TDR unit

The advantage of this method is that they can operate reliably in a variety of difficult soils, including saline soils, heavy clay soils, soils with high organic content, and soils having extremely nonuniform vertical profiles. Time domain reflectometry systems can provide high accuracy at all water contents without any soil calibration. Part of the TDR instrument provides the voltage step and another part, essentially a fast oscilloscope, captures the reflected waveform. Fig. 4.3 shows a waveform that represents the waveguide.

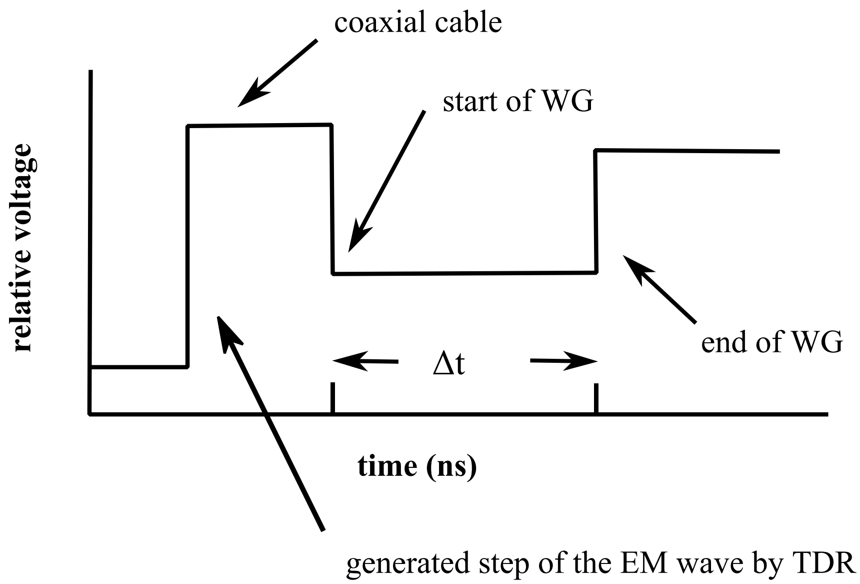


Fig. 4.3. Plot of the waveform (*Source*: Weblink – 4)

The TDR is the most accurate, but expensive when compared to other existing systems of soil moisture content measurements. However, caution must be exercised in interpreting the results as the air gaps in the soil and humidity levels of the atmosphere influence the TDR readings.

<u>Thetaprobe</u>

Thetaprobe is a soil moisture sensor commonly used by soil engineers and soil scientist in the field. It consists of input/output cable, probe body, which is a waterproof housing, and a sensing head. The cable provides connection for a suitable power supply and for an analogue signal output. The water-proof synthetic housing (probe body) contains an oscillator, a specially designed internal transmission line, and measuring circuitry. The sensing head consists of a group of four rods. The center one is a signal rod and the outer three rods form an electrical shield around the signal rod. The sensor is connected to the soil moisture meter from cable to a datalogger. The datalogger displays the measurements and can even store them in the memory.

Thetaprobe measures the volumetric moisture content of the soil as a percentage by measuring the changes in dielectric constant. The sensor measures the variations in dielectric constant by Frequency Domain technique (FDT). The measurement of water content of soils is done by determination of the apparent dielectric constant using the following equation (*Source*: Macaulay Land Use Research Institute website)

$$\theta_v = \left(\sqrt{\varepsilon - A}\right) \Big/ B$$

where ε is the apparent dielectric constant, and A and B are constants dependent on soil type.

The sensor requires calibration by establishing a relationship between square root of dielectric constant and volumetric water content. In directly, it predicts the soil dependent constants in the aforementioned equation. The sensor can measure the volumetric moisture contents in the range of 5 – 55% with an accuracy of 5% with standard calibration. The advantage of this instrument is that it is relatively cheaper when compared to TDR.

Lecture 2

Moisture content measurement

<u>Neutron probe</u>

Neutron probe is a soil moisture meter utilizing the concept of neutron scattering. Fast neutrons with very high energy (million electron volts) are injected into the soil from a source of radioactive material at the measurement location. The neutrons are slowed by collision with nuclei of hydrogen atoms in the soil. These hydrogen atoms present in the soil in terms of pore water and vapors. The average energy loss due to collision of neutrons with hydrogen atoms is considerable for the measurement. This loss is directly a measure of soil moisture content. It is measured in terms of number of slow neutrons counted per unit of time averaged over a volume of soil.

It contains an electronic gauge, a connecting cable, and a source tube containing both nuclear source and detector tube as shown in the Fig. 4.4.

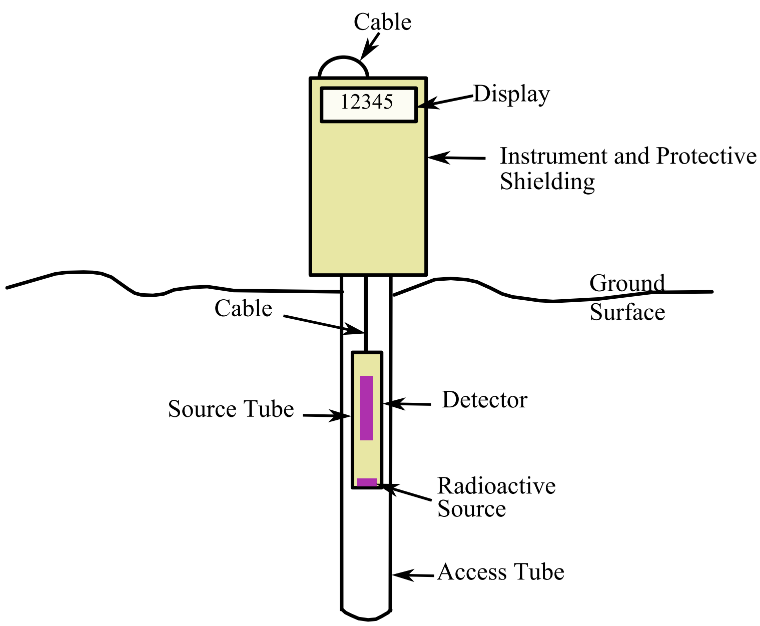


Fig. 4.4. Neutron probe unit in the field

An access tube is installed in the ground, and the source tube is lowered into the tube to the desired depths of measurement. High speed neutrons from radioactive pellets would colloid with hydrogen atoms in the soil moisture, when the source tube is lowered in the ground. The slow neutrons, after collision, would reflect back to the source tube and provide a measurement in the neutron detector. The measured counts give the volumetric soil moisture content using the calibration equation.

Advantage of this method is that it samples higher volumes (around the size of volleyball) of soil sample compared to other methods. This poses a limitation for measurement at shallow depths as some neutrons escape from the soil surface into the air instead of being detected. The major disadvantage is the use of radioactive source for the measurement. This can pose a threat to the inmates.

<u>Moisture tester based on gas pressure</u>

This method is based on generating gas pressure and correlating the measured gas pressure with the moisture content of a given soil. The free moisture in the soil reacts with the calcium carbide in the tester to form acetylene gas. This gas registers percentage of moisture in the soil on the dial. It gives direct readings of moisture content in the sample expressed as a percentage of total wet weight.

Even though this method is found to be efficient on several different materials, the accurate estimation of moisture content of fine-grained soils is uncertain. As part of the moisture in clays is held in the form of diffuse double-layer around the particle surfaces and which is not freely available for chemical reaction, the measured gas pressure under-predicts the moisture content of clays. Further, the instrument requires frequent calibration.

Measurement of Suction

<u>Matric suction</u>

Matric suction is an important variable in unsaturated soil mechanics for establishing hydraulic characteristics and in defining the state of stress in unsaturated soils. Therefore, control or measurement of matric suction becomes essential in order to evaluate physical and mechanical behaviors of the unsaturated soils under changing stress conditions. However, it is difficult to measure and control the negative pore water pressure as cavitation occurs in free water when the pressure approaches – 1 atm. Cavitation is the process of vapor nucleation in a liquid when the absolute pressure falls below the vapor pressure. Even though the boiling refers to the same phase transformation, in the process the vapor nucleation in a liquid occurs when the temperature is raised above the saturated vapor/liquid temperature. The water-phase in both soil and measurement system becomes discontinuous due to cavitation and, thus, making the measurements undependable. However, in nature, the matric suction exists in several hundred folds of magnitude to the atmospheric pressure and this influences the behavior of soils. This is the major limitation of the tensiometers that directly measure negative pore water pressure. Thus, alternative measurement techniques have been developed.

Several commonly used techniques are described, in this section, for measuring the matric suction of soils that are given below

- Tensiometers
- Axis Translation Technique
- Osmotic technique
- Electrical/Thermal Conductivity Sensors
- Filter paper techniques

Tensiometers are used for the direct measurement of negative pore water pressures in the soils by establishing a continuous connection between soil pore water and the measuring system. Similarly, axis-translation techniques rely on controlling the difference between the pore air pressure and pore-water pressure and measuring the corresponding water content of soil in equilibrium with the applied matric suction. On the other hand, electrical or thermal conductivity sensors are the indirect methods of measuring techniques that correlate matric suction to the electrical or thermal conductivity of a porous medium embedded in a mass of unsaturated soil. In contrary, filter-paper technique relies on measuring the equilibrium water content of small filter papers in direct contact with unsaturated soil specimens. The suction measurements use these techniques are combined with the previously described measurement techniques for soil moisture content for establishing soil-water characteristic curve.

In many field applications and research applications in geotechnical engineering, measurement of total suction (i.e., sum of matric suction and osmotic suction) is important. Thus, the following techniques for measuring total suction are also described in this section.

- Humidity control techniques (thermocouple psychrometers, chilled-mirror hygrometers, and polymer resistance/capacitance sensors)
- Non-contact filter paper method. Humidity measurement devices include.

Humidity control techniques described here are using the salt solutions. The noncontact filter paper method is an indirect humidity measurement technique that relies on determining the equilibrium water content of small filter papers sealed to establish the vapor phase connection of unsaturated specimens.

Lecture 3

Suction measurement

<u>Tensiometers</u>

Tensiometers consist of a water-filled tube with a high-air-entry (HAE) ceramic tip at one end and a sensor, for measuring negative water pressure at the other. The ceramic tip is used to create a hydraulic connection between the soil pore water, the water in the saturated HAE material, and the pressure sensor. A schematic diagram for a commonly used laboratory/field tensiometer is given in Fig. 4.8.

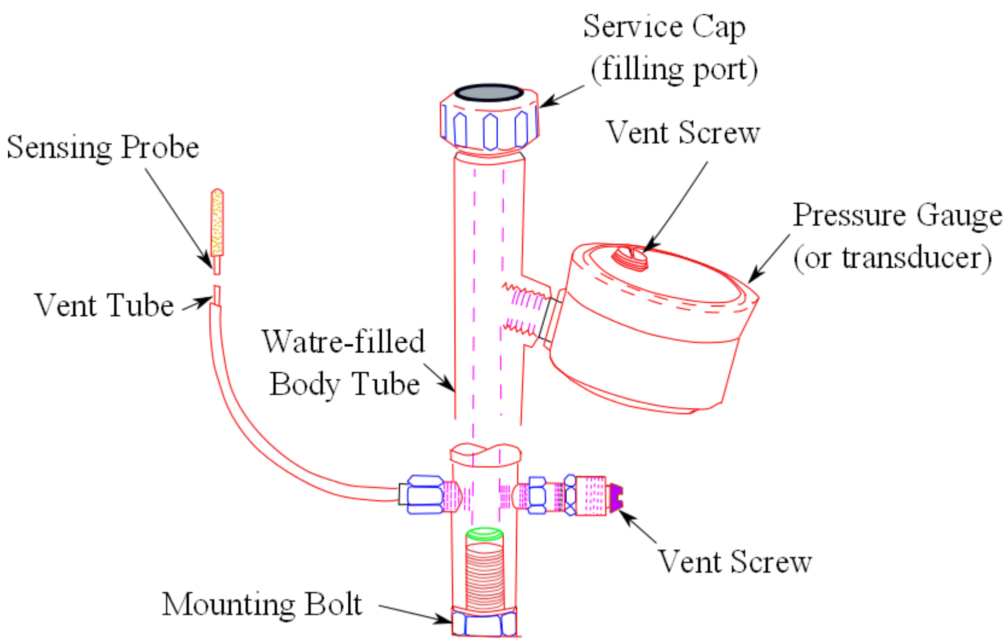


Fig. 4.8. Schematic drawing of small-tip laboratory tensiometer (after Lu and Likos, 2004)

The suction measurements in Tensiometers are based on the characteristics of high-air-entry (HAE) material. It consists of microscopic pores of relatively uniform size as shown in Fig. 4.9.

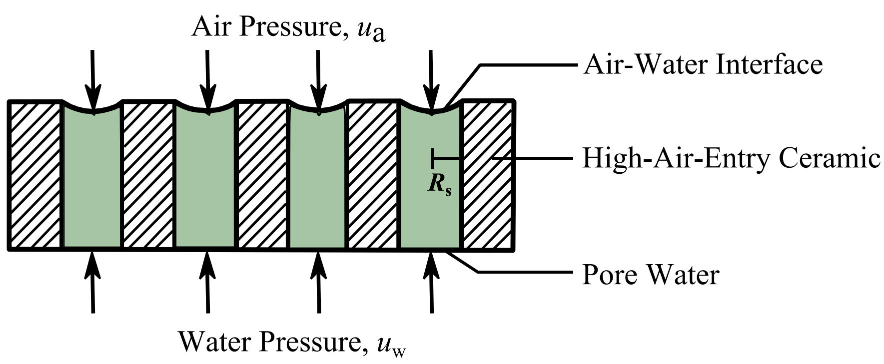


Fig. 4.9 Operating mechanism of HAE ceramic disk (after Lu and Lukos, 2004)

The surface tension at the air-water interfaces of the water- saturated HAE material provides a sustainable pressure difference in the micro-pores. Thus, the saturated HAE material can restrict the advection of atmospheric air while allowing the free flow of water. The water film at the air-water interface, formed due to surface tension, separates the two phases and makes it possible for negative pressure measurement. It requires a relatively high air-pressure to break the water film and, thus, the material is called HAE material. The maximum sustainable interfacial pressure ($u_a - u_w$) can be calculated using Young-Laplace equation and is proportional to the maximum pore size of the material as

$$\left(u_a - u_w\right)_{HAE} = \frac{2T}{r_{\max}} \qquad (4.1.)$$

where $(u_a - u_w)_b$ is the air-entry pressure of the HAE material, T the surface tension, and $r_{\max}$ the effective radius of the maximum pore size of the HAE material. The ceramic tip is brought in contact with the soil sample for matric suction measurements as shown in Fig. 4.10.

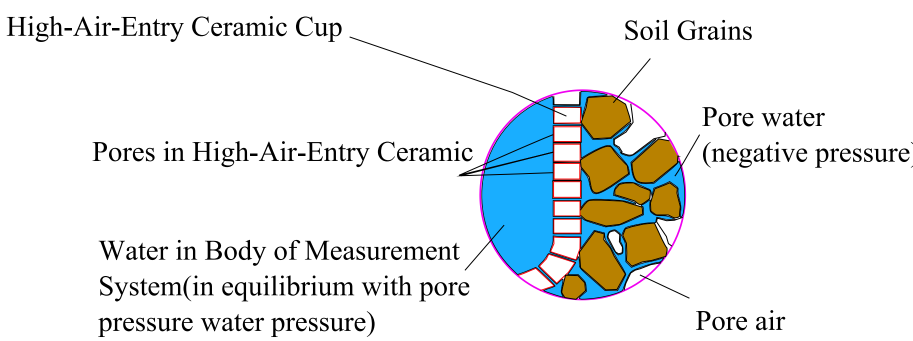


Fig. 4.10 Enlarged schematic showing porous ceramic tip in contact with unsaturated soil grains (after Lu and Likos, 2004)

The negative pressure will be transmitted through the saturated pores of the ceramic tip by drawing the water from tensiometer until the water pressure in the body tube is same as the matric potential of the soil water. This is read by the sensor which is connected to the other end of the water-filled tube. As the ceramic tip is permeable to dissolved solutes, the osmotic potential of the pore water has no effect on the pressure measurement. Thus, the matric suction can be measured directly if the gravitational potential is corrected for the difference in elevation between the sensor probe and the pressure gauge. The response time of the tensiometer measurements is function of hydraulic conductivities of HAE ceramic material and the soil, and thickness of the ceramic tip. The hydraulic conductivity of the soils is a function of their matric suction. Response time of tensiometers generally vary between 1 – 10 min.

Precautions:

- It is suggested to maintain a good contact between pore water of soil and the measurement system for accurate results
- The tensiometer system is required to be saturated with de-aired water and the air bubbles are removed from the system prior to testing
- The system must be periodically resaturated when the measurements are taken for prolonged testing periods

Limitations:

The major drawback with the present technique of suction measurement is its inability to measure negative pressures beyond 1 atm. The range of matric suction measurements is limited by the air-entry pressure of the HAE material. The characteristics of several commercially available HAE ceramic materials are given in the following Table. It is important to note that the hydraulic conductivity decreases with increasing air-entry pressure, thus, an increase in the measurement time.

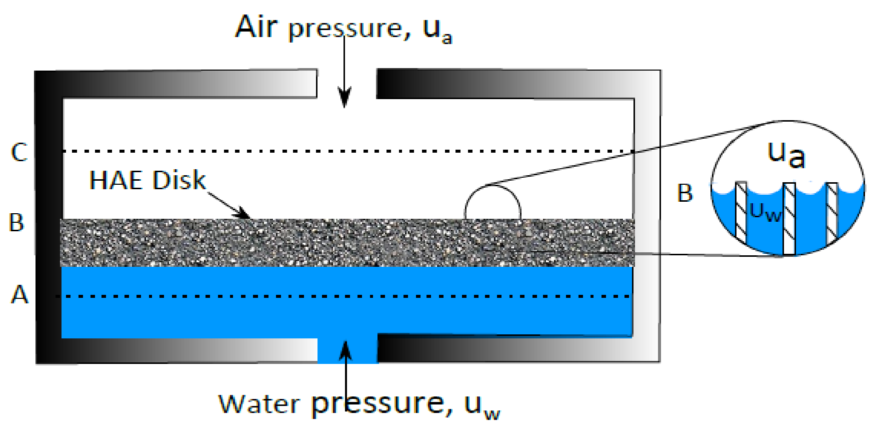
Table 4.1 Air-entry pressure and hydraulic conductivity of several commercially available HAE disks (adopted from Lu and Likos, 2004)

HAE Ceramic disc	Approx. pore diameter (mm)	K_s (m/sec)	Air-entry value (kPa)
½ bar	6×10^{-3}	3.11×10^{-7}	48-62
1 bar	1.7×10^{-3}	7.56×10^{-9}	138-207
2 bar	1.1×10^{-3}	6.30×10^{-9}	262-310
3 bar	0.7×10^{-3}	2.50×10^{-9}	317-483
5 bar	0.5×10^{-3}	1.21×10^{-9}	550
15 bar	0.16×10^{-3}	2.59×10^{-11}	1520

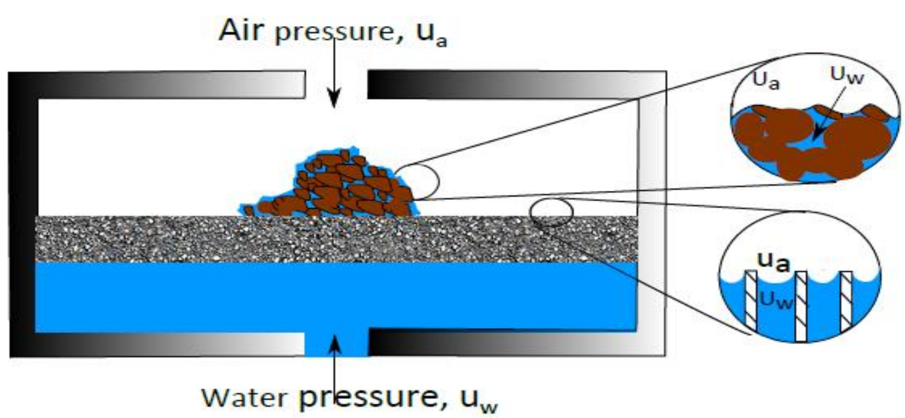
High-capacity tensiometers incorporating extremely small pore size of the HAE material have been developed to alleviate the limitations. However, the maximum measurable matric suction approaches only about 1500 kPa even by using the modified Tensiometers.

Axis-translation technique:

As it was mentioned previously that cavitation occurs when the absolute pressure falls below the vapor pressure and measurement system becomes discontinuous. Thus, axis-translation technique elevates the pore air pressure in unsaturated soil while maintaining the pore water pressure at atmospheric pressure. Thus, the matric suction (i.e., u_a–u_w) is "translated" from the condition of atmospheric air pressure and negative water pressure to the condition of atmospheric water pressure and positive air pressure. This technique makes the system feasible to control the matric suction over a wide range of suctions that are far greater than cavitation limit as the positive air pressure may be easily controlled and measured. Similar to the Transiometers, the present technique also uses HAE material to restrict the advection of air while allowing free flow of water. However, in this case, a positive air pressure is applied to the soil pore air while allowing the pore water to drain freely through the material under atmospheric pressure maintained on the other side as shown in Fig. 4.11.



(a)



(b)

Fig. 4.11. Illustration of testing on axis-translation technique before and after placing the soil in the unit

The separation between the air and water pressure can be maintained up to the value of air-entry pressure of the HAE material. The soil-water characteristic curve of a sample, from a saturated state, can be obtained by increasing the air pressure, u_a, gradually while drainage of soil pore-water is allowed through the material at every pressure increment to establish equilibrium with the atmospheric pressure ($u_w = 0$). The increase in air pressure (matric suction) causes the soil pore-water to expel into the porous disk which decreases the water content of the sample. After equilibrium is achieved at each suction increment, the sample is weighed by quickly withdrawing from the cell for water content determination. This technique is the most commonly used technique for measuring suction in the range of 0 to 1500 kPa using sintered ceramic as HAE material or 10 MPa using special cellulose membranes.

Pros and Cons:

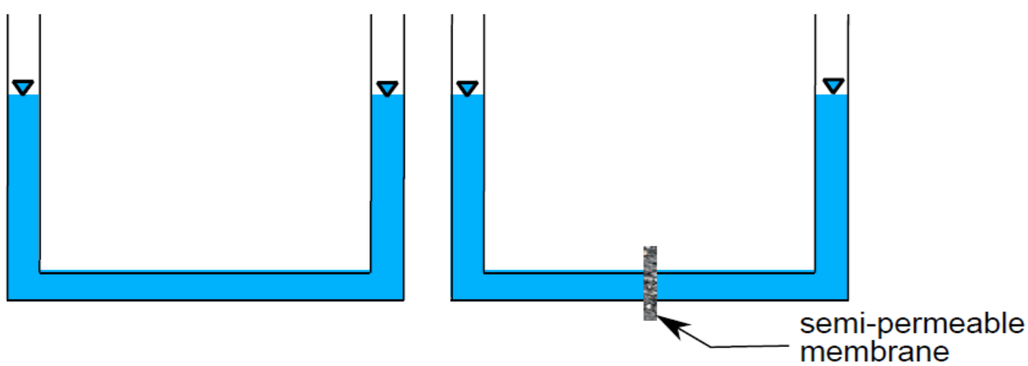
Even though axis-translation technique is widely used in geotechnical engineering for the measurement of matric suction of soils, few difficulties are encountered for its use. It requires maintaining elevated air pressures in the system in order to increase the pore air pressure equivalent to matric suction. Additionally, the soil samples containing occluded air (the degree of saturation is in excess of 80%), the employment of this technique can result in an overestimation of the value of suction prevailing in the soil.

Lecture 4

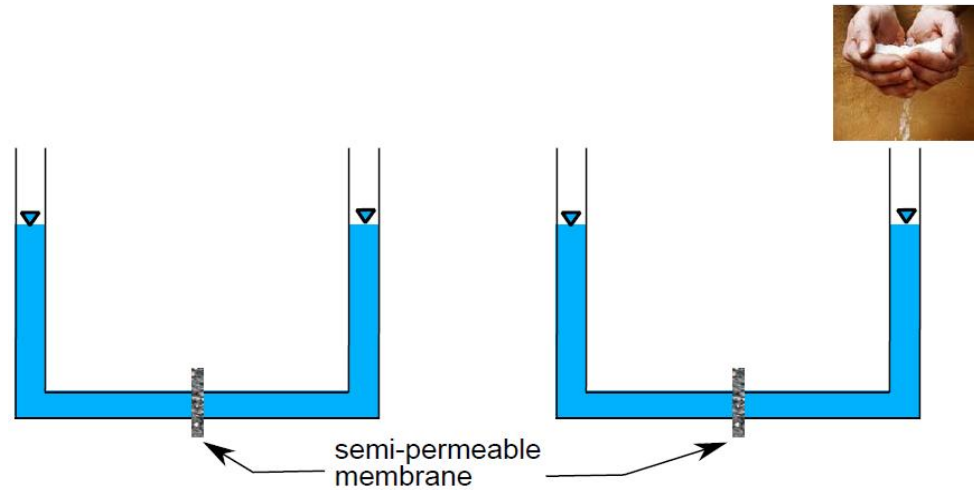
Suction measurement (Continued)

Osmotic Technique

The osmotic technique is an alternative and uncommon suction measurement technique in geotechnical engineering practice for the measurement of matric suction. The phenomenon of osmosis can be observed when a solvent and solute are separated by a semi-permeable membrane, i.e., a membrane that allows diffusion of only solvent molecules. Let us consider a 'U-shaped' tube filled with water and having a semi-permeable membrane at the middle of the tube as shown in the Fig. 4.12(a). Initially the water levels in both sides of the tube are same as the membrane allows the free diffusion of water molecules from one side to another. An aqueous solution is added, which contains larger-size (when compared to the pore size of the membrane) solute molecules, as shown in Fig. 4.12(b), on the right hand-side of the tube. The addition of solution on one side results in free diffusion of solute molecules in the absence of membrane. However, as the diffusion of solute molecules is restricted towards the left-hand side of the tube, the solvents (water molecules) diffuse towards the high-concentrated solute to restore the thermodynamic imbalance existing across the membrane due to chemical concentration, as shown in Fig. 4.12(c). It develops an osmotic head (decrease in the chemical potential) against the downward gravitational pull. This pressure is called osmotic pressure.



(a)



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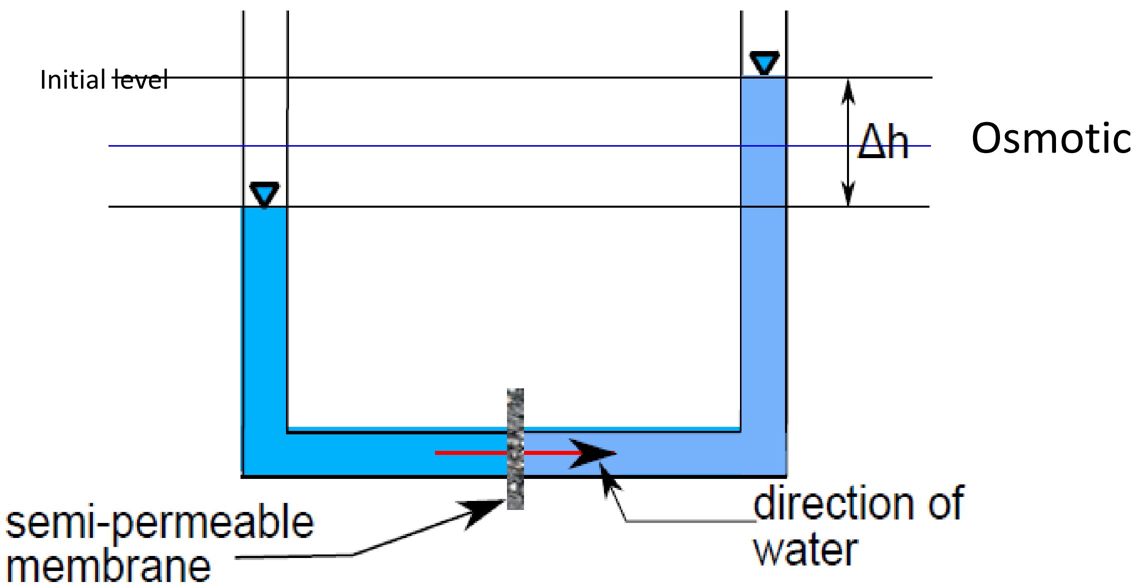


(c)

Fig. 4.12. Conceptual illustration of osmosis and osmotic pressure

Similarly, the osmotic pressure develops when a semi-permeable membrane is placed between soil sample and an aqueous solution. The basic arrangement of the osmotic set-up is given in Fig. 4.13. Generally, polyethyleneglycol (PEG) is used as osmotic solution because it contains larger size PEG molecules. These molecules are filtered in the solution compartment by the membrane to develop osmotic pressure across the membrane. To counter-balance the excess concentration of the osmotic solution, water diffuses from the soil sample into the osmotic container to dilute the solution concentration. The amount of diffusion is directly proportional to the matric suction of the sample.

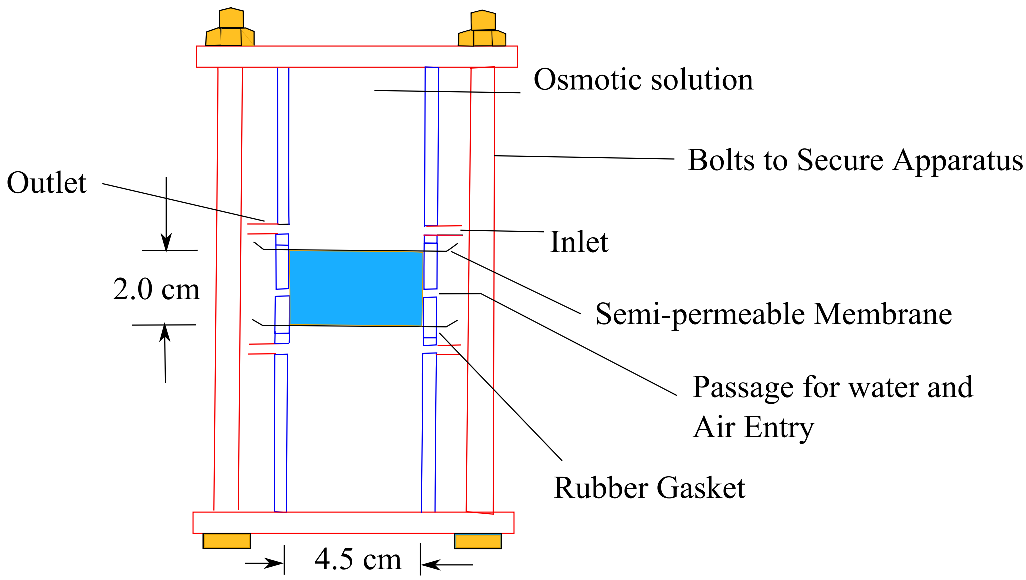


Fig. 4.13 Apparatus for determining water content-water potential relationship using osmotic solutions (after Willimas and Shaykewich, 1969)

Modified form of experimental set-up based on osmotic technique for measuring the matric suction is shown in Fig. 4.14 as presented by Monroy et al. (2007). The use of different varieties of osmotic solution has now been explored for long-term testing of matric suction using this method.

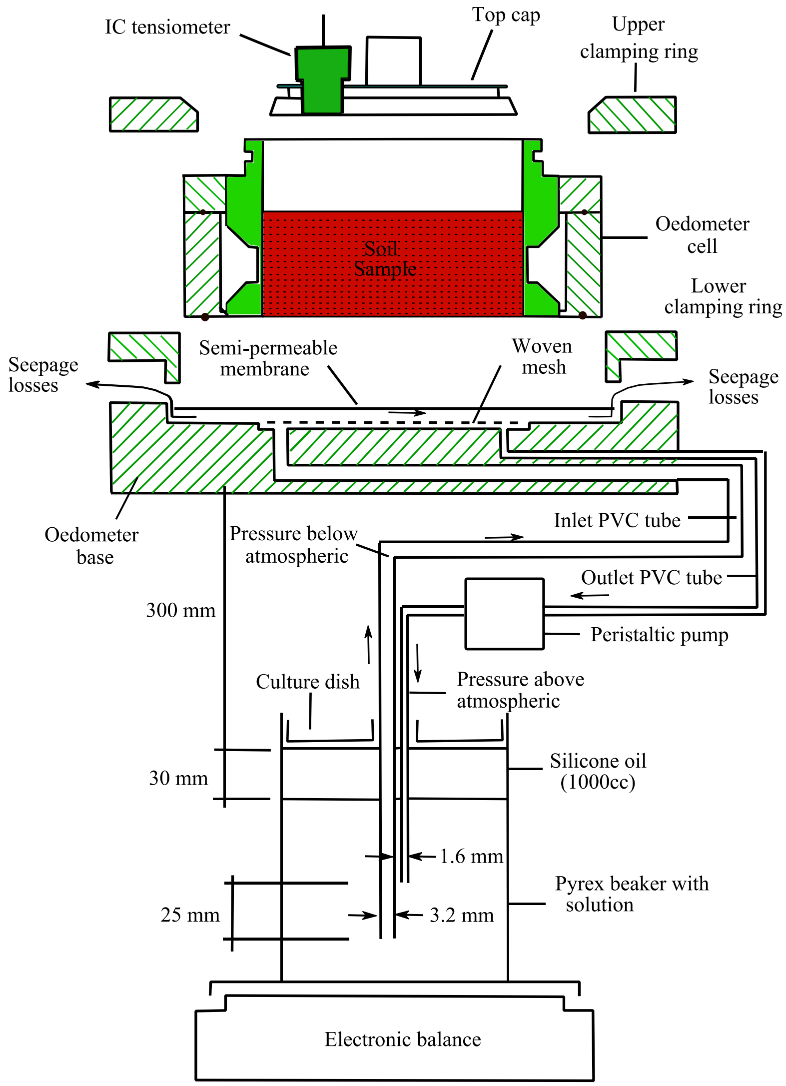


Fig. 4.14 Osmotic technique for measuring the matric suction (after Monroy et al., 2007)

The major advantage of this method is that it requires no elevated air-pressure. Moreover, the matric suctions as high as 10 MPa can easily be achieved by this technique. However, the fragility and vulnerability of the membrane to bacteria is the main concern. Further, the longevity of the membrane for time-dependent and continuous measurements is questionable and makes it expensive.

Thermal conductivity sensors or Gypsum blocks

Another simple way of measuring soil moisture content is by monitoring/measuring the thermal conductivities. The principle mechanism is fairly attractive as the thermal conductivities in unsaturated soils are directly proportional to the amount of moisture in the pore-water. However, the suction values may not be unique for corresponding moisture content measurements to establish the characteristic curve. This is because the characteristic curve depends on the soil type and yield different suction values for given moisture content. Moreover, a discrepancy in predicted suction values from moisture content also occurs due to the effect of hysteresis. Thus, in this method a calibrated porous medium is embedded in unsaturated soil sample and allowed to reach moisture equilibrium in a closed environment. Porous ceramic, polymer synthetics, sintered metal, and gypsum plaster are some of the commonly used porous materials in thermal conductivity sensors, these days. The porous medium is characterized by a unique soil water retention curve. The subsequent changes in the moisture content of the sample during the equilibrium process are characterized by its SWCC. The thermal conductivity of the porous medium is measured and the corresponding matric suction is correlated from the established calibration curves. The measurement of conductivity consists of determining the internal heat dissipation due to applied heat pulse. However, the main drawback is their suction measurement range. Most commercially available thermal conductivity sensors can measure suctions in the range of about 0 to 400 kPa.

Lecture 5

Suction measurement (Continued)

Contact filter paper technique

Filter paper technique is an alternative to complex testing techniques described in the previous sections. This technique is very simple, cheap, and reasonably accurate for the practical measurement purposes. There are two types of measurement procedures using filter paper technique, viz. contact and non-contact technique, as described in the American Society for Testing and Materials (ASTM) - D5298. The former technique is used for the measurement of matric suction and, alternatively, former technique for total suction measurement. Only the contact filter paper technique is described here for the measurement of matric suction and the other method is explained later sections. The filter paper technique is based on measuring the amount of moisture transfer from the soil sample to an initially dry filter paper when the filter paper is placed in contact with soil sample as shown in Fig. 15.

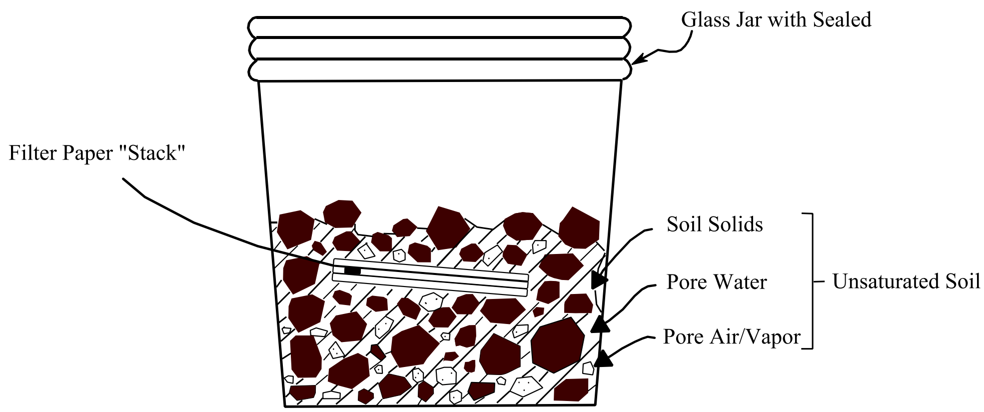


Fig. 4.15 Illustration of contact filter paper technique (after Lu and Lukos)

In general, one filter paper is sandwiched between two sacrificial papers to prevent fouling or contamination of the paper used for the measurement. After attaining the equilibrium between the filter paper and the unsaturated soil sample, the moisture content of the filter paper is measured gravimetrically. The measured gravimetric moisture content is related to matric suction of the soil using predetermined calibration curve for the filter paper. The moisture distribution between the paper and soil sample depends on the pore-size of the papers. The calibration curves vary from paper to paper.

Whatman #42, Schleicher and Schuell #589, White Ribbon, and Fisher 9-790A are the most commonly used types of papers filter papers for suction measurement. Standard calibration curve for filter papers are either acquired or obtained by measuring the SWCC of representative papers using the pressure-plate or any commonly known techniques prior to the testing. This measurement is accomplished in the similar manner the soil specimens are tested. Alternatively, the papers may be brought in contact to the moist soil and the corresponding matric suction is measured using a tensiometers. Representative filter papers are initially oven-dried and then allowed to cool to room temperature in desiccators prior to the suction measurement of soil samples. In situ measurement of matric suction measurements using the contact filter paper method is less commonly used in geotechnical application, albeit the technique is simpler and cheaper to other methods of testing.

Total Suction Measurement

Most of the measurement techniques for total suction measurement are based on

(i) measuring RH and then relating to total suction
(ii) controlling RH and measuring moisture content for obtaining total suction

The above described techniques are based on the following Kelvin's equation which relates relative humidity and pore water potential as

$$\psi_t = -\frac{RT}{v_{w_0} w_v} \ln(RH) \qquad (4.2)$$

where R is the universal gas constant (8.314 J/mol.K), T is absolute temperature (K), v_{w_0} is the specific volume of water or inverse of the density of water (m^3 /kg), w_v is the molecular mass of water vapor (18.016 kg/kmol), and RH is the relative humidity. This equation considers the effect due to all the mechanisms that decrease the chemical potential of water. Thus, the equation is applicable to study the total suction. The following figure (Fig. #) shows the variation of total suction with RH at T = 293.16 K using the Kelvin's equation. The advantage of the Kelvin's formula is that the total suction can be indirectly measured by establishing the equilibrium between relative humidity of the pore water for an unsaturated soil specimen with the controlled ambient atmosphere. The total suction is zero when the RH of the pore water vapor is 100%. The RH value less than its full saturation (RH < 100%) indicates the reduction in the pore water potential in soils and, thus, the presence of negative pressure in the soil pore water.

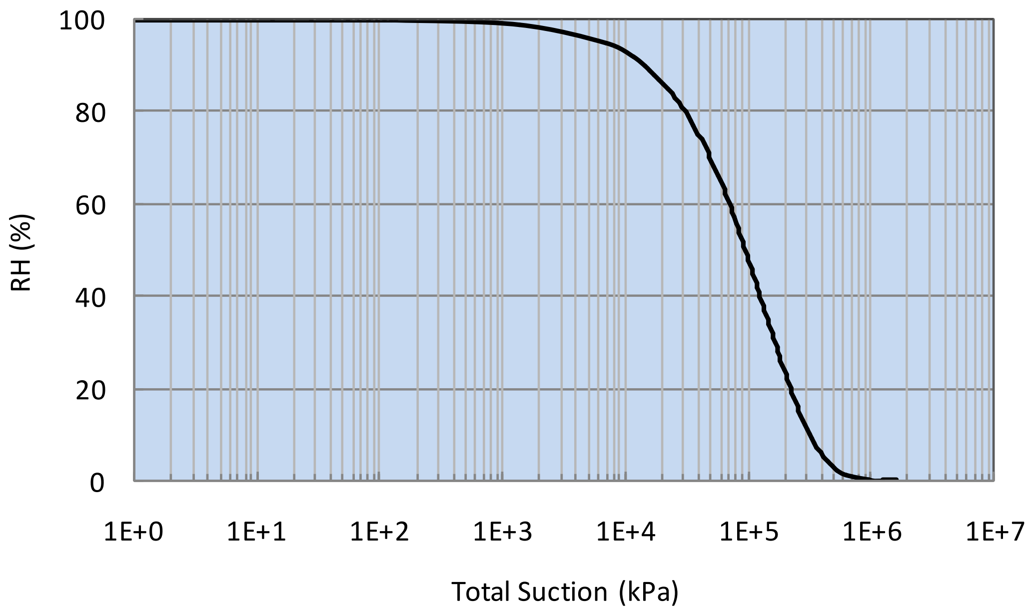


Fig. 4.16 The dependency of total suction with RH using Kelvin's equation

The commonly used indirect measuring techniques for total suction are

(i) Thermocouple psychrometers
(ii) Chilled-mirror hygrometers
(iii) Polymer resistance/capacitance sensors
(iv) Contact filter paper technique
(v) Isopiestic (constant pressure) Humidity Control
(vi) Two-Pressure Humidity Control

The first four techniques are based on measuring the RH by controlling the moisture content of the soil samples. Other two techniques, humidity control techniques, control the ambient RH in a controlled environment while monitoring the changes in moisture contents. The applicability of humidity control techniques is generally in the dry side of water retention curve. Thus, these techniques are used in combination with other methods of total suction measurement on wet side of retention curve. The advantage of the later techniques is that a continuous distribution of total suction vs. gravimetric water content can be generated in a single test. The former techniques require manual measurement of RH values at given moisture contents for suction vs. water content distribution. The following sections describe some of the commonly used measurement techniques.

Lecture 6

Suction measurement (Continued)

<u>Chilled-mirror hygrometers</u>

Chilled-mirror hygrometer (CMH) has been used for the past several decades in humidity transfer applications. The hygrometer is based on the measurements of the dew point temperature of a gas in a controlled environment. It condenses the reflective condensation surface by cooling and sensing the small condensed water droplets optically. Thermoelectric coolers are used to condense the gas.

A metallic reflective surface, chilled-mirror, is maintained in dynamic equilibrium with the water vapor pressure of the surrounding gas sample by maintaining a critical temperature. The critical temperature is at which the rate of dew condensation is equal to the rate of the evaporation of dew layer from mirror. This critical temperature is the dew point temperature of the sample where the mass of the dew layer is maintained constant. The intensity of a reflected beam of LED light from the chilled-mirror is measured by photodetector. The schematic diagram of CMH is given in Fig. 17. The dew point temperature is related to ambient RH and then to total suction using Kelvin's formula.

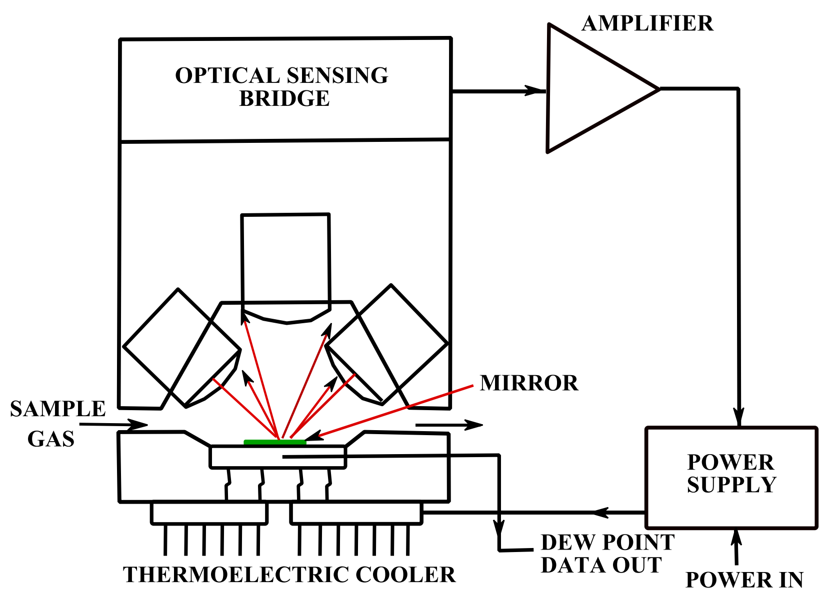


Fig. 4.17 Working mechanism of Chilled-mirror hygrometer

The application of this sensor for total suction measurement of unsaturated soils has come to light only a couple of decades ago. WP4 dew-point potentiometer is commonly used in unsaturated soil testing, which is based on the principle of CMH.

The soil specimen is placed in the sensing sealed chamber and brought to vapor pressure equilibrium prior to cooling. The instrument is provided with an internal fan in the sample chamber to reduce the equilibrium time. At equilibrium, the water potential of the ambient air in the sealed chamber is equal to the water potential of the sample. The mirror temperature is controlled by a thermoelectric cooler. Photoelectric cell is used to detect the condensation on the chilled-mirror. The thermocouple records the temperature at which condensation occurs. The water potential in kPa and temperature in degree Celsius of the soil sample are automatically displayed on the screen. Additional advantage of this instrument is that the sample temperature can also be controlled and, thus, thermal studies can be carried out.

Resistive humidity sensors

These sensors consist of a polymer film or polymer treated substrate which separates two electrodes in a porous probe. The probe is brought into equilibrium with a gas. The substrate either adsorbs or desorbs moisture depending on the changes in RH value. Changes in the electrical impedance of the hygroscopic substrate medium are then measured by the resistive humidity sensors. A well-defined relationship exists between RH with the sensor capacitance and the sensor resistance for a known polymer sensors. The laboratory measurements of either sensor capacitance or resistance could be used to back analyze the RH value using the well established calibration curves. The calibration curves for these sensors as reported by Lu and Likos (2004) are shown in Fig. 4.19.

The advantage of these sensors is that they are applicable for wide range of RH variations and exhibit low hysteresis, albeit inexpensive to use. Further, these sensors are known for quick measurements as the response time is typically less than 30 seconds. It was reported by Lu and Likos (2004) through few research citations that these sensors exhibited good measurement precision and have excellent practical applications in the bench scale study of alternative earthen fill cover.

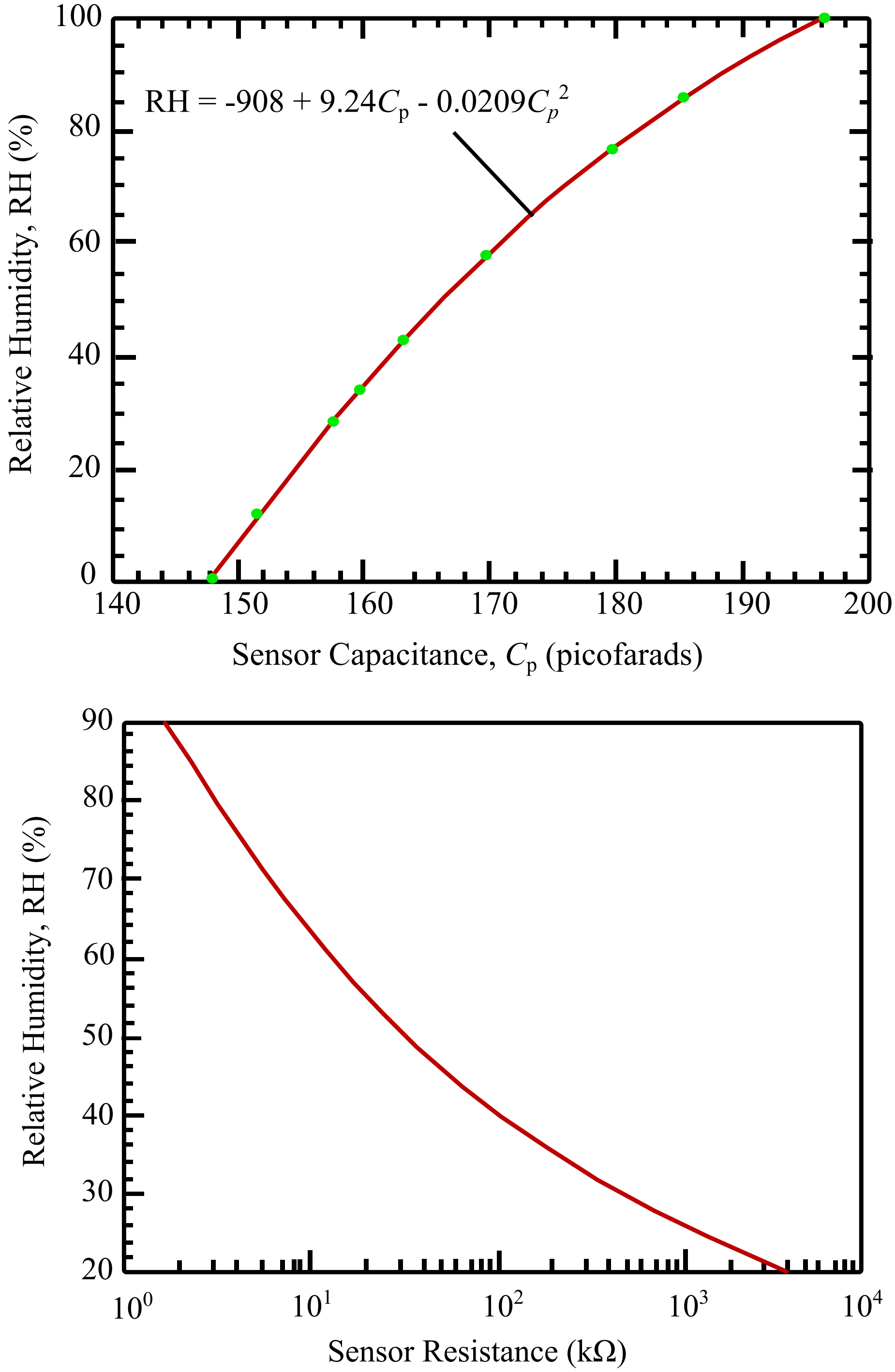


Figure 4.19. Calibration curves for relative humidity sensors: (*a*) relationships between sensor capacitance and relative humidity (after, Albrecht et al. 2003) and (*b*) relationship between sensor resistance and relative humidity (after, Lu and Likos 2004).

Lecture 7

Measurement of total suction and hydraulic conductivity

<u>Non-contact filter paper technique</u>

Similar to the previously described contact filter paper technique, the non-contact filter paper technique also estimates the soil water potential indirectly from the measurement of moisture transfer from an unsaturated soil specimen to an initially dry filter paper under vapor equilibrium. The moisture content of the filter paper is measured gravimetrically at equilibrium and related to soil suction. Fig. 4.20 illustrates the testing setups for non-contact filter paper technique for total suction measurement of unsaturated soils.

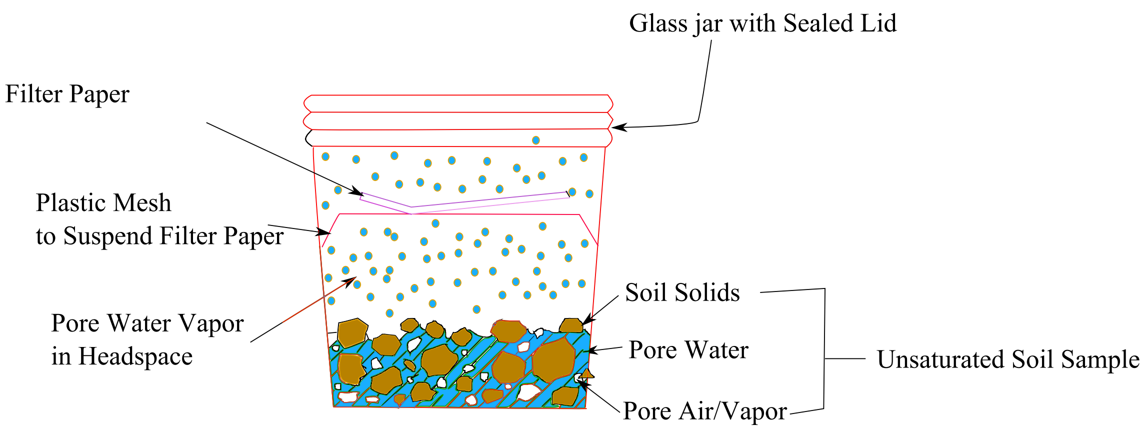


Fig. 4.20. Illustration of non-contact filter paper technique (after, Lu and Likos 2004)

In this technique, a filter paper is brought into vapor equilibrium with the soil specimen as shown in the Fig 4.20. The moisture transfer in the vapor phase occurs during the equilibrium process. The equilibrium amount of adsorbed moisture is a function of the pore-air RH and can be calibrated for different filter papers. Several salt solutions are commonly used to calibrate the filter papers. The pore air RH is then related to total suction using Kelvin's formula. This method has found wider applicability in unsaturated soil engineering practice. The calibration and testing procedure is similar. The filter paper is over-dried and allowed to cool down to room temperature in desiccators. The oven-dry paper is brought into vapor equilibrium with carefully prepared salt solutions at known concentrations in a sealed container. The filter paper is weighted to greater precision for moisture content determination after the equilibration period, typically 7 days. Filter paper moisture content is plotted against total suction for each salt solution for establishing calibration curve. The testing procedure for soil suction is identical to the calibration.

Two-Pressure Humidity Control

As described earlier, the two-pressure humidity control (TPHC) technique controls the RH of the soil pore air while measuring the moisture content of the unsaturated soil sample. There are other control techniques such as "isopiestic humidity control". However, only TPHC technique is described here. In this method, a vapor saturated gas is brought in equilibrium with the soil sample in a closed chamber. The RH values of the gas are manipulated/controlled by varying either pressure or temperature. The moisture content of the sample changes to new value when this humid gas stream, with known RH value, is brought into vapor equilibrium with the soil sample. The variables such as RH and temperature of the sealed chamber are continuously monitored using one of the earlier discussed methods (ex: Resistive humidity sensors). At the same time, the moisture content of the soil sample is monitored by any of the moisture sensors. This data yield a continuous distribution of total suction and moisture content for a given soil sample, after converting the RH values into water potential using Kelvin's equation. Of late, Lu and Likos (2003) had conducted laboratory tests for measuring water potential using divide-flow method, which is the modified form of two-pressure humidity control technique. They used proportioned mixing of vapor saturated gas with dry gas as an equilibrate gas in the environmental chamber. The mass flow controllers regulate mixing of the 'wet' and 'dry' gases to maintain a predetermined RH value of the equilibrate gas. RH and temperature of the soil pore-air, which is equal to the equilibrate gas, are continuously monitored using polymer capacitance probes. An electronic balance is used as a bottom plate of the environmental chamber and, thus, the moisture contents of the sample are continuously monitored. The test set-up developed by Lu and Likos (2003) shown here in Fig. 4.21 as given in Lu and Likos (2004).

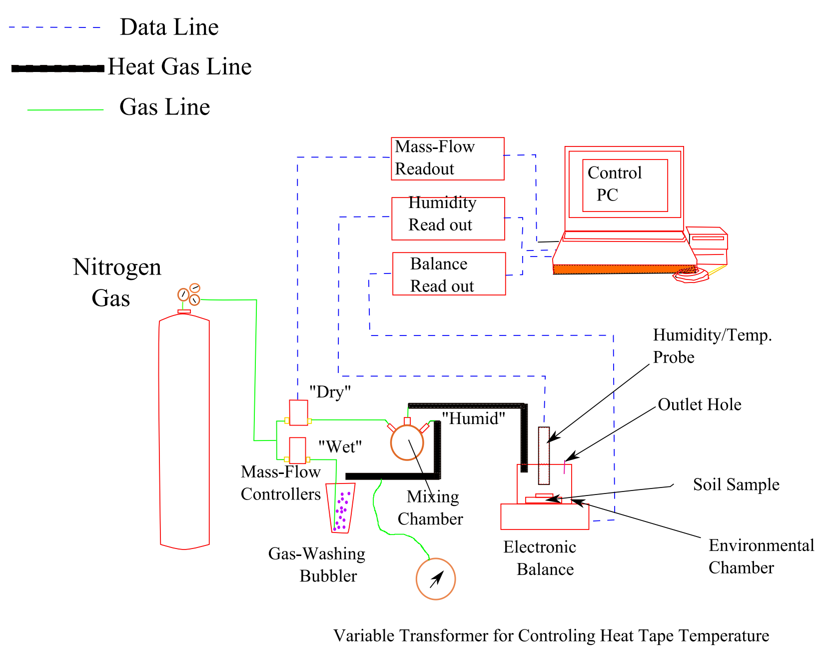


Fig. 4.21 Flow humidity control system for measurement of total suction characterstic curves (after, Likos and Lu 2003)

The RH values of the equilibrate gas are incrementally changed to measure the continuous distribution of soil water potential curve. The accuracy of the RH measurement is same as the polymer capacitance probes.

Measurement of hydraulic conductivity

The hydraulic conductivity of the unsaturated soils is directly related to the amount of moisture in the pore-water, similar to the electrical/thermal conductivity. The measurement techniques for hydraulic conductivity are generally classified as laboratory and field methods, and steady-state and transient techniques. The methods can conveniently be classified as direct and indirect techniques.

Steady-state hydraulic conductivity testing techniques described in this chapter include the constant-head method, the constant-flow method, and the centrifuge method.

Direct

1. Constant-head method
2. Constant-flow method
3. Centrifuge method

Indirect

4. Horizontal infiltration method
5. Multi-step outflow method
6. Instantaneous profile method
7. Method of back analysis

Constant-Head Method

This method is well-known and commonly used steady-state technique for geotechnical engineers for several decades for measuring hydraulic conductivity of saturated soils. This technique is modified for measuring unsaturated soils by controlling the matric suction in conventional constant head method used for saturated soils. Similar to the conventional test, the hydraulic head and water flow across the soil sample are maintained constant to conduct the test under steady-state condition. Axis translation is a commonly used to technique to maintain constant matric suction by sustaining positive air pressure in the system during testing operation. A representative saturated soil sample is sandwiched between initially-saturated HAE disks for establishing hydraulic connection between the soil pore water and fluid reservoirs. The selection of the HAE disks is based on the requirement of maximum desirable matric suction to be maintained. The air-entry values of different HAE disks have been mentioned earlier. The pore air pressure in the system is now increased by using external air-pressure facility to achieve desired matric suction in the soil sample. Hydraulic gradient is measured using external ports along the length of the specimen. The hydraulic gradients can be measured using Tensiometers when the pore water pressures in the sample are negative. The hydraulic conductivity function (i.e. Matric suction vs. hydraulic conductivity) along the drying path can be obtained by incrementally increasing the matric suction and computing the conductivity from the measurements of steady-state flux at each increment. The assumption of validity of Darcy's law is invoked for calculating unsaturated hydraulic conductivity at controlled suction values from the measured steady-state flux. The hydraulic conductivity can be computed from the measured flux as

$$k(\psi_m)=\frac{Q}{A}\left(\frac{\Delta L}{\Delta h}\right) \tag{4.3}$$

where Q is the steady-state flux, ψ_m is matric suction, and Δh the head loss along the length ΔL. However, measurement of conductivity function along the wetting path is less common due to the difficulty in maintaining uniform pore pressure in the sample. The water content corresponding to the computed conductivity can be obtained from the Tensiometer and TDR measurements along the length of the sample. The general laboratory set-up of constant-head method as cited by Lu and Likos (2004) is given in Fig. 4.22.

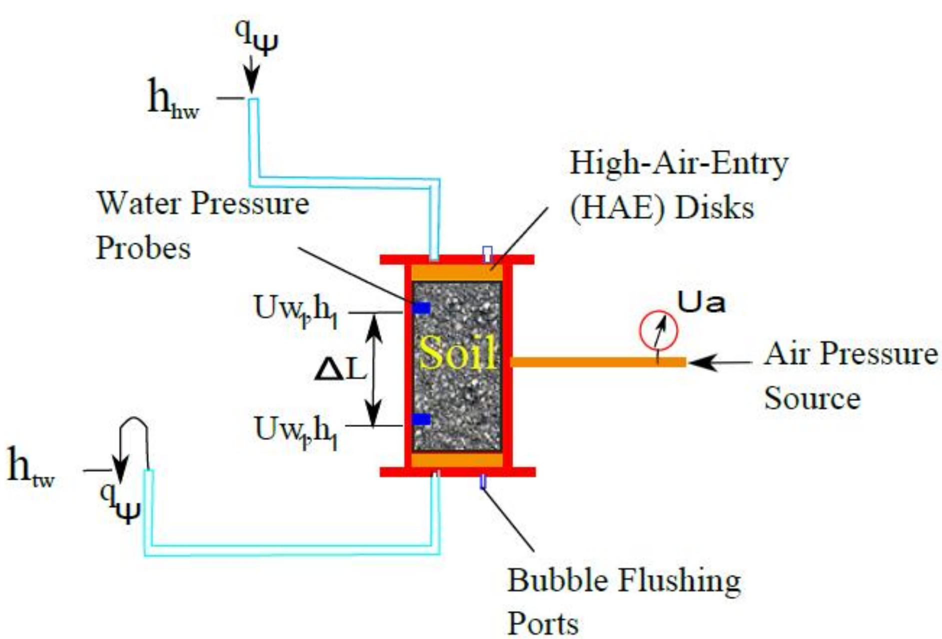


Fig. 4.22. Constant head testing method

This method has been well recognized for conductivity measurement for unsaturated soils due to its simplicity. Further, the conductivity test can be conducted in modified triaxial cell, stress conditions in the field can be maintained. However, it is highly time taking as it involves establishing steady-state condition.

Lecture 8

Measurement of hydraulic conductivity

Constant-flow technique

This technique is similar to the constant-head method which was discussed just before. In contrast to earlier technique, flow rate is controlled through soil specimen by maintaining representative hydraulic gradients. Thus, the field condition can better be represented in the laboratory experiment. The flow rates can be controlled to as low as up to 0.01 cm^3/day using motorized flow pumps. Olsen et al. (1994) used additional pump to control the water content of the soil sample to simultaneously determine SWCC. The set-up for this technique is similar to the constant-head technique. However, flow pumps and pressure transducers attached to the set-up to control the flow rate and to monitor water and pore-air pressures respectively. The soil water content is reduced from initially saturated condition by pumping-out a known volume of pore water through HAE discs. The equilibrium moisture content and matric suction are measured using transducers. The predetermined water flow is circulated through the soil sample. The head loss in the steady-state condition is measured. Hydraulic conductivity value at measured suction/moisture content can be calculated. A continuous conductivity function is obtained by measuring head loss and computing conductivity at each incremental matric suctions.

Centrifuge Method

Even though the earlier laboratory conductivity measurement techniques are simple, they are highly time-consuming for establishing steady-state conditions, especially in case of unsaturated fine-grained soils. Centrifuge testing is a powerful way of reaching steady-state conditions quickly by creating high gravitational acceleration. Nimmo et al. (1987) conducted hydraulic conductivity tests on unsaturated soils using this method. A schematic diagram of the geotechnical centrifuge (bucket-type) is given in Fig. 4.23.

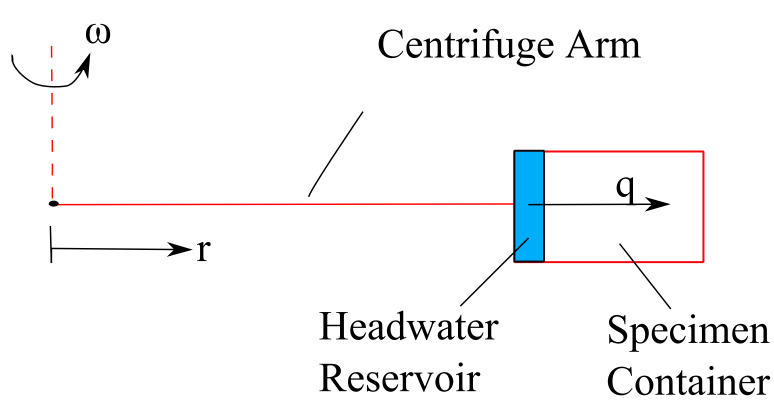


Fig. 4.23 Illustration of centrifuge testing technique (after Lu and Lukos, 2004)

A scale model of the steady-state conductivity testing set-up is loaded into the specimen container of the centrifuge. The cross-sectional view of the experimental set-up used by Nimmo et al. (1987) is presented herein.

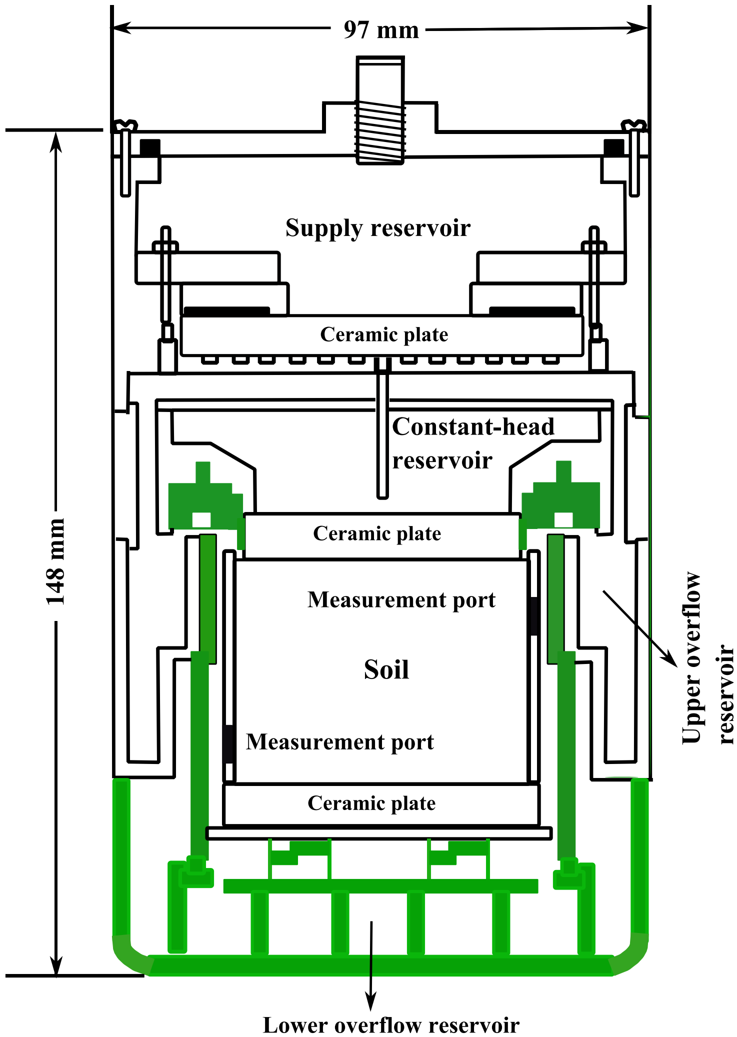


Fig. 4.24 Experimental apparatus (after, Nimmo et al. 1987)

The model is then spun to reach desired centrifugal acceleration. The centrifugal gradients on the sample increase the rate of moisture movement by keeping all the other parameters constant. This establishes a one-dimensional water flow through specimen when earth gravity field is negligible to applied centrifugal gravity field. The following Darcy's one-dimensional equation can be used for computing hydraulic conductivity of unsaturated soil sample (Nimmo et al., 1987)

$$q = -k(\psi)\left[\frac{d\psi}{dr} - \rho\omega^2 r\right] \tag{4.4}$$

where q is flux density, ω angular speed of rotation, and r distance from the axis. The hydraulic conductivity at any given matric suction is calculated using the above equation from measurements of q and dψ/dr. The flow of water can be supplied by either housed in the model set-up or external pump source. The description of test set-up, detailed experimental procedure, and can be found in the original article of Nimmo et al. (1987). Requirement for relatively short testing time is the primary advantage of the technique. However, the centrifugal testing of unsaturated hydraulic conductivity is scarce due to its high cost.

Indirect methods

The transient flow of water through unsaturated soils is a diffusion process and is controlled by diffusivity, $D(\theta)$. The diffusivity of unsaturated soil is a function of water content of the soil. It is defined as (van Genuchten, 1980)

$$D(\theta)=k(\theta)\left|\frac{dh}{d\theta}\right|=\frac{k(\theta)}{C(\theta)} \tag{4.5}$$

where h is the pressure head, θ volumetric moisture content, and C is the slope of the SWCC. The slope of the SWCC is also called specific moisture capacity. The unsaturated hydraulic conductivity can be expressed, after rearranging the above equation, as

$$k(\theta)=D(\theta)\times C(\theta) \tag{4.6}$$

Thus, hydraulic conductivity function can be obtained indirectly from the measurements of hydraulic diffusivity and soil-water characteristic curve. A variety of transient techniques have been used to determine hydraulic conductivity function. The following commonly used techniques in soil science for measuring conductivities are described here.

<u>Multi-step outflow method</u>

Outflow method is a transient laboratory conductivity measurement technique similar to the previous method. This method uses conventional axis-translation testing apparatus for analyzing the pore water outflow data and determines the hydraulic diffusivity. The diffusivity is determined by monitoring time-dependent pore water outflow from soil specimens under the applied matric suction. There are several variations in the testing and analysis. The commonly used multi-step outflow method alone is described here. The test involves subjecting the soil sample to incremental changes in the matric suction by maintaining the difference between pore air and pore water pressures in the soil sample. The changes in the matric suction induce drainage of pore water through HAE disc. The outflow data is monitored for each suction increment to calculate diffusivity for given moisture content. Assuming that the suction increment is small enough to maintain the hydraulic conductivity of the specimen constant, specific moisture capacity is constant for a given suction increment, and validity of one-dimensional flow, the following linear diffusion equation can be used

$$\frac{\partial\psi}{\partial t}=D\frac{\partial^2\psi}{\partial z^2} \tag{4.7}$$

where D is the hydraulic diffusivity, ψ the matric suction, and Z the spatial variable vary between 0 to L, length of the sample. The diffusion equation is Fickian-type and is commonly used for contaminant transport through saturated soils and is analogous to heat conduction through solids and one-dimensional consolidation phenomena. This diffusion equation has analytical solutions for varieties of initial and boundary conditions. For given initial and boundary conditions for the present test conditions, Gardner (1956) proposed the following analytical solution for pore water outflow volume as

$$\ln\left(\frac{V_\infty - V_t}{V_\infty}\right) = \ln\left(\frac{8}{\pi^2}\right) - \frac{D\pi^2 t}{4L^2} \tag{4.8}$$

where V_∞ is the total volume of pore water expelled for the applied suction increment which is very difficult to measure and V_t the outflow volume at time t. The diffusivity can be calculated using the above equation and multiplied it with specific moisture capacity to obtain the hydraulic conductivity.

Outflow methods are widely used as these techniques allow simultaneous determination of the hydraulic conductivity function and the soil-water characteristic curve in a single test without the use of probes. Further, they are advantageous as they are conducted using conventional axis-translation equipment such as pressure plate or Tempe cell systems. However, the testing is limited to only relatively coarse-grained soil where drainage occurs relatively rapidly. Moreover, the practical range of these measurements is limited by the air-entry value of the HAE discs.

Lecture 9

Measurement of hydraulic conductivity (Continued)

<u>Instantaneous profile method</u>

This is another commonly used indirect method of hydraulic conductivity estimation in both laboratory and field. The soil moisture content and matric suction distribution along the depth of the soil profile are measured at different time instants during the transient water infiltration or drainage process. The measurements at different time instants and multiple spatial locations provide redundant data for better analysis. Soil columns with either disturbed or undisturbed specimens are used in the laboratory and, on the other hand, an instrumented vertical soil profiles are used in the field for estimating hydraulic conductivity using this technique. The time dependent water diffusion and hydraulic gradients across the soil column are estimated from the observed time dependent profiles of either moisture content or matric suction. At least one profile is required and other one can be inferred from SWCC. In both the laboratory and field applications, Darcy's law is assumed to be valid for ease in the theoretical analysis. Laboratory and field application of the present technique is described in the following sections.

<u>Laboratory Application</u>

The laboratory instantaneous profile method was used by many researchers (Richards and Weeks, 1953; Chiu and Shackelford, 1998). The soil samples are confined in rigid-wall columns of 10 - 30 cm of length. The gravitational influence on the water flow is ignored for horizontally oriented column testing. The following schematic diagram in Fig 4.26 shows the typical laboratory soil column for water flow testing.

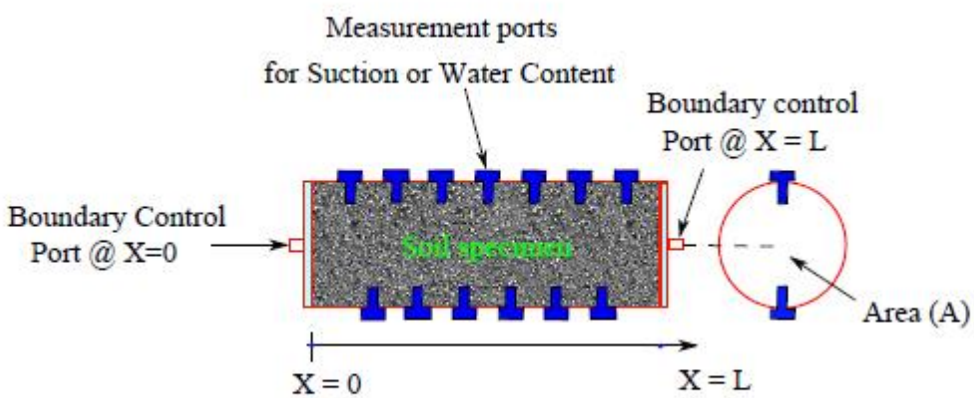


Fig. 4.26 Schematic diagram for laboratory column testing (after Lu and Lukos, 2004)

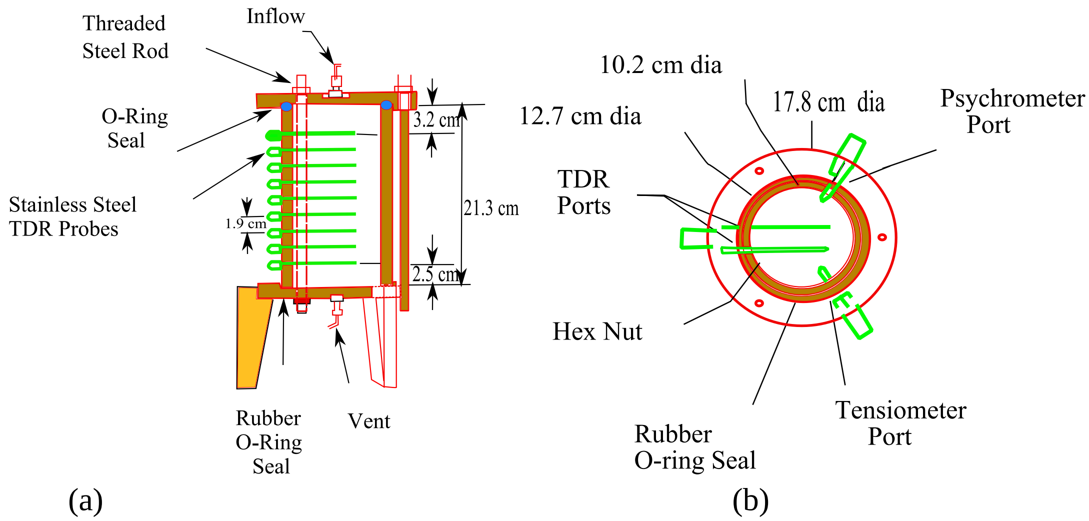


Fig. 4.27 Apparatus for unsaturated Hydraulic conductivity Tests: (a) Side View (b) Top View (after, Meerdink et al. 1996)

It consists of suction and moisture content measurement ports along the column length, and inlet and outlet ports on either of the boundaries. The ports along the length of column are used to monitor changes in the moisture content and suction by respective probes, such as TDR, and suction measurement techniques, such as tensiometers, thermocouple psychrometers, etc. The boundary ports are used for controlling the flow of water. Initially, air-dried uniform soil is compacted in columns and is instrumented for measuring the moisture content and suction along the spatial length of the column as shown in Fig. 4.27. The boundary port is connected to water reservoir for actuating the flow of water into the column. The initial moisture content of the air-dry specimen, at time t_0, is very small and the suction head is designated with h_0.The water is steadily injected through the left boundary using a flow pump and is made sure that the water distribution at the soil-water interface is uniform. The right boundary is exposed to the atmosphere pressure. Care must be taken to avoid any evaporation losses from this boundary during the testing process. The water diffuses in the soil specimen and yields transient moisture content profiles in the soil. The conceptual distribution of water content and suction along the column length are shown in Fig 4.28 for different time instants.

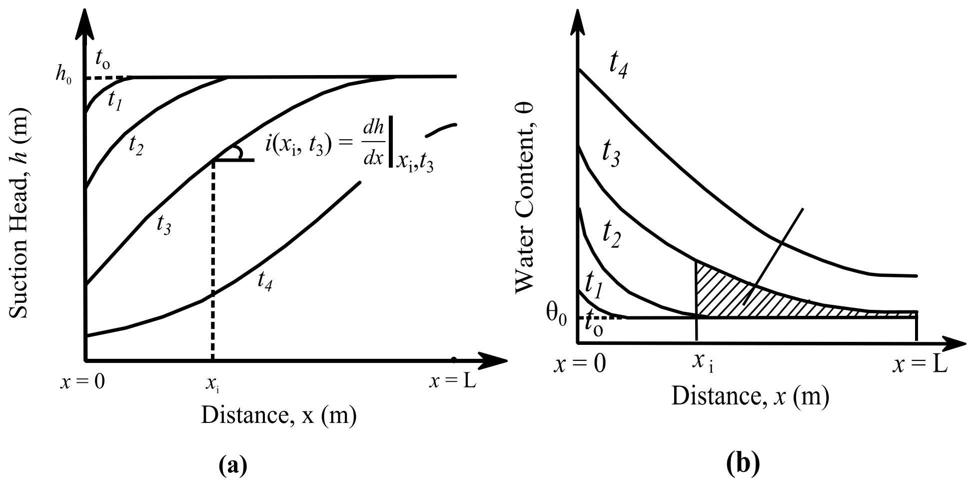


Fig. 4.28 Conceptual profiles (after Lu and Likos, 2004)

The slope of the suction head profiles at any time instant and spatial location is the suction head gradient and is given by $i(x_i, t_i) = \left.\frac{dh}{dx}\right|_{x_i, t_i}$

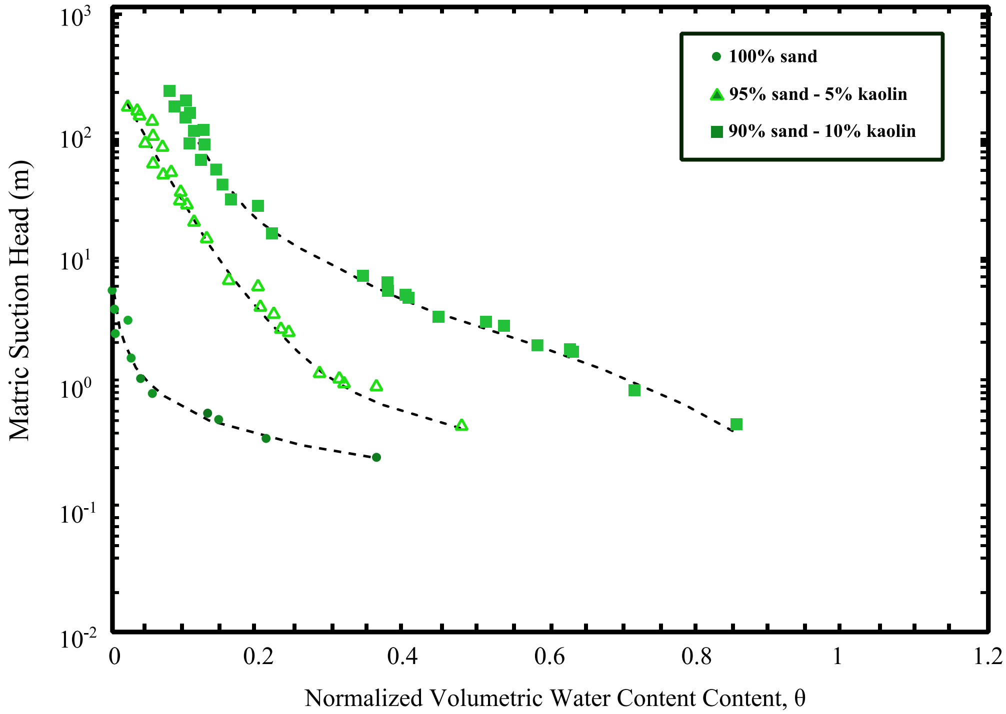
Semi-infinite boundary condition can be invoked in the analysis, if the test is terminated before water flow reaches the right boundary of the soil column. The water flux at any given cross-sectional area and time increment is equal to the change in volume of water between the cross-section under consideration and right boundary of the column. Thus, the change in volume of water at any given time instant between the cross-section under consideration and right boundary is

$$\Delta V_w = A\left(\int_{x_i}^{L} \theta(x, t_1)dx - \int_{x_i}^{L} \theta(x, t_2)dx\right) \qquad (4.9)$$

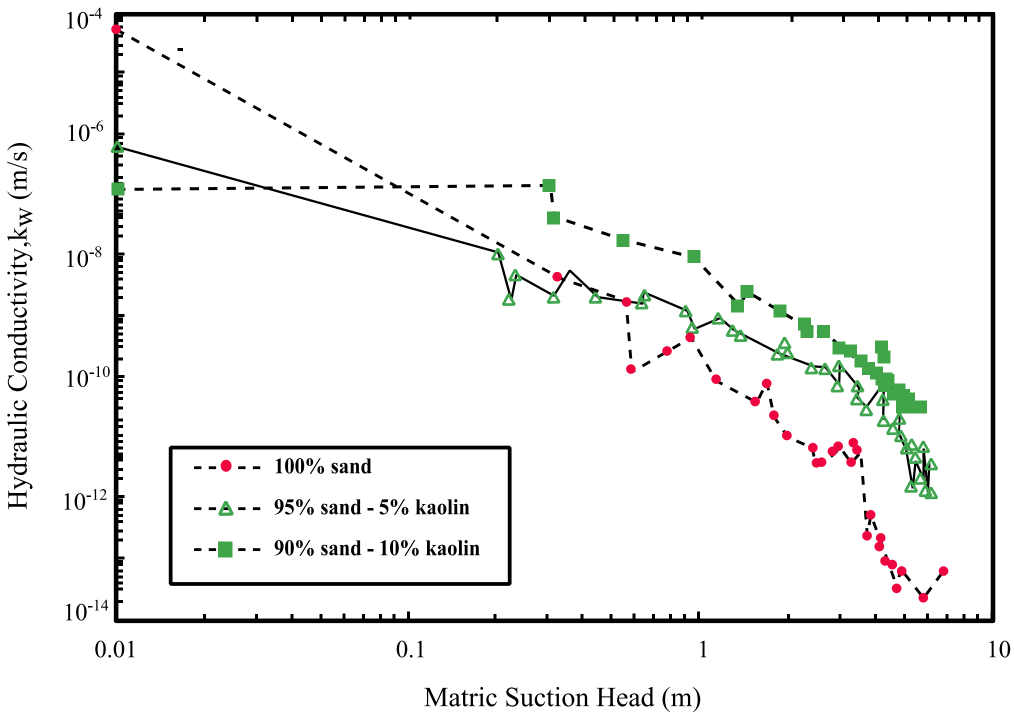
where A is the cross-sectional area of the column; t_1 and t_2 are two different time instants; ΔV_w is the change in volume of water. This can be easily computed using the shaded area between t_1 and t_2 of the temporal water content profile. Hydraulic conductivity can be calculated using Darcy's law described as a ratio of apparent velocity to hydraulic gradient, when the apparent flow velocity is computed using the following expression

$$v = \frac{\Delta V_w}{A \times |t_1 - t_2|} \qquad (4.10)$$

The same procedure is adopted for the data from different spatial locations and at different time instants for obtaining entire hydraulic conductivity function. The following figure gives SWCC and hydraulic conductivity data obtained on Kaolinite clay using the current procedure by Chiu and Shackelford (1998).



(a)



(b)

Fig. 4.29 Measured hydraulic functions of Sand-Kaolin mixtures (after, Chiu and Shackelford 1998)

Field Application

The field method is similar to the laboratory technique described above. However, instrumentation of the field soil is complicated, especially for estimating the conductivities of soils in natural condition. Nevertheless, the estimation of hydraulic conductivities has been widely used across several groups of soil science and engineering. Watson (1966) and Hillel et al. (1972) described the method based on water draining/desorption under the gravitational gradient while the infiltration and evaporation at the surface are restricted. In contrary, Meerdink et al. (1996) studied the retention and hydraulic behavior of fine-grained soils due to evaporative fluxes from the soil surface. A schematic diagram of the testing scenario is given in Fig 4.30, in which the field soil is instrumented with set of tensiometers for suction measurement at different spatial location and neutron probe access tube for measuring water content profile along the vertical depth. The instrumentation is done within berm boundary or infiltration ring within which the water is ponded.

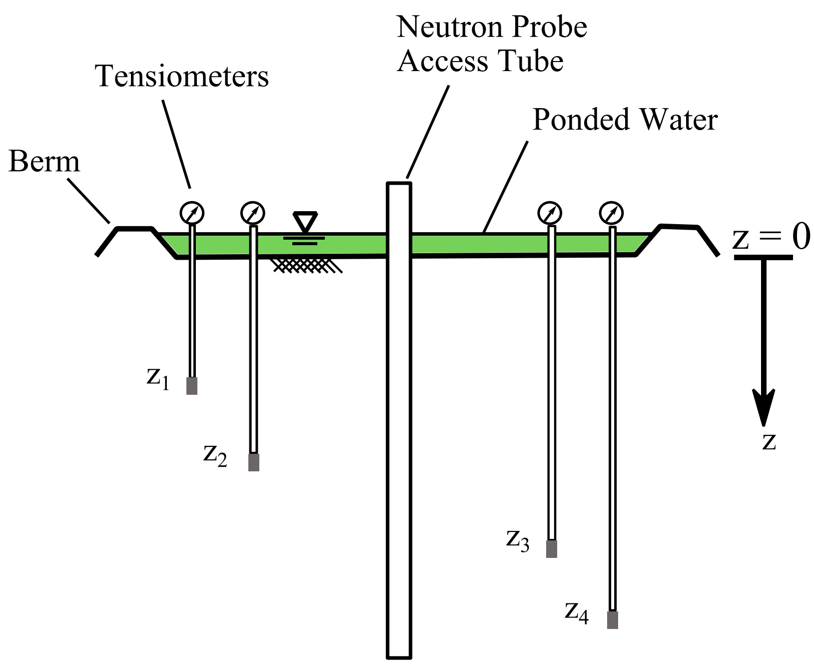


Fig. 4.30 Hydraulic conductivity testing in the field (after Benson and Gribb, 1997)

Infiltration is ceased once the flow reaches steady-state and allowed drainage under gravitational gradient. The temporal variation of suction and moisture content at different depths in the soil during drainage can be plotted for obtaining hydraulic gradients.

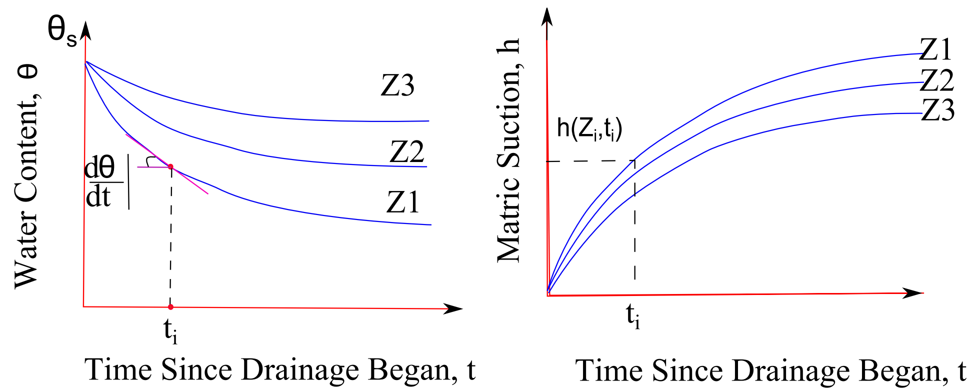


Fig. 4.31 Conceptual illustrations of (a) Transient water content profiles and b) Suction profiles measured during field instantaneous profile test (after Lu and Likos, 2004)

At the moment drainage begins in soil, the moisture content is θ_s and the corresponding suction is zero. As the drainage continues the water content decreases with time and vice-versa for matric suction. Further, the soil layers at greater depth have smaller rates of moisture loss. Thus, the soil layers at greater depths maintain relatively high water content for longer period of time. Similar to earlier analysis, the water flux through any layer at depth z_j and time t_j is

$$q\left(z_i, t_i\right) = -dz\left|\frac{d\theta}{dt}\right|_{z_i, t_i} \tag{4.11}$$

This slope can be obtained from the illustration in Fig 4.31. The cumulative flux is obtained by summing the water fluxes through all the overlying layers. The hydraulic gradient is then calculated from Darcy's law from the above computed flux and the hydraulic gradient, $(dh_t/dz)|_{zi,ti}$.

Lecture 10

Modeling of soil hydraulic characteristics

Modeling soil hydraulic characteristics

As discussed in the introduction lectures that the simulation of vadose zone flow is important in many geotechnical engineering applications, namely seepage through embankment dams, groundwater recharge analysis, and analysis of rainfall induced slope failures. The simulation requires the solution to Richards' equation with appropriate boundary conditions. One-dimensional, isothermal, and Darcian flow of water in variably saturated soils is simulated using the following form of Richards' equation

$$\frac{\partial \theta(\psi)}{\partial t} = \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z}+1\right)\right] \qquad (4.12)$$

where θ is the volumetric water content (cm^3/cm^3), ψ the matric suction (kPa), K the hydraulic conductivity of the soil (m/d), z a spatial variable (m) and t the time (day). The numerical solution of Richards' equation requires two well-defined hydraulic functions viz. soil water characteristic curve (θ vs. ψ) and hydraulic conductivity function and hydraulic conductivity function (K vs. ψ). Furthermore, an accurate modeling of unsaturated flow depends on the precise estimation and description of these two soil hydraulic functions namely, water retention curve (θ-ψ relation) and hydraulic conductivity function (K-ψ relation). The water retention data of an unsaturated soil can be obtained using measured moisture content data with pressure plate data in the laboratory and tensiometer data in the field. On the other hand, both the laboratory and field measurement of unsaturated hydraulic conductivity is highly time-consuming. This is because spatial variability of these values and also the need for measured hydraulic conductivity over a wide range of matric suctions. Consequently, conductivity functions are commonly estimated from water retention data using predictive models. This requires a continuous description of water retention data over a wide range of matric suctions. Water retention models are useful for describing observed retention data using smooth mathematical functions over a wide range of soil suctions. These mathematical models are also helpful in simplifying the numerical analysis of the Richards' equation. As a result, water retention models are crucial in the modeling of flow through unsaturated soil. Several retention and conductivity models have been used for describing volumetric water content as a function of soil suction. Some of the models use normalized volumetric water content Θ to represent the functional form. The normalized volumetric water content is

$$\Theta = \frac{\theta-\theta_r}{\theta_s-\theta_r} \qquad (4.13)$$

where θ_r is the residual water content and θ_s is the saturated water content.

Some of the commonly used soil water retention models are presented below:

S.No.	Model	Equation	Fitting parameter(s)
1.	Brooks and Corey (1964)	$\Theta = \begin{cases} 1 & \psi < \psi_b \\ \left(\dfrac{\psi_b}{\psi}\right)^{\lambda} & \psi \geq \psi_b \end{cases}$	λ (pore-size distribution index)
2.	van Genuchten (1980)	$\Theta = \left[\dfrac{1}{1+(\alpha\psi)^n}\right]^m$	*a*, *m*, *n*
3.	Fredlund & Xing (1995)	$\theta = \left(1 - \dfrac{\ln(1+\psi/\psi_r)}{\ln(1+10^6/\psi_r)}\right)\dfrac{\theta_s}{\left\{\ln\left[e+(\psi/a)^n\right]\right\}^m}$	*a*, *m*, *n*, ψ_r

Similarly, varieties of hydraulic conductivity models have been developed to model the unsaturated hydraulic conductivity over a wide range of matric suction based on the limited data set. Mualem (1986) classified the existing models into: empirical models, macroscopic models, and statistical models. The first two-types of equations require measured conductivity data for obtaining a continuous conductivity function. On the other hand, statistical models are used to predict the conductivity function from the knowledge of soil water characteristic curves. The existing models are summarized in the following table

S. No.	Model	Equation	Fitting parameters	Model type
1.	Richards (1931)	$k(\psi) = a\psi + b$	*a*, *b*	Empirical
2	Gardner (1958)	$k(\psi) = \dfrac{k_s}{1+a\psi^n}$	*a*, *n*	Empirical
3.	Brooks and Corey (1964)	$k = \begin{cases} k_s & \psi \leq \psi_b \\ k_s\left(\dfrac{\psi_b}{\psi}\right)^n & \psi > \psi_b \end{cases}$	*n*	Macroscopic
4.	Campbell (1973)	$k(\theta) = k_s\left(\dfrac{\theta}{\theta_s}\right)^n$	*n*	Macroscopic
5.	Jackson (1972)	$k(\theta_i) = k_s\left(\dfrac{\theta_i}{\theta_s}\right)\dfrac{\sum_{j=1}^{m}\left[(2j+1-2i)h_j^{-2}\right]}{\sum_{j=1}^{m}\left[(2j-1)h_j^{-2}\right]}$	-	Statistical

6.	Burdine (1953)	$\frac{k(\Theta)}{k_s}=k_r(\Theta)=\Theta^2\int_0^{\Theta}\frac{1}{h^2(x)}dx\Big/\int_0^1\frac{1}{h^2(x)}dx$	-	Statistical
7.	Mualem (1976)	$k_r(\Theta)=\sqrt{\Theta}\left[\int_0^{\Theta}\frac{1}{h(x)}dx\Big/\int_0^1\frac{1}{h(x)}dx\right]^2$	-	Statistical

The statistical (predictive) models are very attractive as they can be used to predict the entire hydraulic conductivity function from a well-defined and continuous water retention curves. The statistical models are required to combine with SWCCs. A simple expression can be obtained for hydraulic conductivity function if we assume van Genuchten equation represents the water retention data. The van Genuchten equation can be written in terms of suction head in functional form as

$$h=\frac{1}{\alpha}\left(\Theta^{-1/m}-1\right)^{1/n} \tag{4.14}$$

where α is the air-entry head and Θ is the normalized volumetric water content.
Substitution of van Genuchten equation in Burdine's model results

$$k_r(\Theta)=\Theta^2\frac{f(\Theta)}{f(1)}$$

where $f(\Theta)=\int_0^{\Theta}\left(\frac{x^{1/m}}{1-x^{1/m}}\right)^{2/n}dx$, x is the variable of integration.

Substitution of $x^{1/m}=y$ ($dx=my^{m-1}dy$) yields,

$$f(\Theta)=m\int_0^{\Theta^{1/m}}y^{m-1+2/n}(1-y)^{-2/n}dy \tag{4.15}$$

which is a particular form of incomplete beta function. van Genuchten (1980) felt that this general equation does not have an analytical solution. Therefore, analytical solutions were derived for a particular case i.e., $o = m-1+1/n$, where o is integer. The analytical solution when $o = 1$ gives,

$$f(\Theta)=m\int_0^{\Theta^{1/m}}(1-y)^{m-1}dy=-(1-y)^m\Big]_0^{\Theta^{1/m}}=\left[1-\left(1-\Theta^{1/m}\right)^m\right]$$

Therefore,

$$k_r(\Theta)=\Theta^2\left[1-\left(1-\Theta^{1/m}\right)^m\right]\quad\begin{matrix}(m=1-2/n)\\(0<m<1,n>2)\end{matrix} \tag{4.16}$$

which is a particular solution and the model parameters should be limited in the ranges as given in the eq. 4.16. However, it was realized later by van Genuchten and Nielsen (1985) that eq. 4.15 has an analytical solution in its general form which is

$$k_r(\Theta)=\Theta^2 \mathrm{I}_{\zeta}(r,s) \tag{4.17}$$

where I is the incomplete beta function, $\zeta = \Theta^{1/m}$, $r = m + 2/n$, and $s = 1\text{-}2/n$.

Similarly, the Mualem's model yields the following particular solution when the van Genuchten equation is used:

$$k_r(\Theta) = \sqrt{\Theta}\left[1 - \left(1 - \Theta^{1/m}\right)^m\right]^2 \quad \begin{matrix} (m = 1 - 1/n) \\ (0 < m < 1) \end{matrix} \tag{4.18}$$

The analytical solution in its general form yields

$$k_r(\Theta) = \sqrt{\Theta}\left[\mathrm{I}_\zeta(p,q)\right]^2 \tag{4.19}$$

where $p = m + 1/n$ and $q = 1\text{-}1/n$. The model parameter n should be restricted to unity ($n > 1$) in the above equation for the stability of the solution.