Module 2 : Theory of Earth Pressure and Bearing Capacity

Lecture 10 : Point of Application of Passive Earth Pressure [Section10.1 Rankine's & Coulomb's Theory]

Objectives

In this section you will learn the following

- Rankine’s theory
- Coulomb’s theory
- Method of horizontal slices given by Wang (2000)
- Distribution of the earth pressure
- Height of application of the resultant earth pressure
- Earth pressure distribution in seismic case given by Choudhury et al. (2002)
- Expression for the seismic passive earth pressure

10 POINT OF APPLICATION OF PASSIVE EARTH PRESSURE

1 Rankine's theory

In Rankine's earth pressure theory the intensity of earth pressure at each depth is known. So, point of application of the passive earth pressure is known at any depth as shown in fig. 2.36.

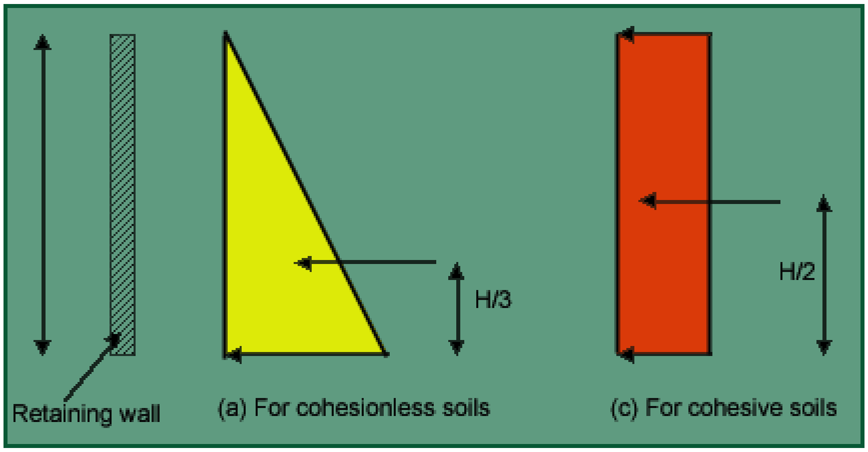

Fig. 2.36 Point of application of passive earth pressure (Rankine's theory)

2 Coulomb's theory

In the coulomb's method only the total earth pressure value acting on the retaining structure can be calculated. The point of application of the earth pressure can be calculated from coulomb's assumption that all points on the back of the retaining wall are essentially considered as feet of the failure surface. So, for each failure surfaces as shown in fig. 2.37, earth pressure intensity can be calculated. Let the earth pressure intensity for failure surface 1-1' is p a at depth z from top of retaining wall and the earth pressure intensity for failure surface 2-2' is p a' at depth (z +dz) from top of retaining wall.

Then, $dp_a = p_a' - p_a$,

Therefore, (dp_a / dz) can be estimated. This gives the variation of the earth pressure intensity along the depth. As the variation of earth pressure is known, earth pressure diagram can be plotted and point of application of the resultant earth pressure can be found.

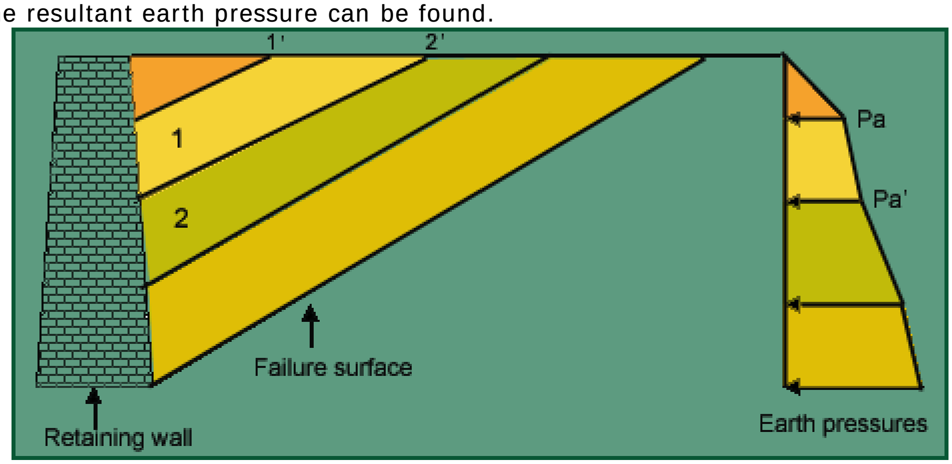

Fig. 2.37Point of application of passive earth pressure (Coulomb's theory)

The assumption made by Coulomb is justified in case of retaining walls, because no retaining wall can fail without yielding in a manner that satisfies the deformation condition for the plastic state. Coulomb, however, did not satisfy this deformation condition. As a consequence, the theory was commonly used for computing the earth pressures against the lateral supports that did not satisfy the deformation condition, such as bracing in open cut.

Method of horizontal slices given by Wang (2000)

The approximate distribution of the earth pressure can be determined numerically by computing resultant earth pressure at various depths along the wall. The unit earth pressure calculated on the basis of Coulomb's and Rankine's theories varies linearly with depth. But, from experimental data it is found that the unit earth pressure is curvilinearly distributed on the back of the retaining wall. Wang (2000) has considered the Coulomb's concept as the basis for determining the intensity of earth pressure by the method of horizontal slices. A sliding wedge considered for the analysis is shown in fig.2.38 (a). An element of thickness dy is considered from the wedge at depth y below the surface. The forces on the element are shown in fig.2.38 (b).

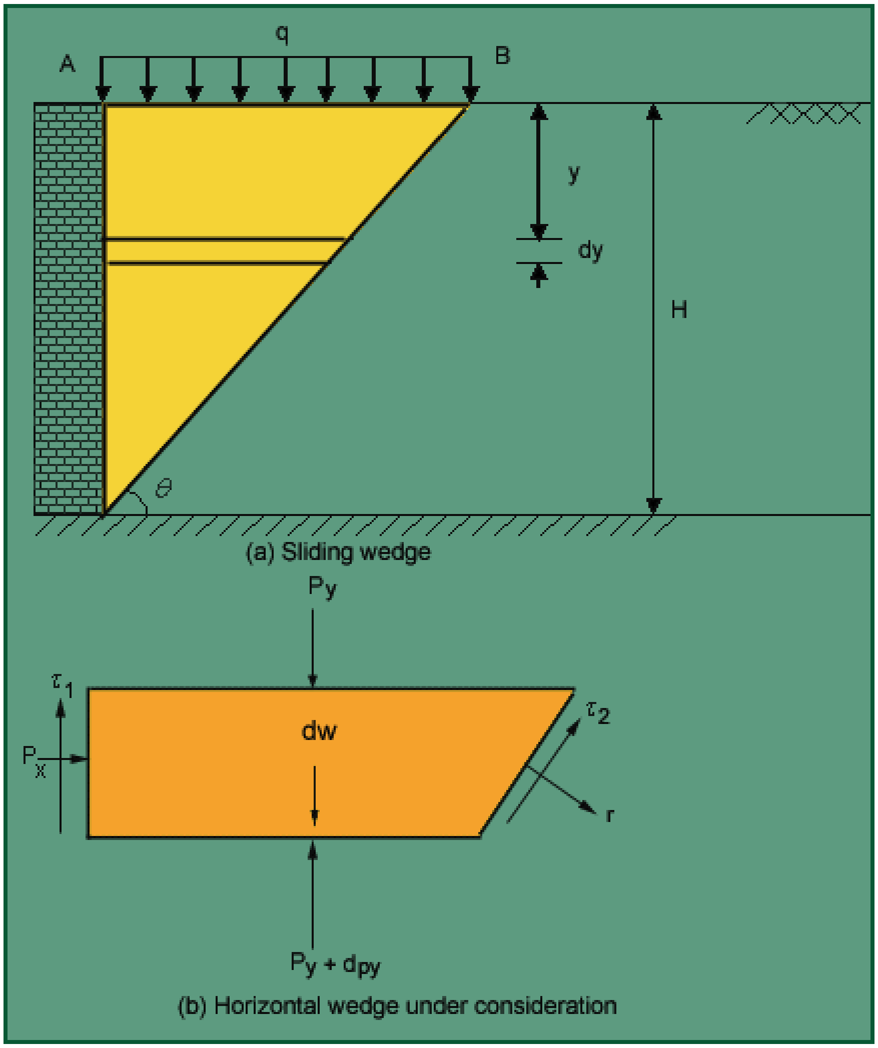

Fig. 2.38 Analytical model

Where,

p_y is the vertical pressure on the top of the element,

(p_y +d p_y) is the vertical reaction on the bottom of the element,

p_x is the horizontal reaction on the retaining wall,

τ_1 is shear between the back fill and the back of retaining wall,

τ_2 is shear between the sliding backfill and the remaining backfill at rest,

dw is the weight of the element.

The shearing forces on the top and bottom of the element are neglected, considering that the wedge slides as a whole. Considering the equilibrium condition of the vertical forces on the element, the basic equation for the unit earth pressure on the retaining wall is given by,

$$\frac{dp_y}{dy} = \left[1 - \frac{\cos(\theta - \phi - \delta)\tan\theta}{\sin(\theta - \phi)\cos\delta}\right]\frac{p_y}{H - y} + \gamma$$

where,

δ is the friction angle between the back of retaining wall and the backfill,

ϕ is the internal friction angle of the backfill,

H is the height of the retaining wall,

γ is density of the backfill material.

Then,

$$p_y = \left(q - \frac{\gamma H}{aK-2}\right)\left(\frac{H-y}{H}\right)^{aK-1} + \frac{\gamma H}{aK-2}\frac{H-y}{H}$$

where,

$$a = \left[1 - \frac{\cos(\theta-\phi-\delta)\tan\theta}{\sin(\theta-\phi)\cos\delta}\right],$$

K is the lateral pressure coefficient.

$$p_x = K\left\{\left(q - \frac{\gamma H}{aK-2}\right)\left(\frac{H-y}{H}\right)^{aK-1} + \frac{\gamma H}{aK-2}\frac{H-y}{H}\right\}$$

Resultant earth pressure on the wall

Total horizontal earth pressure can be given by,

$$P_x = \int_0^H p_x dy = \frac{\sin(\theta-\phi)\cos\delta\cos\theta}{\cos(\theta-\phi-\delta)}\left(qH + \frac{1}{2}\gamma H^2\right)$$

The total shearing force on the wall can be given as,

$$T_1 = \int_0^H \tau_1 dy = \frac{\sin(\theta-\phi)\cos\delta\cos\theta}{\cos(\theta-\phi-\delta)}\left(qH + \frac{1}{2}\gamma H^2\right)$$

The resultant force on the wall is as,

$$P = \frac{\sin(\theta - \phi)\cot\theta}{\cos(\theta - \phi - \delta)}\left(qH + \frac{1}{2}\gamma H^2\right)$$

If the surcharge q =0, then,

$$P = \frac{\sin(\theta - \phi)\cot\theta}{\cos(\theta - \phi - \delta)}\left(\frac{1}{2}\gamma H^2\right)$$

The above equation is same as given by Coulomb's theory.

Distribution of the earth pressure

The unit earth pressure is given as,

$$p = \frac{K}{\cos\delta}\left\{\left(q - \frac{\gamma H}{aK-2}\right)\left(\frac{H-y}{H}\right)^{aK-1} + \frac{\gamma H}{aK-2}\frac{H-y}{H}\right\}$$

This is curvilinearly distributed and related to the lateral pressure coefficient K.

When K=1/a, then,

$$p = (q - \gamma y)\frac{1}{a\cos\delta}$$

This represents linearly distributed earth pressure.

Height of application of the resultant earth pressure

In case where the earth pressure is linearly distributed, the height of application of the resultant earth pressure (H_p) from the wall bottom is,

$$H_p = \frac{1}{3} H \, \frac{3q - \gamma H}{2q - \gamma H}$$

and when q=0, then H_p = H/3.

For the curvilinearly distributed earth pressure,

$$H_p = \frac{M}{p_x} = \left[\frac{1}{3} + \frac{aK - 1}{3(aK+1)}\right] H \, \frac{3q + \gamma H}{2q + \gamma H}$$

where,

M is the resultant moment of the earth pressure about the wall bottom

$$= \int_0^H (H - y) p_x dy = \frac{KH^2}{aK+1}\left(q + \frac{1}{3}\gamma H\right)$$

Earth pressure distribution in seismic case given by Choudhury et al. (2002)

A method of horizontal slices has been suggested for obtaining seismic passive earth pressure distribution by considering seismic forces in a pseudo-static manner. Only planar rupture surfaces have been considered and hence wall friction angle has been restricted upto one-third the soil friction angle. This approach results in the same seismic passive earth pressure coefficients as that obtained by Mononobe-Okabe approach, besides giving additional information about the distribution of earth pressures. It has been found that in the seismic case, passive resistance acts at a point other than at 1/3rd from the base of the wall. Under seismic conditions, the extension of failure zone is more than that under static conditions.

This method is an extension to Wang's approach and is suggested for determining the seismic passive earth pressure distribution for a rigid inclined retaining wall supporting cohesionless backfill.

Analytical Model

The method of horizontal slices is considered in the analysis. In Fig.2.39a, a rigid retaining wall of vertical height H, supporting dry, homogeneous cohesionless backfill material with horizontal ground is shown. The seismic forces are considered as pseudo-static forces along with other static forces. The equilibrium of each elemental slice is considered. It is assumed that the occurrence of earthquake does not affect the basic soil parameters: soil friction angle f and soil unit weight γ .

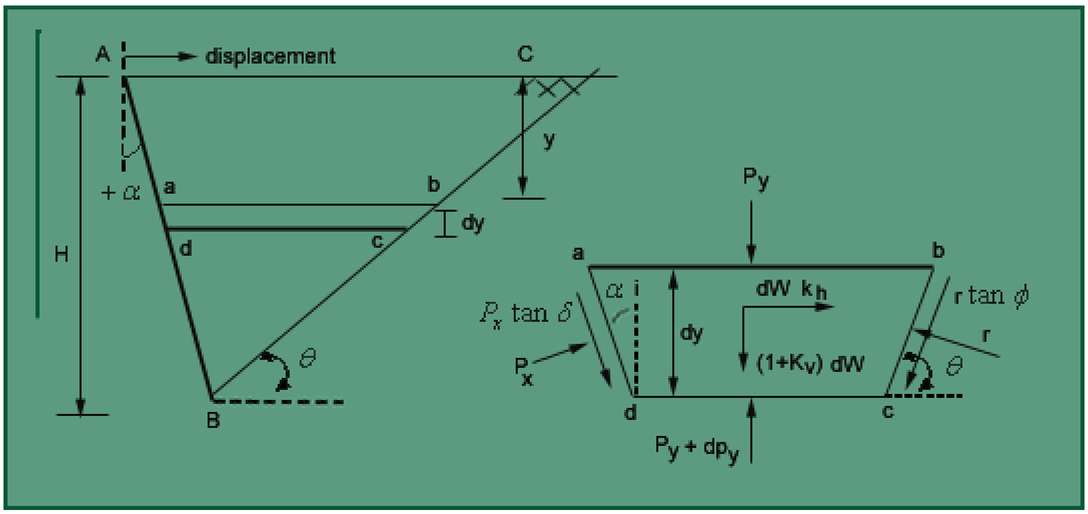

Fig. 2.39 (a)Analytical model (b) Free body diagram

Uniform seismic accelerations $K_h g$ and $K_v g$ are assumed in the horizontal and vertical directions respectively in the domain under consideration. As a first step, only planar rupture surfaces have been considered and to keep this assumption valid, wall friction angle δ has been restricted to less than or equal to $\phi/3$ as shown by Terzaghi (1943). In Fig.2.39b, the free body diagram of an elemental slice shows the action of different forces. The thickness of the slice is dy, at a depth of y from the top ground surface. The vertical pressure p_y is acting on the top of the element and (p_y + d p_y) on the bottom of the element. The reaction p_x normal to the wall and the shear force p_x tan δ are acting on the interface between the retaining wall and the backfill material. The normal force r and the shear force for tan ϕ act on the sliding surface. The other forces are, the weight dW of the element, the seismic forces dWK_h in the horizontal direction and dWK_v in the the vertical direction. The critical directions of these seismic forces are as shown in Fig.4.9b. The horizontal slip planes are assumed as principal planes.

Unit earth pressure on the retaining wall

Considering the equilibrium condition of the vertical forces on the element, the basic equation for the unit earth pressure on the retaining wall is given by,

$$\frac{dp_y}{dy} = \frac{p_y}{H-y}(1+K) + \gamma m$$

Expression for the seismic passive earth pressure

Resolving all the forces in the vertical and horizontal directions, from the boundary condition p_y = 0 at y = 0, and ignoring higher order differential terms and upon simplification, the expression for the seismic passive earth pressure at any depth y is obtained as,

$$p_x = K\left[\frac{\gamma n}{2+aK}\left\{\frac{H^{(2+aK)}}{(H-y)^{(1+aK)}} - (H-y)\right\}\right]$$

where p_x = seismic passive pressure at any depth y from top acting normal to the wall

K = lateral earth pressure coefficient

$$n = (1 - K_v - K_x b)$$

$$b = \cot(\theta + \phi)$$

$$a = \frac{(\tan\phi - \cot\theta)(1 + \tan\alpha\tan\delta)}{(\tan\alpha + \cot\theta)(1 + \tan\phi\cot\theta)} + \frac{(\tan\delta - \tan\alpha)}{(\tan\alpha + \cot\theta)}$$

d = wall friction angle

θ = angle of inclination of the failure plane with horizontal

Integrating over the height of the wall, the total passive resistance p_x is given by,

$$P_x = \int_0^H p_x dy = \frac{\gamma n H^2}{a(2+aK)}\left(1000^{aK} - 1\right) - \frac{K\gamma n H^2}{2(2+aK)}$$

The value of coefficient K has been obtained from the static analysis by comparing the results with that of Coulomb. The equivalent seismic passive earth pressure coefficient with respect to normal to the wall is found out as,

$$K_{pd} = \frac{2P_x}{\gamma H^2}$$

The critical value of θ is obtained by minimizing p_x with respect to θ keeping all other parameters constant, and it is found to exactly match with the Coulombic values for static case.

Height of application of the resultant earth pressure

M is the resultant moment of the earth pressure about the wall bottom

$$= \int_0^H (H-y) p_x dy = \frac{\gamma n K H^2}{aK+2}\left(\frac{1}{(aK-1)}\left(1000^{(aK-1)} - 1\right) - \frac{1}{3}\right)$$

The point of application of the passive earth pressure from base $= h = \dfrac{M}{p_x}$

Recap

In this section you have learnt the following

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