

## **Module 3 : Method of Analyses**

### **Lecture 13 : Limit Analysis [ Section 13.1 : Introduction ]**

#### **Objectives**

**In this section you will learn the following**

- Introduction

## **Module 3 : Method of Analyses**

### **Lecture 13 : Limit Analysis [ Section 13.1 : Introduction ]**

#### **LIMIT ANALYSIS**

In Limit analysis method, three basic considerations are needed in the solution, namely, the stress equilibrium equation, the stress-strain relations and the compatibility equations relating strain and displacement. The complete solution by this approach is likely to be cumbersome for all but the simplest problems, and the methods are needed to furnish the load-carrying capacity in a more direct manner. Limit analysis is the method which enables definite statement to be made about the collapse load without carrying out the step by step elastic-plastic analysis. This method considers the stress-strain relationship of the soil in an idealized manner. This idealization, termed normality, establishes the limit theorems on which the limit analysis is based. Results obtained from this analysis give the upper bound value.

The following examples have been worked out based on Limit Analysis method.

## **Module 3 : Method of Analyses**

### **Lecture 13 : Limit Analysis [ Section 13.1 : Introduction ]**

#### **Recap**

**In this section you have learnt the following**

- Limit Analysis

## **Module 3 : Method of Analyses**

### **Lecture 13 : Limit Analysis [ Section 13.2 : Problems ]**

#### **Objectives**

**In this section you will learn the following**

- Problems

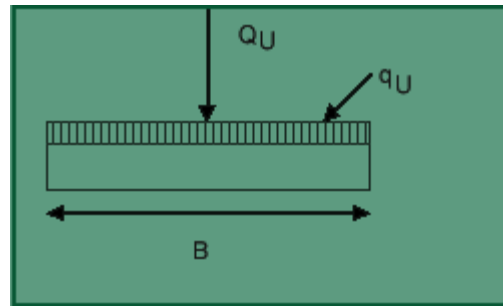
### Module 3 : Method of Analyses

#### Lecture 13 : Limit Analysis [ Section 13.2 : Problems ]

The following examples have been worked out based on Limit Analysis method.

##### **Footing problem**

Calculate the ultimate load bearing capacity of footing (  $Q_U$  )



##### **Trial I**

Assume the circular slip surface CD as shown in the figure 3.3

**Fig. 3.3 Footing with slip surface (trial I)**

In the diagram AE represents the footing, A is the center of the circle of failure. CAD is the deflected shape. The arc length CE is  $B \theta$ . At the center of the footing the arc length is  $B \theta/2$ . According to the direction of the movement of the footing, shear forces act along the failure surface as shown. This is called **velocity profile**.

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#### Lecture 13 : Limit Analysis [ Section 13.2 : Problems ]

Consider clay ( $\phi=0$ ) soil beneath the footing.

Equating, External work = Internal work.

$$(q_c) \cdot (B) \cdot (B\theta/2) = (\pi B) \cdot (C_u) \cdot (B\theta)$$

$C_u$  is calculated from the velocity profile.

$B\theta$  is the displacement from the periphery.

$$q_c = 6.28 c_u$$

$$N_c = 6.28.$$

The values of  $N_c$  calculated by Terzaghi and Meyerhoff were 5.7 and 5.14 respectively from the Limit equilibrium method. Therefore from the Limit analysis we get the upper bound. However with a revised failure surface this value of  $N_c$  can be minimized. This exercise is done in Trial II.

#### Trial II

Consider the circular failure surface as shown in the figure. 3.4

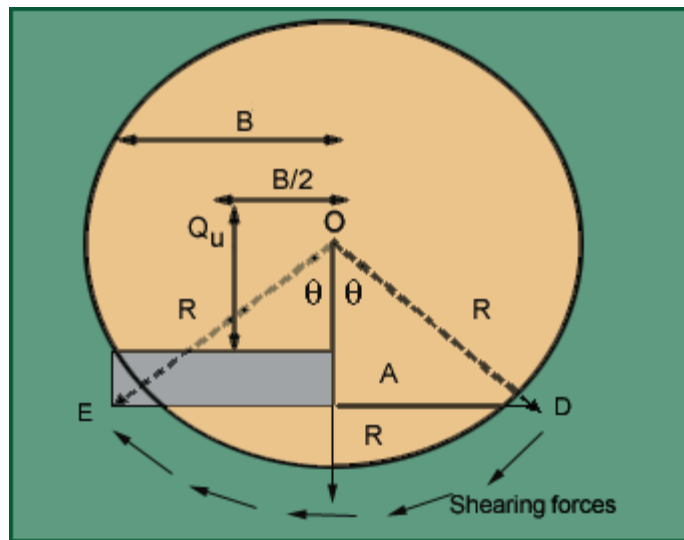


Fig. 3.4 Footing with slip surface (trial II)

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#### Lecture 13 : Limit Analysis [ Section 13.2 : Problems ]

In the diagram AE represents the footing, O is the center of the circle of failure. ED is the deflected shape. According to the direction of the movement of the footing, shear forces act along the failure surface as shown. This is called **velocity profile**.

$$B = R \sin \theta$$

Equating, Overturning and the Resisting Moments,

$$(q_c) \cdot (B) \cdot (B/2) = C_u (2R\theta) \cdot R$$

$$q_c = 4\theta C_u / \sin^2 \theta$$

To get  $(q_c)_{\min}$ , we substitute  $dq_c / d\theta = 0$ .

After solving we get,  $\theta = 66^\circ$ .

$$q_c = 4\theta C_u / \sin^2 \theta = 5.5 C_u$$

$N_c = 5.5$  (which is much lesser than limit equilibrium value of 5.7)

Therefore, the choice of failure surface is most important.

## **Module 3 : Method of Analyses**

### **Lecture 13 : Limit Analysis [ Section 13.2 : Problems ]**

#### **Recap**

**In this section you have learnt the following**

- Problems

**Congratulations, you have finished Lecture 13. To view the next lecture select it from the left hand side menu of the page**