

## **Module 2 : Theory of Earth Pressure and Bearing Capacity**

### **Lecture 6 : Introduction [ Section 6.1 Different Theories of Earth Pressure ]**

#### **Objectives**

**In this section you will learn the following**

- Introduction
- Different Theories of Earth Pressure
- Lateral Earth Pressure For At Rest Condition
- Movement of the Wall
- Different Movements

## Module 2 : Theory of Earth Pressure and Bearing Capacity

### Lecture 6 : Introduction [ Section 6.1 Different Theories of Earth Pressure ]

#### 6 Introduction

A soil mass is stable when the slope of the surface of the soil mass is flatter than the safe slope. At some locations, due to limitation of space, it is not possible to provide flat slope & the soil is to be retained at a slope steeper than the safe one. In such cases, a retaining structure is required to provide lateral support to the soil mass.

A retaining structure is a permanent or temporary structure which is used for providing lateral support to the soil mass or other materials. Some of the examples of retaining structures used in soil & foundation engineering are: Retaining wall, Sheet piles, Anchored Bulkheads, Sheet piling & Basement wall, etc. Generally, the soil masses are vertical or nearly vertical behind the retaining structure. Thus, a retaining structure maintains the soil at different elevations on its either side. In the absence of a retaining structure, the soil on the higher side would have a tendency to slide and may not remain stable.

The design of the retaining structure requires the determination of the magnitude & line of action of the lateral earth pressure. The magnitude of the lateral earth pressure depends upon a number of factors, such as the mode of movement of the wall, the flexibility of the wall, the properties of the soil, the drainage conditions. For convenience, the retaining wall is assumed to be rigid & the soil structure interaction effect is neglected which arises due to the flexibility of the wall.

The lateral earth pressure is usually computed using the classical theories proposed by Coulomb (1773) & Rankine (1857). The general wedge theory proposed by Terzaghi (1943) is more general and is an improvement over the earlier theories. The general equations developed for both theories are based on the fundamental assumptions that

the retained soil is **cohesionless** (no clay component), **homogeneous** (not a varying mixture of materials), **isotropic** (similar stress-strain properties in all directions or in practical terms, not reinforced), **semi-infinite** (wall is very long and soil goes back a long distance without bends or other boundary conditions), and **well drained** to avoid consideration of pore pressures.

The pressure or force exerted by soil on any boundary is called the earth pressure. When the earth pressure acts on the side (back or face) of a retaining wall, it is known as the Lateral earth pressure. The magnitude of the lateral earth pressure depends upon the movement of the retaining wall relative to the backfill & upon the nature of the soil.

## Module 2 :Theory of Earth Pressure and Bearing Capacity

### Lecture 6 : Introduction [ Section 6.1 Different Theories of Earth Pressure ]

#### 6.1 Different Theories of Earth Pressure

There are two classical theories of earth pressure. They are

- Coulomb's earth pressure theory.
- Rankine's earth pressure theory.

Coulomb published the first rigorous analysis of lateral earth pressure problem in 1776. Rankine proposed a different approach to the problem in 1857. These theories propose to estimate the magnitudes of two pressures called *Active earth pressure* and *Passive earth pressure* . All the theories proposed by Coulomb have been discussed in the later part of the chapter.

#### Assumptions

Most of the theories of earth pressure are based on the following assumptions: the backfill of the wall is isotropic and homogenous; the deformation of the backfill occurs exclusively parallel to the vertical plane at right angles at the back of the wall, and the neutral stresses in the backfill material are negligible.

- The soil mass is semi infinite, homogeneous, dry & cohesion less.
- The ground surface is plane which may be horizontal or inclined.
- The back of the wall is smooth & vertical. In other words, there are no shearing stresses or frictional stresses between the wall & the soil. The stress relationship for any element adjacent to the wall is the same as for any other element farther away from the wall.
- The soil mass is in a state of plastic equilibrium i.e. at the verge of failure.

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### Lecture 6 : Introduction [ Section 6.1 Different Theories of Earth Pressure ]

The earth pressures are defined as follows:

Consider a retaining wall with a plane vertical face, as shown in Fig. 2.1, which is backfilled with cohesionless soil. If the wall does not move even after filling the materials, the pressure exerted on the wall is known as pressure for the *at rest condition* of the wall. If suppose the wall gradually rotates about the point A and moves away from the backfill, the unit pressure on the wall gradually gets reduced and after a particular displacement of the wall at the top, the pressure reaches a constant value. This pressure is the minimum possible. The pressure is termed as the active earth pressure since the weight of the backfill is responsible for the movement of the wall. If the wall is smooth, the resultant pressure acts normal to the face of the wall. If the wall is rough, the resultant pressure acts at an angle of  $\delta$  to the normal to the face. This angle  $\delta$  is known as *the angle of wall friction*. As the wall moves away from the backfill, the soil also tends to move forward. When the wall movement is sufficient, a soil mass of weight  $W$  ruptures along a surface AC'C shown in fig. This surface is slightly curved. If the surface is assumed to be plane surface AC, analysis would indicate that this surface would make an angle of  $45^\circ + \phi/2$  with the horizontal.

If the wall is now rotated about A towards the backfill, the actual failure plane AC'C is also a curved surface. However, if the failure surface is approximated to a plane AC, this makes an angle  $45^\circ + \phi/2$  with the horizontal and the pressure on the wall increases from the value of at rest condition to a maximum possible value. The maximum pressure  $p_p$  that is developed is termed as passive earth pressure. The pressure is called passive earth pressure because the weight of the backfill opposes the movement of the wall. It also makes an angle of  $\delta$  with the normal if the wall is rough.

The gradual increase or decrease of the pressure of the wall with the movement of the wall from the "at rest condition" may be depicted as shown in Fig.2.2. The movement  $\Delta_p$  required to develop passive state is considerably larger than the  $\Delta_a$  required for active case.

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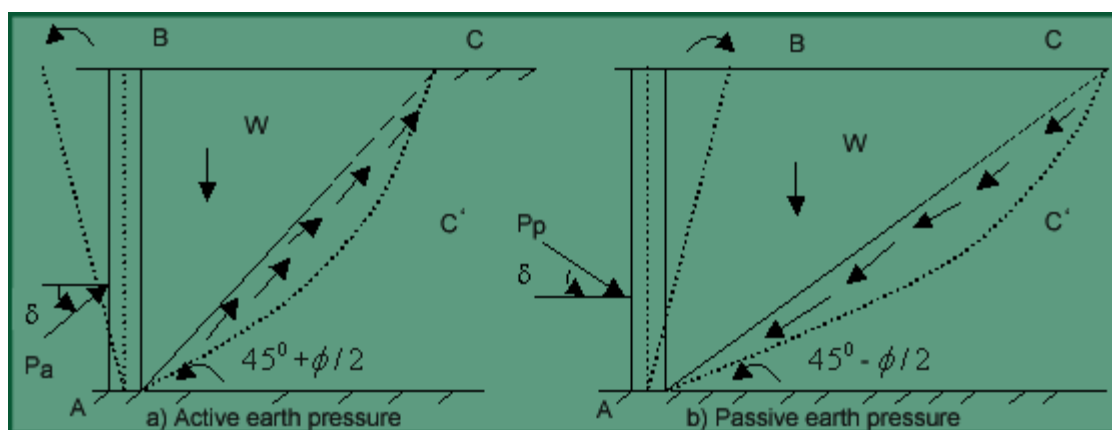


Fig 2.1. Wall movement for the development of active and passive earth

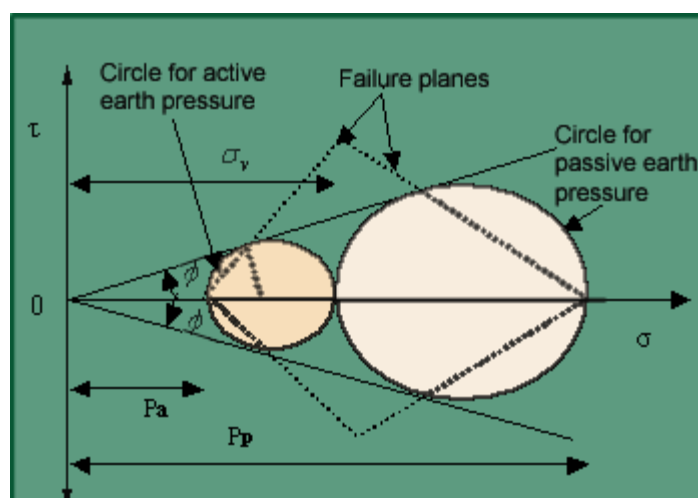


Fig.2.2. Mohr stress diagram

## Module 2 : Theory of Earth Pressure and Bearing Capacity

### Lecture 6 : Introduction [ Section 6.1 Different Theories of Earth Pressure ]

#### 1. Lateral Earth Pressure For At Rest Condition

The earth retained behind retaining walls may be natural earth or filled up soil. These backfill materials exert certain lateral pressures on the wall. If the wall is rigid and does not move with the pressure exerted on the wall, the soil behind the wall will be in a state of *elastic equilibrium* .

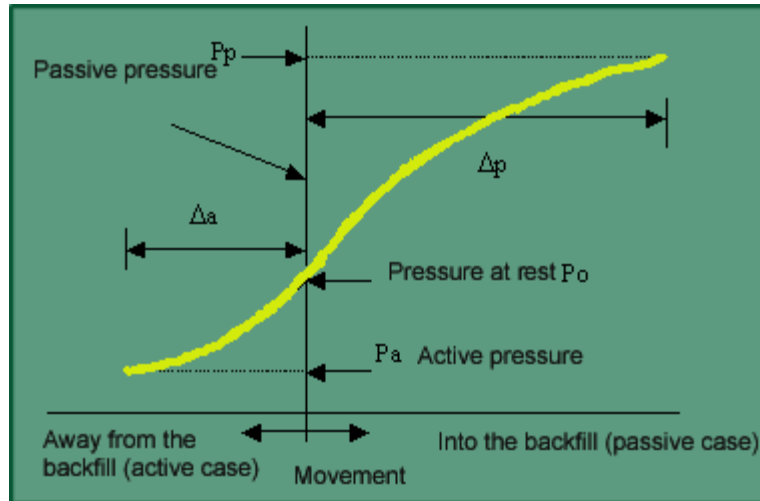


Fig.2.3 Development of active and passive earth pressures

To reach the plastic state, for  $p_a$  to develop we need a minimum displacement  $\delta_a$  . If the displacement developed is less than  $\delta_a$  , the pressure developed is called partially mobilized earth pressure which is more than  $p_a$  .

Similarly for passive case the wall moves towards the soil and the soil tries to resist the movement of the wall. To reach the passive case a very high movement is required,  $\delta_p$  .

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When we are measuring the passive pressure we are measuring the resistance of the soil against wall movement.

Hence for all practical purposes we get fully mobilised active earth pressure but partially mobilised passive earth pressure as such a large movement does not occur usually. Hence it is more acceptable to design with active earth pressure.

Thus when  $\delta_a$  is reached active pressure is fully mobilised but passive pressure is partially mobilised.

Therefore we should design with (  $p_a$  – a small % of  $p_p$  ), as, we are making an unsafe design when we are designing with

(  $p_a$  – fully mobilised  $p_p$  ) and an uneconomic one when we are designing with only  $p_a$ .

✓To get fully mobilised active and passive earth pressures the following wall movement are required approximately.

$\delta_a = (0.1 - 0.4)\%$  of the height of the wall.

$\delta_p = (5-10)\%$  of the height of the wall.

For dense sand  $p_a$  mobilised at 0.1% strain and  $p_p$  at 5% strain.

For loose sand  $p_a$  mobilised at 0.4% strain and  $p_p$  at 10% strain.

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### Lecture 6 : Introduction [ Section 6.1 Different Theories of Earth Pressure ]

#### 2 Movement of the Wall

Considering a more generalised case when wall is not smooth.

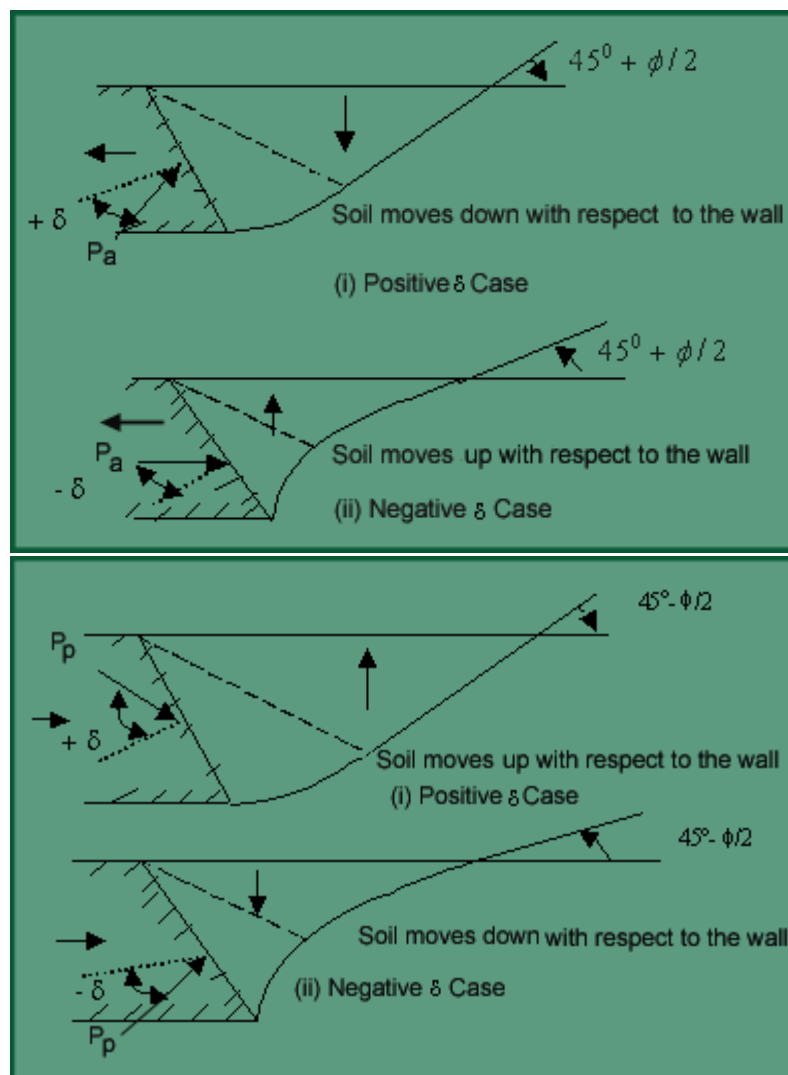


Fig. 2.4 Different movements of the wall



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3 Different Movements

There are basically three ways a wall can move. They are as follows:-

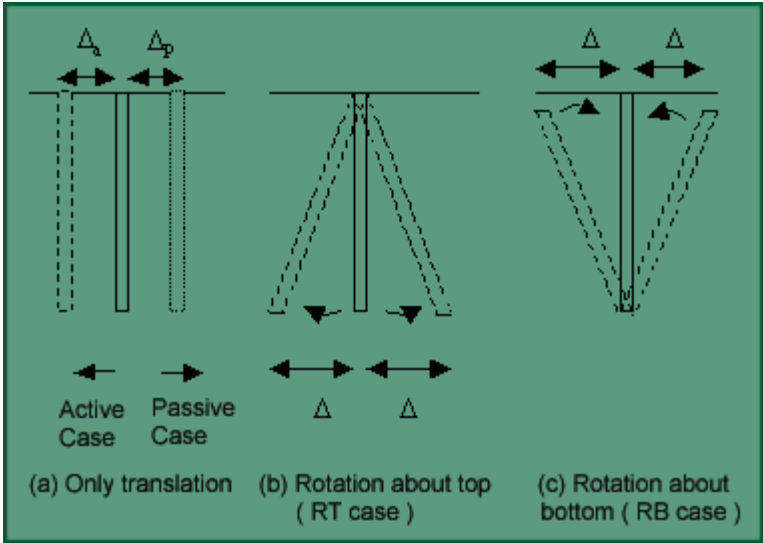


Fig. 2.5 Movements of the wall for both active and passive cases

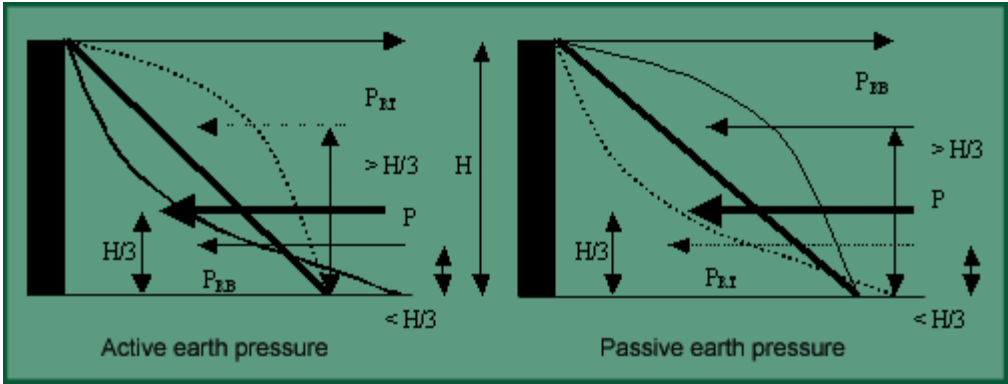
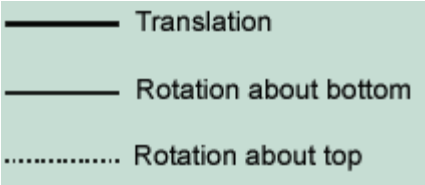


Fig. 2.6 Earth pressure distribution



## **Module 2 : Theory of Earth Pressure and Bearing Capacity**

### **Lecture 6 : Introduction [ Section 6.1 Different Theories of Earth Pressure ]**

#### **Recap**

**In this section you have learnt the following**

- Introduction
- Different Theories of Earth Pressure
- Lateral Earth Pressure For At Rest Condition
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## **Module 2 : Theory of Earth Pressure and Bearing Capacity**

### **Lecture 6 : Introduction [ Section 6.2 Displacement Related Earth Pressure]**

#### **Objectives**

##### **In this section you will learn the following**

- Displacement Related Earth Pressure
- Displacement-related active earth pressure
- Types of movements
- Proposed method
- Translation
- Rotation about top (RT)
- Rotation about the bottom (RB)

## Module 2 : Theory of Earth Pressure and Bearing Capacity

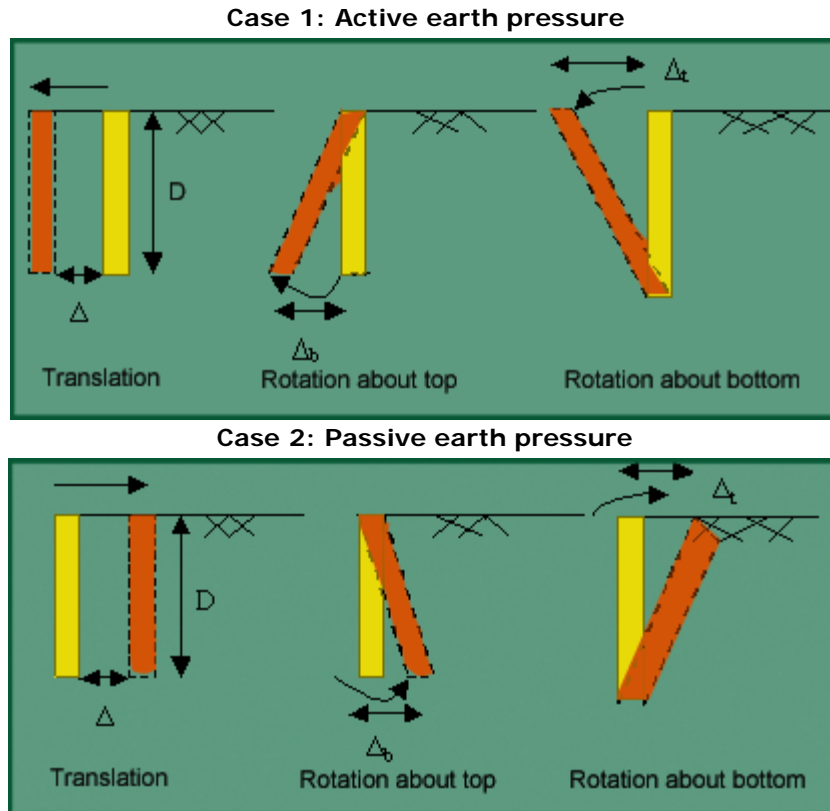
### Lecture 6 : Introduction [ Section 6.2 Displacement Related Earth Pressure]

#### 6.2 Displacement Related Earth Pressure

##### 1. Displacement-related active earth pressure

###### Types of movements

Three types of rigid body movements are considered in the analysis. These are translation, rotation about top (RT) and rotation about bottom (RB) modes as shown in Fig. 2.7 below.



**Fig 2.7 different movements of the wall under active and passive conditions**

$\Delta$  is the displacement at any depth  $Z$  and  $\Delta_a$  is the amount of displacement required for full mobilization of active earth pressure.  $\Delta_a$  is found to be 0.1% to 0.4% of  $D$  for dense to loose sands where  $D$  is the total height of the retaining wall. The displacement at the base of the wall is denoted by  $\Delta_b$  for RT mode and the displacement at the top of the wall is denoted by  $\Delta_t$  for RB mode.

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### Lecture 6 : Introduction[ Section 6.2 Displacement Related Earth Pressure]

#### 2. Proposed method:

In the proposed semi-empirical approach, the assumptions made are:

- For translation mode, when  $\Delta = \Delta_a$  full active pressure gets mobilized at all depths. So  $\Delta$  is assumed to be constant throughout the depth.
- For rotation about top (RT) mode,  $\Delta$  is a linear function of depth  $z$ . At the bottom when  $\Delta_b = \Delta_a$ , the mobilized friction angle  $\phi_m$  will be zero at the top and full value of  $\phi$  at the bottom. For  $\Delta_t < \Delta_a$   $\phi_m < \phi$  at the bottom and  $\phi_m = 0$  at the top.
- For rotation about bottom mode,  $\Delta$  is also a linear function of depth  $Z$ . At top when  $\Delta_t = \Delta_a$  the mobilized friction angle  $\phi_m$  will be zero at bottom and full value of  $\phi$  at the top. For  $\Delta_t < \Delta_a$   $\phi_m < \phi$  at the top and  $\phi_m = 0$  at the bottom.
- The mobilized wall friction angle  $\delta_m$  increases with the increase in mobilized friction angle resulting in a constant value of  $\delta_m / \phi_m$

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### Lecture 6 : Introduction[ Section 6.2 Displacement Related Earth Pressure]

#### I. Translation

The fig. 2.5 (a) shows a rigid retaining wall of height  $D$ . The amount of displacement is denoted by  $\Delta$ , which is independent of depth  $z$ . The steps followed are,

- The experimental observations by Sherif et al (1984) and Fang et al (1986) are used. At various depths, for different wall movements, normal active earth pressure  $\sigma_{ax}$  is noted from experimental curves.

The mobilized active earth pressure coefficient  $(K_a)_m$  at any depth  $z$  is calculated as  $(K_a)_m = \frac{\sigma_{ax}}{\gamma Z}$  Where

$\sigma_{ax}$  is the measured normal pressure at any depth  $z$  and  $\gamma$  is unit weight of soil.

- Along the depth, average  $(K_a)_m$  is calculated and denoted by  $(K_a)_{m,avg}$ . Some abnormal values very near to ground are neglected because of the arching action and also some experimental error near the ground as reported by the experimentalists in their paper.
- Using Coulomb's active earth pressure coefficient (Bowles, 1996), mobilized friction angle  $\phi_m$  is obtained corresponding to  $(K_a)_{m,avg}$ .
- The above procedure is repeated for various values of  $\Delta/\Delta_a$ . In table 2.1, a typical calculation is shown.

**Table 2.1 : Typical calculation to obtain mobilized friction angle in translation mode**

$(\phi = 34^\circ, \delta / \phi = 0.5, \gamma = 15.4 \text{ kN/m}^3 \text{ and } \Delta / \Delta_a = 1.0 \text{ with } \Delta_a = 0.0005D)$  (Data from Fang et al, 1986),

Depth $z$ in m	Horizontal active earth pressure $\sigma_{ax}$ in $\text{kN/m}^2$	$(K_a)_m$ values $\sigma_{ax} / \gamma z$	$(K_a)_{m,avg}$ (neglecting 1 <sup>st</sup> value)	Calculated $\phi_m$ (Degrees)	Corrected $\phi_m$ (Degrees)
0	0	-	-	-	-
0.153	1.166	0.497	-	-	-
0.305	1.114	0.237	-	-	-
0.467	1.531	0.213	-	-	-
0.628	2.353	0.243	0.204	38.29 <sup>0</sup>	34 <sup>0</sup>
0.793	2.281	0.187	-	-	-
0.958	2.541	0.172	-	-	-
1.016	2.645	0.169	-	-	-

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### Lecture 6 : Introduction[ Section 6.2 Displacement Related Earth Pressure]

Now in case of full mobilization ( $\Delta / \Delta_a$ ),  $\phi_m = \phi$ , hence a correction factor of (34/38.29) is applied. In this way all other data for partial mobilization case are also calculated. In other words, for any other ( $\Delta/\Delta_a$ ), the corrected  $\phi_m = (\text{the calculated } \phi_m \text{ for any } \Delta) * (\text{the correction factor})$ .

Finally all results are expressed in terms of two parameters viz.

$$x = \frac{\Delta}{\Delta_a} \quad y = [(\phi_m / \phi) / (\Delta / \Delta_a)]$$

$$y = \frac{1}{x^{0.65}} \quad \text{valid for } 0 < x \leq 1.$$

## II Rotation about top (RT)

Consider fig. 2.5 (b) a rigid retaining wall of height D is rotating about the top and the bottom horizontal displacement is denoted by  $\Delta_b$ . Let,

$$\xi = \Delta_b / \Delta_a$$

assuming linear variation with depth,

$$x = \Delta / \Delta_a = \xi (Z / D)$$

$$y = [(\phi_m / \phi) / (\Delta / \Delta_a)]$$

The values of x and y thus obtained for various cases are shown in table 2.2 and it is found that all the points are falling within a narrow band with each point as an average of multiple points in one experiment.

A lower bound equation will provide a proper safe estimate of active earth pressure and hence the proposed relationship for translation mode is expressed as given below.

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### Lecture 6 : Introduction[ Section 6.2 Displacement Related Earth Pressure]

The following Table 2.2 shows a typical calculation. In the full mobilization case, at the base  $\Delta_b = \Delta_a$ , so  $\phi_m = \phi$  at the base and at the top  $\phi = 0^\circ$ . The correction factor at the base is therefore  $(40.4/71.47)=0.565$ , and at the top it is zero. By assuming a linear variation of correction factor from the top, the calculated values of  $\phi_m$  are corrected as shown in table For other cases with  $\Delta_b < \Delta_a$ , the correction factor of 0.565 obtained at the base for the case of  $\Delta_b = \Delta_a$  is maintained, with again a linear variation from zero at the top.

Some ill-points very near to the ground are neglected by considering that experimental errors may have been caused due to some arching action near ground as also mentioned by experimentalists in their papers.

**Table 2.2 :Typical calculation to obtain mobilized friction angle in RT mode**

(  $\phi = 40.4^\circ$ ,  $\delta/\phi = 0.5$ ,  $\gamma = 16.1 \text{ kN/m}^3$  and  $\Delta/\Delta_a = 1.0$  with  $\Delta_a = 0.001D$ ) (Data from Fang et al, 1986),

Depth z in m	Horizontal active earth pressure $\sigma_{ax}$ in $\text{kN/m}^2$	$(K_a)_m$ values $\sigma_{ax}/\gamma z$	Calculated $\phi_m$ (Degrees)	Z/D values (= $\Delta/\Delta_b$ )	Corrected $\phi_m$ (Degrees)	$x = \Delta/\Delta_a$ [= $1.0(\Delta/\Delta_b)$ ]	$y = [(\phi_m/\phi)/(\Delta/\Delta_a)]$
0	0	-	-	0	0	0	-
0.153	4.790	1.951	-	0.15	-	0.15	-
0.305	1.294	0.263	32.54	0.3	23.12	0.3	1.91
0.467	1.444	0.192	39.50	0.46	25.22	0.46	1.36
0.628	1.87	0.185	40.32	0.62	21.1	0.62	0.84
0.793	1.583	0.124	48.51	0.78	24.25	0.78	0.77
0.958	0.766	0.05	63.32	0.94	34.02	0.94	0.9
1.016	0.298	0.018	71.47	1.00	40.40	1.00	1.00

The proposed equation as a lower bound is given by,

$$y = \frac{1}{x^{0.5}} \quad \text{-----(1)}$$

and eq(1) is valid for  $0 < x \leq 1$  and  $0 < \xi < 1$ . However at depth  $z=0$ , for any value of  $\xi$ , active earth pressure will be zero (ground surface).



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### Lecture 6 : Introduction[ Section 6.2 Displacement Related Earth Pressure]

#### III Rotation about the bottom (RB)

Consider Fig.2.5 (c), a rigid retaining wall of height  $D$ , rotating about the bottom and the horizontal displacement at top is given by  $\Delta_t$ , Let,

$$\eta = \Delta_t / \Delta_a \quad \text{-----}(2)$$

and assuming linear variation with depth,

$$X = (1 - \Delta / \Delta_a) = 1 - \eta(1 - Z / D) \quad \text{-----}(3)$$

$$Y = [(\phi_m / \phi) / (1 - \Delta / \Delta_a)] \quad \text{-----}(4)$$

**Table 2.3 : Typical calculation to obtain mobilized friction angle in RB mode**

( $\phi = 35^\circ$ ,  $\delta / \phi = 0.58$ ,  $\gamma = 15.5 \text{ kN/m}^3$  and  $\Delta / \Delta_a = 1.0$  with  $\Delta_a = 0.0003D$ ) (Data from Fang et al, 1986),

Depth $z$ in m	Horizontal active earth pressure $\sigma_{ax}$ in kN/m <sup>2</sup>	$(K_a)_m$ values $\sigma_{ax} / \gamma z$	Calculated $\phi_m$ (Degrees)	1-Z/D values ( $\Delta / \Delta_t$ )	Corrected $\phi_m$ (Degrees)	$X = 1 - \Delta / \Delta_a$ $[= 1 - (\Delta / \Delta_t)]$	$Y = \left[ \frac{(\phi_m / \phi)}{(1 - \Delta / \Delta_a)} \right]$
0	0	-	36.3	1.00	35.00	0.00	-
0.153	0.559	0.237	34.39	0.83	27.55	0.17	4.72
0.302	1.331	0.284	30.10	0.67	17.82	0.33	1.54
0.467	1.663	0.23	34.98	0.49	16.72	0.51	0.94
0.628	2.461	0.253	32.93	0.31	8.79	0.69	0.37
0.793	2.994	0.244	33.76	0.13	3.63	0.87	0.12
0.915	3.326	0.235	34.57	0.00	0.00	1.00	0.00

Table 2.3 shows a typical calculation. In the full mobilization case at top ( $\Delta_t / \Delta_a = 1$ ),  $\phi_m = \phi$  at the top and at the base  $\phi = 0^\circ$ . Considering this criteria, correction has been made using a correction factor of (35/36.3) at the top. Corrected values  $\phi_m$  are shown in table 2.3. For other cases with  $\Delta_t < \Delta_a$ , the same maximum correction factor is applied at top to a linear variation of zero at the base.

The proposed equation as a lower bound to the points is given by,

$$Y = (1/x) - 1 \quad \text{-----}(5)$$

And Eq.(5) is valid for  $0 < X \leq 1$  and  $\eta$  not equal to zero.

## **Module 2 : Theory of Earth Pressure and Bearing Capacity**

### **Lecture 6 : Introduction [ Section 6.1 Different Theories of Earth Pressure ]**

#### **Recap**

**In this section you have learnt the following**

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**Congratulations, you have finished Lecture 6. To view the next lecture select it from the left hand side menu of the page**