Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 8 : Development of Bearing Capacity Theory [Section 8.1: Terzaghi's & Meyerhof's Bearing Capacity Theory]

Objectives

In this section you will learn the following

- Development of Bearing Capacity Theory
- Terzaghi's Bearing Capacity Theory
- Assumptions in Terzaghi's Bearing Capacity Theory.
- Meyerhof's Bearing Capacity Theory
- Bearing capacity of square and circular footings

8.1 Development of Bearing Capacity Theory

Application of limit equilibrium methods was first done by Prandtl on the punching of thick masses of metal. Prandtl's methods was adapted by Terzaghi to bearing capacity failure of shallow foundations. Vesic and others improved on Terzaghi's original theory and added other factors for a more complete analysis.

1. **Terzaghi's Bearing Capacity Theory:**

Assumptions in Terzaghi's Bearing Capacity Theory.

- Depth of foundation is less than or equal to its width.
- Base of the footing is rough.
- Soil above bottom of foundation has no shear strength; it is only a surcharge load against the overturning load
- Surcharge upto the base of footing is considered.
- Load applied is vertical and non-eccentric.
- The soil is homogenous and isotropic.
- L/B ratio is infinite.

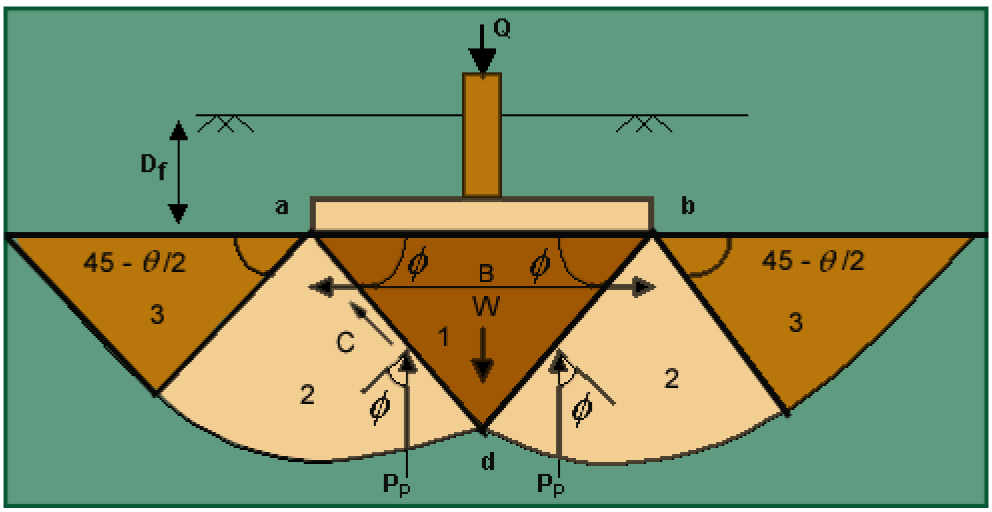

Fig. 2.25 Terzaghi's Bearing Capacity Theory

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Consider a footing of width B and depth D f loaded with Q and resting on a soil of unit weight γ. The failure of the zones is divided into three zones as shown below. The zone1 represents an active Rankine zone, and the zones 3 are passive zones. The boundaries of the active Rankine zone rise at an angle of $45 + \phi/2$, and those of the passive zones at $45 - \phi/2$ with the horizontal. The zones 2 are known as zones of radial shear, because the lines that constitute one set in the shear pattern in these zones radiate from the outer edge of the base of the footing. Since the base of the footing is rough, the soil located between it and the two surfaces of sliding remains in a state of equilibrium and acts as if it formed part of the footing. The surfaces ad and bd rise at ϕ to the horizontal. At the instant of failure, the pressure on each of the surfaces ad and bd is equal to the resultant of the passive earth pressure P_p and the cohesion force C_a. Since slip occurs along these faces, the resultant earth pressure acts at angle ϕ to the normal on each face and as a consequence in a vertical direction. If the weight of the soil adb is disregarded, the equilibrium of the footing requires that

$$Q_d = 2P_p + 2C_a \sin\phi = 2P_p + Bc\tan\phi \qquad \text{---------- (1)}$$

The passive pressure required to produce a slip on *def* can be divided into two parts, $P_p^{'}$ and $P_p^{''}$. The force $P_p^{'}$ represents the resistance due to weight of the mass *adef.* The point of application of $P_p^{'}$ is located at the lower third point of *ad*. The force $P_p^{''}$ acts at the midpoint of contact surface *ad.*

The value of the bearing capacity may be calculated as :

$$Q_d = 2(P_p + P_C + Pq + \frac{1}{2}Bc\tan\phi) \qquad \text{----------(2)}$$

by introducing into eqn(2) the symbols,

$$N_C = \frac{2P_C}{Bc} + \tan\phi$$

$$N_q = \frac{2P_q}{B\gamma D_f}$$

$$N\gamma = \frac{4P_p^{'}}{B^2\gamma}$$

we obtain, $Q_d = B(cN_C + \gamma D_f N_q + \frac{1}{2}\gamma B N_\gamma)$ ---------- (3)

the quantities N_c, N_q, N_γ are called bearing capacity factors.

$$N_q = \frac{a^2}{2\cos^2(45 + \phi/2)}$$

$$N_c = \cot\phi\left[\frac{a^2}{2\cos^2(45 + \phi/2)} - 1\right]$$

$$N_\gamma = \frac{1}{2}\tan\phi\left[\frac{K_p}{\cos^2\phi} - 1\right]$$

where *K p* = passive earth pressure coefficient

$$a = \exp\left[\left(\frac{3\pi}{4} - \frac{\phi}{2}\right)\tan\phi\right]$$

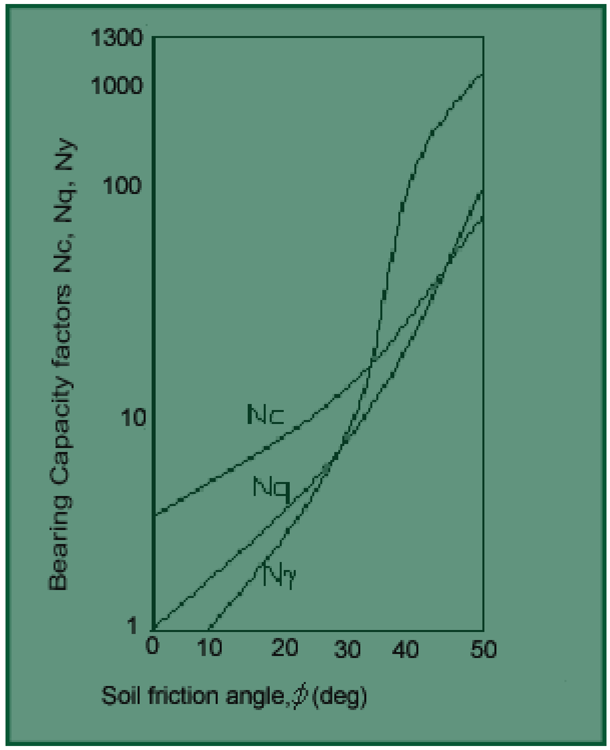

Fig. 2.26 Variation of bearing capacity factors with soil friction angle.

The use of chart figure (2.27) facilitates the computation of the bearing capacity. The results obtained by this chart are approximate.
Loaded strip, width B, Total load per unit length of footing

General shear failure : $Q_d = B(cN_c + \gamma D_f N_q + \frac{1}{2}\gamma B N_g)$

Local shear failure : $Q'_d = B(1/2cN'_c + \gamma D_f N'_q + 1/2\gamma BN'_g)$

Square footing, width B

Total critical load : : $Q_{ds} = B^2(1.3cN_c + \gamma D_f N_q + 0.4\gamma BN_g)$

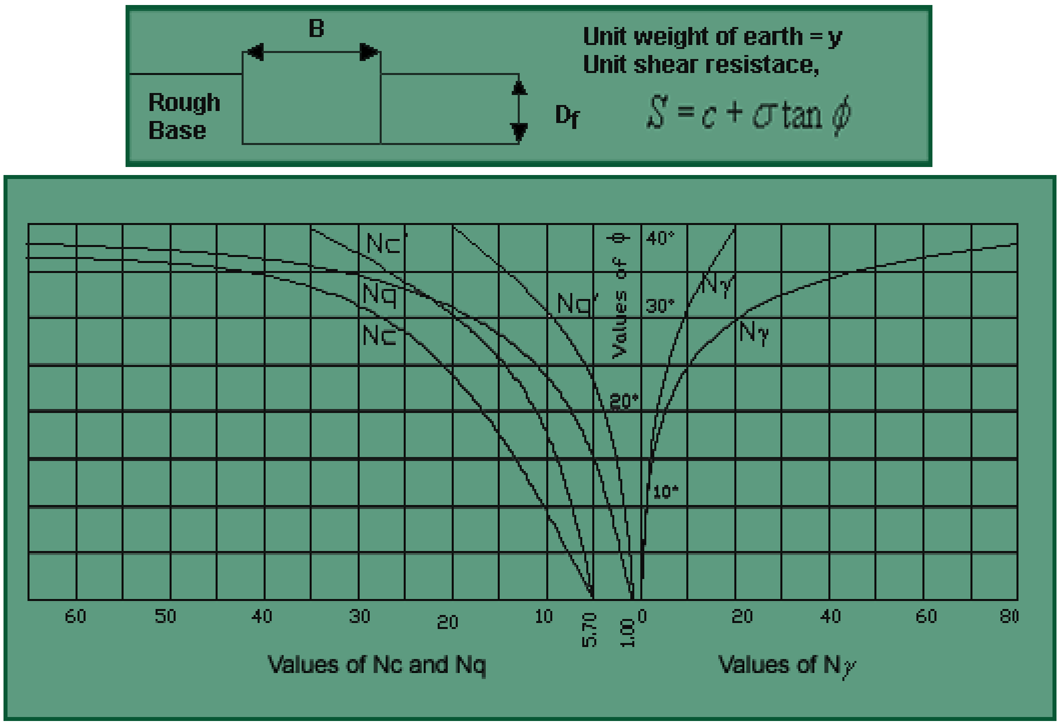

Fig. 2.27 Chart Showing Relation between Angle of Internal Friction and Bearing Capacity Factors.

Table 2.5 : Terzaghi's bearing capacity factors

ϕ^o	N_c	N_q	N_γ	ϕ^o	N_c	N_q	N_γ
0	5.7	1.0	0.0	35	57.8	41.4	42.4
5	7.3	1.6	0.5	40	95.7	81.3	100.4
10	9.6	2.7	1.2	45	172.3	173.3	297.5
15	12.9	4.4	2.5	48	258.3	287.9	780.1
20	17.7	7.4	5.0	50	347.5	415.1	1153.2
25	25.1	12.7	9.7	-	-	-	-
30	37.2	22.5	19.7	-	-	-	-
34	52.6	36.5	30.0	-	-	-	-

2. Meyerhof's Bearing Capacity Theory

The form of equation used by Meyerhof (1951) for determining ultimate bearing capacity of symmetrically loaded strip footings is the same as that of Terzaghi but his approach to solve the problem is different. He assumed that the logarithmic failure surface ends at the ground surface, and as such took into account the resistance offered by the soil and surface of the footing above the base level of the foundation. The different zones considered are shown in fig. 2.28

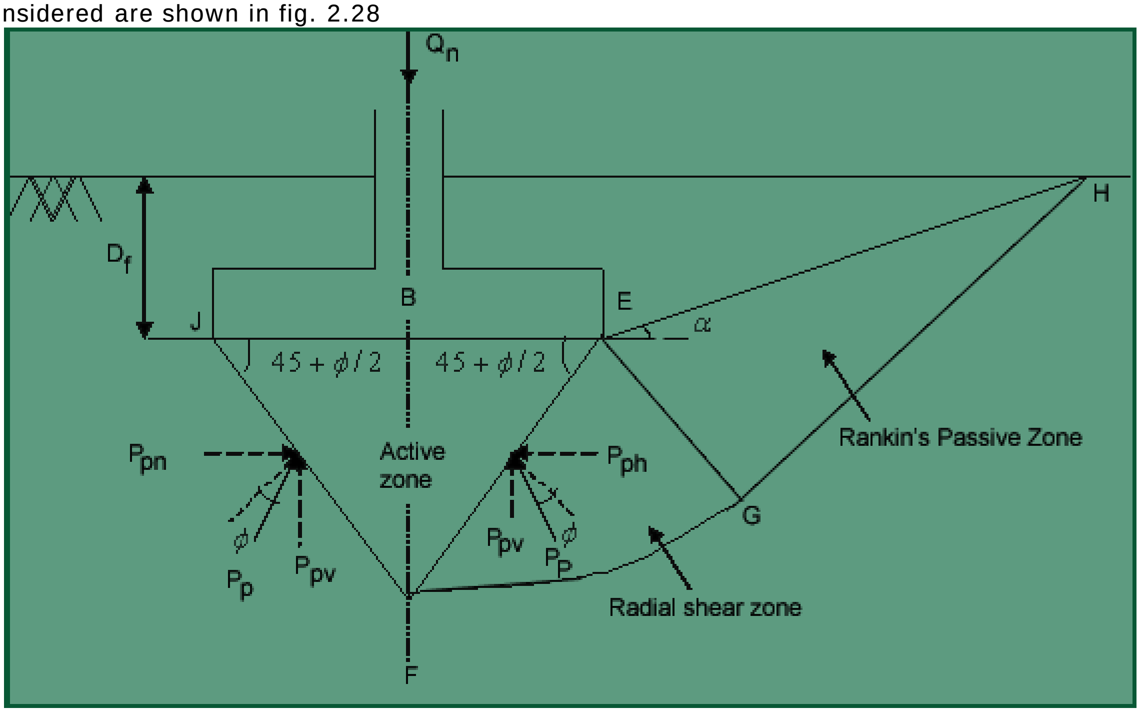

Fig. 2.28 Failure zones considered by Meyerhof

In this, EF failure surface is considered to be inclined at an angle of $(45+\phi/2)$ with the horizontal followed by FG which is logspiral curve and then the failure surface extends to the ground surface (GH).

EF is considered as a imaginary retaining wall face with failure surface as FGH. This problem is same as the retaining wall with the inclined backfill at an angle of a. For this case the passive earth pressure acting on the retaining wall P_p is given by Caqnot and Kerisel (1856). Considering the equilibrium of the failure zone,

$$\sum F_v = 0$$

$$Q_u + W = 2P_{pv}$$

where,

Q_u is the load on the footing,

W is the weight of the active zone and,

P_{pv} is the vertical component of the passive pressure acting on walls JF and EF.

Then the ultimate bearing capacity (q_u) is given as,

$$q_u = \frac{Q_u}{Bx1} = \frac{2P_{pv} - W}{B} = \frac{2(P_{pcv} + P_{pqv} + P_{p\gamma v}) - W}{B}$$

Where,

B is the width of the footing.

Comparing the above equation with,

$$q_u = CN_c + qN_q + \frac{1}{2}\gamma BN_\gamma$$

We get ,

$$N_c = \frac{2P_{pcv}}{cB}$$,

$$N\gamma = \frac{2P_{p\gamma v} - W}{\frac{1}{2}\gamma B^2}$$

$$N_q = \frac{2P_{pqv}}{qB}$$

The form of equation proposed by Meyerhof (1963) is,

$$q_u = cN_c d_c s_c i_c + \bar{\gamma} D N_q d_q s_q i_q + \frac{1}{2}\gamma B N_\gamma d_\gamma s_\gamma i_\gamma$$

where,

N_q, N_c, N_γ = Bearing capacity factors for strip foundation,

c = unit cohesion,

s_q, s_c, s_γ = Shape factors,

i_q, i_c, i_γ = inclination factors for the load inclined at an angle a 0 to the vertical,

d_q, d_c, d_γ = Depth factors,

$\bar{\gamma}$ = effective unit weight of soil above base level of foundation,

γ = effective unit weight of soil below foundation base,

D = depth of the foundation.

Table 2.6 : Shape, depth and inclination factors for Meyerhof bearing capacity

Factors	Value	For
Shape	$s_c = \left(1 + 0.2K_p \frac{B}{L}\right)$	Any ϕ
	$s_q = s_\gamma = \left(1 + 0.1K_p \frac{B}{L}\right)$	ϕ > 10
	$s_q = s_\gamma = 1$	ϕ = 0
Depth	$d_c = \left(1 + 0.2\sqrt{K_p}\frac{D}{B}\right)$	Any ϕ
	$d_q = d_\gamma = \left(1 + 0.1\sqrt{K_p}\frac{D}{B}\right)$	ϕ > 10
	$d_q = d_\gamma = 1$	ϕ = 0
Inclination	$i_c = i_q = \left(1 - \frac{\theta}{90^0}\right)^2$	Any ϕ
	$i_\gamma = \left(1 - \frac{\theta}{\phi}\right)^2$	ϕ > 10
	$i_\gamma = 0$	ϕ = 0

In the above table,

$K_p = \tan^2(45 + \phi/2),$

ϕ = angle of resultant measured from vertical without sign,

B = width of footing,

L = length of footing,

D = depth of footing.

3. Bearing capacity of square and circular footings

If the soil support of a continuous footing yields due to the imposed loads on the footings, all the soil particles move parallel to the plane which is perpendicular to the centre line of the footing. Therefore the problem of computing the bearing capacity of such footing is a plane strain deformation problem. On the other hand if the soil support of the square and circular footing yields, the soil particles move in radial and not in parallel planes. Terzaghi has proposed certain shape factors to take care of the effect of the shape on the bearing capacity. The equation can be written as,

$$q_u = cN_c s_c + \bar{\gamma} D N_q s_q + \frac{1}{2}\gamma B N_\gamma s_\gamma$$

where,

s_q, s_c, s_γ are the shape factors whose values for the square and circular footings are as follows,

For long footings: $s_c = 1, s_q = 1, s_y = 1,$

For square footings: $s_c = 1.3, s_q = 1, s_y = 0.8,$

For circular footings: $s_c = 1.3, s_q = s_\gamma = 0.6,$

For rectangular footing of length L and width B : $s_c = \left(1+0.3\dfrac{B}{L}\right)$, $s_q = 1, s_y = \left[1-0.2\dfrac{B}{L}\right],$

Skempton has given the values of N c for purely cohesive soils, as given below:
For (D_f / B) < 2.5,

(N_c) for rectangular footing = $5\left(1+\dfrac{0.2D_f}{B}\right)\left(1+\dfrac{0.2B}{L}\right) \le 9$

(N_c) for circular and rectangular footing = $6\left(1+\dfrac{0.2D_f}{B}\right)\left(1+\dfrac{0.2B}{L}\right) \le 9$

For (D_f / B) >= 2.5,

(N_c) for rectangular footing = $7.5\left(1+\dfrac{0.2B}{L}\right) \le 9$

Then,

$$q_u = c_u N_c + \gamma D_f,$$

Where,

C_u is the undrained cohesion of the soil.

Recap

In this section you have learnt the following

- Development of Bearing Capacity Theory
- Terzaghi's Bearing Capacity Theory
- Assumptions in Terzaghi’s Bearing Capacity Theory.
- Meyerhof's Bearing Capacity Theory
- Bearing capacity of square and circular footings

Congratulations, you have finished Lecture 8. To view the next lecture select it from the left hand side menu of the page